Problem 20.1

(5.3 #8 - Introduction to Linear Algebra)

Suppose

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

Find its cofactor matrix C and multiply AC^T to find $\det(A)$

$$C = \begin{bmatrix} 6 & -3 & 0 \\ -3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 6 & -3 & -6 \\ -3 & 1 & 2 \\ & & \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & 1 \end{bmatrix}$$

$$AC^T = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & -3 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3I$$

$\Rightarrow$ Since $AC^T = \det(A)\, I$, we have $\det(A) = 3I$

If 4 is changed to 100, $\det(A)$ is unchanged because the cofactor of that entry 0, and thus its value does not contribute to the determinant.