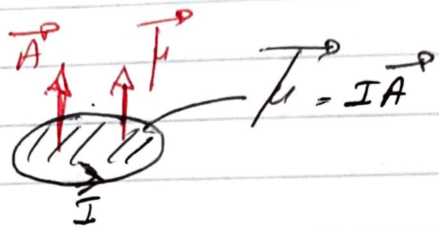
Lect 21

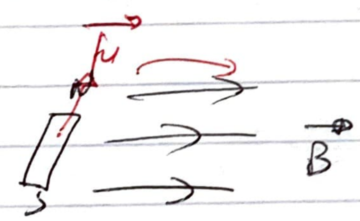
- Magnetic Materials, Dia-Para-& ferromagnetism

* External field = vaccumm field.

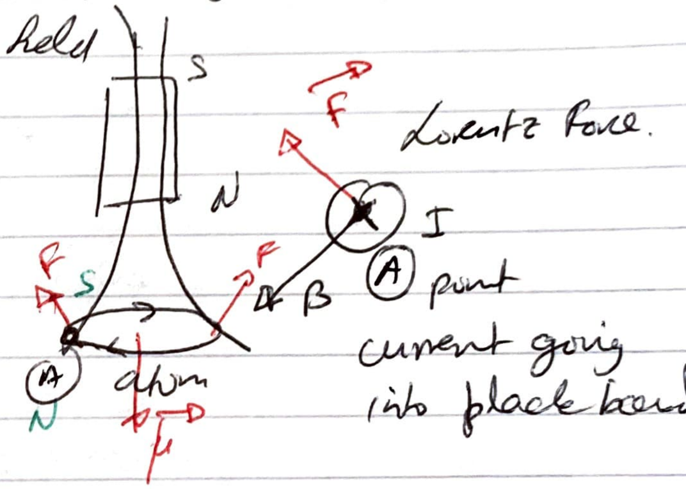
* Magnetic dipole



- diamagnetism ↗ induce dipoles oppose vacuum field (nothing to do with Lenz's law) ↳ lower than external field
- paramagnetism (small)



↳ hair bigger than vacuum field



(∴ Net Force ↑ upwards)

* Ferromagnetism (permanent dipole moments)
↳ Domains

- $\vec{B} = \underbrace{K_M}_{\text{relative permeability}} \vec{B}_{vac}$

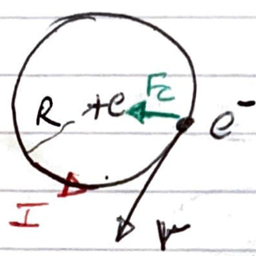
$K_M = 1 + \underbrace{\chi_M}_{\text{magnetic susceptibility}}$ susceptibility.

Lect 22

- Maxwell's Equations, 600 Daffodil
- Field inside material - Eg ferromagnetic, paramagnetic.

$\vec{B} = K_M \vec{B}_{vac\,(ext)}$

-



$m_e = 9.11 \times 10^{-31}\,kg$

$q_e = 1.6 \times 10^{-19}\,C$

$R = 5 \cdot 10^{-11}\,m.$ (Bohr radius)

(Magnetic dipole moment) $\mu = IA$, $A = \pi R^2 = 8 \times 10^{-21}\,m^2$

$F_c = \dfrac{e^2}{4\pi\varepsilon_0 R^2} = \dfrac{mv^2}{R}$ } centrifugal force

$v = \sqrt{\dfrac{e^2}{m 4\pi\varepsilon_0 R}} \approx v = 2.3 \times 10^6\,m/s$

(time) $T = \dfrac{2\pi R}{v} = 1.4 \times 10^{-16}\,s$

$I = \dfrac{dQ}{dt} = \dfrac{e}{T} \approx 1.1 \times 10^{-3}\,A. \therefore \underline{\mu \approx 9.3 \times 10^{-24}\,Am^2}$

Bohr magneton

- $\vec{B} = \vec{B}_{vac} + \underbrace{\vec{B}'}_{\text{allign dioples}}$ depends on Bvac & Temperature

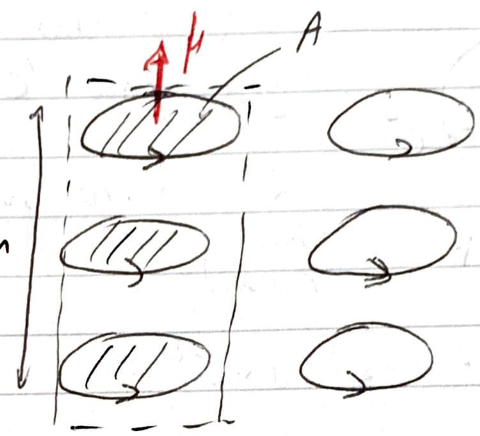
IF
$\vec{B}' \propto \vec{B}_{vac}$

$\therefore \vec{B}' = \chi_M \vec{B}_{vac}$ $\quad \vec{B} = (1 + \chi_M)\vec{B}_{vac}$

$\cancel{B_{vac}} = K_M \vec{B}_{vac}$

>

$\mu = 2\mu_{Bohr}$



Solenoid: $\vec{B} = \mu_0 \frac{IN}{l}$ $= \mu_0 I \underset{\sim}{A} N$

$= \mu_0 2\mu_{Bohr} N$

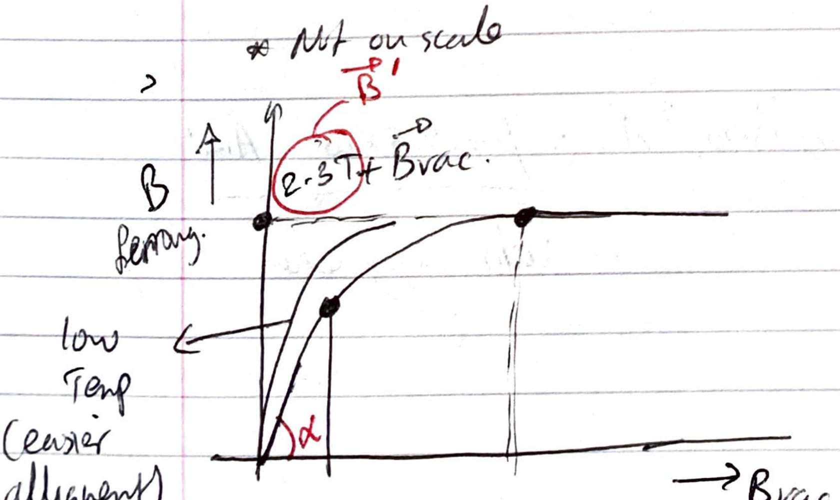
$\approx 2.3T$

$N = 10^{29} m^{-3}$ (# density of atoms/ molecules)

