Problem 29.1

(6.7 #4)

- $A = U\Sigma V^T$ of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A^T A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

- Find eigen values & eigen vectors

$\det(A - \lambda I) = \det\begin{pmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{pmatrix}$

$= (2-\lambda)(1-\lambda) - 1$

$= 2 - 3\lambda + \lambda^2 - 1$

$= \lambda^2 - 3\lambda + 1$ $\rightsquigarrow$ using Quad. Formula

$\lambda_1 = \frac{3+\sqrt{5}}{2}$, $\lambda_2 = \frac{3-\sqrt{5}}{2}$

$\therefore$

$\sigma_1^2 = \frac{3+\sqrt{5}}{2}$ & $\sigma_2^2 = \frac{3-\sqrt{5}}{2}$

Check $\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$

since: we already know:

$$\Sigma = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{\sqrt{5}-1}{2} \end{bmatrix}$$

Check:

$$\left(\frac{1+\sqrt{5}}{2}\right)^2 = \frac{1+2\sqrt{5}+5}{4} = \frac{3+\sqrt{5}}{2} \checkmark$$

$$\left(\frac{\sqrt{5}-1}{2}\right)^2 = \frac{5-2\sqrt{5}+1}{4} = \frac{3-\sqrt{5}}{2} \checkmark$$

Problem 29.2
(6.7 #11)

Since A is orthogonal $\therefore A^TA$ is a diagonal matrix

$$A^TA = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix}$$

$\therefore A^T A = V \underset{\sim}{\Sigma}^2 V^T$

$\therefore \Sigma = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix}$

$\rightarrow$ We can also conclude that $V = I$

The equation, $A = U\Sigma V^T$ then tells us that U must be the matrix whose columns are $\frac{1}{\sigma_i} w_i$