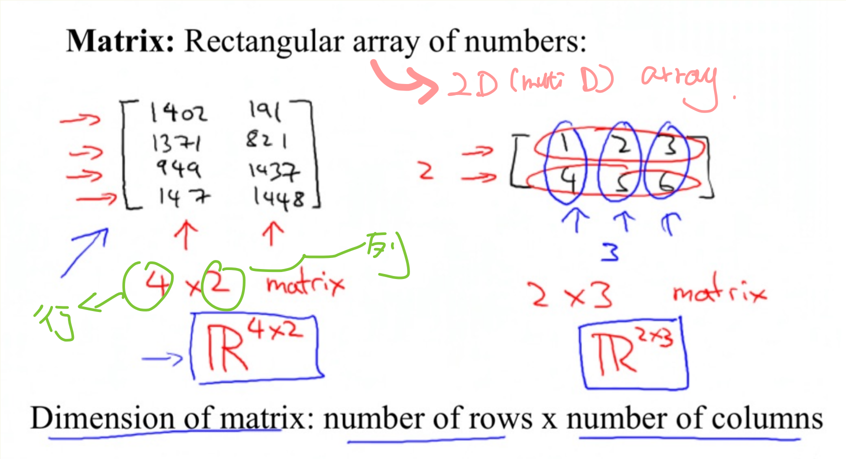
Matrices and Vectors.



Question

Which of the following statements are true? Check all that apply.

- [x] $\begin{bmatrix} 1 & 2 \\ 4 & 0 \\ 0 & 1 \end{bmatrix}$ is a 3×2 matrix.
- [ ] $\begin{bmatrix} 0 & 1 & 4 & 2 \\ 3 & 4 & 0 & 9 \end{bmatrix}$ is a 4×2 matrix.
- [x] $\begin{bmatrix} 0 & 4 & 2 \\ 3 & 4 & 9 \\ 5 & -1 & 0 \end{bmatrix}$ is a 3×3 matrix.
- [x] $\begin{bmatrix} 1 & 2 \end{bmatrix}$ is a 1×2 matrix.

Matrix Elements (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

$A_{ij} =$ "i, j entry" in the i^{th} row, j^{th} column.

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

$\cancel{A_{43}}$ = undefined (error)

Question

Let A be a matrix shown below. A_{32} is one of the elements of this matrix.

$$A = \begin{bmatrix} 85 & 76 & 66 & 5 \\ 94 & 75 & 18 & 28 \\ 68 & 40 & 71 & 5 \end{bmatrix}$$

What is the value of A_{32}?

- ○ 18
- ○ 28
- ○ 76
- ◉ 40

✓ Correct

保证它是 1列

Vector: An n x 1 matrix

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$n = 4$

← 4-dimensional vector. $\mathbb{R}^4$

~~$\mathbb{R}^{3\times 2}$~~

y: Vector.

$y_i = i^{th}$ element

$y_1 = 460$

$y_2 = 232$

$y_3 = 315$

$\boxed{A, B, C, X}$ ← Upper for matrix

$\boxed{a, b, x, y}$ ← lower for Vector / num.

1-indexed vs 0-indexed:

y[1]

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

1-indexed

y[0]

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

0-indexed

Matrices and Vectors

Matrices are 2-dimensional arrays:

$$[a \quad b \quad cd \quad e \quad fg \quad h \quad ij \quad k \quad l]$$

The above matrix has four rows and three columns, so it is a 4 x 3 matrix.

A vector is a matrix with one column and many rows:

$$[wxyz]$$

So vectors are a subset of matrices. The above vector is a 4 x 1 matrix.

Notation and terms:

- A_{ij} refers to the element in the ith row and jth column of matrix A.
- A vector with 'n' rows is referred to as an 'n'-dimensional vector.
- v_i refers to the element in the ith row of the vector.
- In general, all our vectors and matrices will be 1-indexed. Note that for some programming languages, the arrays are 0-indexed.
- Matrices are usually denoted by uppercase names while vectors are lowercase.
- "Scalar" means that an object is a single value, not a vector or matrix.
- $\mathbb{R}$ refers to the set of scalar real numbers.
- $\mathbb{R}^n$ refers to the set of n-dimensional vectors of real numbers.

