Find the conditions on a & b that make the matrix A invertible, and find A^{-1} when it exists

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

---

* Spot if matrix is not invertible
- column or row of $\emptyset$
- 2 rows/columns that are the same

- A is not invertible if $a=0$.
- or $a=b$

$\therefore$ We need to find the inverse of a matrix

S1 S2 S3

$$[A \mid I] \xrightarrow{\text{elimination}} [I \mid A^{-1}]$$

$\therefore$

$$\left[\begin{array}{ccc|ccc} \boxed{a} & b & b & 1 & 0 & 0 \\ a & a & b & 0 & 1 & 0 \\ a & a & a & 0 & 0 & 1 \end{array}\right] \; \ominus$$

$$\to \left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ 0 & a-b & 0 & -1 & 1 & 0 \\ 0 & a-b & a-b & -1 & 0 & 1 \end{array}\right] \; \ominus$$

$$\to \left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \end{array}\right.$$

$$\left[\begin{array}{ccc|ccc} 0 & a-b & 0 & -1 & 1 & 0 \\ 0 & 0 & a-b & 0 & -1 & 1 \end{array}\right]$$

/a (divide by a) ∴ $a \neq 0$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & b/a & b/a & 1/a & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ 0 & 0 & 1 & 0 & \frac{-1}{a-b} & \frac{1}{a-b} \end{array}\right]$$

∗ $a-b \neq 0$

$\rightarrow$ $row_1 - \frac{b}{a}(row_2 + row_3)$

to go into row 1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a-b} & 0 & \frac{-b}{a(a-b)} \\ 0 & 1 & 0 & -1/a-b & \frac{1}{a-b} & 0 \\ 0 & 0 & 1 & & & \end{array}\right]$$

$\therefore$

$$A^{-1} = \frac{1}{a-b}\begin{bmatrix} 1 & 0 & -b/a \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$