Feynman Lectures on Physics - Vol III

- QUANTUM MECHANICS

Chapter 1 - Quantum Behaviour

* Quantum behaviour of atomic objects (electrons, protons, neutrons photons ...) is the same for all, they are all 'particle waves'

* Double-slit experiment with waves
→ interfearence ⇝ waves are in phase. ⇝ large amplitude & intensity. (constructive) / whole # of wavelengths : $\pi, 2\pi, 3\pi$ ...

→ interfearence of π ⇝ waves are not in phase (out of phase) (destructive) ⇝ low amplitude & intensity.
* odd number of half wavelengths : $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ ...

* Quantitative relationship of I_1, I_2, I_{12} ∴ h_1 : complex number
> I_1 can be written as : $h_1 e^{i\omega t}$. h_1 : amplitude.
(instantaneous height of water from hole 1)
∴ Intensity ∝ mean squared height ∴ $|h_1|^2$

- Same for I_2

∴
> Total (I_{12}) when both holes are open:

$$I_{12} = (h_1 + h_2)e^{i\omega t}.$$
$$|I_{12}| = |h_1 + h_2|^2$$

In general:

$$I_1 = |h_1|^2 ; \quad I_2 = |h_2|^2 , \quad I_{12} = |h_1 + h_2|^2$$

* If we expand $|h_1 + h_2|^2$

> $|h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2| \cos\delta$

where δ: phase difference between h_1 & h_2

* In terms of intensities

> $I_{12} = I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\delta}_{\text{"interfearence term"}}$

* Double slit experiment with electrons

- Electron gun consist of a tungsten wire heated by an electric current & surrounded by a metal box with a hole
(Fig. 1-3 pg)
* Wire is at negative voltage with respect to the box.
↳ ∴ electrons will accelerate towards the walls & some will pass through the hole.

* All the electrons which come out of the gun will have nearly the same energy.

* Detector can be a geiger counter, or electron multiplier connected to a loudspeaker.

* Loudness stays the same
↳ if we lower the temp of the tungesten wire, the rate of clicking slows down, but still each clicks sound the same.

The interference of electron waves

- Similar ~~wave~~ graph from wave-experiment ∴
Interfearence : $P_{12} \neq P_1 + P_2$

∴ $P_1 = |\phi_1|^2$ where ϕ : function of x.
$P_2 = |\phi_2|^2$
∴ $P_{12} = |\phi_1 + \phi_2|^2$

*> Concluded that:
Electrons arrive in lumps, like particles, and the probability of arrival of these lumps is distributed like the distribution of intensity of a wave.

.. Watching the electron (SOS!!)

- Placing a light-source behind the wall & between the 2 holes
*We know that electric charges scatter light. (movement.)

- Photon: $p = \frac{h}{\lambda}$ (momentum).

- Heisenberg uncertainty principle.

* Very interesting
↳ Looking the electron & not looking
(particle) (wave)

First principles of quantum mechanics

- An ideal experiment is one which all of the initial & final conditions of the (specific) experiment are completely specified.
  * An event : set of initial & final conditions.

SUMMARY

1. The probability of an event in an ideal experiment is given by the square of the absolute value of a complex number $\phi \rightarrow$ probability amplitude.

P = probability
ϕ = propability amplitude

$$P = |\phi|^2$$

2. when an event can occur in ~~specific~~ several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered seperately. There is interfearence

$$\phi = \phi_1 + \phi_2$$
$$P = |\phi_1 + \phi_2|^2$$

3. If an experiment is performed which is ~~cat~~ capable of determining whether one or another alternative is actually taken the probability of the event is the sum of the probabilities for each alternative. The interfearence is lost

$$P = P_1 + P_2.$$

The uncertainty principle

- If you make the measurement on any object, and you can determine the x-component of its momentum with an uncertainty of Δp, you cannot, at the same time, know its x-position more accurately than $\Delta x \geq \hbar / 2 \Delta p$ where $\hbar$ is a definite fixed number given by nature.
$\hbar$ = "reduced Planck constant" $\approx 1.05 \times 10^{-34}$ J s

* The more general statement was that one cannot design equipment in any way to determine which of two alternatives is taken, without, at the same time, destroying the pattern of interfearence.

