

Vectorul de moduri :

$$t = 0,5, 0,8, 1,4, 2,1, 2,4, 2,9, 4,0, 4,5, 4,9.$$

$$t_0 = 0,5$$

$$t_1 = 0,8$$

$$t_2 = 1,4$$

$$t_3 = 2,1$$

$$t_4 = 2,4$$

$$t_5 = 2,9$$

$$t_6 = 4,0$$

$$t_7 = 4,5$$

$$t_8 = 4,9$$

si punctele de control :

$$d_1(-2, -3)$$

$$d_2(-1, 2)$$

$$d_3(2, 2)$$

$$d_4(3, 0)$$

$$d_5(1, -3)$$

a) Determinati vectorul de moduri si punctele de control ale derivatei curbei

$$d_1' = \frac{(3-1+1)(d_2-d_1)}{t_4-t_1} = \frac{3 \cdot ((-1, 2) - (-2, -3))}{2,4 - 0,8} =$$

$$= (1,875; 9,375)$$

$$d_2' = \frac{3 \cdot (d_3 - d_2)}{t_5 - t_2} = 3 \cdot \frac{(2,2) - (1,2)}{2,9 - 1,4} = 3 \cdot \frac{(3,0)}{1,5} = 6,0.$$

$$d_3' = 3 \cdot \frac{d_4 - d_3}{t_6 - t_3} = 3 \cdot \frac{1,4 - 2}{1,9} = (1,578; -3,157)$$

$$d_4' = 3 \cdot \left(\frac{d_5 - d_4}{t_7 - t_4} \right) = 3 \cdot \frac{(-2, -3)}{2,1} = (-2,857; -4,285).$$

b) Folositi algoritmul lui de Boole pentru a deriva pentru a calcula $x'(2,8)$.

$$M_m^i(t)' = \frac{m}{t_{i+m-1} - t_{i-1}} M_{m-1}^i(t) - \frac{m}{t_{i+m} - t_i} M_{m-1}^{i+1}(t)$$

$$x^{(k)}(t) = \sum_{i=1}^{m+M-k} d_i^{(k)} M_{m-k}^{i(k)}(t)$$

$$M_0^i(t) \begin{cases} 1, & t \in [t_{i-1}, t_i] \\ 0 & \text{altfel} \end{cases}$$

$$m=3 \Rightarrow M=2$$

$$a_c = 4$$

$$2,8 \notin [t_0, t_1]$$

$$M_0'(t) = 0$$

$$M_0^2(t) = 0$$

$$M_0^3(t) = 0$$

$$\mu_1'(t) = 0$$

$$\mu_1^2(t) = 0$$

$$\mu_1^3(t) = 0 + \frac{(t_4 - t)}{t_4 - t_3} \cdot 1 = \frac{2,4 + 2,8}{0,3} = 1/3$$

$$\mu_0^4(t) = 1$$

$$\mu_1^4(t) = \frac{t - t_3}{t_4 - t_3} \cdot 1 + 0 = \frac{2,8 + 2,4}{0,3} = 1/3$$

$$\mu_0^5(t) = 0$$

$$\mu_1^5(t) = 0$$

$$\mu_0^6(t) = 0$$

$$\mu_1^6(t) = 0$$

$$\mu_0^7(t) = 0$$

$$\mu_1^7(t) = 0$$

$$\mu_2'(t) = 0 + \frac{t_2 - t}{t_3 - t_1} \cdot \mu_1^2 = 0$$

$$\mu_2^2(t) = 0 + \frac{t_4 - t}{t_4 - t_2} \cdot 1/3$$

$$\mu_2^3(t) = \frac{t - t_2}{t_4 - t_2} \mu_1^3(t) + \frac{t_5 - t}{t_5 - t_3} \mu_1^4(t)$$

$$\mu_2^4(t) = \frac{t - t_3}{t_5 - t_3} \mu_1^4(t) + \dots - 1 - \underbrace{\mu_1^5(t)}_{=0}$$

