MATH 260, Homework 8, Spring '14 $\,$

Due: March 28, 2014

Honor Code: Name:

1) (12 pts) Determine if the following sets of "vectors" are linearly independent. Briefly explain your reasoning.

a)
$$\left\{ \begin{bmatrix} 2\\4\\6 \end{bmatrix}, \begin{bmatrix} 1\\2\\3 \end{bmatrix} \right\}$$

b)
$$\left\{ \begin{bmatrix} 1\\1\\1 \end{bmatrix}, \begin{bmatrix} 0\\1\\1 \end{bmatrix}, \begin{bmatrix} 0\\0\\-1 \end{bmatrix} \right\}$$

c)
$$\{2t^2+1, t, 3t+4, -t^2\}$$

$$\mathrm{d}) \, \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

2) (16 pts) For each vector space below, determine if the accompanying set is a basis for it. If it is not, add or remove vectors from the set until your new set is a basis for the vector space. State the dimension of the vector space.

a)
$$\mathbb{R}^2$$
; $\left\{ \begin{bmatrix} 1\\2 \end{bmatrix}, \begin{bmatrix} 3\\4 \end{bmatrix}, \begin{bmatrix} 5\\6 \end{bmatrix} \right\}$

b)
$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} | x + y = 0 \right\}; \quad \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

c) the set of diagonal
$$3 \times 3$$
 matrices;
$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

d) the set of all upper triangular 2×2 matrices; $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\}$