Find the dimensions ② of the vector space spanned by vectors:

$(1, 1, -2, 0, -1)$

$(1, 2, 0, -4, 1)$

$(0, 1, 3, -3, 2)$

$(2, 3, 0, -2, 0)$

and find a basis for that space. ①

Basis for the vector space:

↪ Need to check if vectors are linearly independent!

Ⓢ① Add vectors to matrix form

Ⓢ② Perform elimination

Ⓢ③ Columns with pivots → independent!

$$\begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 1 & 2 & 0 & -4 & 1 \\ 0 & 1 & 3 & -3 & 2 \\ 2 & 3 & 0 & -2 & 0 \end{bmatrix}$$

$\to$

$$\begin{bmatrix} \boxed{1} & 1 & -2 & 0 & -1 \\ 0 & \boxed{1} & 2 & -4 & 2 \\ 0 & 1 & 3 & -3 & 2 \\ 0 & 1 & 4 & -2 & 2 \end{bmatrix}$$

$\to$

$$\begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 \end{bmatrix}$$

→
$$\begin{bmatrix} \boxed{1} & 1 & -2 & 0 & -1 \\ 0 & \boxed{1} & 2 & -4 & 2 \\ 0 & 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix} \Big\} \text{ basis}$$

∴ basis : $(1, 1, -2, 0, -1)$

$(0, 1, 2, -4, 2)$

$(0, 0, 1, 1, 0)$

← dimension : 3

T

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 3 \\ -2 & 0 & 3 & 0 \\ 0 & -4 & -3 & -2 \\ -1 & 1 & 2 & 0 \end{bmatrix} \quad \xrightarrow{\cdot \cdot \cdot} \quad \begin{bmatrix} \textcircled{1} & 1 & 0 & 2 \\ 0 & \textcircled{1} & 1 & 1 \\ 0 & 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$\uparrow \quad \uparrow \quad \uparrow$

basis.

X X X

* But , can no longer use ↑ elements

of the basis:

<u>because</u>, when we did eliminations we changed the column space of the original matrix