$$\mu_2^5(t) = 0$$

$$\mu_2^6(t) = 0$$

$$M_3^1(t) = 0 + \frac{t_4 - t}{t_4 - t_2} M_2^2(t)$$

$$M_3^2(t) = \frac{t - t_1}{t_4 - t_1} M_2^2(t) + \frac{t_5 - t}{t_5 - t_2} M_2^3(t)$$

$$M_3^3(t) = \frac{t - t_2}{t_5 - t_2} M_2^3(t) + \frac{t_6 - t}{t_6 - t_3} M_2^4(t)$$

$$M_3^4(t) = \frac{t - t_3}{t_6 - t_3} M_2^4(t) + \frac{t_7 - t}{t_7 - t_4} M_2^5(t)$$

$$M_3^5(t) = \frac{t - t_4}{t_7 - t_4} M_2^5(t) + 0.$$

$$M_4^1(t) = \frac{t - t_0}{t_4 - t_0} M_3^1(t) + \frac{t_5 - t}{t_5 - t_1} M_3^2(t)$$

$$M_4^2(t) = \frac{t - t_1}{t_5 - t_1} M_3^2(t) + \frac{t_6 - t}{t_6 - t_2} M_3^3(t)$$

$$M_4^3(t) = \frac{t - t_2}{t_6 - t_2} M_3^3(t) + \frac{t_7 - t}{t_7 - t_3} M_3^4(t)$$

$$M_4^4(t) = \frac{t - t_3}{t_7 - t_3} M_3^4(t) + \frac{t_8 - t}{t_8 - t_4} M_3^5(t)$$

$$M_5^1(t) = \frac{t - t_0}{t_5 - t_0} M_4^1(t) + \frac{t_6 - t}{t_6 - t_1} M_4^2(t)$$

$$M_5^2(t) = \frac{t - t_1}{t_6 - t_1} M_4^2(t) + \frac{t_7 - t}{t_7 - t_2} M_4^3(t)$$

$$M_5^3(t) = \frac{t-t_1}{t_7-t_1} M_5^2(t) + \frac{t_8-t}{t_8-t_1} M_6^2(t)$$

$$M_4^1(t) = \frac{t-t_0}{t_7-t_0} M_6^1(t) + \frac{t_8-t}{t_8-t_1} M_6^2(t)$$

$$M_6^1(t) = \frac{t-t_0}{t_6-t_1} M_5^1(t) + \frac{t_2-t}{t_7-t_1} M_5^2(t)$$

$$M_6^2(t) = \frac{t-t_1}{t_7-t_1} M_5^2(t) + \frac{t_8-t}{t_8-t_2} M_5^3(t)$$

$$x'(t) = \sum_{i=1}^4 d_i' (M_2^i)'(t) \quad i=1,2,3,4$$

$$= d_1' (M_2^1)'(t) + d_2' (M_2^2)'(t) + d_3' (M_2^3)'(t) + d_4' (M_2^4)'(t)$$

$$d_1' = (\dots, \dots)$$

$$d_2' = (\dots, \dots)$$

$$d_3',$$

$$d_4'$$

$$(M_2^1)'(t) = \frac{2}{t_2-t_0} \cdot M_1^1(t) - \frac{2}{t_3-t_1} \cdot M_1^2(t) = 0$$

$$(M_2^2)'(t) = \frac{2}{t_3-t_1} \cdot M_1^2(t) - \frac{2}{t_4-t_2} M_1^3(t) = 2\sqrt{6}$$

$$(M_2^3)'(t) = \frac{2}{t_4-t_2} M_1^3(t) - \frac{2}{t_5-t_3} M_1^4(t) = -5\sqrt{9}$$

$$(M_2^4)(t) = \frac{2}{t_5 - t_3} M_1^4(t) - \frac{2}{t_6 - t_4} M_1^5(t) = 3(3)$$

$$\begin{aligned} x'(t) &= 0 + (6, 0) \cdot (-2, 6) + (1, 578; -3, 157) \cdot (6, 9) + \\ &+ (-2, 857; -4, 285) \cdot (3, 13) = (15, 996; 0) + \\ &(9, 466; 18, 938) + (9, 522; -14, 281) = \\ &= (34, 984; 4, 657) \end{aligned}$$

$$x'(t) = (34, 984; 4, 657)$$