KEY

MATH 260, Linear Systems and Matrices, Summer I '14
Activity 2: Matrix Operations

Matrices for today's problems:

$$\mathbf{A} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 2 & -5 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 3 & 8 & 2 \\ -1 & x & x^2 \end{bmatrix}$$

Warmup: Constants and Matrices

Find $k\mathbf{C}$ where k is a constant. What is $k\mathbf{C}$ if $k = 3$?

$$kC = \begin{bmatrix} k & 4k \\ -3k & 5k \end{bmatrix} \qquad 3C = \begin{bmatrix} 3 & 12 \\ -9 & 15 \end{bmatrix}$$

Problem 1: Vectors, and Vectors with Matrices

a) Find the expected dimension of $\mathbf{AB}$, $\mathbf{BI}_3$, and $\mathbf{BE}$.

$$A_{3\times 1} \; B_{1\times 3} = (AB)_{3\times 3}$$

$$B_{1\times 3} \; I_{3\times 3} = (BI)_{1\times 3}$$

$$B_{1\times 3} \; E_{2\times 3} \rightarrow$$ NOT DEFINED since inner dimensions do not match

$\neq$

b) Now calculate each product.

$$AB = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -5 \end{bmatrix} = \begin{bmatrix} 6 & 4 & -10 \\ 12 & 8 & -20 \\ -3 & -2 & 5 \end{bmatrix}$$

$$BI_3 = \begin{bmatrix} 3 & 2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -5 \end{bmatrix}$$

BE undefined

Problem 2: Matrix on Matrix

a) Find **CD**.

$$\begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 18 & 13 \\ 14 & 12 \end{bmatrix}$$

b) Make a prediction (don't calculate yet): Is the statement: **CD** = **DC** true or false?

It's false

c) Find **DC**. Were you correct?

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 13 \\ -5 & 31 \end{bmatrix}$$

So CD ≠ DC, so correct, it's false.

d) Can you find any two matrices (**A** and **B**) for which **AB** = **BA**? Note that here, **A** and **B** are NOT the matrices given on page 1. *Hint: This is a SPECIAL circumstance.*

There are many possibilities, but an easy example is one matrix is the identity matrix I.

Example: $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Problem 3: More Matrix on Matrix

a) Which of these are true for any generic matrices where the operations are well-defined (give some reasoning, you don't *have* to prove them):

(**AB**)**C** = **A** + **BC**   **A**(**B** + **C**) = **AB** + **AC**   (**B** + **C**)**A** = **AB** + **CA**

false

mult. can't turn into addition.

true, distributive property

false, mult. order switched which we saw was false.

b) Under certain circumstances, we can raise a matrix to a power. Consider the $m \times n$ matrix **A**. Discuss at your table what some requirements on **A** might be, and what the resultant dimensions of $\mathbf{A}^k$ would be.

A must be square, so $m = n$. (A has dimensions $n \times n$).

c) Find **CC**. This is equivalent to $\mathbf{C}^2$.

$$\begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -11 & 24 \\ -18 & 13 \end{bmatrix}$$

$$C \cdot C \qquad C^2$$

d) Find $\mathbf{C}^3$. *Hint: Use your result from (c) to reduce your work*

$$C^3 = CC^2$$

$$\begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} -11 & 24 \\ -18 & 13 \end{bmatrix} = \begin{bmatrix} -83 & 76 \\ -57 & -7 \end{bmatrix}$$

e) Could you find $\mathbf{E}^3$? If you can, give its dimensions. If you can't find it, explain why not?

Not possible, E isn't square.

Problem 4: Transposing Matrices

We can find what's called the "transpose" of a matrix by swapping the matrix's rows and columns. More technically, to find the transpose of **A**, denoted by $\mathbf{A}^{\mathrm{T}}$ for all i, j $[a_{ij}]^{\mathrm{T}} = [a_{ji}]$.

Find $\mathbf{E}^{\mathrm{T}}$.

$$E^T = \begin{bmatrix} 3 & 8 & 2 \\ -1 & x & x^2 \end{bmatrix}^T = \begin{bmatrix} 3 & -1 \\ 8 & x \\ 2 & x^2 \end{bmatrix}$$