Run the cell below to get familiar with the commands in Octave/Matlab. Feel free to create matrices and vectors and try out different things.

```
% The ; denotes we are going back to a new row.
A = [1, 2, 3; 4, 5, 6; 7, 8, 9; 10, 11, 12]

% Initialize a vector
v = [1;2;3]

% Get the dimension of the matrix A where m = rows and n = columns
[m,n] = size(A)

% You could also store it this way
dim_A = size(A)

% Get the dimension of the vector v
dim_v = size(v)

% Now let's index into the 2nd row 3rd column of matrix A
A_23 = A(2,3)

```

Run

Reset

```
A =

    1    2    3
    4    5    6
    7    8    9
   10   11   12

v =

   1
   2
   3

m =  4
n =  3
dim_A =

   4   3

dim_v =

   3   1

A_23 =  6
```

Addition & scalar multiplication.

Matrix Addition

行和列数目相等才能相加

$$\underbrace{\begin{bmatrix}1 & 0\\2 & 5\\3 & 1\end{bmatrix}}_{3\times2 \text{ matrix}} + \underbrace{\begin{bmatrix}4 & 0.5\\2 & 5\\0 & 1\end{bmatrix}}_{3\times2} = \underbrace{\begin{bmatrix}5 & 0.5\\4 & 10\\3 & 2\end{bmatrix}}_{3\times2}$$

$$\cancel{\underbrace{\begin{bmatrix}1 & 0\\2 & 5\\3 & 1\end{bmatrix}}_{3\times2} + \underbrace{\begin{bmatrix}4 & 0.5\\2 & 5\end{bmatrix}}_{2\times2}} = \text{error}$$

Question

What is

$$\begin{bmatrix}8 & 6 & 9\\10 & 1 & 10\end{bmatrix} + \begin{bmatrix}3 & 10 & 2\\6 & 1 & -1\end{bmatrix}?$$

- ○ $\begin{bmatrix}5 & -4 & 7\\4 & 0 & 11\end{bmatrix}$
- ✓ $\begin{bmatrix}11 & 16 & 11\\16 & 2 & 9\end{bmatrix}$
- ○ $\begin{bmatrix}14 & 7 & 8\\13 & 11 & 12\end{bmatrix}$
- ○ $\begin{bmatrix}8 & 6 & 9\\10 & 1 & 10\end{bmatrix}$

Scalar Multiplication

real number

$$3 \times \underbrace{\begin{bmatrix}1 & 0\\2 & 5\\3 & 1\end{bmatrix}}_{3\times2} = \underbrace{\boxed{\begin{bmatrix}3 & 0\\6 & 15\\9 & 3\end{bmatrix}}}_{3\times2} = \begin{bmatrix}1 & 0\\2 & 5\\3 & 1\end{bmatrix} \times 3$$

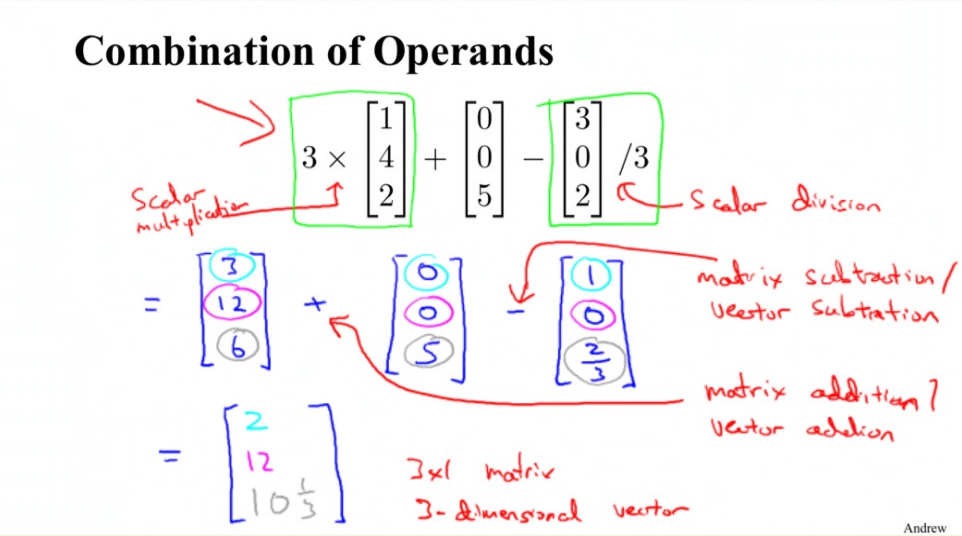
$$\boxed{\begin{bmatrix}4 & 0\\6 & 3\end{bmatrix}/4} = \frac{1}{4}\begin{bmatrix}4 & 0\\6 & 3\end{bmatrix} = \begin{bmatrix}1 & 0\\\frac{3}{2} & \frac{3}{4}\end{bmatrix}$$

