Problem 27.1

(6.5 #33)

$$AB x = \lambda x$$
$$(AB)^T Bx = (\lambda x)^T Bx$$
$$(Bx)^T A^T Bx = \lambda x^T Bx$$
$$(Bx)^T A(Bx) = \lambda (x^T Bx)$$

where $A^T = A$ because A is symmetric. Since A is positive definite we know $(Bx)^T A(Bx) > 0$ since B is positive definite $x^T Bx > 0$.

Hence, λ must be positive as well.

Problem 27.2

$$A = \begin{bmatrix} 1 & 5 \\ 7 & 9 \end{bmatrix}$$

→ To find the quadratic form, compute $x^T A x$

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 7 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= x(x+5y) + y(7x+9y)$$

$$= x^2 + 12x + 9y^2 \quad //$$

This expression can be positive, eg $y=0$ and $x \neq 0$

→ This expression will sometimes be negative. because A is not positive definite.

eg $f(2,-2) = -8$

$\det(A) = -26$ } not positive definite.