Problem 5.1:

(2.7 #13. Introduction to Linear Algebra: Strang).

(a) Find a 3 by 3 permutation matrix with $P^3 = I$ (but not $P = I$)

(b) Find a 4 by 4 permutation $\hat{P}$ with $\hat{P}^4 \neq I$

Solution:

Let P move the rows in a cycle: the first to the second, the second to the third, the third to the first.

So:

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

$$P^3 = I$$

(b) Let $\hat{P}$ be the block diagonal matrix with 1 and P on the diagonal, $\hat{P} = \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix}$.

Since $P^3 = I$, also $\hat{P}^3 = I$

So $\hat{P}^4 \neq I$.

Problem 5.2:

- Suppose A is a four by four matrix

How many entries of A can be chosen independently if:

(a) A is symmetric?

(b) A is skew-symmetric? ($A^T = -A$)

Solution:

(a) The most general form of a four by four symmetric matrix.

$$A = \begin{bmatrix} a & e & f & g \\ e & b & h & i \\ f & h & c & j \\ g & i & j & d \end{bmatrix}$$

$\therefore$ 10 entries can be chosen independently.

(b) The most general form of a four by four skew-symmetric matrix is:

$$A = \begin{bmatrix} 0 & -a & -b & -c \\ a & 0 & -d & -e \\ b & d & 0 & -f \\ c & e & f & 0 \end{bmatrix}$$

Problem 5.3:

(3.1 #18)

True or false (check addition or

give a counterexample):

(a) The symmetric matrices in M. (with $A^T = A$) form a subspace

<u>True</u>: $A^T = A$ & $B^T = B$ lead to:

$(A+B)^T = A^T + B^T = A + B$, and $(cA)^T = cA$.

(b) The skew-symmetric matrices in M (with $A^T = -A$) form a subspace.

<u>True</u>: $A^T = -A$ & $B^T = -B$ lead to:

$(A+B)^T = A^T + B^T = -A - B$

$= -(A+B)$

$\cdot (cA)^T = c\,{-A} = -cA$

(c) The unsymmetric matrices in M (with $A^T \neq A$) form a subspace.

False:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$