$Vol = Am^3$

AN = # of windings/m

>

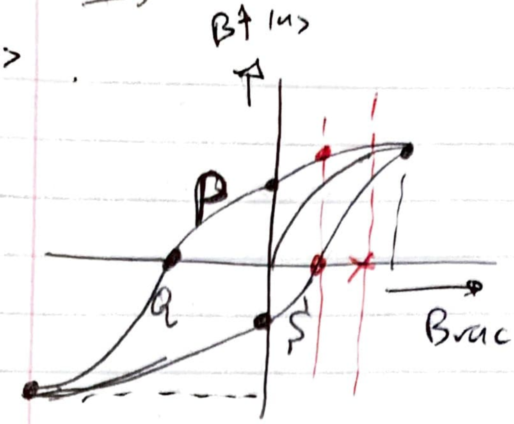


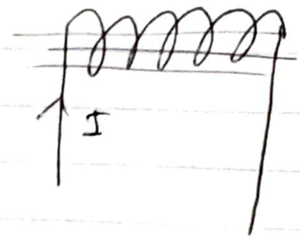
$B_{vac} = \mu_0 I N/l$

$K_M = 10^3$

$\tan\alpha = K_m = 10^3$

(Hysteresis Curve)





I_{cw} $B_v \rightarrow$

I_{cc} $B_v \leftarrow$

(CW) At location P

Bvac = 0

$B' \rightarrow$

(CCW) Q Bvac ←

B = 0

(Total field) } technically added ∴ ϕ

★ Effect of Magnetic Materials on Maxwell equations??

Maxwells equations

- $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{free}}{K\varepsilon_0}$

- $\oint \vec{B} \cdot d\vec{A} = 0$ | * magnetic monopoles !=

- $\oint \vec{E} \cdot d\vec{l} = \frac{-d\Phi_B}{dt}$ ~ Faraday's Law

Ampere's Law - $\oint \vec{B} \cdot d\vec{l} = K_M \mu_0 \left(I_{pen} + \varepsilon_0 K \frac{d\Phi_E}{dt} \right)$

↓ material.

eg ferromagnetic...

Why use an inductor instead of resistor? power

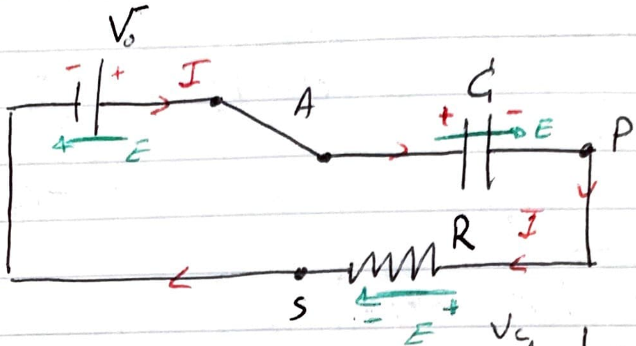
Lect 24

- Transformers, Car coils, RC Circuits

> RC circuits

$V_C = V_A - V_P$

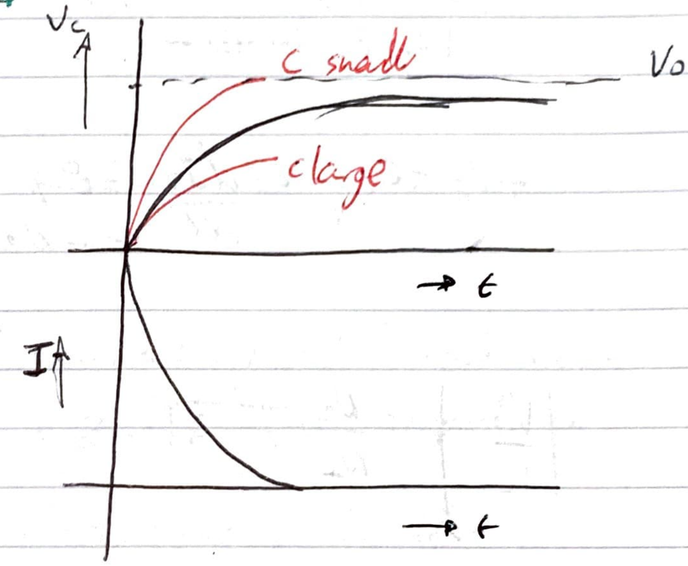
$V_R = IR = V_P - V_S$



At $t=0$, $V_C = 0$

$t > 0$, $V_C \uparrow$, $I \downarrow$

$t \to \infty$, V_0, $I = 0$



* $\oint \vec{E} \cdot \vec{dl} = 0$ $\qquad I = \frac{dQ}{dt} \qquad V_C = \frac{Q}{C}$

$\therefore \quad +V_C + IR - V_0 = 0$

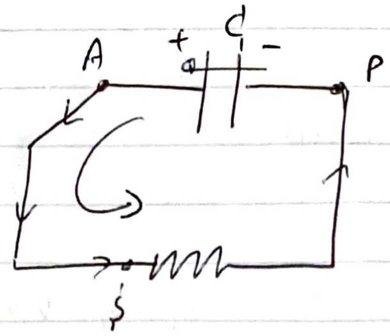
$\frac{Q}{C} + \frac{dQ}{dt}R - V_0 = 0$ → differential equation.

