Session Overview

- A major application of linear algebra is to solving systems of linear equations.
- 3 ways:
  - "Row method": focuses on individual equations
  - * "Column method": focuses on combining the colums
  - "Matrix method"

Example: [n linear equations, n unknowns]

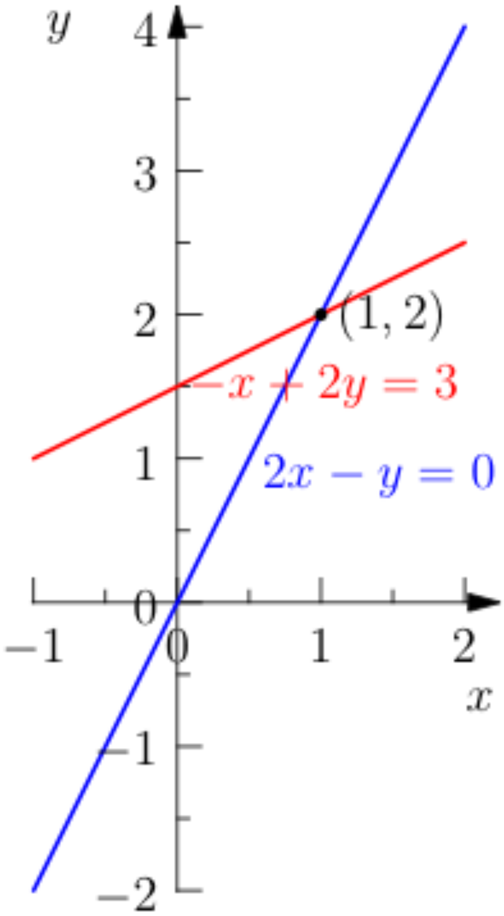
