Solving $Ax=b$ and row reduced form

Problem 8.1: (3.4 # 13(a,b,d)
Introduction to Linear Algebra: Strang)

Explain why these are all false:

a) The complete solution is any linear combination of x_p and x_n

⇒ No, the coefficient of x_p must be 1

b) The system $Ax=b$ has at most one particular solution.

⇒ If $x_n \in N(A)$ is in the nullspace of A and x_p is one particular solution, then $x_p + x_n$ is also a particular solution.

(c) If A is invertible there is no solution x_n in the null space

$\Rightarrow$ There is always $x_n = 0$.

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* Problem 8.2 *

Let

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix} \quad \& \quad c = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

: Use Gauss-Jordan elimination

to reduce the matrices $[U \ 0]$ and $[U \ c]$ to $[R \ 0]$ and $[R \ d]$

- Solve $Rx = 0$ and $Rx = d$
- Check your work by plugging your values into the equations $Ux = 0$ and $Ux = c$

<u>Solution</u>:

(S1) First we transform $[U \ 0]$ into $[R \ 0]$:

$$[U \ 0] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$= [R \quad 0].$$

- We now solve $Rx = 0$ via back substitution:

$$\begin{array}{c}
\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\qquad\qquad\quad \text{free}
\end{array}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x_1 + 2x_2 = 0 \\ x_3 = 0 \end{bmatrix} \rightarrow x = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

where we used the free variables

$x_2 = -1$ (cx is a solution for all c)

*Check if correct, by plugging into $Ux = 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \checkmark$$

Next we transform $[U \ c]$ into $[R \ d]$

$$[U \ c] = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 0 & 4 & 8 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$= [R \ d]$

- We now solve $Rx = d$ via back substitution:

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x_1 + 2x_2 = -1 \\ x_3 = 2 \end{bmatrix} \rightarrow x = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

where we used the free variable $\underline{x_2 = 1}$

Finally, we check that this is correct.

$\underline{Ux = c}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \checkmark$$

Problem 8.3: (3.4 #36)

Suppose $Ax = b$ and $Cx = b$ have the same (complete) solution, for every b. Is it true that $A = C$?

Yes,