3.4 - Drift of Carriers in Electric and Magnetic fields

• Conductivity & Mobility

• Charge carriers in a solid are in constant motion, even at thermal equilibrium

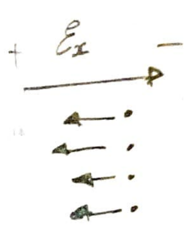
↳ At room temp eg... thermal motion of an individual electron is visualised as random scattering from lattice vibrations, other electrons & defects

↳ Since scattering is random
No net motion of the groups of n electrons/cm^3 over any period of time

→ The probability of the electron in the figure above returning to its starting point after some time (t) is negligible small

HOWEVER:
eg n-type (10^{16} cm^{-3} electrons), there will be no prefered direction of motion for the group of electrons ∴ no net current flow.

• If electric field ($\mathcal{E}_x$) is applied in the x-direction



$F_e = -q\mathcal{E}_x$ → May be insufficient to alter the random path of an individual electron.

(*) BUT:
when the effect is averaged over all electrons is a net motion of the group in the ($-x$ direction)

∴ The force on the field on the n electrons/cm^3 is:

$$\boxed{-nq\mathcal{E}_x = \left.\frac{dp_x}{dt}\right|_{\text{field}}}$$

• where p_x is the total momentum of the group

↳ This indicates continuous acceleration of the electron → NOT THE CASE (*)
because the net acceleration is balanced by the deceleration of the (*) collision process.

• While the steady field $\mathcal{E}_x$ produces a net momentum P_{-x}
The rate of change of momentum when collisions are included must be zero
in the case of steady state current flow

• Find the total rate of momentum change from collisions :
↳ If collisions are truly random, there will be a constant probability of
collisions at any time for each electron

Consider:

• Group of N_0 electrons at time $t=0$
where $N(t)$ as the number of electrons that have NOT undergone a
collision by time t.

constant of probability

$$\therefore \boxed{-\frac{dN(t)}{dt} = \frac{1}{\bar{t}} N(t)}$$

rate of decrease in $N(t)$ is proportional to the number left unscattered at t

Solution :

• $N(t) = N_0 e^{-t/\bar{t}}$ where $\bar{t}$: mean time between scattering events.
(mean free time).

• The probability that any electron has a collision in the time
interval dt is $dt/\bar{t}$

$$\therefore \quad dp_x = -p_x \frac{dt}{\bar{t}} \quad \Big\} \text{ differential change in } p_x.$$

$$\bullet \quad \left.\frac{dp_x}{dt}\right|_{\text{collisions}} = -\frac{p_x}{\bar{t}} \quad \Big\} \text{ due to the decelerating effect of collisions}$$

• The sum of acceleration & deceleration effects must be zero for
a steady state

$$\therefore \boxed{-\frac{p_x}{\bar{t}} - nq\mathcal{E}_x = 0}$$

∴ Average momentum per electron is:

$$\boxed{\langle p_x \rangle = \frac{p_x}{n} = -q\,\bar{t}\,\mathcal{E}_x.} \quad ①$$

∴ Constant net velocity in the negative x-direction

$$\boxed{\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = \frac{-q\bar{t}}{m_n^*}\mathcal{E}_x} \quad ②$$

↳ Shows the net drift of an average electron in response to the electric field (drift speed)

① The drift speed is smaller than the random speed due to thermal motion v_{TH}. (individual electrons move in many directions by thermal motion, during a time period).

• The current density:

$$\boxed{J_x = -q\,n\,\langle v_x \rangle} \quad ③ \quad \left[\frac{\text{Ampere}}{\text{cm}^2} = \frac{\text{columb}}{\text{electron}} \cdot \frac{\text{electron}}{\text{cm}^3} \cdot \frac{\text{cm}}{\text{s}} \right].$$

↳ Number of electrons crossing a unit area per unit time ($n\langle v_x \rangle$) multiplied by the charge of electron ($-q$)

(sub ② into ③)

∴ Average velocity:

$$\boxed{J_x = \frac{nq^2\bar{t}}{m_n^*}\mathcal{E}_x}$$

∴ Current density is proportional to the electric field as excpected.

from Ohm's law:

$$\boxed{J_x = \sigma\,\mathcal{E}_x} \quad \text{where } \sigma = \frac{nq^2\bar{t}}{m^*n}$$

& conductivity $\sigma\,(\Omega\text{-cm})^{-1}$ can be written as:

$$\boxed{\sigma = q\,n\,\mu_n} \quad \text{where } \mu = \frac{q\bar{t}}{m_n^*}$$

(*)

• μ_n is the electron mobility.
describes the ease which electrons drift in the material.

