FACULTY OF COMPUTING

SEMESTER 1 2024/2025

SECI 1013 DISCRETE STRUCTURE

SECTION 03

ASSIGNMENT 1

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ASSIGNMENT 1

CHAP 1 :

10 - 30

1\. $U = \{n \mid n \in$ whole numbers, $10 <= n <= 30\}$

$G = \{g \mid g \in$ even numbers$\}$ gencp

11 - 29

$F = \{f \mid f \in$ natural numbers, $f > 10$ and $f < 30\}$

$G \subseteq F$

$F \subseteq U$

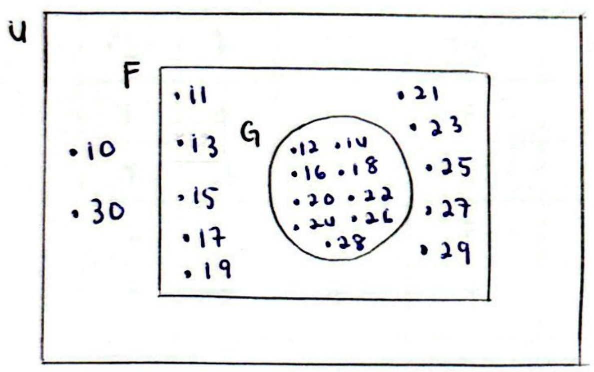
a) $F = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$|F| = 19$

b) $G = \{12, 14, 16, 18, 20, 22, 24, 26, 28\}$

$|G| = 9$

c)



$U = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20,$
$21, 22, 23, 24, 25, 26, 27, 28,$
$29, 30\}$

d) $G \oplus F = (G \cup F) - (G \cap F)$

$= (G - F) \cup (F - G)$

$= \{\} \cup \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$= \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$|G \oplus F| = 10$

2. $A = \{ s, u, b \}$

$B = \{ s, e, t \}$

$C = \{ n, e, t \}$

a) $|A| = 3$

$|P(A)| = 2^3$

$= 8$

b) $A \cap B \cup C$

$A \cap B = \{ s \}$

$A \cap B \cup C = \{ e, n, s, t \}$

c) $A - B = \{ b, u \}$

d) $B \times C = \{ s, e, t \} \times \{ n, e t \}$

$= \{ (s, n), (s, e), (s, t),$

$(e, n), (e, e), (e, t),$

$(t, n), (t, e), (t, t) \}$

3. a) True

b) True

c) True

d) False

e) True

) $(p \to q) \wedge (\neg p \leftrightarrow \neg q)$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \leftrightarrow \neg q$	$(p \to q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b) $(p \leftrightarrow q) \vee (\neg p \to \neg q)$

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \leftrightarrow q) \vee (\neg p \to \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

5. $A = \neg p \wedge (\neg q \vee \neg r)$

$B = p \vee (q \wedge r)$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \vee \neg r$	$\neg p \wedge (\neg q \vee \neg r)$
T	T	T	F	F	F	F	F
T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	F
T	F	F	F	T	T	T	F
F	T	T	T	F	F	F	F
F	T	F	T	F	T	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	T	T	T

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$\therefore \neg p \wedge (\neg q \vee \neg r) \not\equiv p \vee (q \wedge r)$

$A \not\equiv B$

6. $A = p \wedge (p \vee q)$

$B = p \vee (p \wedge q)$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$\therefore p \wedge (p \vee q) \equiv p \vee (p \wedge q)$

$A \equiv B$

7. $P(x) = x$ is a student

$Q(x) = x$ is smart

$R(x) = x$ is shy

a) $\exists x \, (P(x) \land R(x))$

b) $\forall x \, (Q(x) \to \neg R(x))$

8. $x < 0 \to x$ is negative number

$$(-x)(-x) = (-1)(-1)(-x)(-x)$$
$$= +x^2$$

∴ Hence, it proves that a square of any negative numbers is positive.