Question

What is $2 \times \begin{bmatrix}4 & 5\\1 & 7\end{bmatrix}$?

- ◉ $\begin{bmatrix}8 & 10\\2 & 14\end{bmatrix}$
- ○ $\begin{bmatrix}8 & 5\\1 & 7\end{bmatrix}$
- ○ $\begin{bmatrix}8 & 10\\1 & 7\end{bmatrix}$
- ○ $\begin{bmatrix}4 & 5\\1 & 14\end{bmatrix}$

✓ Correct



Question

What is $\begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} / 2 - 3 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$?

- ○ $\begin{bmatrix} 4 \\ 0 \\ 5 \end{bmatrix}$
- ○ $\begin{bmatrix} -4 \\ -1 \\ 3 \end{bmatrix}$
- ◉ $\begin{bmatrix} -4 \\ 0 \\ 3.5 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 \\ 2 \\ 3.5 \end{bmatrix}$

✓ Correct

Addition and Scalar Multiplication

Addition and subtraction are **element-wise**, so you simply add or subtract each corresponding element:

$$\begin{bmatrix} a & bc & d \end{bmatrix} + \begin{bmatrix} w & xy & z \end{bmatrix} = \begin{bmatrix} a+w & b+xc+y & d+z \end{bmatrix}$$

Subtracting Matrices:

$$\begin{bmatrix} a & bc & d \end{bmatrix} - \begin{bmatrix} w & xy & z \end{bmatrix} = \begin{bmatrix} a-w & b-xc-y & d-z \end{bmatrix}$$

To add or subtract two matrices, their dimensions must be **the same**.

In scalar multiplication, we simply multiply every element by the scalar value:

$$\begin{bmatrix} a & bc & d \end{bmatrix} * x = \begin{bmatrix} a*x & b*xc*x & d*x \end{bmatrix}$$

In scalar division, we simply divide every element by the scalar value:

$$\begin{bmatrix} a & bc & d \end{bmatrix} / x = \begin{bmatrix} a/x & b/xc/x & d/x \end{bmatrix}$$

Experiment below with the Octave/Matlab commands for matrix addition and scalar multiplication. Feel free to try out different commands. Try to write out your answers for each command before running the cell below.

```
% Initialize matrix A and B
A = [1, 2, 4; 5, 3, 2]
B = [1, 3, 4; 1, 1, 1]

% Initialize constant s
s = 2

% See how element-wise addition works
add_AB = A + B

% See how element-wise subtraction works
sub_AB = A - B

% See how scalar multiplication works
mult_As = A * s

% Divide A by s
div_As = A / s

% What happens if we have a Matrix + scalar?
add_As = A + s

```

Run

Reset

```
A =

   1   2   4
   5   3   2

B =

   1   3   4
   1   1   1

s = 2
add_AB =

   2   5   8
   6   4   3

sub_AB =

   0  -1   0
   4   2   1

mult_As =

    2    4    8
   10    6    4

div_As =

   0.50000   1.00000   2.00000
   2.50000   1.50000   1.00000

add_As =

   3   4   6
   7   5   4
```

Matrix and Vector Multiplication

Example

$$\underbrace{\begin{bmatrix}1 & 3\\ 4 & 0\\ 2 & 1\end{bmatrix}}_{3\times 2}\underbrace{\begin{bmatrix}1\\ 5\end{bmatrix}}_{2\times 1}=\underbrace{\begin{bmatrix}16\\ 4\\ 7\end{bmatrix}}_{3\times 1 \text{ matrix}}$$

