Which of the following statements are true? Explain.

a) If A & B are similar matrices, then $2A^3 + A - 3I$ and $2B^3 + B - 3I$

b) If A & B are 3×3 matrices with eigenvalues $1, 0, -1$, then A and B are similar

(c) The matrices $J_1 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ and $J_2 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ are similar

(a) True,

Know: M s.t $MAM^{-1} = B$

$$M(2A^3 + A - 3I)M^{-1}$$

$$= 2(MAM^{-1}MAM^{-1}MAM^{-1}) + MAM^{-1} - 3MIM^{-1}$$

$$= 2B^3 + B - 3I \quad //$$

(b) True,

$A = S\Lambda S^{-1}$, where $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$B = T\Lambda T^{-1}$.

$$(TS^{-1})A(TS^{-1})^{-1} = B$$

(c) $J_1 + I = \begin{pmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix}$

$J_2 + I = \begin{pmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{pmatrix}$

$\dim N(J_1 + I) = 1$

$\dim N(J_2 + I) = 2$