9. $C \cap (D \cap C') = \{\}$

Assume $C \cap (D \cap C') \neq \{\}$

let $x \in C \cap (D \cap C')$

therefore, $x \in C$ and $x \in (D \cap C')$

$$C \cap (D \cap C') = ((C \cap C') \cap D$$
$$= \emptyset \cap D$$
$$= \emptyset$$

$\emptyset = \{\}$

∴ ~~He~~ Hence, the assumption is wrong

So, $C \cap (D \cap C') = \{\}$ is true

CHAP 2

-reflexive
-irreflexive
-symmetric
-symmetric
-antisymmetric
-transitive

10. a R b if and only if $|a-b| = 2$

Z = set of integer number

$Z = \{1, 2, 3, 4\}$

$a=1,\ b=1\ \ |1-1| = 0$ $\quad a=2\ b=1\ |2-1| = 1$ $\quad a=3\ b=1\ |3-1| = 2$ $\quad a=4\ b=1\ |4-1| = 3$

$a=1,\ b=2\ \ |1-2| = 1$ $\quad a=2\ b=2\ |2-2| = 0$ $\quad a=3\ b=2\ |3-2| = 1$ $\quad a=4\ b=2\ |4-2| = 2$

$a=1,\ b=3\ \ |1-3| = 2$ $\quad a=2\ b=3\ |2-3| = 1$ $\quad a=3\ b=3\ |3-3| = 0$ $\quad a=4\ b=3\ |4-3| = 1$

$a=1,\ b=4\ \ |1-4| = 3$ $\quad a=2\ b=4\ |2-4| = 2$ $\quad a=3\ b=4\ |3-4| = 1$ $\quad a=4\ b=4\ |4-4| = 0$

$R = \{(\overset{x}{1},\overset{y}{3}), (\overset{x}{2},\overset{y}{4}), (\overset{x}{3},\overset{y}{1}), (\overset{x}{4},\overset{y}{2})\}$

$$M_r = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 \end{array}$$

$$M_r \otimes M_r = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 \end{array} \otimes \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 \end{array}$$

$$= \begin{vmatrix} 0\vee0\vee1\vee0 & 0\vee0\vee0\vee0 & 0\vee0\vee0\vee0 & 0\vee0\vee0\vee0 \\ 0\vee0\vee0\vee0 & 0\vee0\vee0\vee1 & 0\vee0\vee0\vee0 & 0\vee0\vee0\vee0 \\ 0\vee0\vee0\vee0 & 0\vee0\vee0\vee0 & 1\vee0\vee0\vee0 & 0\vee0\vee0\vee0 \\ 0\vee0\vee0\vee0 & 0\vee0\vee0\vee0 & 0\vee0\vee0\vee0 & 0\vee1\vee0\vee0 \end{vmatrix}$$

$$= \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

$M_r \otimes M_r \neq M_r$

R is irreflexive because $(1,1), (2,2), (3,3), (4,4) \notin R$

R is symmetric because $(2,4) \in R$ then $(4,2) \in R$.

R is not transitive because $M_r \otimes M_r \neq M_r$.

R is not reflexive because $(a,a) \in R$, $|a-a| \neq 2$

R is not asymmetric because if $(a,b) \in R$, then $(b,a) \notin R$,

R is not antisymmetric because if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$ but $a \neq b$.

11. $A = \{a, b, c, d\}$

$R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)\}$

$$M_r = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 1 & 1 \\ d & 1 & 0 & 0 & 1 \end{array}$$

$$M_r \otimes M_r = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 1 & 1 \\ d & 1 & 0 & 0 & 1 \end{array} \otimes \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 1 & 1 \\ d & 1 & 0 & 0 & 1 \end{array}$$

$$= \begin{vmatrix} 1\vee0\vee0\vee1 & 1\vee1\vee0\vee0 & 0\vee1\vee0\vee0 & 1\vee0\vee0\vee1 \\ 0\vee0\vee0\vee0 & 0\vee1\vee0\vee0 & 0\vee1\vee1\vee0 & 0\vee0\vee1\vee0 \\ 0\vee0\vee0\vee1 & 0\vee0\vee0\vee0 & 0\vee0\vee1\vee0 & 0\vee0\vee1\vee1 \\ 1\vee0\vee0\vee1 & 1\vee0\vee0\vee0 & 0\vee0\vee0\vee0 & 1\vee0\vee0\vee1 \end{vmatrix}$$

$$= \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 0 & 1 & 1 & 1 \\ c & 1 & 0 & 1 & 1 \\ d & 1 & 1 & 0 & 1 \end{array}$$

R is reflexive relations.