Chapter 2:

- ## The Relation of Waves & Particle Viewpoints

- ## Probability wave amplitudes

- Way of representing anything in the quantum world, is to give an amplitude for every event that can occur.
  - If the event involves the reception of 1 particle.
    ↳ give the amplitude & found that one particle at different places & at different times.

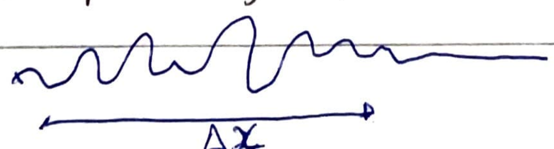
- The probability of finding the particle is proportional to the absolute square of the amplitude

- Particle's energy: $E = \hbar\omega$ where: k: wavenumber
- Particle's momentum: $p = \hbar k$ $\hbar = h/2\pi$

For example:
If the amplitude to find the particle at different places is given by $e^{i(\omega t - k \cdot r)}$, whose absolute square is a constant
∴ probability of finding ~~the~~ a particle is the same at all points.
That means we do not know where it is. - can be anywhere - great uncertainty in its location.

- If position of particle is more or less well known, & predict it fairly accurately. → probability of finding it in different places must be confined to a certain region, whose length is Δx.

Outside this region, probability is zero. ∴ amplitude is zero (~~as~~ probability is the absolute square).



wave-packet ~~Δx~~ of length Δx.

* Distance between the nodes corresponds to particles momentum

Note:

* → We cannot define a unique wavelength for a short wave train.
Such a wave train does not have a definite wavelength.
There is indefiniteness in the wave number that is related to the finite length of the train, & therefore there is indefiniteness in the momentum.

- Measurement of position & momentum

① * Single-slit experiment *

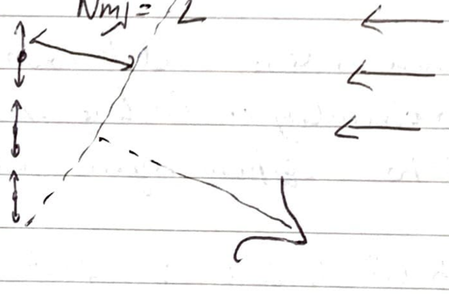
↳ $\Delta y \, \Delta p_y \geq \hbar/2$ refers to the predictability of a situation.

② Measure the wavelengths of waves. (fig. 2-3).

$\frac{\Delta_\lambda}{\lambda} = \frac{1}{N_m}$ (relative uncertainty in the wavelength.

$\frac{\Delta_\lambda}{\lambda^2} = \frac{1}{N_m \lambda} = \frac{1}{L}$

$N_m \lambda = L$



Where d:
- distance between the total distance that the particle or wave or whatever it is has to travel if it is reflected from the bottom of the grating, and the distance that is has to travel if it is reflected from the top of the grating

* Narrower d → simultaneous scattering from all parts of the grating.
Thus the wave train must be of length d in order to have an uncertainty in the wavelength less than that given previously:

$$\frac{\Delta\lambda}{\lambda^2} = \Delta\left(\frac{1}{\lambda}\right) = \frac{\Delta k}{2\pi} \quad \therefore \Delta k = \frac{2\pi}{d}$$

d: length of the wave train.

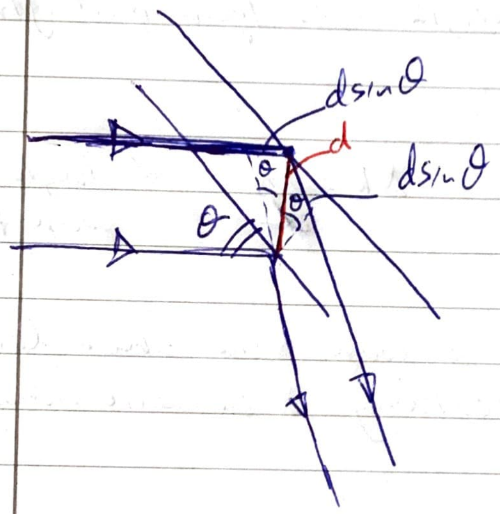
This means that if we have a wave train whose length is less than d, the uncertainty in the wave number must <u>exceed</u> $\frac{2\pi}{d}$.