$1\times 1 + 3\times 5 = 16$

$4\times 1 + 0\times 5 = 4$

$2\times 1 + 1\times 5 = 7$

Question

Consider the product of these two matrices:

$$\begin{bmatrix}1 & 2 & 1 & 5\\ 0 & 3 & 0 & 4\\ -1 & -2 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\ 3\\ 2\\ 1\end{bmatrix}$$

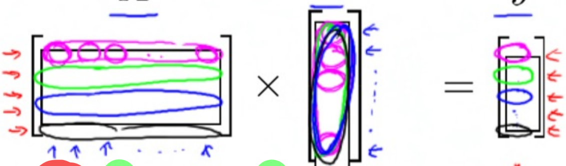
What is the dimension of the product?

- (●) 3×1
- ( ) 3×4
- ( ) 1×3
- ( ) 4×4

✓ Correct

Details:

$$\underline{A} \times \underline{x} = \underline{y}$$



To get y_i, multiply A's i^{th} row with elements of vector x, and add them up.

Andrew

$$\underbrace{\text{num of columns}}_{1^{st}} = \underbrace{\text{num of rows}}_{2^{nd}}$$

才能相乘。

Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{14} \\ \boxed{13} \\ \boxed{-7} \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

$\boxed{3} \times 4$ $\quad 4 \times 1 \quad 3 \times 1$

$1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14$

$0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13$

$-1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7$

Question

What is $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 5 \\ 3 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$?

- ○ $\begin{bmatrix} 5 \\ 10 \\ 1 \end{bmatrix}$
- ○ $\begin{bmatrix} 7 \\ 12 \\ 7 \end{bmatrix}$
- ◉ $\begin{bmatrix} 7 \\ 18 \\ 13 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ 18 \\ 13 \end{bmatrix}$

✓ Correct

Example:

House size:

2104
1416
1534
852

$h_\theta(x) = -40 + 0.25x$

matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -40 \times 1 + 0.25 \times 2104 \\ \vdots \end{bmatrix}$$

$h_\theta(2104)$

prediction (4x1) = DataMatix * Parameters

for i = 1:4 ,

Prediction (i) = - - -

Matrix-Vector Multiplication

We map the column of the vector onto each row of the matrix, multiplying each element and summing the result.

$$\begin{bmatrix} a & bc & de & f \end{bmatrix} * \begin{bmatrix} xy \end{bmatrix} = \begin{bmatrix} a*x+b*yc*x+d*ye*x+f*y \end{bmatrix}$$

The result is a **vector**. The number of **columns** of the matrix must equal the number of **rows** of the vector.

An **m x n matrix** multiplied by an **n x 1 vector** results in an **m x 1 vector**.

Below is an example of a matrix-vector multiplication. Make sure you understand how the multiplication works. Feel free to try different matrix-vector multiplications.

```
% Initialize matrix A
A = [1, 2, 3; 4, 5, 6;7, 8, 9]

% Initialize vector v
v = [1; 1; 1]

% Multiply A * v
Av = A * v
```

Run

Reset

```
A =

   1   2   3
   4   5   6
   7   8   9

v =

   1
   1
   1

Av =

    6
   15
   24
```

Matrix-Matrix Multiplication

Example

$$\begin{bmatrix}1 & 3 & 2\\4 & 0 & 1\end{bmatrix}\begin{bmatrix}\boxed{\begin{matrix}1\\0\\5\end{matrix}} & \boxed{\begin{matrix}3\\1\\2\end{matrix}}\end{bmatrix} = \begin{bmatrix}11 & 10\\9 & 14\end{bmatrix}$$

$\underbrace{\qquad}_{2\times 3}$ $\underbrace{\qquad}_{3\times 2}$ $\quad 2\times 2$

$$\begin{bmatrix}1 & 3 & 2\\4 & 0 & 1\end{bmatrix}\times\begin{bmatrix}1\\0\\5\end{bmatrix} = \begin{bmatrix}11\\9\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\4 & 0 & 1\end{bmatrix}\times\begin{bmatrix}3\\1\\2\end{bmatrix} = \begin{bmatrix}10\\14\end{bmatrix}$$

Details:

$A \times B = C$

m x n matrix (m rows, n columns) $\qquad$ n x o matrix (n rows, o columns) $\quad n\times 1$ $\qquad$ m x o matrix $\qquad$ ~~o x l~~

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B. (for i = 1,2,…,o)

Andrew

$\underbrace{\text{num of col}}_{\text{1st}}$ must = $\underbrace{\text{num of row}}_{\text{2nd.}}$

Example

$$\underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} \boxed{\begin{matrix}0\\3\end{matrix}} & \boxed{\begin{matrix}1\\2\end{matrix}} \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times 0 + 3\times 3 \\ 2\times 0 + 5\times 3 \end{bmatrix} = \cancel{\begin{bmatrix} 9 \\ 15 \end{bmatrix}} \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 3\times 2 \\ 2\times 1 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\begin{Bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{Bmatrix}$$

Have 3 competing hypotheses:

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

Matrix

Prediction of first h_θ

Predictions of 2nd h_θ

predict of 3rd h_θ.

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Question

In the equation $\begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ a & b \\ c & d \end{bmatrix}$, what is a?

Hint: Compute $\begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.

- ○ 7
- ○ 12
- ◉ 10
- ○ 6

✓ Correct

Matrix-Matrix Multiplication

We multiply two matrices by breaking it into several vector multiplications and concatenating the result.

$$[a \quad bc \quad de \quad f] * [w \quad xy \quad z] = [a*w+b*y \quad a*x+b*zc*w+d*y \quad c*x+d*ze*w+f*y \quad e*x+f*z]$$

An **m x n matrix** multiplied by an **n x o matrix** results in an **m x o** matrix. In the above example, a 3 x 2 matrix times a 2 x 2 matrix resulted in a 3 x 2 matrix.

To multiply two matrices, the number of **columns** of the first matrix must equal the number of **rows** of the second matrix.

For example:

```
% Initialize a 3 by 2 matrix
A = [1, 2; 3, 4;5, 6]

% Initialize a 2 by 1 matrix
B = [1; 2]

% We expect a resulting matrix of (3 by 2)*(2 by 1) = (3 by 1)
mult_AB = A*B

% Make sure you understand why we got that result
```

Run

Reset

```
A =

   1   2
   3   4
   5   6

B =

   1
   2

mult_AB =

    5
   11
   17
```

Matrix Multi Properties

- Matrix is Not Commutative $(A \times B = B \times A)$

$3 \times 5 = 5 \times 3$ "Commutative"

Let A and B be matrices. Then in general,
$A \times B \neq B \times A$. (not commutative.)

E.g. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$\neq$

$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$

$\underset{m \times n}{A} \times \underset{n \times m}{B}$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

- Matrix is Associative $(A \times (B \times C) = (A \times B) \times C)$

$3 \times 5 \times 2$ $\quad 3 \times (5 \times 2) = (3 \times 5) \times 2$ $\quad \boxed{\text{"Associative"}}$

$3 \times 10 = 30 = 15 \times 2$

$A \times (B \times C)$

$(A \times B) \times C$

$A \times B \times C$.

Let $D = B \times C$. Compute $A \times D$.

Let $E = A \times B$. Compute $E \times C$.

$A \times (B \times C)$

$(A \times B) \times C$

Same answer.

Identity Matrix

1 is identity. $\boxed{1 \times z = z \times 1 = z}$ for any z

Denoted I (or $\boxed{I_{n \times n}}$).

Examples of identity matrices:

$[1]$ 1×1 $\quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2 x 2 $\quad$ 3 x 3 $\quad$ 4 x 4

Informally:

$$\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & \ddots & 1 \end{bmatrix}$$

For any matrix A,

$$\underset{m\times n}{A} \cdot \underset{n\times n}{\boxed{I}} = \underset{m\times m}{\boxed{I}} \cdot \underset{m\times n}{A} = \underset{m\times n}{A}$$

$I_{n\times n}$

Note:

$AB \neq BA$ in general

$AI = IA$ ✓

$$AI = IA = A$$

(I 相当于 real num 里的 "1")

Question

What is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$?