$$2x - y = 0$$
$$-x + 2y = 3$$

vector of unknowns

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A \quad x \quad = \quad b$$

- Row picture:



- Column picture

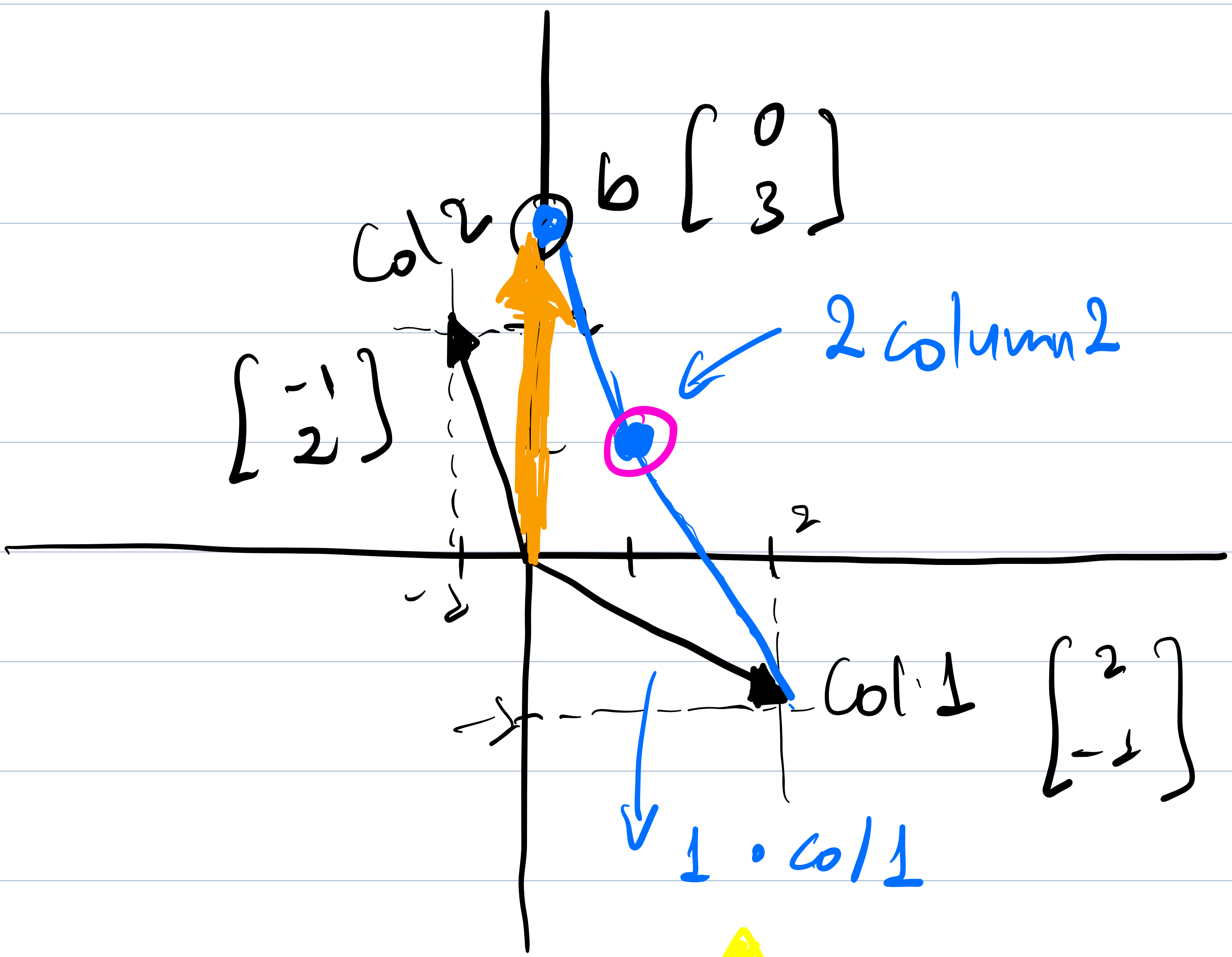
$$\begin{bmatrix} 2x \\ -x \end{bmatrix} + \begin{bmatrix} -y \\ 2y \end{bmatrix} = \begin{matrix} 0 \\ 3 \end{matrix}$$

