Solve using the method of elimination

$$x - y - z + u = 0$$
$$2x + 2z = 8$$
$$-y - 2z = -8$$
$$3x - 3y - 2z + 4u = 7$$

pivot

$$\begin{bmatrix} \boxed{1} & -1 & -1 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & -1 & -2 & 0 \\ 3 & -3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -8 \\ 7 \end{bmatrix}$$

(-2) (-3)

(S1) pivot : To get rid of all the numbers under it

$$\begin{bmatrix} \boxed{1} & -1 & -1 & 1 & 0 \\ 0 & \boxed{2} & 4 & -2 & 8 \\ 0 & -1 & -2 & 0 & -8 \\ 0 & 0 & 1 & 1 & 7 \end{bmatrix}$$

(+1/2)

$$\left[\begin{array}{cccc|c} 0 & -1 & -2 & 0 & -8 \\ 0 & 0 & 1 & 1 & 7 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & -1 & 1 & 0 \\ 0 & \boxed{2} & 4 & -2 & 8 \\ 0 & 0 & \boxed{0} & -1 & -4 \\ 0 & 0 & 1 & 1 & 7 \end{array}\right]$$

switching

* But we cannot use 0 as a pivot

:

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & -1 & 1 & 0 \\ 0 & \boxed{2} & 4 & -2 & 8 \\ 0 & 0 & \boxed{1} & 1 & 7 \\ 0 & 0 & 0 & \boxed{-1} & -4 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

Starting from the bottom: back-sub.

$$-u = -4 \Rightarrow \boxed{u = 4}$$

$$z + u = 7 \Rightarrow \boxed{z = 3}$$

$$2y + 4z - 2u = 8 \Rightarrow \boxed{y = 2}$$

$$x - y - z - u = 0 \Rightarrow \boxed{x = 1}$$