MATH 260, Linear Systems and Matrices, Fall '14
Activity 5: Matrix Inverses

Warmup: RREF Review

Find the RREF of the following matrix:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}\right]$$

Congratulations! You've found your first matrix inverse.

1 The Identity Matrix and Inverses

Notice that the right-hand side (RHS) of the warmup started as the identity matrix. The warmup exercise could be stated as: $[\mathbf{A}|\mathbf{I}]$. Reducing this combined matrix into RREF is the simplest way to find the inverse of a matrix. The inverse of the matrix $\mathbf{A}$ is the RHS after getting the left-side into RREF. That is, you start with $[\mathbf{A}|\mathbf{I}]$ then row reduce to $[\mathbf{I}|\mathbf{A}^{-1}]$.

a) One trait of the inverse is that the statement: $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$. Verify this is true for the warmup.

b) Another trait if a matrix has an inverse, then its transpose has an inverse. Let's check that:

(i) Find $\mathbf{A}^T$. Recall that the transpose is defined as: $[a_{ij}]^T = [a_{ji}]$

(ii) Find the inverse of $\mathbf{A}^T$. *Hint: start with the matrix* $\left[A^T|I\right]$

c) Another practice problem: Obtain the inverse of the following matrix using row operations and RREF.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

2 A Use for the Inverse

a) How did we use RREF to find solutions to the matrix-vector equation: $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$?

b) If we do the same operations to find a RREF, can't we use the inverse to find an answer too...?

(i) Solve the following system by finding the RREF of the ***augmented*** matrix:

$$\begin{array}{rcrcr} x & + & y & = & -1 \\ 4x & + & y & = & -7 \end{array}$$

(ii) Now multiply the inverse you found in the warmup by $\vec{\mathbf{b}} = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$

c) Consider your results and write down observations. Here's a little guidance... Left multiply each side of the equation $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ by $\mathbf{A}^{-1}$ and compare to answers you've obtained so far...

3 More Solutions...

a) Use the algorithm we discussed to find the inverse of the matrix:

$$\mathbf{M} = \begin{bmatrix} 3 & 0 & 3 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

b) What happened? Based on your exploration in Section 2, if this matrix described a system, do you think it has a unique answer? Justify your answer.

4 Challenge

For what value(s) a, b and c make each of the following matrices invertible. If no such constant exists, say so and explain why.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ b & b & b \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & c & 0 \end{bmatrix}$$