Objectives

Section 2.7 (Book) [PA = LU]

Section 3.1 [Vector spaces & subspaces]

Permutations:

Permutations P: execute row exchanges

What happen to $A = LU$??

→ $PA = LU$ // Any invertible A

↳ does the row exchanges.

- Permutations :

↳ P = identity matrix with re-ordered rows.

[How many possibilities : $n!$]

$n! = n(n-1)\ldots(3)(2)(1)$

↳ counts reorderings.

counts all $n \times n$ permutations.

$$\boxed{P^{-1} = P^T} \quad \neq \quad // \quad \boxed{P^T P = I}$$

Transposes

e.g. row → column // column → row.

T

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

3×2 $\quad\quad$ 2×3

$\hookrightarrow R^T$ $\quad\quad$ $\hookrightarrow R$ (rectangular matrix)

General:

Transpose $(A^T)_{ij} = A_{ji}$

Symmetric matrix $\Rightarrow A^T = A$

eg

$$\begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 9 \\ 7 & 9 & 4 \end{bmatrix}$$

$R^T R \Rightarrow$ is always symmetric.

$\Leftrightarrow$

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 11 & 7 \\ 11 & 13 & 11 \\ 7 & 11 & 17 \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 3$

Why? $\rightarrow$ TAKE TRASPOSE!

• $(R^T R)^T = R^T (R^T)^T = R^T R = I$

(order gets reversed!)

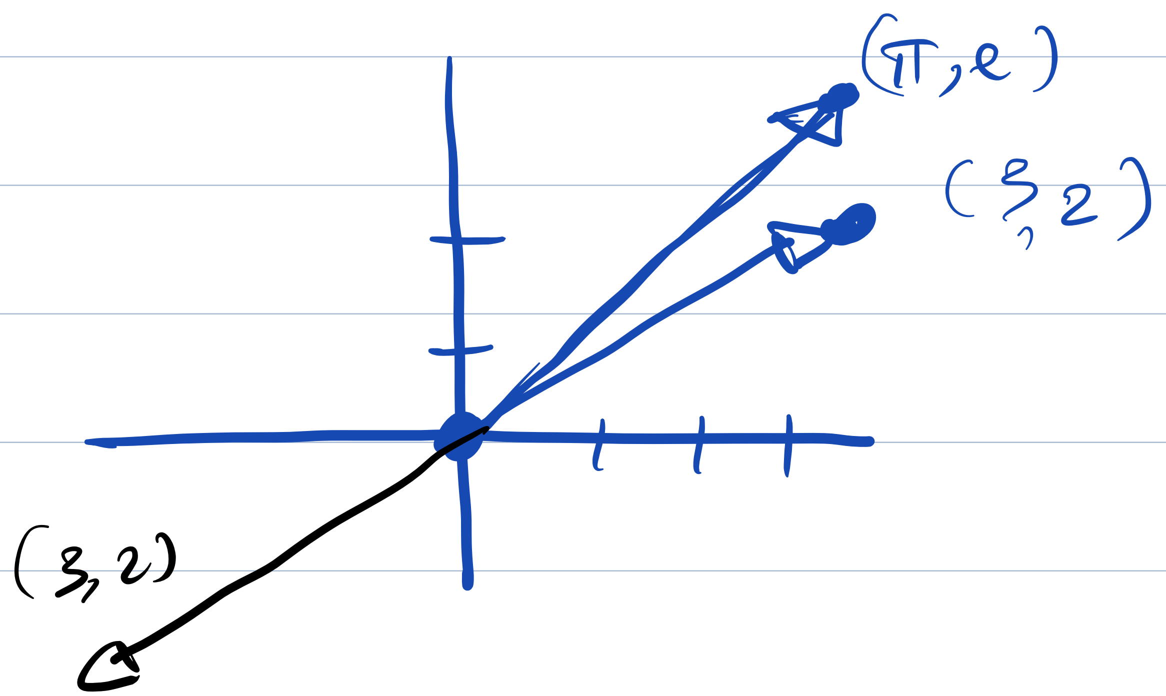
Chapter 3.

