MATH 260, Homework 12, Fall '14
Due: November 14, 2014 at 2:20 PM
Honor Code: **Name:**
Section:

As in Activity 12, we will use the matrix: $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ with eigenvalues $\lambda_1 = 3$ and $\lambda_2 = -1$ with corresponding eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively.

Powering a diagonal matrix is easy: $\mathbf{D}^k = \begin{bmatrix} d_1^k & 0 & \dots & 0 \\ 0 & d_2^k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_n^k \end{bmatrix}$. However, powering any non-diagonal square matrix isn't quite as easy. Let's see if we can abuse the diagonalizability of $\mathbf{A}$ to make it easier, though.

1) (5 pts) Compute $\mathbf{PD}^2\mathbf{P}^{-1}$ and $\mathbf{A}^2$. What do you notice?

2) (5 pts) Compute $\mathbf{PD}^3\mathbf{P}^{-1}$. Is it equal to $\mathbf{A}^3 = \begin{bmatrix} 13 & 7 \\ 28 & 13 \end{bmatrix}$?

3) (10 pts) Give a general equation to compute $\mathbf{A}^k$. This works for every diagonalizable matrix $\mathbf{A}$. Why does this work? Recall that $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.