MATH 260, Linear Systems and Matrices, Fall '14
Activity 2: Matrix Operations

Matrices for today's problems:

$$\mathbf{A} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 2 & -5 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 3 & 8 & 2 \\ -1 & x & x^2 \end{bmatrix}$$

Warmup: Constants and Matrices

Find $k\mathbf{C}$ where k is a constant. What is $k\mathbf{C}$ if $k = 3$?

Problem 1: Vectors, and Vectors with Matrices

a) Find the expected dimension of $\mathbf{AB}$, $\mathbf{BI}_3$, and $\mathbf{BE}$.

b) Now calculate each product.

Problem 2: Matrix on Matrix

a) Find $\mathbf{CD}$.

b) Make a prediction (don't calculate yet): Is the statement: $\mathbf{CD} = \mathbf{DC}$ true or false?

c) Find $\mathbf{DC}$. Were you correct?

d) Can you find any two matrices ($\mathbf{A}$ and $\mathbf{B}$) for which $\mathbf{AB} = \mathbf{BA}$? Note that here, $\mathbf{A}$ and $\mathbf{B}$ are NOT the matrices given on page 1. *Hint: This is a SPECIAL circumstance.*

Problem 3: More Matrix on Matrix

a) Which of these are true for any generic matrices where the operations are well-defined (give some reasoning, you don't *have* to prove them):
$(\mathbf{AB})\mathbf{C} = \mathbf{A} + \mathbf{BC}$ $\quad$ $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$ $\quad$ $(\mathbf{B} + \mathbf{C})\mathbf{A} = \mathbf{AB} + \mathbf{CA}$

b) Under certain circumstances, we can raise a matrix to a power. Consider the $m \times n$ matrix $\mathbf{A}$. Discuss at your table what some requirements on $\mathbf{A}$ might be, and what the resultant dimensions of $\mathbf{A}^k$ would be.

c) Find $\mathbf{CC}$. This is equivalent to $\mathbf{C}^2$.

d) Find $\mathbf{C}^3$. *Hint: Use your result from (c) to reduce your work*

e) Could you find $\mathbf{E}^3$? If you can, give its dimensions. If you can't find it, explain why not?

Problem 4: Transposing Matrices

We can find what's called the "transpose" of a matrix by swapping the matrix's rows and columns. More technically, to find the transpose of $\mathbf{A}$, denoted by $\mathbf{A}^\intercal$ for all i, j $[a_{ij}]^\intercal = [a_{ji}]$.

Find $\mathbf{E}^\intercal$.