$\Updownarrow$

$$x\begin{bmatrix} 2 \\ \end{bmatrix} + y\begin{bmatrix} -1 \\ \end{bmatrix} = \begin{bmatrix} 0 \\ \end{bmatrix}$$

$[-1]$ $[2]$ $[3]$

∴ Now we need to find the amount of x & y that satisfies the outcome. In other words: linear combination of the columns.



Correct x & y

$$1\begin{bmatrix}2\\-1\end{bmatrix}+2\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}$$

represented by blue.

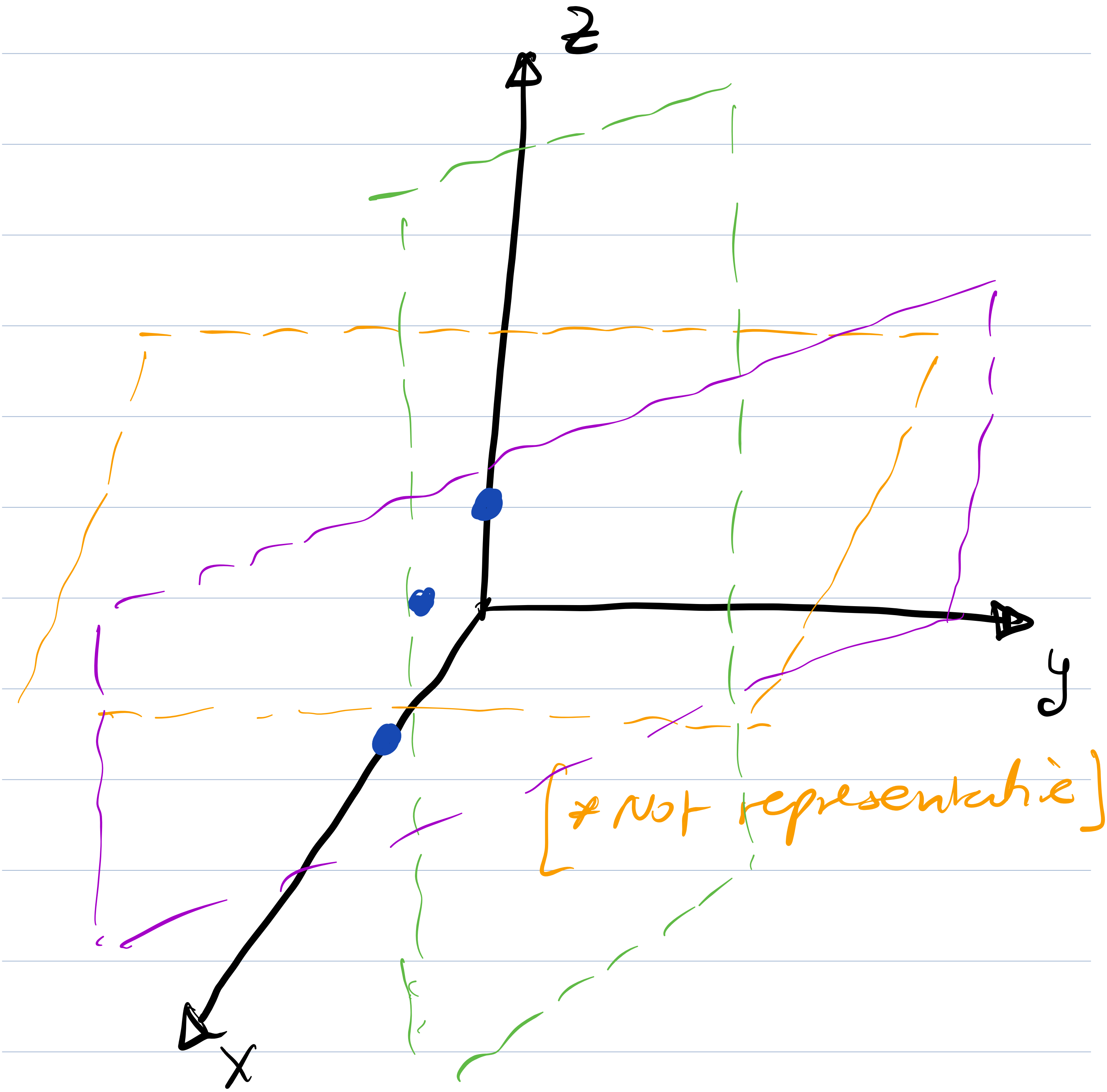
3x3 example

$$\begin{aligned}2x - y &= 0\\ -x + 2y - z &= -1\\ -3y + 4z &= 4\end{aligned}$$

Matrix Form

$$A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -3 & 4\end{bmatrix} \qquad b=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$