⇒ Wave number = measure of the particles momentum. (Quantum) ∴ $p = \hbar k \Leftrightarrow \Delta p \simeq \frac{\hbar}{\Delta x}$.

Crystal diffraction

- Reflection of particle waves from a crystal.
  * crystal: solid which has lots of similar atoms arranged in an array.

→ In order to show maximum interfearence
  ↳ Find regions of constant phase
They are planes which make equal angles with the initial & final directions.



* 2 parallel planes
→ wave scattered from the two planes will be in phase provided the difference in distance travelled by a wave front is an integral number of wavelengths

* coherent reflection: $2d\sin\theta = n\lambda$ $(n=1,2,3,4\ldots)$

The size of an atom

* Every time we look at the electron, it's is somewhere, but it has an amplitude to be in different places so there is a probability of it being found in different places.
- Hence these places cannot all be at the nucleus
  ↳ spread in position of order α (a) → distance of electron from nucleus
- Determine α (a) by minimizing the total energy of the atom

* Spread in momentum $\approx \frac{\hbar}{a}$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{\hbar^2}{2ma^2}$$

$$U = \frac{e^2}{a}$$

(potential energy)

Total Energy

$$E = \frac{\hbar^2}{2ma^2} - \frac{e^2}{a}$$

$$\rightarrow \frac{dE}{da} = \frac{-\hbar^2}{ma^3} + \frac{e^2}{a^2} \qquad // \quad \frac{dE}{da} = 0$$

↳ $a_0 = \frac{\hbar^2}{me^2} = 0.528$ angstroms $= 0.528 \times 10^{-10}$ m

Bohr radius

Plug in α in E :. $-13.6\,eV$ → Rydberg of energy
(ionisation energy of hydrogen)

* Negative energy means that the electron has less energy when it is in the atom ~~that~~ than free

- Energy-Levels

* $\omega = \frac{(E_3 - E_2)}{\hbar}$

* Ritz combination principle.

Philosophical implications

→ Uncertainty principle → making an observation affects a phenomenon
→ Unless a thing ~~can~~ be defined by measurement, it has no place in a theory

Probability Amplitudes

- The laws for combining amplitudes

* Superposition of probability amplitudes.
  ↳ Double slit experiment → interfearence of electrons.

- The probability that the particle will arrive at x (position of detector) → can be represented quantitatively by the ~~abso~~ absolute square of a complex number → [probability amplitude].

Notation:

$$\left.\underbrace{\langle \text{Particle arrives at } x}_{\text{starting condition}} \mid \underbrace{\text{particle leaves } s \rangle}_{\text{final condition}}\right\} = \langle x \mid s \rangle$$

$\langle\ \rangle$ → "amplitude that."

*amplitude is a single complex number

- The probability that an electron arrives at the detector when both paths are open is:

$$P_{12} = |\phi_1 + \phi_2|^2$$

② Principle of quantum mechanics
→ When a particle can reach a given state by 2 possible routes, the total amplitude for the process is the sum of the amplitudes for the 2 routes considered separately

$$\langle x \mid s \rangle_{\text{both holes open}} = \langle x \mid s \rangle_{\text{through 1}} + \langle x \mid s \rangle_{\text{through 2}}$$

③ General principle.

- When a particle ~~goes~~ goes by some particular route the amplitude for that route can be written as the product of the amplitude to go part way with the amplitude to go the rest of the way.

$$\langle x | s \rangle_{\text{via } 1} = \langle x | 1 \rangle \langle 1 | s \rangle$$

→ The amplitude to go from $s \to x$ by way of hole 1 is equal to the amplitude to go from $s \to 1$ × the amplitude to go from $1 \to x$

Note:
Notice that the equation above is written in the reverse order (Read from right to left)
The electron goes from $s \to 1$ & then $1 \to x$

Resultant amplitude.

$$\langle x | s \rangle_{\text{both}} = \langle x | 1 \rangle \langle 1 | s \rangle + \langle x | 2 \rangle \langle 2 | s \rangle$$

* Pg 3-3 // Last paragraph !

