MATH 260, Linear Algebra, Spring '14
Activity 5: Matrix Inverses
Honor Code: **Names:**

Directions: Everyone should work on the assignment and should fill out their paper. You are expected to make corrections based on what is presented on the board.

The worksheet is due by the end of the day (5pm).

In-Class Learning Goals:

1. Be able to find the inverse of a matrix.
2. Be able to identify several characteristics of an invertible matrix.
3. Be able to state the number of solutions to a matrix-vector equation, based on invertibility.

Warmup: RREF Review

Find the RREF of the following matrix:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}\right]$$

Congratulations! You've found your first matrix inverse.

1 The Identity Matrix and Inverses

Notice that the right-hand side (RHS) of the warmup started as the identity matrix. The warmup exercise could be stated as: $[\mathbf{A}|\mathbf{I}]$. Reducing this combined matrix into RREF is the simplest way to find the inverse of a matrix. The inverse of the matrix $\mathbf{A}$ is the RHS after getting the left-side into RREF.

a) One trait of the inverse is that the statement: $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$. Verify this is true for the warmup.

b) Another trait if a matrix has an inverse, then its transpose has an inverse. Let's check that:
(i) Find $\mathbf{A}^T$. Recall that the transpose is defined as: $[a_{ij}]^T = [a_{ji}]$

(ii) Find the inverse of $\mathbf{A}^T$. *Hint: start with the matrix* $\left[A^T|I\right]$

c) Another practice. Obtain the inverse of the following matrix using row operations and RREF.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

2 A Use for the Inverse

a) How did we use RREF to find solutions to the matrix-vector equation: $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$?

b) If we do the same operations to find a RREF, can't we use the inverse to find an answer too...?
(i) Solve the following system by finding the RREF of the ***augmented*** matrix:

$$\begin{array}{rcrcr} x & + & y & = & -1 \\ 4x & + & y & = & -7 \end{array}$$

(ii) Now multiply the inverse you found in the warmup by $\vec{\mathbf{b}} = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$

c) Discuss your results at your table, and write down observations. Here's a little guidance...
Left multiply each side of the equation $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ by $\mathbf{A}^{-1}$ and compare to answers you've obtained so far...

3 More Solutions...

a) Find the inverse of the matrix:

$$\mathbf{C} = \begin{bmatrix} 3 & 0 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

b) What happened? Based on your exploration in Section 2, if this matrix described a system, do you think it has a unique answer? Support your response, and discuss at your table. (This is a good place to check in once you've discussed)

Invertibility and Solutions (pg 151)

Notes:

4 Challenge

Find a constant a so each of the following matrices are invertible. If no such a exists, says so and explain why. (Problems 32 & 33 from pg 155)

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ a & a & a \end{bmatrix}$$