R is antisymmetric because $(a,b) \in R$ but $(b,a) \notin R$

R is not transitive because $M_r \otimes M_r \neq M_r$.

∴ R is not an equivalence relation

$y = (x + 4$

$f(x) = 2x - y$

$f(y) = x - 2y$

12. $f(x,y) = (2x - y,\ x - 2y)\ ;\ (x,y) \in R \times R$,

R = real numbers

a) $2x_1 - y_1 = 2x_2 - y_2$ -①

$x_1 - 2y_1 = x_2 - 2y_2$ -②

① ÷ 2 $x_1 - \frac{y_1}{2} = x_2 - \frac{y_2}{2}$ -③

② -③ $-\frac{3}{2}y_1 = -\frac{3}{2}y_2$

$y_1 = y_2$

$x_1 - 2y_1 = x_2 - 2y_2$

$x_1 - 2y_1 + 2y_1 = x_2$

$x_1 = x_2$

∴ This shows that f is one to one.

b) f^{-1} $f(x,y) = (2x - y,\ x - 2y)$

$p = 2x - y$ -①

$q = x - 2y$ -②

$y = 2x - p$ -③

③ in ②

$q = x - 2(2x - p)$

$q = x - (4x - 2p)$

$q = x - 4x + 2p$

$q = -3x + 2p$

$3x = 2p - q$

$x = \frac{2p - q}{3}$ -④

④ in ③

$y = 2\left(\frac{2p - q}{3}\right) - p$

$y = \frac{4p - 2q}{3} - p$

$y = \frac{4p - 2q - 3p}{3}$

$y = \frac{p - 2q}{3}$

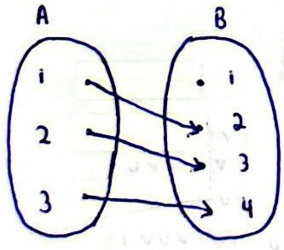
$\therefore f^{-1}(x,y) = \left(\frac{2x - y}{3},\ \frac{x - 2y}{3}\right)$

13. $A = \{1,2,3\}$

$B = \{1,2,3,4\}$

$C = \{1,2\}$

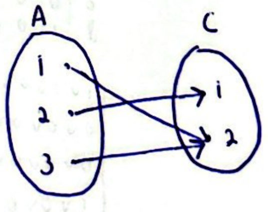
a)



$f = \{(1,2), (2,3), (3,4)\}$

is a one-to-one function because each element in B ~~that~~ has at most one arrow but not onto $B = \{1,2,3,4\}$ because no arrow pointing at 1.

b) $g : A \rightarrow C$



$g = \{(1,2), (2,1), (3,2)\}$

is not a one-to-one function because has two arrows pointing at 2 and is onto $C = \{1,2\}$ because each element in C has at least one arrow.

14. $f(x)=x^3, \; g(x)=x-1$

i) $g.f = g(f(x))$

$= x^3-1$

$f.g = f(g(x))$

$= (x-1)^3$

$= (x-1)(x-1)(x-1)$

$= (x^2-2x+1)(x-1)$

$= x^3-x^2-2x^2+2x+x-1$

$= x^3-3x^2+3x-1$

ii) $g.f = x^3-1$

$f.g = (x-1)^3$

$= x^3-3x^2+3x-1$

$g.f \neq f.g$

15. a_n = number of strings that do not contain 01.

case 1 : number of strings that all numbers are zero - 1 way

case 2 : number of strings that have all one before zero - $(n-1)$ ways

case 3 : number of strings that end with 1 - 1 way.

$a_n = (n-1)+1+1 = n+1$ ways

$a_n = n+1$

$a_{n-1} = n-1+1$

$= n$

$a_n = a_{n-1}+1, \; n \geq 2, a_1 = 2$

16.
```
input : n
output : C(n)
  Cn
{ if (n=1)
    return 0
else if (n =2 or n =3)
    return 1
else
  return C(n-1)+C(n-2)
}
```