Objectives

- Bases of new vector spaces
- Rank one matrices
- Small world graphs

(M) M = all 3x3 matrices

symmetric 3x3 (S)

upper triangular (3x3) (U)

Reminder

We've talked a lot about $\mathbb{R}^n$, but we can think about vector spaces made up of any sort of "vectors" that allow addition and scalar multiplication.

A Basis for M = all 3x3

dim M = 9

dim S = 6

dim U = 6

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\ldots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{bmatrix}$$

$\begin{bmatrix} \cdots \\ 0 & 0 & 1 \end{bmatrix}$

3 - symmetric matrices (S)

$S \cap U$ = symmetric & upper triangular

↳ diagonal 3x3's $\dim(S \cap U)$

$\Rightarrow 3$

Why we are not interested in $S \cup U$

↳ Not a subspace !!

∴ ↳ We instead need $S + U$

(all linear combinations)

[SUM]

$\Rightarrow$ any elements of S + any elements U

$\Rightarrow 9$

$\star$ $\text{Dim } S + \text{Dim } U = \dim S \cup U + \dim S \cap U$

Differential Equations

- Another example of a vector space that's not $\mathbb{R}^n$ appears in differential equations:

$\Rightarrow \dfrac{d^2y}{dx^2} + y = 0$ → as element of null space

Solutions:

$y = \cos x$, $y = \sin x$, $y = e^{ix}$

BASIS

- What are all the complete solution?

↳ Combination)

∴

$\Rightarrow y = c_1 \cos x + c_2 \sin x$

→ what is the dimension & basis ?

↳ <u>basis</u> :

↳ <u>dimension</u> : (solution space)

↳ 2 [sin & cos]

↳ $2^{\underline{nd}}$ ODE

<u>Rank of matrices</u>

eg To have <u>Rank = 1</u>

A $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix}$ * Not independent

2×3

↳ basis:

$[1\ 4\ 5]$ row space

& $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ column space

dim C

=

rank

=

$C(A^T)$ → row space

$\Rightarrow 1$

$$\Rightarrow \underset{2\times 1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} \underset{1\times 3}{\begin{bmatrix} 1 & 4 & 5 \end{bmatrix}} = \underset{2\times 3}{A}$$

Note:

Every rank 1 matrix has the form $A = UV^T$

↳ column vectors.

If we have eg 5×17 matrix
↳ Rank 4

Will it form a subspace??

→ M = all 5×17 matrices.
→ subset of rank 4 matrices
↳ subset of rank 1 matrix } Not a subspace

eg

In $\mathbb{R}^4$, the set of all vectors $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ for which $v_1 + v_2 + v_3 + v_4 = 0$ is

S = all vector in $\mathbb{R}^4$ with
$v_1 + v_2 + v_3 + v_4 = 0$ $\quad (Av = 0)$

↳ is it a subspace? → <u>YES</u>

↳ dim: 3

↳ basis (N(A)) = special sol

→ free variables

null space of A = $\begin{array}{cccc} & 2 & 3 & 4 \\ \end{array}$
$$\begin{bmatrix} \boxed{1} & 1 & 1 & 1 \end{bmatrix}$$

pivot (n=4)

rank = 1 = r

dim(N(A)) = n − r = 3

↳ Basis for S

$$\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

→ column space: $C(A) = \mathbb{R}^1$

[we only have 1 column] ↳ 1

→ row space = 1

∴ 3+1=4.

Small world graphs

Small world graphs

In this class, a *graph* G is a collection of nodes joined by edges:

$$G = \{\text{nodes}, \text{edges}\}.$$

A typical graph appears in Figure 1. Another example of a graph is one in

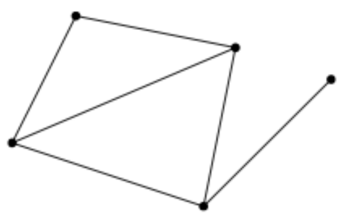

Figure 1: A graph with 5 nodes and 6 edges.

which each node is a person. Two nodes are connected by an edge if the people are friends. We can ask how close two people are to each other in the graph – what's the smallest number of friend to friend connections joining them? The question "what's the farthest distance between two people in the graph?" lies behind phrases like "six degrees of separation" and "it's a small world".

Another graph is the world wide web: its nodes are web sites and its edges are links.

We'll describe graphs in terms of matrices, which will make it easy to answer questions about distances between nodes.