Vector (spaces) $\rightarrow$ space of vectors

What do we do vectors?
→ add them, multiply them by numbers (scalars)

<u>Examples</u>: $\mathbb{R}^{2}$ = all 2-D real vectors
= "x-y plane"

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ e \end{bmatrix}, \ldots$$



- What if we take the <u>origin</u>. away?
↳ very important!

$\mathbb{R}^3$ = all vectors with 3 real components.

eg $\begin{bmatrix}3\\2\\0\end{bmatrix}$

* What is $\mathbb{R}^n$ = all column vectors with n real components.

Suppose not a vector space

1/4

Not closed

↳ Adding ✓

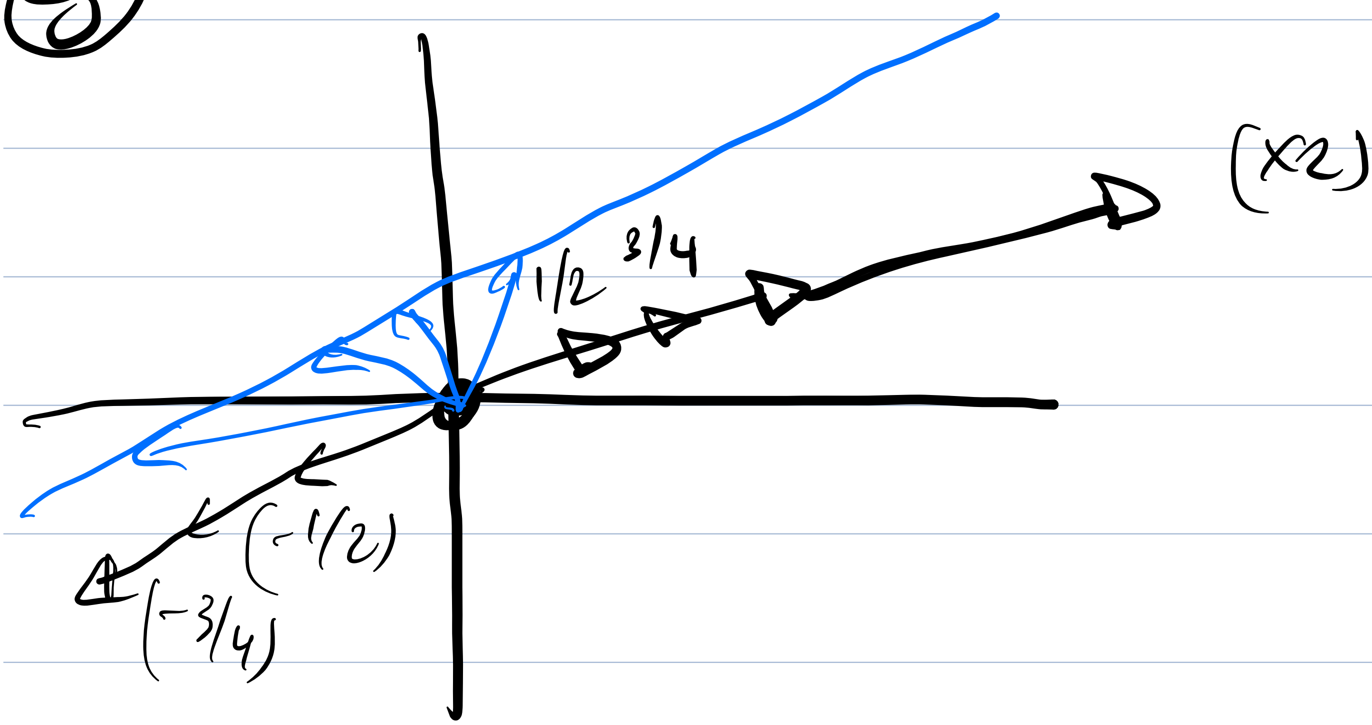
↳ Multiply scalers

by multiplication ... (take outside of the shaded)

* A vector space has to be closed by multiplication & additions of vectors → linear combinations!