- $Q = V_0 C (1 - e^{-t/RC})$

$I = \frac{dQ}{dt} = V_0 \cancel{C} (-)(-) \frac{1}{R\cancel{C}} e^{-t/RC}$

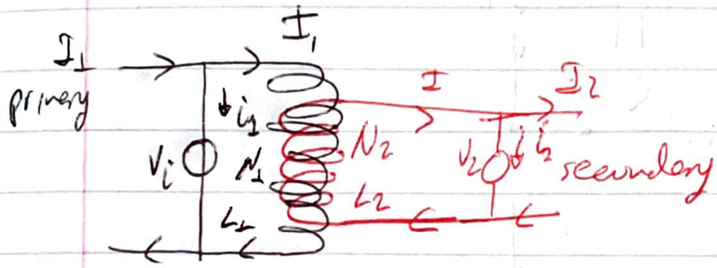
$I = \frac{V_0}{R} e^{-t/RC}$

- $V_C = \frac{Q}{C} = V_0 (1 - e^{-t/RC})$ //



> Transformers (AC)

$$0 - V_1 = -L_1 \frac{dI_1}{dt} = \mathcal{E}_{ind\,1}$$
$$= -N_1 \frac{d\Phi_B}{dt}$$



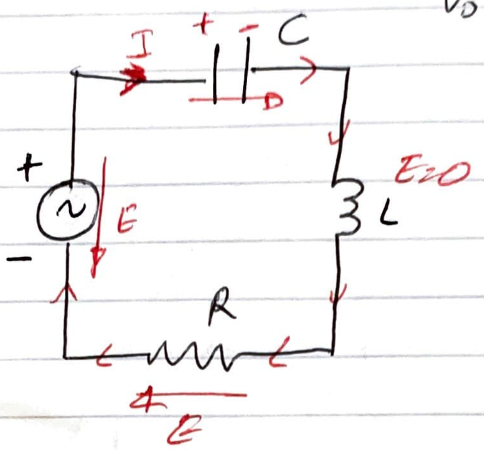
$$+V_2 + 0 = -L_2 \frac{dI_2}{dt} = \mathcal{E}_{ind\,2}$$
$$= -N_2 \frac{d\Phi_B}{dt}.$$

$$\left| \frac{V_2}{V_1} \right| = \frac{N_2}{N_1}$$

Lect 25

- Driven LRC Circuits, Metal Detectors

> LRC



$V_0 = V_0 \cos\omega t$

$$\boxed{\oint E \cdot dl \neq \phi}$$

$$-V_C + 0 + IR - V_0 \cos\omega t = -L\frac{dI}{dt}$$

$$I = \frac{dQ}{dt}$$

$$\Rightarrow L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \cos\omega t \quad //$$

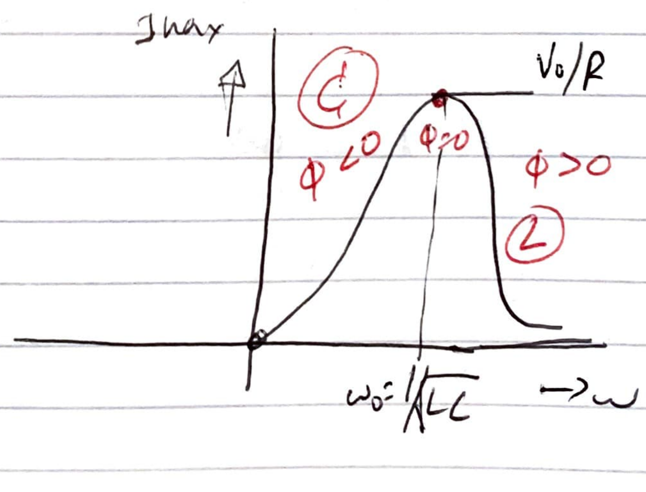
$$I = \frac{V_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \cos(\omega t - \phi) \quad \text{(effective resistance)}$$

$\rightarrow Z = \sqrt{R^2 + X^2}$ = impedance

$$\tan\phi = \frac{\omega L - 1/\omega C}{R} \rightarrow X_{\text{reactance}} = \omega L - 1/\omega C$$

$\left(\omega_0 = \frac{1}{\sqrt{LC}} = \text{reasonance}\right)$ $\therefore Z = R$

$$I_{max} = \frac{V_0}{R} \qquad \phi = 0$$



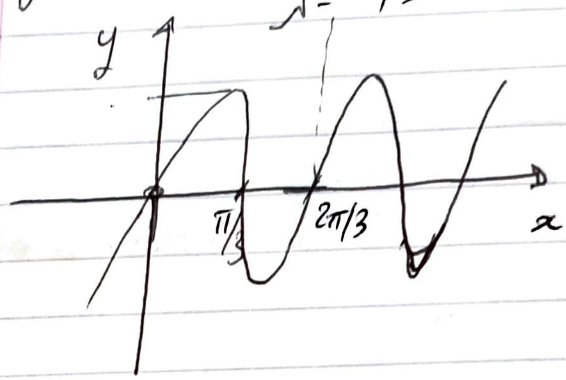
Lect 26

- Travelling waves, Standing Waves, Musical Instruments

v $+x \rightarrow x - vt$

$-x \rightarrow x + vt$

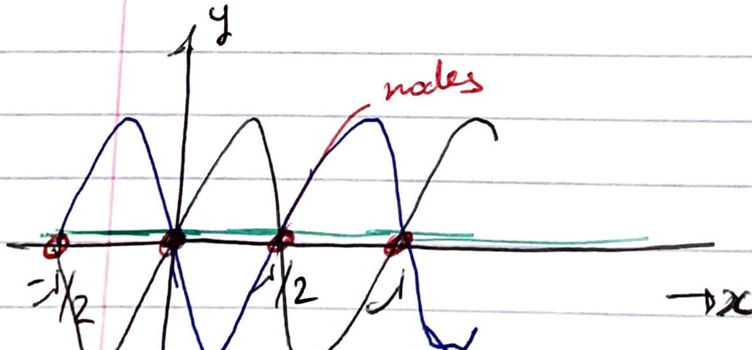
$y = 2\sin 3x$ $\lambda = 2\pi/3$ $k = \frac{2\pi}{\lambda} = 3 \rightarrow$ wave number



