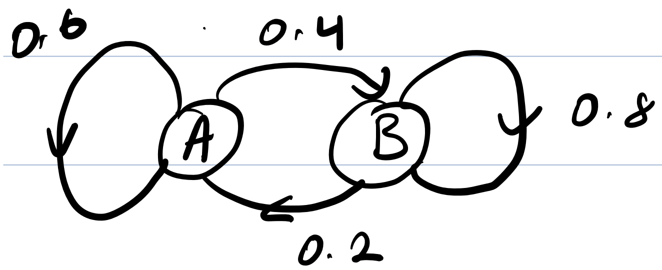
A particle jumps between positions A & B with the following probabilities:



If it starts at A, what is the probability it is at A and B after

i) 1 step
ii) n steps
iii) ∞ - steps

We describe graph using matrix

A ↓ ↓ B

$$A = \begin{pmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{pmatrix} \begin{matrix} \leftarrow A \\ \leftarrow B \end{matrix}$$

starts at A

$$p_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{matrix} \leftarrow A \\ \leftarrow B \end{matrix}$$

i) 1-step:

$$p_1 = A\,p_0 = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$$

ii) n-steps

$$p_1 = Ap_0 \,,\quad p_2 = Ap_1 = A^2 p_0$$

$$p_n = A^n p_0$$

Recall $A = U D U^{-1}$

$\lambda = 1 \quad u_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda = 0.4 \quad u_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$U = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0.4 \end{pmatrix}$

$U^{-1} = -\frac{1}{3} \begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

$$p_n = A^n p_0 = U D^n U^{-1} p_0$$
$$= \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0.4^n \end{pmatrix} \frac{1}{3} \begin{pmatrix} 1/2 \end{pmatrix}$$

(iii) $p_n = 1 \ (\ 1$

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