Explain why each of the following is true:

a) Every positive definite matrix is invertible

b) The only positive definite projection matrix is $P = I$

c) D is diagonal with positive entries is positive definite.

d) S symmetric with $\det S > 0$ might not be positive definite

a) A invertible $\Leftrightarrow \det A \neq 0$

$\therefore \det A = \lambda_1 \lambda_2 \lambda_3 \dots \lambda_n$

$\lambda_1, \dots, \lambda_n$ - eigenvalues of A.

- If A is positive definite $\Leftrightarrow$ $\lambda_1, \lambda_2 \dots, \lambda_n > 0$

$\therefore \det A = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \dots \lambda_n > 0$
$\neq 0$

b) - P is projection $\Rightarrow$ Eig. values of $P = 0$ or 1

- P is pos. definite $\Rightarrow$ Eig values > 0

$\Rightarrow$ Eig. values of $P = 1$.

Only matrix $P = I$

If P diagonalizable.

$P = U \cdot I \, U^{-1}$

$P = U U^{-1} = I$

(c) $D = diag(d_1 \, d_2 \ldots d_n)$

For any vector x, $x \neq 0$

$x^T D x > 0$

$x^T = (x_1 \, x_2 \ldots x_n)$

$x^T D x = d_1 x_1^2 + d_2 x_2^2 + \ldots + d_n x_n^2$

$\therefore > 0$

d)

$$S = \begin{pmatrix} \boxed{-3} & 1 \\ 1 & -2 \end{pmatrix}$$

$$\det S = 6 - 1 = 5 > 0$$

$$x^T S x \Rightarrow x = (1 \quad 0)^T$$

$$x^T S x = -3$$