• m_n^* is the conductivity effective mass for electrons
which is different from the density-of-states-effective mass. } used for charge transport problems

$$\to \frac{1}{m_n^*} = \frac{1}{3}\left(\frac{1}{m_\ell} + \frac{2}{m_t}\right)$$

eg

Calculate the conductivity effective mass of electron in Si

$Si \Rightarrow m_\ell = 0.98\,m_0$

$m_t = 0.19\,m_0$

$$\therefore \frac{1}{m_n^*} = \frac{1}{3}\left(\frac{1}{m_x} + \frac{1}{m_y} + \frac{1}{m_z}\right) = \frac{1}{3}\left(\frac{1}{m_\ell} + \frac{2}{m_t}\right)$$

$$\frac{1}{m_n^*} = \frac{1}{3}\left(\frac{1}{0.98\,m_0} + \frac{2}{0.19\,m_0}\right)$$

$$m_n^* = 0.26\,m_0$$

• Mobility can also be expressed by:

$$\mu_n = \ominus\frac{\langle v_x \rangle}{\mathcal{E}_x}$$ } average particle drift velocity per unit electric field

$\left[\frac{cm/s}{V/cm}\right]$

$\to$ results in a positive value of mobility, since electrons drift opposite to the electric field.

$\therefore$ Current density can be written in terms of mobility:

$$\boxed{J_x = q n \mu_n \mathcal{E}_x}$$

(Derivation has been based on the assumption that the current is carried primarily by electrons).

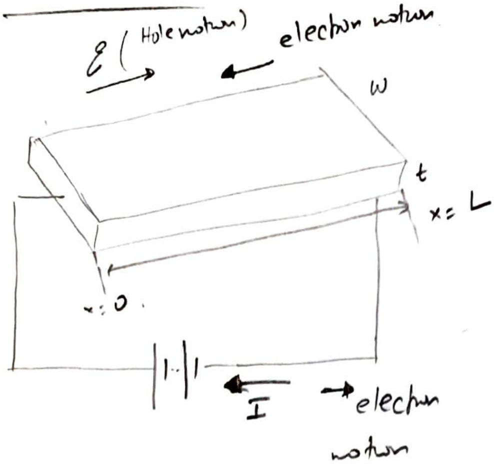
→ For holes

$$J_x = +q p \mu_p \mathcal{E}_x \qquad \therefore \mu_p = \frac{+\langle v_x \rangle}{\mathcal{E}_x}.$$

$\therefore$ Both together:

$$\boxed{J_x = q(n\mu_n + p\mu_p)\mathcal{E}_x = \sigma \mathcal{E}_x}$$

• Drift & Resistance



• $R = \frac{\rho l}{A} \Rightarrow R = \frac{\rho L}{wt} = \frac{L}{wt} \times \frac{1}{\sigma}$ //

• Mechanism of carrier drift requires <u>holes</u> to move as a group <u>in the direction</u> of the <u>electric field</u> ($\mathcal{E}$) & <u>electrons</u> to move as a group in <u>the opposite direction</u> of the electric field ($\mathcal{E}$).

• Conventional current is +ve in the hole flow direction
-ve in the electron flow direction

∴ Drift current is constant throughout the bar.

• For <u>electrons</u>:
→ For every electron leaving the left end ($x = 0$) there is a corresponding electron entering at ($x = L$) ∴ electron concentration remains constant (n).

• For <u>holes</u>:
→ As hole reaches the <u>ohmic</u> contact at $x = L$ → recombines with an electron supplied from the external circuit.
→ As the hole dissapears a corresponding hole must appear at $x = 0$ to maintain space charge neutrality.
↳ Reasonable to consider the source of this hole generation of an (EHP at $x = 0$), with the hole flowing into the bar & the electron flowing into the external circuit.