A vector space inside R^2 (subspace of R^2)

eg



→ A line in R^2 must go through the 0-vector

↳ every subspace has to go through 0.

Subspaces of R^2

1. all of R^2
2. any line through $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
3. zero vector only. $[Z]$.

The subspaces of $\mathbb{R}^3$ are:

1. all of $\mathbb{R}^3$,
2. any plane through the origin,
3. any line through the origin, and
4. the zero vector alone (Z).

Column space

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$

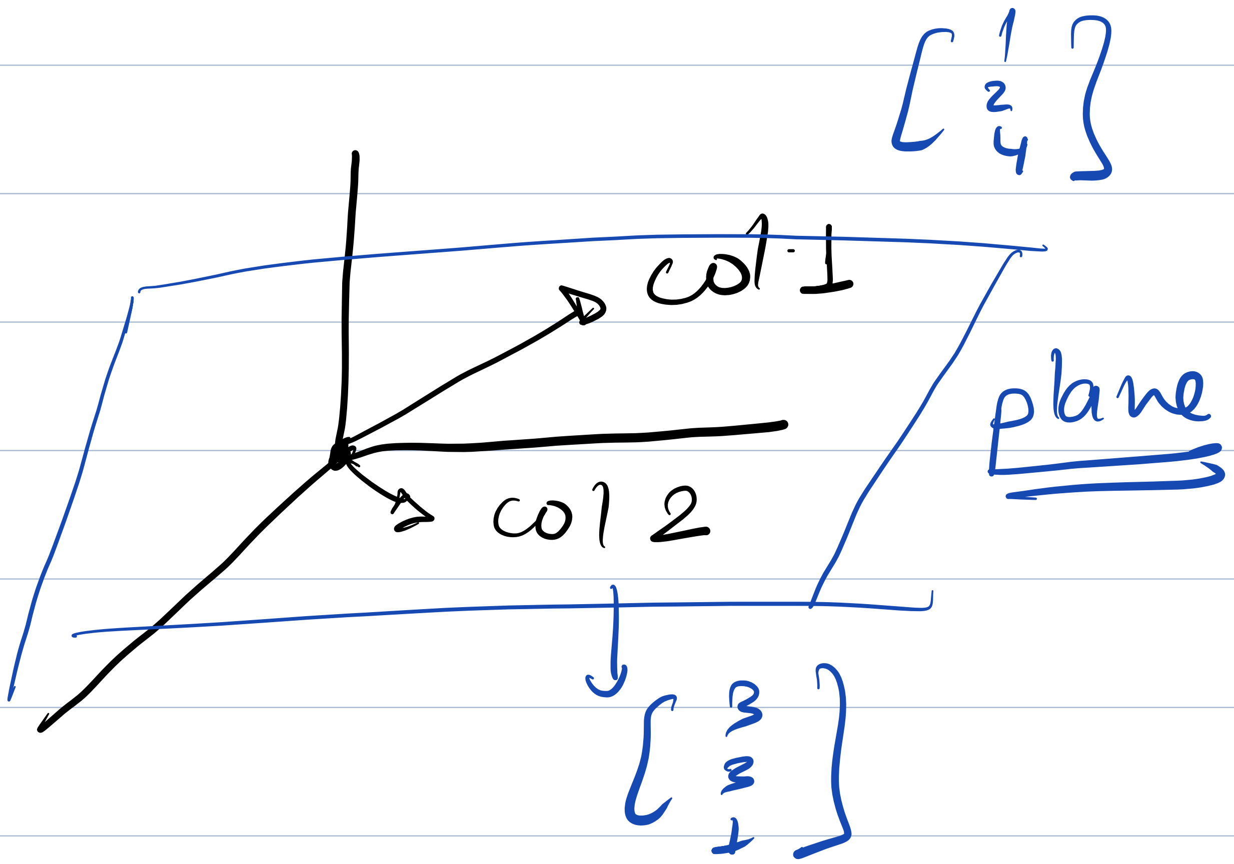
columns are in $\mathbb{R}^3$

all their combinations form a subspace.

↳ called column space.

$C(A)$

eg



All combinations → fill a plane.