

Problem:

S is spanned by $(1 \ 2 \ 2 \ 3)$
and $(1 \ 3 \ 3 \ 2)$

i) Find a basis for S^\perp

ii) Can every v in \mathbb{R}^4 be written uniquely in terms of S and S^\perp ?

(i)

- If x in S^\perp

$$(1 \ 2 \ 2 \ 3) x = 0$$

$$(1 \ 3 \ 3 \ 2) x = 0$$



$$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 1 & 3 & 3 & 2 \end{pmatrix} x = 0$$

→ row reduce

$$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 1 & -1 \end{pmatrix} \underline{x} = 0$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad \begin{array}{l} \text{let } x_4 = b \\ x_3 = a \end{array}$$

$$x_2 = -x_3 + x_4 = \underline{-a + b}$$

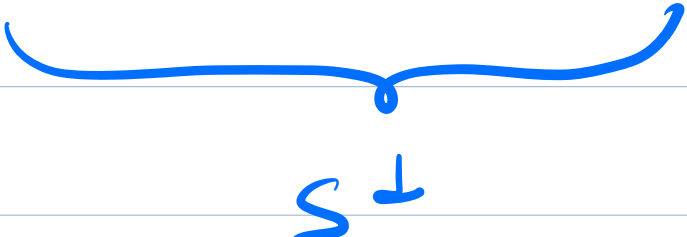
$$x_1 = -2x_2 - 2x_3 - 3x_4$$

$$= -2(a+b) - 2a - 3b$$

$$= \underline{-5b}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -5b \\ -a+b \\ a \\ b \end{pmatrix}$$


$$= a \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} -5 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$



span

ii) Yes,

$$v = c_1 \begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 3 \\ 3 \\ 2 \end{pmatrix} +$$



span

$$c_3 \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + c_4 \begin{pmatrix} -5 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 0 & -5 \\ 2 & 3 & -1 & 1 \\ 2 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = v$$