**MATH 260, Final Test, Fall '14**
**November 20, 2014**

**Name:**

**Section:**

**READ THESE DIRECTIONS:**

- **There are 6 problems, 5 total pages, 100 points on this test.**
- **Write your answers on this copy of the exam. If you need more space, use the back of these sheets but clearly identify what problem it refers to.**
- **You must show your work. Unjustified answers may not receive full credit.**
- **Even if you can't finish a problem, write down what you can! You may receive partial credit.**
- **When you've finished the test, bring it to me.**
- **Authorized aid: Your brain.**
- **Un-authorized aid: Anything not listed as authorized aid. This includes calculators, notes, books, cell phones (FOR ANY REASON), etc.**

**Honor Code:**

**1) (10 pts) Which of the following subsets of $\mathbb{R}^3$ are subspaces? Justify your answer.**

**a)** $S_1 = \{(x, y, z) \,|\, z = 2\}$.

**b)** $S_2 = \{(x, y, z) \,|\, y = x + z\}$.

**2) (15 pts) State whether the following sets of vectors are linearly dependent or independent. Justify your conclusion.**

**a)** $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

**b)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

**c)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

**3) (10 pts) Find a basis for** $S = \left\{ \begin{bmatrix} a \\ b \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$ **and give the dimension of** $S$**.**

**4) Let** $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$**.**

**a) (5 pts) Find the eigenvalues of A.**

**b) (10 pts) Find a basis for each eigenspace of A.**

**c) (5 pts) Is A diagonalizable? Justify.**

**5) (25 pts) Let $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 7 \\ 2 & 2 & 2 \end{bmatrix}$. Find...**

**a) a basis for the null space of A.**

**b) a basis for the column space of A.**

**c) a basis for the column space of $\mathbf{A}^T$.**

**d) the rank(A) and nullity(A).**

**e) Is $\begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$ in the column space of A? Justify.**

**6) (20 pts) Let $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $\mathbf{A} = \mathbf{PDP}^{-1}$. (NOTE: You do NOT need to find $\mathbf{P}^{-1}$ unless you want to check your work.)**