Find the determinant of:

$$A = \begin{bmatrix} x & y & 0 & 0 & 0 \\ 0 & x & y & 0 & 0 \\ 0 & 0 & x & y & 0 \\ 0 & 0 & 0 & x & y \\ y & 0 & 0 & 0 & x \end{bmatrix}$$

$$B = \begin{bmatrix} x & y & y & y & y \\ y & x & y & y & y \\ y & y & x & y & y \\ y & y & y & x & y \\ y & y & y & y & x \end{bmatrix}$$

(-1)

(-1)

(-1)

(-1)

Hint: You may combine two of the following methods:

i) Elimination

ii) $\sum \pm a_{1\alpha} a_{2\beta} \cdots a_{5\gamma}$

iii) By cofactors.

$$\det A = x \cdot \underbrace{x^{4}}_{\text{upper } \Delta \; 4\times 4} + y \cdot (-1)^{5+1} \cdot \underbrace{y^{4}}_{\text{lower } \Delta \; 4\times 4}$$

$$\det A = x^{5} + y^{5}$$

$$\det B =$$

$$\begin{bmatrix} x & y & y & y & y \\ y-x & x-y & 0 & 0 & 0 \\ 0 & y-x & x-y & 0 & 0 \\ 0 & 0 & y-x & x-y & 0 \\ 0 & 0 & 0 & y-x & x-y \end{bmatrix}$$

$\cdot 1$

$\cdot 1$

$\cdot 1$

$$\rightarrow \begin{bmatrix} 4y & 3y & 2y & y \\ x-y & 0 & 0 & 0 \\ 0 & x-y & 0 & 0 \\ 0 & 0 & x-y & 0 \\ 0 & y-x & 0 & x-y \end{bmatrix}$$

(see video again!)

$\det B = (x+4y)(x-y)^4$