(1) Show that the set of 2x3 matrices whose <u>nullspace</u> contains $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ is a vector subspace, and find (2) a basis for it.

(3) What about the set of those column space contains $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

How do we show that something is a vector subspace?

↳ <u>Check</u>

- 2 vectors in that space & their <u>sum</u>
- Multiplication by scalar
  → still in the space

i) $A\begin{bmatrix}2\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$, $B\begin{bmatrix}2\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$$(A+B)\begin{bmatrix}2\\1\\1\end{bmatrix}=A\begin{bmatrix}2\\1\\1\end{bmatrix}+B\begin{bmatrix}2\\1\\1\end{bmatrix}$$

$$=\begin{bmatrix}0\\0\end{bmatrix}$$

$\therefore$ Indeed $A+B$ is in the set.

2) $A\begin{bmatrix}2\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$, c as a scalar

$$(cA)\begin{bmatrix}2\\1\\1\end{bmatrix}=c\left(A\begin{bmatrix}2\\1\\1\end{bmatrix}\right)=c\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

∴ Indeed a vector subspace. ✓

Basis:

Each row of A must be

$$[a\ b\ c]\begin{bmatrix}2\\1\\1\end{bmatrix}=0$$

$$(2a+b+c=0)$$

→ can also be in the form

$$\underbrace{[a\quad b\quad c=-2a-b]}$$

$$=[a\quad 0\quad -2a]+[0\quad b\quad -b]$$

∴ must be a linear combination

of $\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$

Hence a Basis is

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

dimension = 4.

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③ Can check if the ϕ matrix belongs to the set?

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ does not contain $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

* Note: You should always be able to multiply matrix by

scaler[0] and still be in the set.