- ○ $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$
- ○ $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$
- ◉ $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$

✓ Correct

Matrix Multiplication Properties

- Matrices are not commutative: $A*B \neq B*A$
- Matrices are associative: $(A*B)*C = A*(B*C)$

The **identity matrix**, when multiplied by any matrix of the same dimensions, results in the original matrix. It's just like multiplying numbers by 1. The identity matrix simply has 1's on the diagonal (upper left to lower right diagonal) and 0's elsewhere.

$$[1 \quad 0 \quad 00 \quad 1 \quad 00 \quad 0 \quad 1]$$

When multiplying the identity matrix after some matrix (A∗I), the square identity matrix's dimension should match the other matrix's **columns**. When multiplying the identity matrix before some other matrix (I∗A), the square identity matrix's dimension should match the other matrix's **rows**.

```
% Initialize random matrices A and B
A = [1,2;4,5]
B = [1,1;0,2]

% Initialize a 2 by 2 identity matrix
I = eye(2)

% The above notation is the same as I = [1,0;0,1]

% What happens when we multiply I*A ?
IA = I*A

% How about A*I ?
AI = A*I

% Compute A*B
AB = A*B

% Is it equal to B*A?
BA = B*A

% Note that IA = AI but AB != BA
```

Run

Reset

```
A =

   1   2
   4   5

B =

   1   1
   0   2

I =

Diagonal Matrix

   1   0
   0   1

IA =

   1   2
   4   5

AI =

   1   2
   4   5

AB =

    1    5
    4   14

BA =

    5    7
    8   10
```

Inverse and Transpose.

I = "identity." $3\boxed{(3^{-1})} = 1$ $\frac{1}{3}$ $12 \times (12^{-1}) = 1$ $\frac{1}{12}$

Not all numbers have an inverse. $0\,(0^{-1})$ undefined

Matrix inverse: square matrix (# rows = # columns) A^{-1}

If A is an m x m matrix, and if it has an inverse,

$$A(A^{-1}) = A^{-1}A = I.$$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Doesn't hv inverse as well

E.g. $$\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{\substack{2\times 2 \\ A}} \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$$

$A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate"

Andrew

Matrix Transpose

Example: $$A = \underbrace{\begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}}_{2\times 3} \qquad B = A^T = \underbrace{\begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}}_{3\times 2}$$

Let A be an m x n matrix, and let $B = A^T$.
Then B is an n x m matrix, and

$$B_{ij} = A_{ji}.$$

$B_{12} = A_{21} = 2$

$B_{32} = 9 \qquad A_{23} = 9.$

Question

What is $\begin{bmatrix} 0 & 3 \\ 1 & 4 \end{bmatrix}^T$?

- ○ $\begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$
- ○ $\begin{bmatrix} 4 & 3 \\ 1 & 0 \end{bmatrix}$
- ○ $\begin{bmatrix} 4 & 1 \\ 3 & 0 \end{bmatrix}$
- ◉ $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$

✓ Correct

Inverse and Transpose

The **inverse** of a matrix A is denoted A^{-1}. Multiplying by the inverse results in the identity matrix.

A non square matrix does not have an inverse matrix. We can compute inverses of matrices in octave with the $pinv(A)$ function and in Matlab with the $inv(A)$ function. Matrices that don't have an inverse are *singular* or *degenerate*.

The **transposition** of a matrix is like rotating the matrix 90° in clockwise direction and then reversing it. We can compute transposition of matrices in matlab with the transpose(A) function or A':

$$A = \begin{bmatrix} a & bc & de & f \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & c & eb & d & f \end{bmatrix}$$

In other words:

$$A_{ij} = A^T_{ji}$$

```
% Initialize matrix A
A = [1,2,0;0,5,6;7,0,9]

% Transpose A
A_trans = A'

% Take the inverse of A
A_inv = inv(A)

% What is A^(-1)*A?
A_invA = inv(A)*A

```

Run

Reset

```
A =

   1   2   0
   0   5   6
   7   0   9

A_trans =

   1   0   7
   2   5   0
   0   6   9

A_inv =

   0.348837  -0.139535   0.093023
   0.325581   0.069767  -0.046512
  -0.271318   0.108527   0.038760

A_invA =

   1.00000  -0.00000   0.00000
   0.00000   1.00000  -0.00000
  -0.00000   0.00000   1.00000
```