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Abstract

1 Algorithm notes

- Graph convolution:

$$h_{v_i}^{(l+1)} = softmax(\sum_{j \in \hat{A}(i)} h_{v_j}^{(l)}) \tag{1}$$

  with $\hat{A}(i)$ the infection-weighted neighbours of node v_i as derived from the infection-weighted neighbours adjacency matrix $\hat{A}$.

- $\hat{A}$ is constructed from A and I which are the regular adjacency matrix and the infection matrix, respectively.
  - The adjacency matrix A is time dependent, $A(t)$, and inferred from data. In our use case, $A_{ij} = 1$ if nodes v_i and v_j, hence persons i and j, have been in contact. This corresponds to $dist(v_i, v_j) \leq \epsilon$ with $\epsilon = 0$ in the discrete case that we consider here.
  - The infection matrix is constructed as

$$I = \begin{pmatrix} 0 & 0 & p_a & 0 \\ 0 & 0 & p_a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

    with I_{ij} and i is the index of the host state and j is the index of the contact person state. The states that we consider here are ordered as follows: unknown, susceptible, infected, recovered. p_a denotes the probability of infection after contact (also known as attack rate).
  - $\hat{A}$, with $\hat{A}_{ij} \in \{0, 1\}$, is the adjacency matrix that takes the infection interactions into account and is computed as follows

$$\hat{A}_{ij} = A_{ij} \cdot \frac{I_{\texttt{indOfState}(i),\texttt{indOfState}(j)} + I_{\texttt{indOfState}(j),\texttt{indOfState}(i)}}{p_a} \tag{3}$$

with `indOfState`(k) as index of the state of agent k in order to access the elements from I. The sum comes from the fact that both, agent i and j, can act as host during a contact. The division by p_a normalises the factor to one to ensure $\hat{A}_{ij} \in \{0, 1\}$. Since I is not symmetric, p_a is a proper normalisation because the sum is in $\{0, p_a\}$.

- The feature matrix, H, consists of all agents' features and is thereby of dimension $N \times D$ where there are N agents in the population and each agent is described by D features. A four dimensional feature space is used, $D = 4$. The unit vectors of this space are interpreted as following:
  - $\vec{e}_1$: unknown state
  - $\vec{e}_2$: susceptible state
  - $\vec{e}_3$: infected state
  - $\vec{e}_4$: recovered state
- Open modeling aspects:
  - Incorporate that an infected person recovers over time.