

Exercises on similar matrices and Jordan form

Problem 28.1: (6.6 #12. *Introduction to Linear Algebra*: Strang) These Jordan matrices have eigenvalues 0, 0, 0, 0. They have two eigenvectors; one from each block. However, their block sizes don't match and they are *not similar*:

$$J = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ and } K = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right].$$

For a generic matrix M , show that if $JM = MK$ then M is not invertible and so J is not similar to K .

Problem 28.2: (6.6 #20.) Why are these statements all true?

- a) If A is similar to B then A^2 is similar to B^2 .
- b) A^2 and B^2 can be similar when A and B are not similar (try $\lambda = 0, 0$.)
- c) $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$.
- d) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ is not similar to $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$.
- e) Given a matrix A , let B be the matrix obtained by exchanging rows 1 and 2 of A and then exchanging columns 1 and 2 of A . Show that A is similar to B .

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