Standing Wave

$y_1 = y_0 \sin(kx - \omega t)$ $y_2 = y_0 \sin(kx + \omega t)$

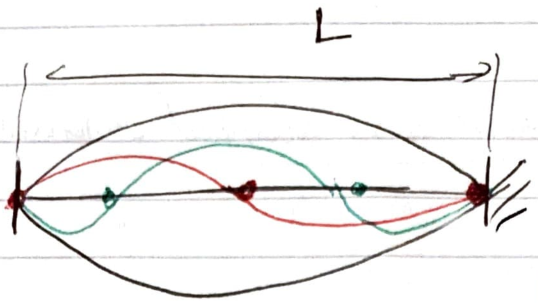
$$\boxed{y = y_1 + y_2 = 2y_0 \underbrace{\sin(kx)}_{\text{spatial information}} \underbrace{\cos(\omega t)}_{\text{time information}}}$$



$t = 0$

$t = 1/4T$

$t = 1/2T$



fundamental $\lambda_1 = 2L$, $f_1 = \frac{v}{2L}$

2nd Harmonic: $\lambda_2 = L$ $f_2 = \frac{v}{L}$

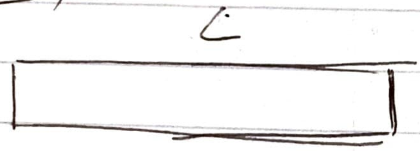
3rd Harmonic

$f_3 = 3f_1$

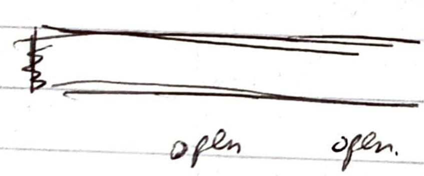
$$\boxed{\lambda_n = \frac{2L}{n} \; ; \; f_n = \frac{nv}{2L}}$$

$$v = \sqrt{\frac{\text{Tension}}{\text{mass/length}}}$$

(wind).



$v = 340 ms^{-1}$ (speed of sound)



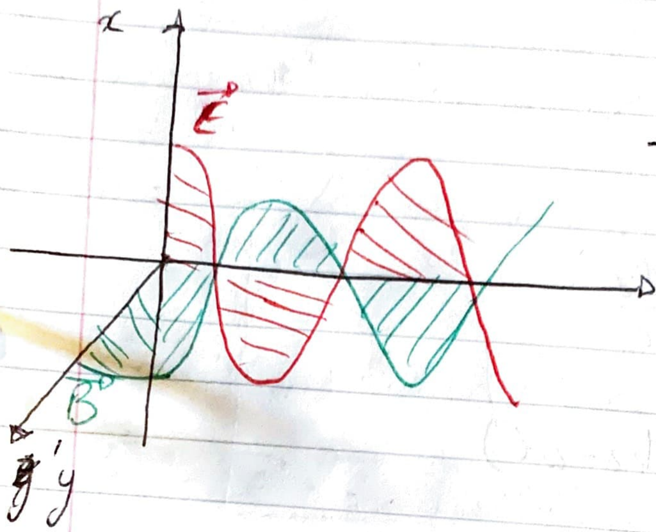
$v = \sqrt{\frac{\text{Temp}}{M}}$ ~ molecular weight

Lect 27

- Destructive Resonance, Electromagnetic Waves, Speed of light

- $\vec{E} = E_0 \hat{x} \cos(kz - \omega t)$
- $\vec{B} = B \hat{y} \cos(kz - \omega t)$

$\hat{x} \times \hat{y} = \hat{z}$



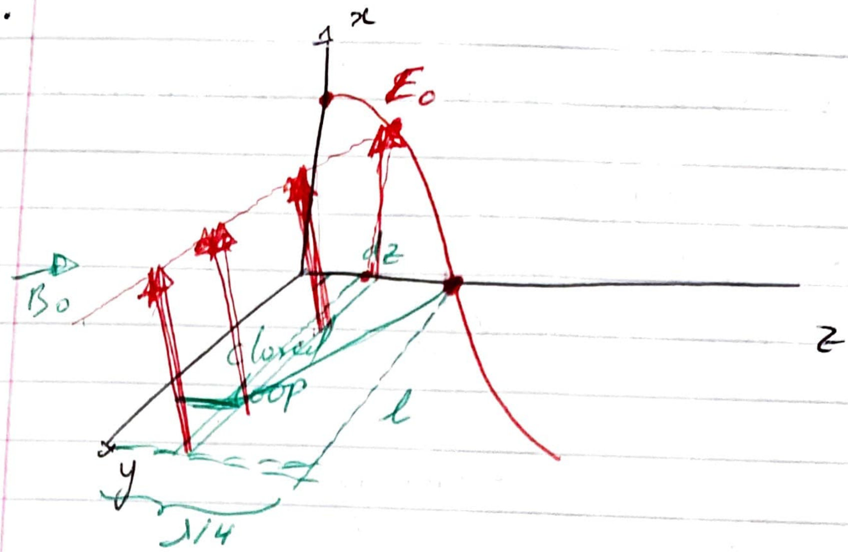
$\rightarrow v = \frac{\omega}{k} = c$

$\lambda = \frac{2\pi}{k}$

$B_0 = \frac{E_0}{c}$ $\qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$\varepsilon_0 = 8.85 \times 10^{-12}$
$\mu_0 = 1.26 \times 10^{-6}$

