For which values of c is

$$B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2+c \end{bmatrix}$$

- positive definite?
- positive semi-definite?

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* determinant Test

(2), $\det \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = 3$,

$\det B = 2 \cdot \begin{vmatrix} 2 & -1 \\ \cdot & \cdot \end{vmatrix} -$

$$\begin{vmatrix} -1 & 2+c \end{vmatrix}$$

$$(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2+c \end{vmatrix} +$$

$$(-1)\begin{vmatrix} -1 & 2 \\ -1 & -1 \end{vmatrix}$$

$$= 2\cdot(4+2c-1) + (-2-c-1)$$
$$-(1+2) = 6+4c-3-c-3 = \underline{3c}$$

$c > 0$

$c \geq 0$

* pivot Test

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2+c \end{bmatrix} \quad 1/2$$

$$\sim \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3/2 & -3/2 \\ 0 & -3/2 & 3/2+c \end{bmatrix} \quad (+)$$

$$\sim \begin{bmatrix} \boxed{2} & -1 & -1 \\ 0 & \boxed{3/2} & -3/2 \\ 0 & 0 & \boxed{c} \end{bmatrix}$$

* energy test / completing the square.

$$\begin{bmatrix} x & y & z \end{bmatrix} \cdot B \cdot \begin{bmatrix} x \\ y \end{bmatrix} \geq 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore \quad 2x^2 + 2y^2 + (2+c)z^2 - 2xy - 2xz - 2yz$$

$$= 2 \cdot \left(x - \tfrac{1}{2}y - \tfrac{1}{2}z^2 \right) + \frac{3}{2} \cdot (y-z)^2 + c \cdot z^2 \geq 0$$

$c \geq 0$

$$\begin{matrix} 2 \\ 3/2 \\ c \end{matrix} \begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$c = 0$

$$2 \begin{bmatrix} \boxed{1} & -1/2 & -1/2 \end{bmatrix} \quad \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$z = 1$$