* Suppose we have a particle, with definite energy going in empty space from a location $\underline{r}_1 \to \underline{r}_2$

(3.7) $$\langle \underline{r}_2 | \underline{r}_1 \rangle = \frac{e^{i\underline{p}\cdot\underline{r}_{12}/\hbar}}{r_{12}}$$

$r_{12} = r_2 - r_1$; p = momentum related to the energy E by the relativistic equation

$$p^2c^2 = E^2 - (m_0c^2)^2$$

or non-relativistic equation

$$\frac{p^2}{2m} = KE$$

→ Equation 3.7 says in effect that the particle has wavelike properties, the amplitude propagating as a wave with a wave number equal to the momentum divided by $\hbar$

* Let say the particle is at a certain place P at a certain time, and you would like to know the amplitude for it to arrive at some location, $\underline{r}$, some time later on

→ $\langle \underline{r}, t = t_1 | P, t = 0 \rangle$

* The function of $\underline{r}$ & t, in general, satisfies a differential equation which is the wave equation.

For non-relativistic → Schrodinger equation

* If we are dealing with more than 1 particle is not a simple wave in 3D space, but depends on the six space variables r_1 & r_2.

- Provided that the 2 particles do not interact, the amplitude that one particle will do one thing and the other one something else is the product of the 2 amplitudes that the two particles would do the two things seperately

Example :
If $\langle a|s_1\rangle$, is the amplitude for particle 1 to go from $s_1 \to a$, & $\langle b|s_2\rangle$ is the amplitude for particle 2 to go from $s_2 \to b$.
The amplitude of both things will happen together is:

$$\langle a|s_1\rangle\langle b|s_2\rangle$$

The two-slit interference pattern

* Fig 3-3 - An experiment to determine which hole the electron goes through

$\phi_1 = \langle x|1\rangle\langle 1|s\rangle$
$\phi_2 = \langle x|2\rangle\langle 2|s\rangle$

These are the amplitudes to go through the 2 holes & arrive at x if there is no light.

* The ~~part~~ amplitude that the electron goes from s to x, via slit 1 & scatters a photon into D_1 is:

$$\langle x|1\rangle a\langle 1|s\rangle = a\phi_1$$

For particle 2:

$$\langle x|2\rangle b\langle 2|s\rangle = b\phi_2$$

* The amplitude to find the electron at x and the photon in D_1 is the sum of the 2 terms, ~~each~~ for one for each possible path for the electron

$$\left\langle \begin{matrix}\text{electron at } x\\ \text{photon at } D1\end{matrix} \middle| \begin{matrix}\text{electron from } s\\ \text{photon from } L\end{matrix}\right\rangle = a\phi_1 + b\phi_2 \quad (3.8)$$

For detector D_2 (symmetry)

$$\left\langle \begin{matrix}\text{electron at } x\\ \text{photon at } D_2\end{matrix} \middle| \begin{matrix}\text{electron from } s\\ \text{photon from } L\end{matrix}\right\rangle = a\phi_2 + b\phi_1. \quad (3.9)$$

* Figure 3-4

* Suppose that we want to know with what probability we get a count in D_1 & an electron at x.

$\Rightarrow |a\phi_1 + b\phi_2|^2$

If $b=0 \Rightarrow |a\phi_1|^2 = a^2|\phi_1|^2 = |a|^2$.

* If the wavelength is very long, the scattering behind hole 2 into D_1 may be just about the same as for hole 1
↳ 2 phases are equal
∴ If x is practically equal to $b \Rightarrow |\phi_1+\phi_2|^2 \times |a|^2$
↳ The probability distribution we would have gotten without the photons at all
∴ show original distribution ~~curve~~ curve which shows interfearence.

* In case, that the detection is partially effective, there is an interfearance between a lot of ϕ_1 and a little of ϕ_2 ⇝ intermediate distribution

* Note: * SOS! (last paragraph) of 3-7)

Suppose that you only want the amplitude that the electron arrives at x, regardless of whether the photon was counted at D_1 or D_2.
Should you add eq 3.8 & 3.9
NO!!, you must never add amplitudes for different and district final states.
Once, the photon is accepted by one of the photon counters we can always determine the alternative.
You do ~~not~~ add the amplitudes for different indistinguishable alternatives inside the experiment before the complete process is finished.