$c = 2.99 \times 10^{8}\ ms^{-1}$



• $\oint \vec{B}\cdot\vec{dl} = \varepsilon_0 \mu_0 \frac{d\phi_E}{dt}$

$$\phi_E = \int_0^{\lambda/4} l\cdot dz\, E_0 \cos(kz-\omega t)$$

$$\frac{d\phi_E}{dt} = l E_0 (\cancel{-}\omega)\int_0^{\lambda/4} (\cancel{-}\sin(kz-\cancel{\omega t}^{\,0}))\, dz \qquad t=0$$

$$\frac{d\phi_E}{dt} = l\, \frac{E_0\, \overset{c}{\omega}}{k}\; -\cos(kz)\Big|_0^{\lambda/4} = l E_0 c$$

• $\boxed{B_0 \cdot \cancel{d} l = \varepsilon_0 \mu_0 = \cancel{l} E_0 c}$

Lect 28

- Poynting vector, Oscillating charges, Polarization, Radiation Pressure

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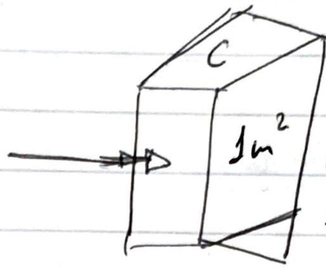
- $u_E = \frac{1}{2}\varepsilon_0 E^2$ [J/m³]

- $u_B = \frac{1}{2\mu_0} B^2$ [J/m³]

$= \frac{1}{2\mu_0}\frac{E^2}{c^2} = \frac{1}{2}\varepsilon_0 E^2$

Vol: cm³



$u_{tot} \cdot c = \varepsilon_0 E B c^2 = \frac{EB}{\mu_0}$

{J/m² sec}

Total $u_{TOT} = \varepsilon_0 E^2 = \varepsilon_0 E B c$

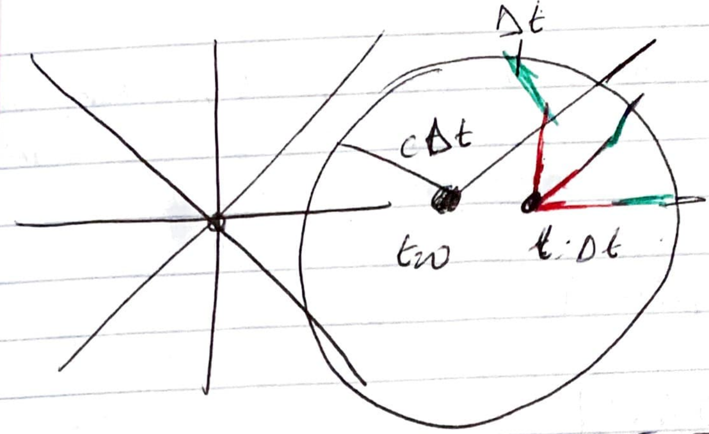
$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ [W/m²]

(Poynting vector)

$$\langle \vec{S} \rangle = \frac{1}{2}\frac{E_0 B_0}{\mu_0} = \frac{1}{2}\frac{E_0^2}{\mu_0 c}$$

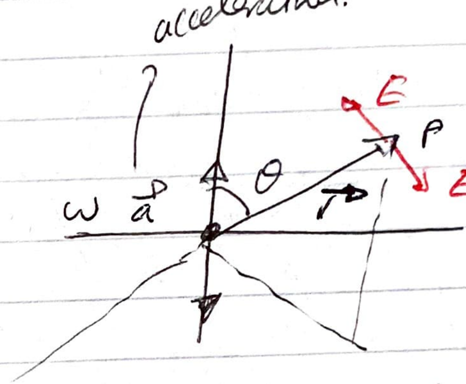
$E_0 = 100$ V/m

$\langle S \rangle = \frac{100^2}{2\mu_0 c} = 13$ W/m²



- Photons:

$p = \frac{\text{energy of photon}}{c}$



$E \propto q \frac{a}{r} \sin\theta$

$S \propto \frac{q^2 a^2 \sin^2\theta}{r^2}$

Lect 29

- Snell's Law, Index of Refraction, Huygen's Principle, Illusion of Color

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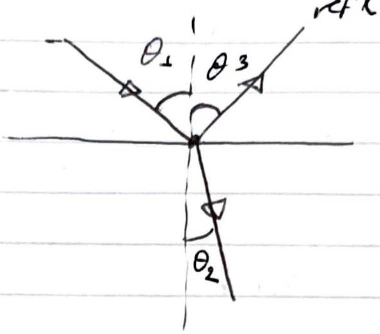
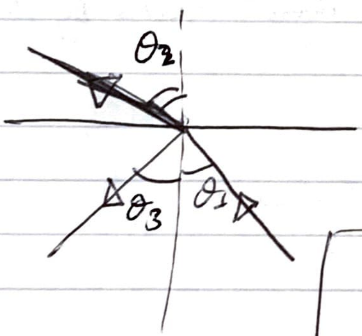


air ①

water ②

Snells' Law: $\frac{\sin\theta_1}{\sin\theta_2} = 1.3$ (air → water) $= \frac{n_2}{n_1}$ → refraction. n_1 → incident



air ②

water ①

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{1}{1.3}$

$\theta_2 = 90°$ → $\theta_1 = 50°$

Total reflection.

$\sin\theta_{cr} = \frac{n_2}{n_1}$ $\quad n_1 > n_2$

* Fibre-optic cables

* Huygen's Principle

→ wave front.

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0 K K_m}} = \frac{c}{\sqrt{K K_m}} = \frac{c}{n}$$

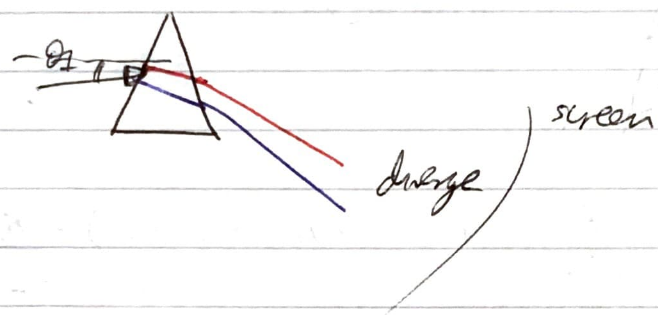
dielectric constant

magnetic permeability.

