MATH 260, Linear Algebra, Spring '14
Activity 9: Eigenvalues and Eigenvectors
Honor Code: **Names:**

Directions: Everyone should work on the assignment and should fill out their paper. You are expected to make corrections based on what is presented on the board.
The worksheet is due by the end of the day (5pm).
If you need more explanations after class, you can read Section 5.3 of your textbook.

In-Class Learning Goals:

1. Be able to calculate the eigenvalues and eigenvectors of a 2x2 matrix.
2. Understand how to calculate the eigenvalues and eigenvectors for ANY matrix.
3. (Stretch) Be able to identify several properites of eigenvalues/vectors.

Warm-up: Matrix Multiplication

Given the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ and the vectors: $\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$, $\vec{v}_4 = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$, $\vec{v}_5 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Find $\mathbf{A}\vec{v}_i$ for $i = 1 \to 5$.

Visualizing Eigenvectors

1. Sketch each $\vec{v}_i$ and the result of $\mathbf{A}\vec{v}_i$ from the warm-up on the x-y plane. You should have 5 pictures when you are done. Be sure to clearly label which was your initial, and which your resultant vector.

2. Two vectors behaved differently, which ones? Describe & discuss at your table how they behaved differently.

3. For the two vectors that behaved differently, you should be able to write the eigenvector equation: $\mathbf{A}\vec{v} = \lambda\vec{v}$. Find λ_i for each pair of vectors.

Practice

Let's practice.

4. For each matrix, take the determinant $|\mathbf{A} - \lambda \mathbf{I}|$ then set the resulting polynomial equal to 0 and solve for λ. (they should all be factorable). Once you have found ALL the eigenvalues (λ) find at least one eigenvector for each matrix.

$$\mathbf{W} = \begin{bmatrix} 1 & 4 \\ -4 & 11 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

5. Go back and look at $\mathbf{R}$ what do you notice about the eigenvalues you found for $\mathbf{R}$?

Some Properties of Eigenvalues

Note: We will find these for 2×2 *matrices, but these properties generalize to any* $n \times n$ *matrix.*

6. We find eigenvalues by solving: $|\mathbf{A} - \lambda\mathbf{I}| = 0$. What happens if $\lambda = 0$? What must be true about the matrix $\mathbf{A}$ for $\mathbf{A}$ to have eigenvalues of 0?

7. (a) Find $\mathbf{W}^T$.

   (b) What is the polynomial from $|\mathbf{W}^T - \lambda\mathbf{I}|$ and how does it compare to the polynomial for $|\mathbf{W} - \lambda\mathbf{I}|$?

   (c) What can you say about the eigenvalues (and vectors) of $\mathbf{A}$ and $\mathbf{A}^T$?

8. (a) Find $\mathbf{W}^{-1}$. (the inverse)

   (b) What is the polynomial from $|\mathbf{W}^{-1} - \lambda\mathbf{I}|$? Set the polynomial equal to 0, and solve for λ

   (c) Look carefully, how do the eigenvalues of $\mathbf{W}^{-1}$ relate to the eigenvalues of $\mathbf{W}$?