Row picture → results to a plane in 3D



* Solving a linear equation with 3 unknowns → linear plane

* Those 2 planes (orange & green)
  ↳ gives a line where the 2 planes meet

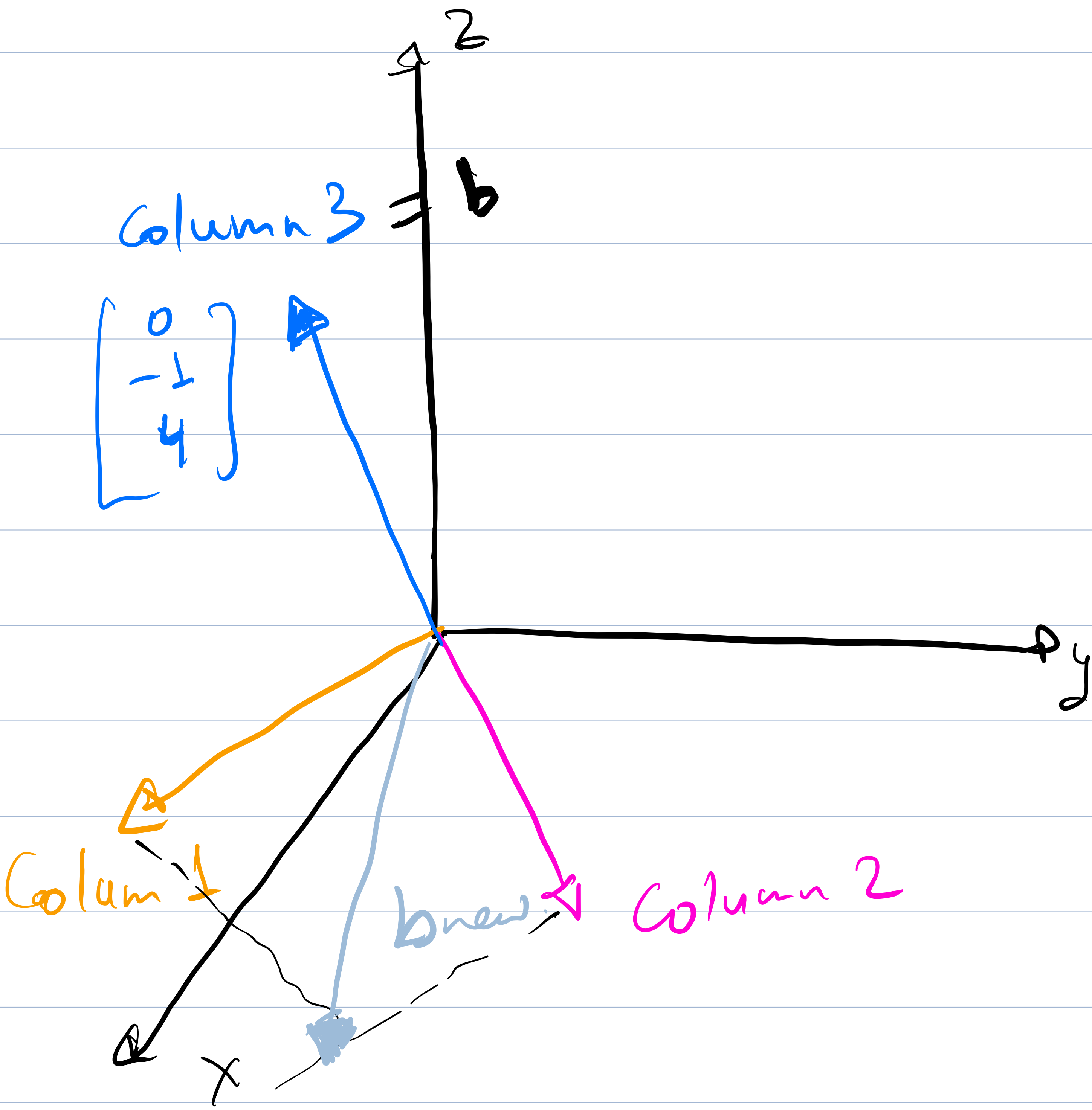
* Those 3 planes (orange & green & purple) → meet at a point

[Note: Row picture consist of 3 planes i.e. 3 planes meet to 1 point [solution]
BUT difficult to sketch]

Column picture

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$

[Linear combination of 3 vectors]



- We can see that the right hand side → already involves the 3rd

Column

$\therefore$ The solution is: $x=0, y=0, z=1$

As a result the point that we have discussed earlier in the row picture could be found easily using column picture

Not always going to be able to see it from the column picture either.

Example #2

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}1\\1\\-3\end{bmatrix}$$

Solution: $x=1, y=1, z=0.$

[b new]

- Can I solve $Ax=b$ for every b?
- Do the linear combination of the columns fill 3-D space

↳ For this matrix column $\hat{A}$ is YES [non-singular matrix, invertable]

↳ **NO**, matrixes lie on the same plane. [do not fill all dimensions].

Thought

Lets say we have 9 dimensions
i.e. 9 equations, 9 unknowns
↳ 9 columns → each one vector in 9-D space

∴ find linear combinations?

Matrix Form:

$$Ax = b$$

Matrix Multiplication

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

[* Column vector]

$$= \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

* Ax is a combination of column of A.