Water

$n_{red} = 1.331$

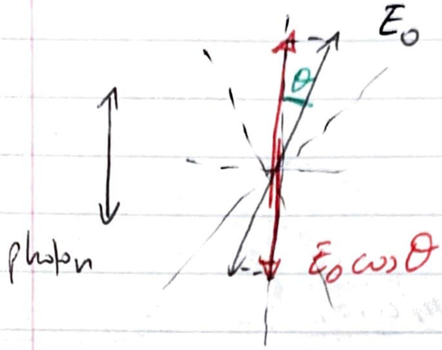
$n_{blue} = 1.343$



* Benham Top. * brain interpret ~~light~~ colors by their different phases (λ)
(interesting).

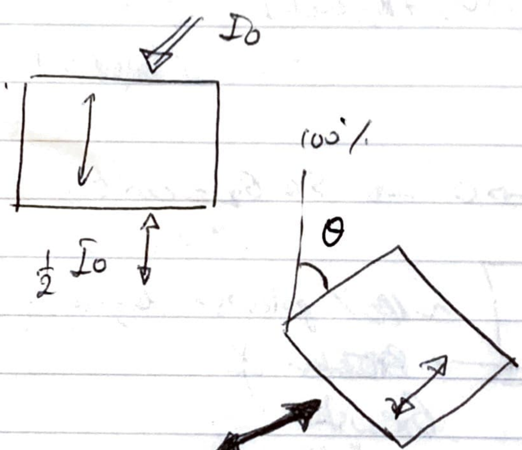
Lect 30

- Polarizers, Malus's Law, Light scattering, Blue Skies, Red Sunsets



$S \propto E_0^2$

I_0 unpol $\longrightarrow \frac{1}{2} I_0$ 100% pol



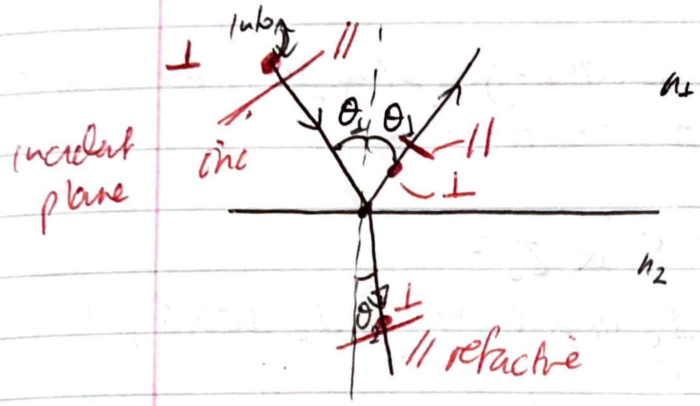
$\frac{1}{2} I_0\cos^2\theta \rightsquigarrow$ Malus's Law

$\theta = 30°$ $\quad \frac{1}{2} I_0 \cos^2(30) =$

$\theta = 0°$ $\quad \frac{1}{2} I_0$

$\theta = 90$ $\quad \emptyset \quad \cos(90) = 0 \rightsquigarrow$ cross polarizers

* All components perpendicular



* $$E_{0\,//\,refl} = E_{0\,//\,inc}\,\frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1} = -E_{0\,//\,inc}\,\frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}$$

(Maxwell's) (Snell's)

$\theta_1 + \theta_2 = 90^\circ$ $\quad E_{//\,refl} \to 0 \to \sin\theta_2 = \cos\theta_1$

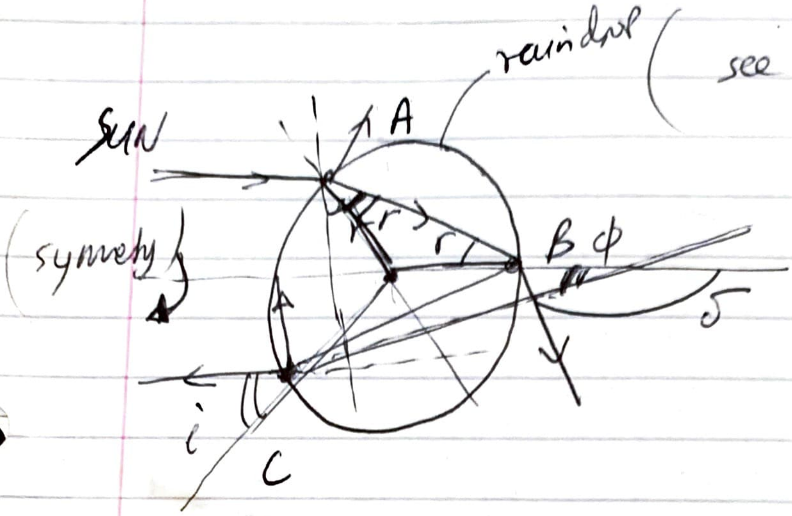
$$\frac{\sin\theta_1}{\sin\theta_2} \; \boxed{\frac{n_2}{n_1} = \tan\theta_1} \quad \sim 100\%\ \text{polarised light}$$

→ ~~Brewster~~ ∡ Brewster

Lect 31

- Rainbows, Fog Bows, Haloes, Glories, Sun Dogs

$\delta = 180^\circ + 2i - 4r$

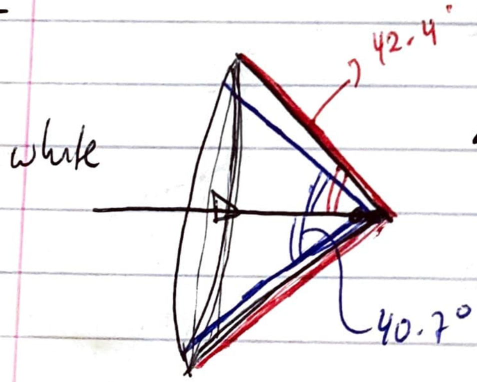


(see sketch on lecture)