• Effects of Temperature & Doping on Mobility

→ Scattering Mechanism that influence electron & hole mobility

- lattice scattering
- Impurity scattering

• Lattice scattering, a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature

(Vibration of atoms in the crystal are called phonons.)

∴ lattice scattering is also known as phonon scattering)

⇒ Frequency of scattering events increases as temperature increases since the thermal agitation of lattice becomes greater ∴ $\boxed{\text{mobility decreases}}$

• Impurity scattering

→ Scattering caused by crystal defects (such as ionized impurities).

become dominant at low temperatures

• Since atoms in lattice are cooler ∴ less agitated, lattice scattering is less important.

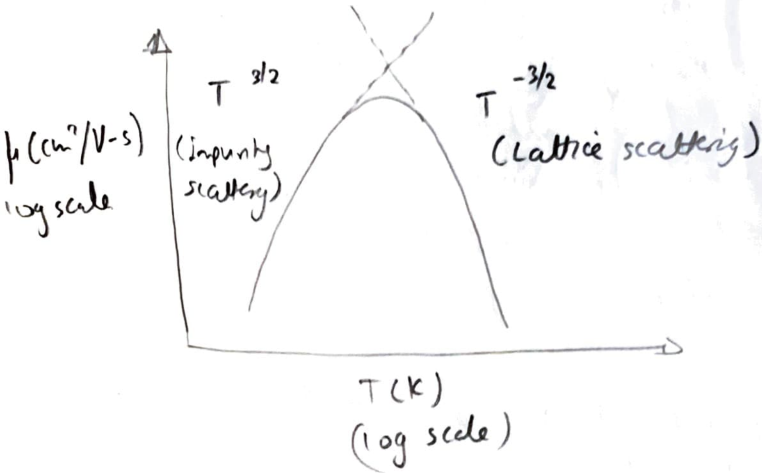
The thermal motion of the carriers is also slower.

• Since slowly moving carrier is likely to be scattered more strongly by an interaction with a charged ion than a carrier with greater momentum → impurity scattering cause a $\boxed{\text{decrease in mobility}}$ as temperature decreases

· According to equation:

$$dp_x = -p_x \frac{dt}{\bar{t}} \qquad \therefore \; p_x \propto \frac{1}{\bar{t}}$$

Since scattering probability is inversely proportional to the mean free-time & therefore mobility



→ Total mobility:

↳ $$\boxed{\frac{1}{\mu_T} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \cdots}$$

∴ the mechanism causing the lowest mobility dominate.

Example: (*)

A Si bar of 0.1 cm long and 100 μm^2 area is doped with 10^{17} cm^{-3} phosphorus.

Find the current at 300K with 10V applied

→ Since voltage applied is only 10V ∴ electric field is weak.

∴ ohmic regime. ($\mu_n = 700 cm^2/Vs$)

$$\sigma = \mu_n q n_o = 700 \times 1.6 \times 10^{-19} \times 10^{17} = 11.2 \, (\Omega . cm)^{-1} = \rho^{-1}$$

$\rho = 0.0893 \, \Omega cm$

$$R = \frac{\rho l}{A} = 0.0893 \times 0.1 / 10^{-6} = 8.93 \times 10^3 \, \Omega$$

$$I = V/R = \frac{10}{(8.93 \times 10^3)} = 1.12 mA \; //.$$

· High-field Effects

$\sigma = \frac{nq^2 t}{m_n^*}$

· For now we assumed that Ohm's Law was valid. [$J_x = \sigma \mathcal{E}_x$]

↳ Assumed that:

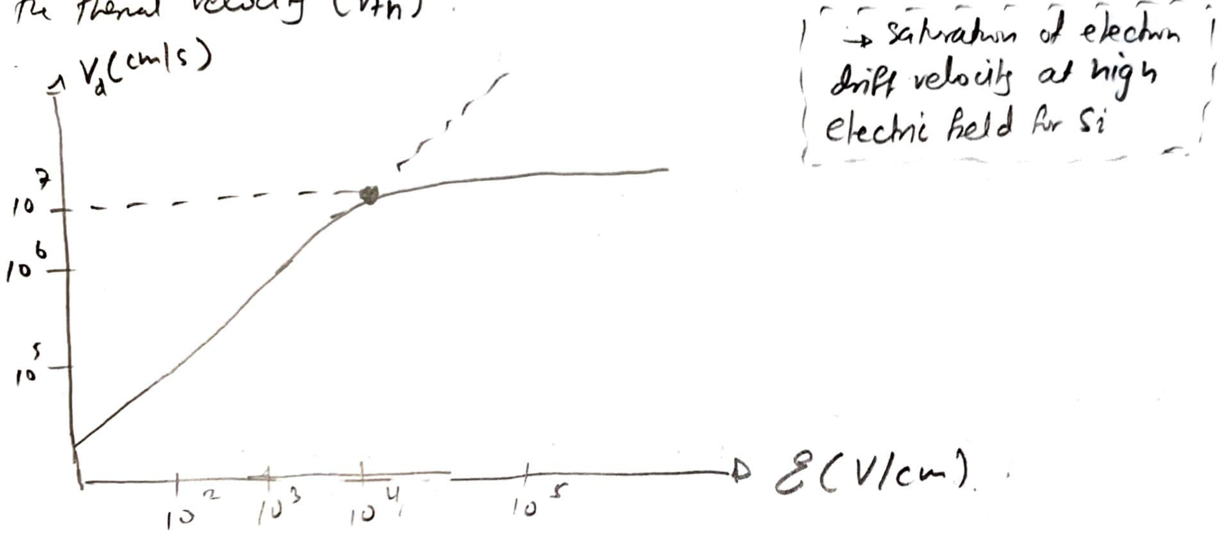
· Drift current is proportional to electric field

& the proportionality constant (σ) is not a function of $\mathcal{E}$.

HOWEVER:

For large electric fields ($>10^3$ V/cm) can cause drift velocity to exhibit dependence on $\mathcal{E}$.
& therefore the current $J = -qnv_d$

↳ This dependence of σ upon $\mathcal{E}$ is an example of a hot carrier effect → (*) carrier drift velocity (v_d) is comparable to the thermal velocity (v_{th}).



· Upper limit is reached for the carrier drift velocity in a high field.
This limit occurs near the mean thermal velocity. ($\simeq 10^7$ cm/s)

↳ This limit occurs near the point at which added energy imparted by the field is transferred to the lattice rather than increasing the carrier velocity.

⇒ Result of this scattering limited velocity is fairly constant current at high field

Hall Effect

- A magnetic field is applied perpendicular to the direction in which holes drift in a p-type bar → the path of the holes tend to be deflected.

→ Using vector notation, the total force on a single hole due to the electric and magnetic field is:

$$\boxed{F = q(\mathcal{E} + v \times B)}$$ (→ cross product)

In y-direction:

$$F_y = q(\mathcal{E}_y - v_x B_z).$$

→ Result, unless an electric field ($\mathcal{E}_y$) is established along the width of the bar, each hole will experience a net force (∴ acceleration) in the -y-direction) due to the $qv_x B_z$ product

∴ To maintain a steady state flow of holes the electric field ($\mathcal{E}_y$) must be balance by the product $v_x B_z$:

$$\boxed{\mathcal{E}_y = v_x B_z}$$

$$\therefore \boxed{F_y = 0}$$

$\therefore V_{AB} = \mathcal{E}_y w$ → Hall voltage

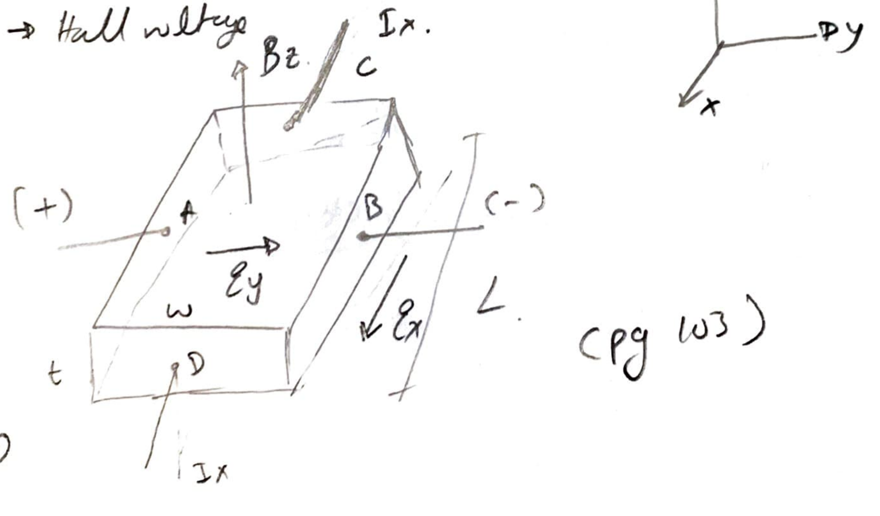


Figure ①

(pg 103)

$$\therefore \mathcal{E}_y = \frac{J_x}{qp_0} B_z = R_H J_x B_z, \quad R_H = \frac{1}{qp_0}.$$

• Thus Hall field is proportional to the product of the current density & the magnetic flux density.

$$\rightarrow \rho = \frac{R w t}{L} = \frac{V_{CD}/I_x}{L/wt}$$

$$R_H = \frac{1}{q p_0} \rightarrow \text{Hall coefficient}$$

$$(\Rightarrow) \ \mu_p = \frac{\sigma}{q p_0} = \frac{1/\rho}{q(1/q R_H)} = \frac{R_H}{\rho}$$

→ Measurement of the Hall coefficient and the resistivity over a range of temperature yields a plot of majority carrier concentration & mobility vs temperature [holes]

[electrons] → $(-q)$; $-V_{AB}$ (Hall voltage) ; $-R_H$ (Hall coeff).

(⇒) Sign of the Hall voltage is common technique to determine if a sample is p-type or n-type

eg Figure ①, $w = 0.1\,mm$, $t = 10\,\mu m$, $L = 5\,mm$

$B = 10\,kG$ in the direction shown ($1\,kG = 10^{-5}\,Wb/cm^2$)

$I = 1\,mA$

$V_{AB} = -2\,mV$

$V_{CD} = 100\,mV$

→ $B_z = 10^{-4}\,Wb/cm^2$

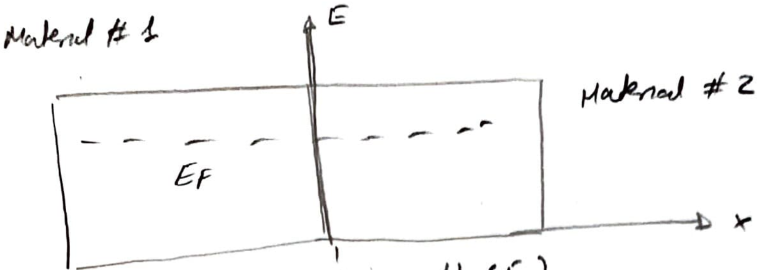
• From the sign of V_{AB} → majority carriers are electrons

$$n_0 = \frac{I_x B_z}{q t(-V_{AB})} = \frac{(10^{-3})(10^{-4})}{(1.6\times10^{-19})(2\times10^{-3})(10^{-3})} = 3.125\times10^{17}\ cm^{-3}$$

$$\rho = \frac{R}{L/wt} = \frac{V_{CD}/I_x}{L/wt} = \frac{0.1/10^{-3}}{0.5/0.01\times10^{-3}} = 0.002\ \Omega cm$$

$$\mu_n = \frac{1}{\rho q n_0} = \frac{1}{(0.002)(1.6\times10^{-19})(3.125\times10^{17})} = 10.000\ cm^2(V\cdot s)^{-1}$$

· Invariance of the Fermi-Level Equilibrium



Density of states: $N_1(E)$ | $N_2(E)$

Fermi-Distribution: $f_1(E)$ | $f_2(E)$

=) No net energy transfer, no net charge transport, no current at thermal equilibrium.

