- Let $T(A) = A^T$, A is 2×2

1) Why is T linear? What is T^{-1}?

2) Write down the matrix of T in

- $v_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$,

  $v_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

- $w_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, $w_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

  $w_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

3) Eigenvalues / eigenvectors of T?

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1) $T(A+B) = (A+B)^T = A^T + B^T$

$= T(A)+T(B)$

$T(cA) = (cA)^T = cA^T = cT(A)$

$T^2 = I \Rightarrow T^{-1} = T$

2)
$$\left.\begin{aligned} Tv_1 &= v_1 \\ Tv_2 &= v_3 \\ Tv_3 &= v_4 \\ Tv_4 &= v_4 \end{aligned}\right\} \Rightarrow M_T = \begin{array}{c} \begin{matrix} Tv_1 & Tv_2 & Tv_3 & Tv_4 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$\begin{tikzcd}
\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \arrow[r] \arrow[d, "T"] & \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \arrow[d, leftrightarrow, "M_T"] \\
\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \arrow[r] & \begin{pmatrix} 1 \\ 3 \\ 2 \\ 4 \end{pmatrix}
\end{tikzcd}$$

$$\left.\begin{array}{l} \cdot\ T w_1 = w_1 \\ T w_2 = w_2 \\ T w_3 = w_3 \\ T w_4 = -w_4 \end{array}\right\} \Rightarrow M_T' = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

(3) $Tv = \lambda v$ (above basis)