$\delta_{min} \simeq 138^\circ$

$\phi_{max} \simeq 42^\circ$

-



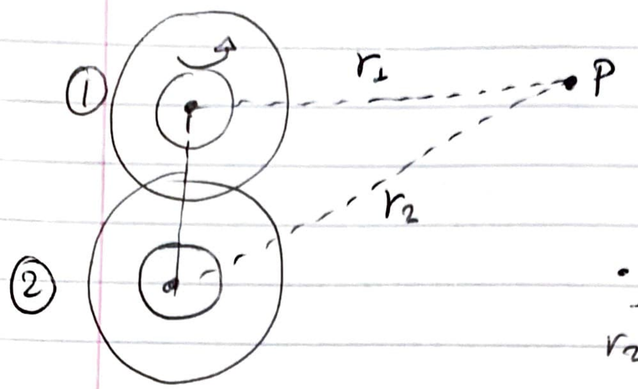
$n_{red} = 1.331$ $\phi_{max} \simeq 42.4^\circ$.

$n_{blue} \simeq 1.343$ $\phi_{max} \simeq 40.7^\circ$

* Nice explanation in lecture video.

Lect 33

- Double-slit interference, Interferometers



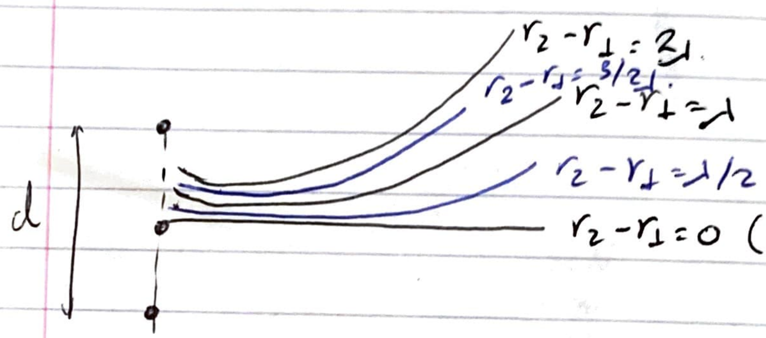
• destructive

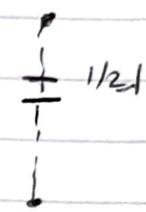
$r_2 - r_1 = \pm \frac{1}{2}\lambda, \pm 3/2\lambda$

$r_2 - r_1 = \frac{(2n+1)\lambda}{2}$ $n = 0, \pm 1, \pm 2 \ldots$

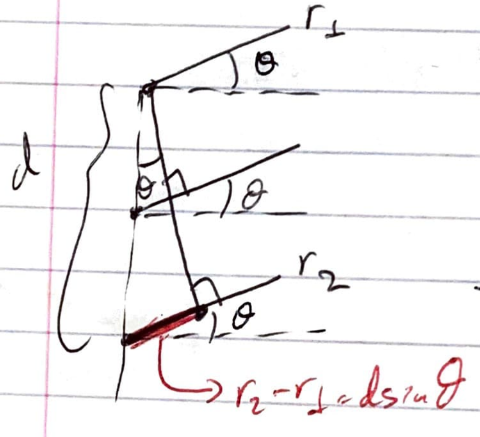
• constructive

$r_2 - r_1 = n\lambda.$ $n = 0, n = \pm 1, n = \pm 2$





$\#max = \#min = \frac{2d}{\lambda}$ ↑ ↓



constructive

$d\sin\theta = n\lambda$

$\sin\theta_n = \frac{n\lambda}{d}$

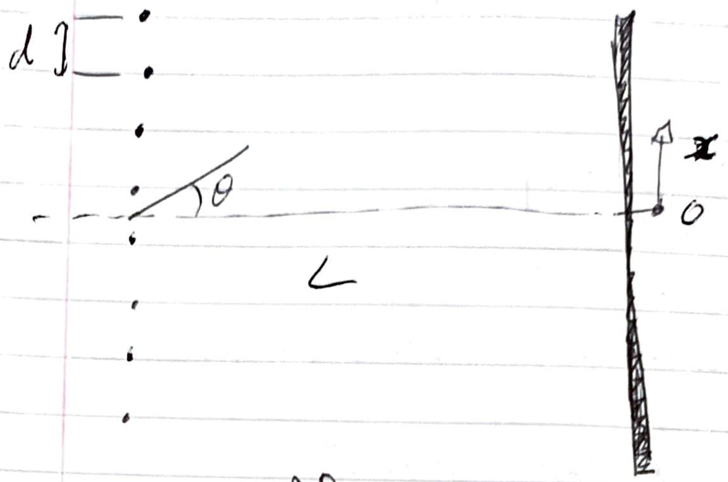
destructive: $d\sin\theta_n = \frac{(2n+1)\lambda}{2}$

$\sin\theta_n = \frac{(2n+1)\lambda}{2d}$

(* Slit-experiment)