→ For any transfer of electrons from Material 1 → 2 must be exactly balanced by the opposite transfer of electron from 2 → 1

∴ At energy E the rate of transfer of electrons from 1 → 2 is proportional to the number of filled states at E in material 1 times the number of empty states at E in material 2.

$$\text{* rate from 1 to 2} \propto N_1(E) f_1(E) \cdot N_2(E)[1 - f_2(E)]$$

where: $f(E)$ → probability of states being filled at E.

$$\text{* rate from 2 to 1} \propto N_2(E) f_2(E) \cdot N_1(E)[1 - f_1(E)]$$

At equilibrium those must be equal

$$(=)\ N_1 f_1 N_2 - N_1 f_1 N_2 f_2 = N_2 f_2 N_1 - N_2 f_2 N_1 f_1$$

which results in:

$f_1(E) = f_2(E)$ that is $[1 - e^{(E-E_{F1})/kT}]^{-1} = [1 + e^{(E-E_{F2})/kT}]^{-1}$

Conclude that $E_{F1} = E_{F2}$ (=) No discontinuity in the equilibrium Fermi Level.

· Fermi level at equilibrium must be constant throughout materials in intimate contact ∴ no gradient exist → $\boxed{\frac{dE_F}{dx} = 0}$

Chapter 4 - Excess Carriers in Semiconductors

- Excess carriers can be created:
  - optical excitation
  - electron bombardment
  - injected accross a forward-biased pn-junction.

- Quasi-Fermi levels.

In summary the Quasi-Fermi levels F_n & F_p are steady state analogues of the equilibrium Fermi level E_F.

- When excess carriers are present, the deviations of F_n & F_p from E_F indicate how far the electron & hole populations are from the equilibrium value n_0 & p_0

The seperation of the quasi-Fermi levels $F_n - F_p$ is a direct measure of the deviation from equilibrium ($F_n = F_p = E_F$).

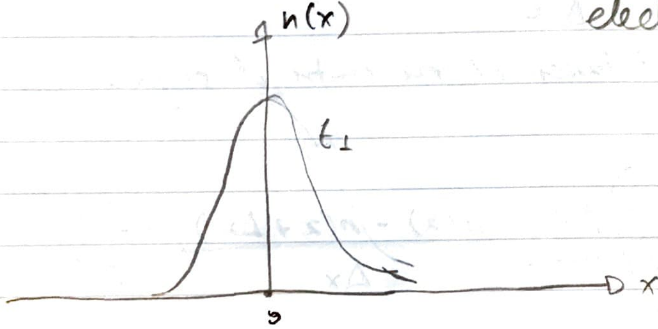
Useful in visualizing minority & majority carriers concentrations in device where these quantities vary in position.

Diffusion of Carriers

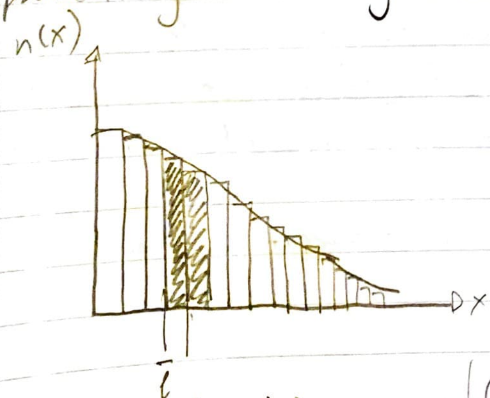
- When excess carriers are created non-uniformly in a semiconductor, the electron & hole concentration vary with position in sample
- ⊛ ↳ Variation (gradient) in n & p → net motion of the carriers from regions of high concentration to regions of low concentration ⟹ diffusion.

- Diffusion Process
  ↳ random motion of the individual molecules.
- Carriers in a semiconductor diffuse in a carrier gradient by random thermal motion & scattering from the lattice & impurities.

spreading of a pulse of electrons by diffusion

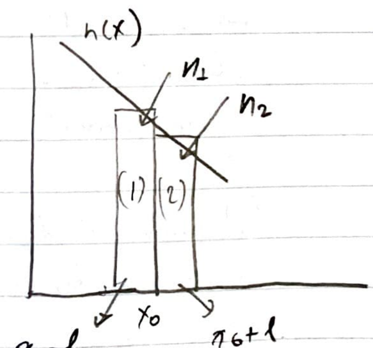


→ Rate at which the electrons diffuse in a one-dimensional problem by considering an arbitrary distribution $n(x)$



(mean free path)
small incremental distance

⟹



⊛ ⟹ Electron in segment (1) have equal chances moving left & right & in mean free time ($\bar{t}$) half of them will move into segment #2.

