

Find the determinant of:

$$A = \begin{bmatrix} x & y & 0 & 0 & 0 \\ 0 & x & y & 0 & 0 \\ 0 & 0 & x & y & 0 \\ 0 & 0 & 0 & x & y \\ y & 0 & 0 & 0 & x \end{bmatrix}$$

$$B = \begin{bmatrix} x & y & y & y & y \\ y & x & y & y & y \\ y & y & x & y & y \\ y & y & y & x & y \\ y & y & y & y & x \end{bmatrix} \begin{array}{l} \downarrow (-1) \\ \downarrow (-1) \\ \downarrow (-1) \\ \downarrow (-1) \end{array}$$

Hint: You may combine two of the following methods:

i) Elimination

ii)  $\sum \pm a_{1\alpha} a_{2\beta} \dots a_{s\sigma}$

iii) By cofactors.

$$\det A = \underbrace{x \cdot x^4}_{\substack{\text{upper } \Delta \\ 4 \times 4}} + y \cdot \overset{s+1}{(-1)} \cdot \underbrace{y^4}_{\substack{\text{lower } \Delta \\ 4 \times 4}}$$

$$\det A = x^5 + y^5$$

$$\det B =$$

$$\begin{bmatrix}
 x & y & y & y & y \\
 y-x & x-y & 0 & 0 & 0 \\
 0 & y-x & x-y & 0 & 0 \\
 0 & 0 & y-x & x-y & 0 \\
 0 & 0 & 0 & y-x & x-y
 \end{bmatrix}$$

Diagram showing row operations on the matrix above. Blue arrows indicate row 1 is added to rows 2, 3, 4, and 5. Orange arrows indicate row 2 is added to rows 3, 4, and 5. Small blue and orange '1' marks are present near the bottom of the matrix.

$$\rightarrow \begin{bmatrix}
 4y & 3y & 2y & y \\
 x-y & 0 & 0 & 0 \\
 0 & x-y & 0 & 0 \\
 0 & 0 & x-y & 0 \\
 0 & y-x & 0 & x-y
 \end{bmatrix}$$

(see video again!)

$$\det B = (x+4y)(x-y)^4$$