Lect 34

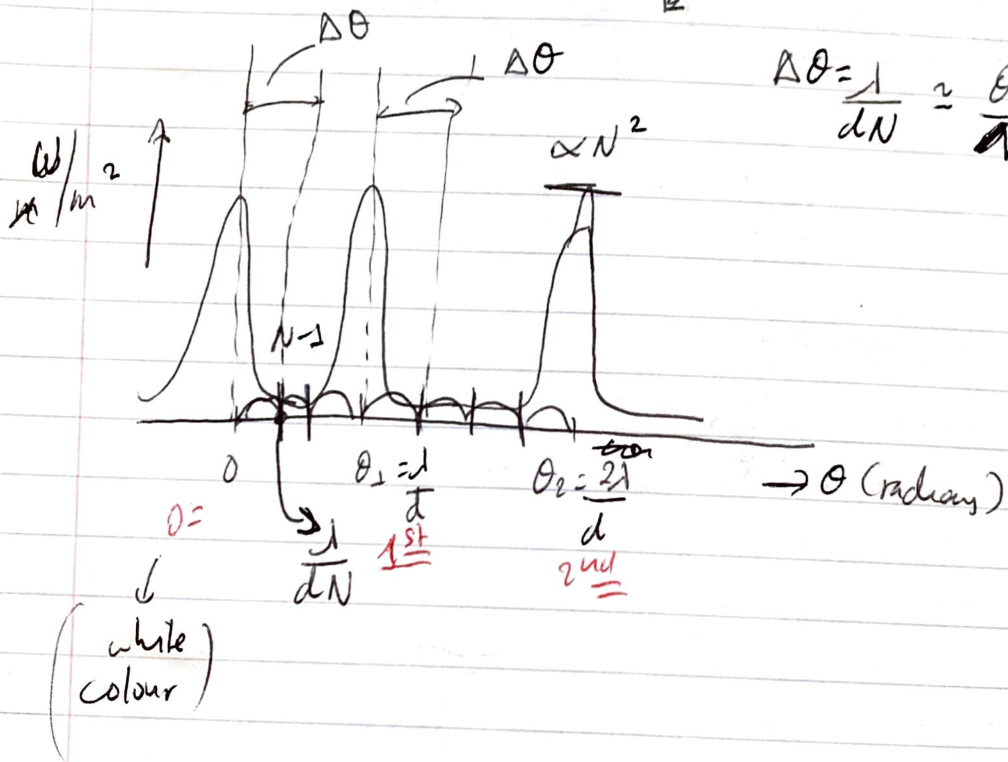
- Diffraction, Gratings, Resolving Power, Angular Resolution



constr = $\sin\theta_n = \frac{n\lambda}{d} \approx \theta_n$ (radians)

$\theta_n = \frac{n\lambda}{d}$ $x_n \approx \frac{Ln\lambda}{d}$

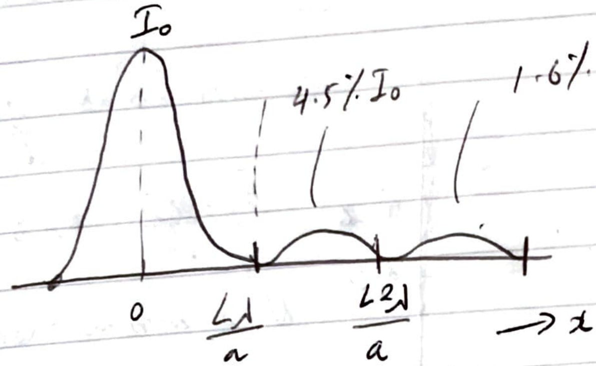
N − 1 minima → destr. interference



$\Delta\theta = \frac{\lambda}{dN} \approx \frac{\theta_1}{N}$

* Single Sources

$x_1 \simeq \frac{L\lambda}{a}$



* smaller (a); the wider the bandwidth of the wave

* Angular resolution $\theta = \frac{1.2\lambda}{a}$ $\lambda = 5000\,\text{Å}$

Lect 35-

Doppler Effect, Big Bang, Cosmology

* Doppler Effect

transm — receiver — $v > 0 \rightarrow$ — $v < 0 \leftarrow$

$\bullet\!\longrightarrow$ v_{tr} — $\bullet\!\longrightarrow$ v_{rec}

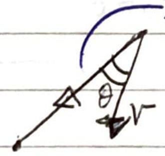
$$\boxed{f'_{(receiver)} = f_{tr} \frac{v_s - v_{rec}}{v_s - v_{tr}}}$$

$v_s = 340\ m/s$

$\rightarrow$ $f'_{received} = f_{tr} \left(\frac{1-\beta}{1+\beta} \right)^{1/2}$ $\quad \beta = \frac{v}{c}$ relative. $\quad \beta > 0$ receding, $\beta < 0$ approaching.

$\lambda = \frac{c}{f} \Rightarrow \lambda'_{rec} = \frac{c}{f'_{rec}}$.

$\rightarrow v\cos\theta$ (use in eq.)



$$\lambda' = \lambda \left(\frac{1+\beta}{1-\beta} \right)^{1/2}$$

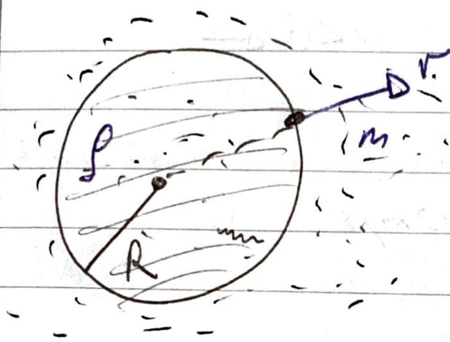
$\lambda' > \lambda \rightarrow$ redshift.
$\lambda' < \lambda \rightarrow$ blueshift

Note: 1 Mpc $= 3.26 \times 10^6$ light years
$= 3.1 \times 10^{19}$ km.

* $d = v\, t_{age}$ × } $t_{age} = \frac{1}{H}$ } universe. ≈ 14 billion years
* $v = \underset{\sim}{H} \cdot d$

↳ Hubble constant

$H_0 = \frac{72 \text{ km/sec}}{\text{Mpc}}$



$M_{ins\ sphere} = \frac{4}{3}\pi R^3 \rho$

$\frac{1}{2}\cancel{m}v^2 = \frac{\cancel{m}MG}{R}$ } Total $E = \phi$

$\frac{1}{2}v^2 = \frac{4/3\,\pi R^{\cancel{3}}\rho}{\cancel{R}}$

$\frac{v}{R} = H$

:(⇒ $\boxed{\rho_0 = \frac{3}{8\pi G} H_0^2}$