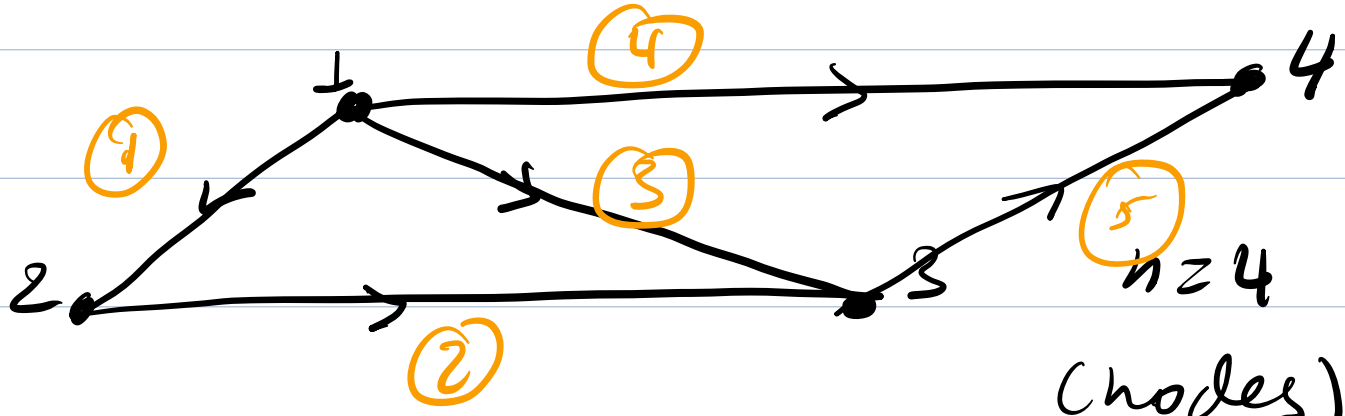
Objectives

- Graphs and Networks
- Incidence Matrices
- Kirchhoff's laws

} Application of Linear Algebra

* Graph: Nodes & Edges



n = 4 (nodes)

m = 5 (edges)

Incidence Matrix

The incident Matrix of this graph:

→ row coresponed to edges

→ column coresponed to nodes

node: 1 2 3 4

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{matrix} \text{edge } 1 \\ \text{edge } 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$

loop

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

are they independent?

rank = 3 [independent].

→ row 1 + row 2 = row 3

∴ loops coresponed to linearly dependent rows

Nullspace? no are columns independent?

↳ only [0]

[columns independent ??]

We can solve $\underline{Ax = 0}$

$$Ax = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_4 - x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x = x_1, x_2, x_3, x_4$ → Potential at nodes

$\downarrow A \quad (e = Ax)$

$x_2 - x_1$, etc → Potential difference

∴ Nullspace vectors

$(A^T y = f)$

$x = c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

→ constant potential

c → current

[OHMS law] y_1, y_2, y_3, y_4, y_5

↳ basis : on edges

$\dim N(A) = 1$

$(y = ce)$ $A^T y = 0$

[KCL] Kirchhoffs. current law

Null space of A^T $[N(A^T)]$

$A^T y = 0$

↓

$n \times m$

4×5

$\dim N(A^T) = m - r$

$5 - 3 = 2$

→ columns

$$\begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \vdots \\ y_4 \\ y_5 \end{bmatrix} \begin{bmatrix} \vdots \\ 0 \end{bmatrix}$$

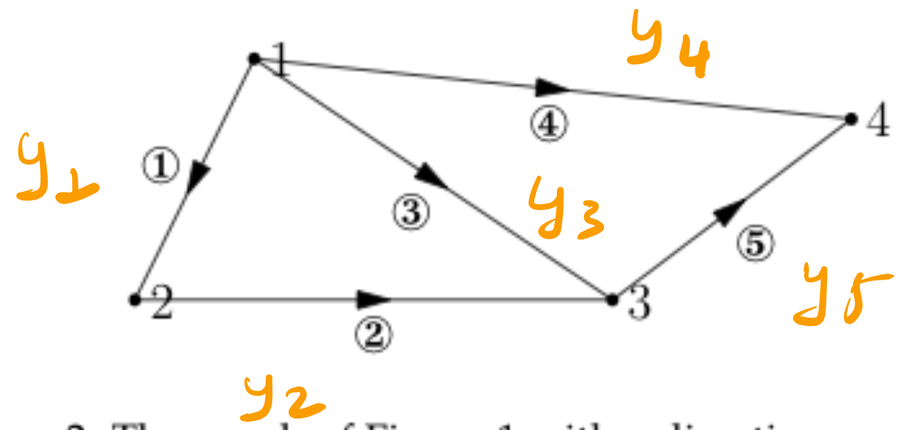


Figure 2: The graph of Figure 1 with a direction on each edge.

⇒ D

- $-y_1 - y_3 - y_4 = 0$
- $y_1 - y_2 = 0$
- $y_2 + y_3 - y_5 = 0$

$y_4 + y_5 = 0$

- Basis for $N(A^T)$ ⇒ 2D

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

- Tree : is a graph with No loops

dim $N(A^T) = m - r$

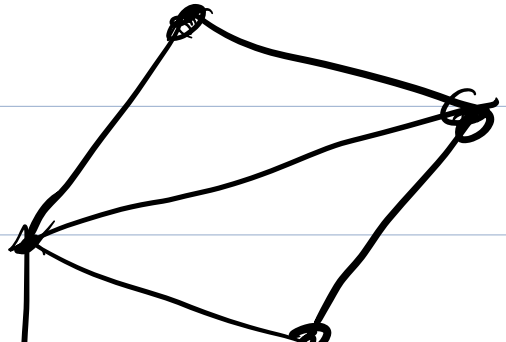
loops = # edge − (# nodes − 1)

(rank $= n - 1$)

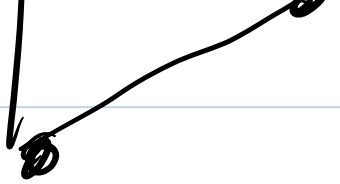
*

nodes − # edges + # loops = 1

Euler's formula



eg $5 - 7 + 3 = 1$



$$\Rightarrow \left(A^T C A x = f \right)$$