

ASSIGNMENT 2

CHAP 2

1. i) q_n = the price of the stock at the end of the day

n = number of days

$$q_0 = 50$$

$$\begin{aligned} q_1 &= q_0 (1.02)(0.98) \\ &= q_0 (0.9996)^1 \end{aligned}$$

$$\begin{aligned} q_2 &= q_0 (1.02)(0.98)(1.02)(0.98) \\ &= q_0 (1.02)^2 (0.98)^2 \\ &= q_0 (0.9996)^2 \end{aligned}$$

$$\begin{aligned} q_3 &= q_0 (1.02)(0.98)(1.02)(0.98)(1.02)(0.98) \\ &= q_0 (1.02)^3 (0.98)^3 \\ &= q_0 (0.9996)^3 \end{aligned}$$

$$q_n = q_0 (0.9996)^n$$

$$\begin{aligned} \text{ii) } q_4 &= q_0 (0.9996)^4 \\ &= 50 (0.9996)^4 \\ &= 49.92 \end{aligned}$$

$$\begin{aligned} 2. \text{ a) } a_1 &= 5 \\ d &= \frac{37}{7} - 5 \\ &= \frac{2}{7} \end{aligned}$$

$$\begin{aligned} a_{n+1} &= a_n + \frac{2}{7} \\ a_n &= a_{n-1} + \frac{2}{7}, \quad n \geq 1 \end{aligned}$$

$$\begin{aligned} \text{b) } a(n) &\{ \\ \text{if } (n=1) & \\ \text{return } 5 & \\ \text{return } a(n-1) + \frac{2}{7} & \\ \} \end{aligned}$$

CHAP 3

3-1

1. 2 fair dice, 1 red, 1 blue

R \ B	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

i) sum of dice rolled is 6 = 5 ways
sum of dice rolled is 10 = 3 ways
number of ways = $5 + 3 = 8$ ways

ii) at least one dice shows the number of 3
= 11 ways

iii) red dice which shows number 3 = 6 ways

2. i) pass through R1 to R3 via R2

$R1 \rightarrow R2 = 2$ ways

$R2 \rightarrow R3 = 3$ ways

number of ways to pass through R1 to R3 via R2 is $2 \times 3 = 6$ ways.

(ii) $6 \times 6 = 36$ routines

3.

i) burger and side = $4 \times 6 = 24$ ways

burger and beverages = $4 \times 5 = 20$ ways

number of ways ^{a meal-deal} can be formed if Nina prefer a set that contain burger :
 $24 + 20 = 44$ ways

ii) burger and side = 24 ways

burger and beverages with peach tea and ^{lemon} tea = $4 \times 2 = 8$ ways

number of ways a meal-deal can be formed if Nina does not like peach tea or lemon tea : $44 - 8 = 36$ ways

iii) number of ways a meal-deal can be formed if Nina prefer main and side only = $4 \times 6 = 24$ ways .

4

7 chocolate cakes

2 cheesecake

6 fruity cake

two layer cakes

$\therefore 7 + 2 + 6 + 1 = 16$ options

3.2 and 3.3

1. i) permutations of 3 letters = $26^3 = 17576$

permutations of 5 digits = $10^5 = 100\,000$

number of different subject codes = $17576 \times 100\,000 = 1757600000$ ways

ii) $\frac{CS}{CS} = \frac{3}{2}$

$$2(2^6(1 \times 10^4)) = 2(260000) = 520000 \text{ ways}$$

iii) permutations of all letters = ${}^{26}P_3 = 15600$

permutations of all digits = ${}^{10}P_5 = 30240$

number of subject codes are possible in which all the letters and the digits are distinct = $15600 \times 36240 = 471744000$ ways

2. i) $(10, 3) = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!} = 120 \text{ ways}$

k) $C(15, 9) = \frac{15!}{9!(15-9)!} = \frac{15!}{9!6!} = 5005 \text{ ways}$

$$\text{iii) } \frac{{}^8P_5}{2!} = \frac{8!}{2!(8-5)!} = 3360 \text{ strings}$$

iv) $C(10, 2) = \frac{10!}{2!8!} = 45$ ways (to select 2 girls from 10 girls)

$${}^7C_2 = \frac{7!}{2!5!} = 21 \text{ ways (select 2 boys from 7 boys)}$$

number of ways to select 2 girls and 2 boys
 $= 45 \times 21 = 945$ ways

$$3. C(20, 3) = \frac{20!}{3!(17!)} = 1140 \text{ ways}$$

$$C(15, 2) = \frac{15!}{2!(13!)} = 105 \text{ ways}$$

number of ways to set up the quiz = $1140 \times 105 = 119700$ ways

3-4

1. pigeons - people ($n=40$)

pigeonholes - months ($m=12$)

$$k = \left\lceil \frac{n}{m} \right\rceil = \left\lceil \frac{40}{12} \right\rceil = 4$$

= at least 4 people

2. pigeons - $n=35$ students

pigeonholes - $m=10$

$$k = \left\lceil \frac{n}{m} \right\rceil = \left\lceil \frac{35}{10} \right\rceil = 4$$

= at least 4 students

3. $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $Y = \text{sum of } 11$

$$Y = \{(10, 1), (4, 2), (8, 3), (7, 4), (6, 5)\}$$

Only 5 pairs of numbers that sum to 11.

By selecting 6th numbers, the Pigeonhole Principle shows that at least one pair will be chosen. So the sum of 2 of them will be exactly 11.

4. pigeons : $n = 115$ different classes

pigeonholes : $m = 53$ different time periods

$$k = \left\lceil \frac{n}{m} \right\rceil = \left\lceil \frac{115}{53} \right\rceil = 3$$

= at least 3 different room.

5. The number of connection of a computer have range 1-24.

If each computer must have connection, it require 25 connection, but only 24 connection available.

By Pigeonhole Principle, at least 2 computers must have the same number of direct connection.