Problem 3.1

Add AB to AC and compare with A(B+C)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 5 & 6 \end{bmatrix}$$

$\therefore$

① $$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$$

② $$AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 12 \\ \end{bmatrix}$$

$$\begin{bmatrix} & \\ 20 & 24 \end{bmatrix}$$

$$\therefore\ AB+AC = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 12 \\ 23 & 24 \end{bmatrix}$$

<u>Now</u> :

$$(B+C) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix}$$

$$A \cdot (B+C) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 12 \\ 23 & 24 \end{bmatrix}$$

Therefore $AB + AC = A(B+C)$

Problem 3.2

[2.5 #24 Introduction to Linear Algebra: Strang]

- Use Gauss-Jordan elimination on $[U \quad I]$ to find the upper triangular U^{-1}:

$$UU^{-1} = I = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} [x_1 x_2 x_3]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* According to Gauss - Jordan Elimination we need to reduce rows:

$$\left[\begin{array}{ccc|ccc} \boxed{1} & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times 1 \\ \times a \\ \times c \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & b-ac & 1 & -a & 0 \\ 0 & 1 & 0 & 0 & 1 & -c \end{array}\right.$$

$$\left[\begin{array}{ccc|ccc} & & & & & \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Note:

$R_1 = R_1 - aR_2$

$R_2 = R_2 - cR_2$

$\therefore R_1 = (R_1 - (cb - ac)R_3)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$= [\, I \; L^{-1} \,]$

$=$