Find the quadratic equation through the origin that is a best fit for the points $(1,1), (2,5), (-1,-2)$.

Since quadratic equation, *through origin.

$$ct + dt^2 = y$$

$$A = \begin{pmatrix} t & t^2 \\ 1 & 1 \\ 2 & 4 \\ -1 & 1 \end{pmatrix}, \quad \hat{x} = \begin{pmatrix} c \\ d \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$$

* We cannot solve $A\hat{x} = b$

$$\Updownarrow$$

$$A^T A\hat{x} = A^T b$$

- $A^T A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 4 \\ -1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 6 & 8 \\ 8 & 10 \end{pmatrix}$$

- $A^T b = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 13 \\ 19 \end{pmatrix}$

$\Rightarrow$ $\begin{matrix} \times 3 \\ \times 4 \end{matrix} \begin{pmatrix} 6 & 8 \\ 8 & 10 \end{pmatrix} \hat{x} = \begin{pmatrix} 13 \\ 19 \end{pmatrix}$

using elimination:

$$\begin{pmatrix} 6 & 8 \\ 0 & -2 \end{pmatrix} \hat{x} = \begin{pmatrix} 13 \\ 5 \end{pmatrix}$$

$d = -5/2$ ", $c = 11/2$ "

$$\boxed{y = \frac{11}{2}t - \frac{5}{2}t^2}$$

Steps:

(S1) Identify the equation

(S2) Matrix format

(S3) Set up Projection Equation

(S4) Computation.