- Same is true for the other side also

∴ Net number of electrons passing x_0 from left to right in a mean time is

$\frac{1}{2}(n_1 \bar{l} A) - \frac{1}{2}(n_2 \bar{l} A)$ where $A \Rightarrow$ area perpendicular to x.

∴

$\phi_n(x_0) = \frac{\bar{l}}{2\bar{t}}(n_1 - n_2) \Rightarrow$ rate of electron flow in +ve x direction per unit Area [electron flux density]

Since the mean free path $\bar{l}$ is small difference in electron concentration $(n_1 - n_2)$. can be written as

$$n_1 - n_2 = \frac{n(x) - n(x+\Delta x)}{\Delta x}\,\bar{l}$$

where x is taken at the centre of segment (1) & $\Delta x = \bar{l}$.

$$\phi_n(x) = \frac{\bar{l}^2}{2\bar{t}} \lim_{\Delta x \to 0} \frac{n(x) - n(x+\Delta x)}{\Delta x} = -\frac{\bar{l}^2}{2\bar{t}} \frac{dn(x)}{dx}$$

called electron diffusion coefficient [D_n] [cm^2/s].

- (−) arrises from the definition of the derivative
↳ Indicates that the net motion of electrons due to diffusion is in the direction of decreasing electron concentration [moving from high concentration to low concentration]

Ⓠ. Show that hole in the hole concentration gradient move with a diffusion coefficient D_p.

∴

$$\phi_n(x) = -D_n \frac{dn(x)}{dx}$$

$$\phi_p(x) = -D_p \frac{dp(x)}{dx}$$

- The diffusion current crossing a unit area (current density) is the particle flux density multiplied by the charge of carrier

$$J_n(diff) = -(-q) D_n \frac{dn(x)}{dx} = +q D_n \frac{dn(x)}{dx}$$

$$J_p(diff) = -(+q) D_p \frac{dp(x)}{dx} = -q D_n \frac{dp(x)}{dx}$$

→ **Note**: Electrons & holes move in a carrier gradient. but resulting currents are in opposite directions. because of the opposite charge of electron & holes

Diffusion and Drift Carriers ; Built-in Fields

- If an electric field is present in addition to carrier gradient. current densities will have:
  - drift component
  - diffusion component.

$$J_n(x) = \underbrace{q\mu_n n(x)\mathcal{E}(x)}_{\boxed{\text{drift}}} + \underbrace{qD_n \frac{dn(x)}{dx}}_{\boxed{\text{diffusion}}}$$

$$J_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

- Total current densities:

$$\boxed{J(x) = J_n(x) + J_p(x)}$$

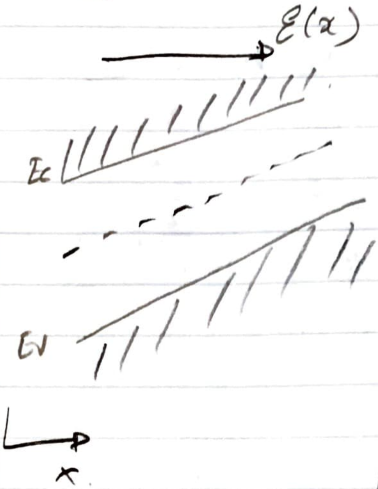
Note: Minority charge carriers can contribute significantly to the current through diffusion.

eg

in an n-type material minority hole concentration (p) may be many orders of magnitude smaller than electron concentration (n).

but → $\frac{dp}{dx}$ may be significant

• Energy band diagram of semiconductors in an electric field



⊛ Since electrons drift in the direction opposite to $\mathcal{E}(x)$ we expect potential energy to increase in the direction of the field.

$$\mathcal{E}(x) = \frac{-d(V_x)}{dx} \quad \Big| \quad V(x) = \frac{E_x}{-q}$$

$$\therefore \mathcal{E}(x) = \frac{-dV(x)}{dx} = \frac{-d}{dx}\left\{\frac{E_i}{-q}\right\} = \frac{1}{q}\frac{dE_i}{dx} \;//$$

④ At equilibrium:

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}$$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right)$$

$$\therefore \boxed{\frac{D}{\mu} = \frac{kT}{q}}$$ Einstein relation