* We first square the amplitudes for all possible final events and then sum.

The correct result for an electron at x, and a photon at either D_1 or D_2 is:

$$\left|\left\langle \begin{matrix} e \text{ at } x \\ ph \text{ at } D_1 \end{matrix} \middle| \begin{matrix} e \text{ from } s \\ ph \text{ from } L \end{matrix} \right\rangle\right|^2 + \left|\left\langle \begin{matrix} e \text{ at } x \\ ph \text{ at } D_2 \end{matrix} \middle| \begin{matrix} e \text{ from } s \\ ph \text{ from } L \end{matrix} \right\rangle\right|^2$$

$$= |a\phi_1 + b\phi_2|^2 + |a\phi_2 + b\phi_1|^2$$

3-3 - Scattering from a crystal

- Scattering of neutrons from a crystal.

Suppose we have a crystal which has a lot of atoms with nuclei at their centres, arranged in a periodic array, and a neutron beam that comes from far away.

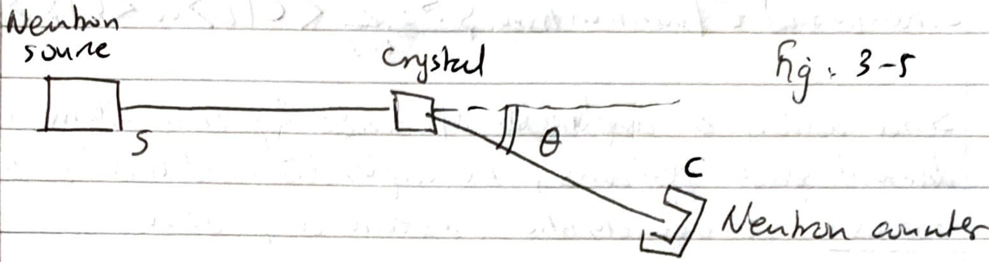


fig: 3-5

- Nuclei in crystal: i ; $i = 1, 2, 3, 4 \dots N$
  N ; = total number of atoms.

Calculate the probability of getting a neutron into a counter with the arrangement shown in fig 3-5

For any particular atom i, ~~the amplitude~~

$\langle$neutron at C | neutron at $S\rangle$ via i $= \langle C | i \rangle a \langle i | S \rangle$

* The amplitude that a neutron arrives at counter C, is the amplitude that the neutron gets from the source S to nucleus i, ($\times$) the amplitude a, that gets scattered there, ($\times$) the amplitude that it gets from i to the counter C

* Assumed that the scattering a, is the same for all atoms.
We have a number of indistinguishable routes, because a low energy neutron is scattered from a nucleus without knocking the atom out of its place in crystal
$\therefore$

$$\langle \text{neutron at } C \mid \text{neutron from } S \rangle = \sum_{i=1}^{N} \langle C \mid i \rangle a \langle i \mid S \rangle$$

Since adding the amplitudes of scattering from atoms with different space positions, the amplitudes will have different phases $\Rightarrow$ characteristic interference pattern.

* Figure 3-6 *
(Electrons have spin $\Rightarrow$ which this will affect scattering)

$\therefore$ Now

$$\langle C\,\text{up}, \text{crystal all down} \mid S\,\text{up}, \text{crystal all down} \rangle = \cancel{\langle C \mid i \rangle a \langle i}$$
$$= \langle C \mid i \rangle a \langle i \mid S \rangle$$

There is another case, where the spin of the detected neutron is down although it started from S with spin-up
lets say the k^{th} atom changed to the up direction.

$$\langle C\,\text{down}, \text{nucleus } k \text{ up} \mid S\,\text{up}, \text{crystal all down} \rangle = \langle C \mid k \rangle b \langle k \mid S \rangle$$

3-4- Identical particles (SOS!) ⋉ nice chapter.

⋉ Figure 3-7. ⋉.

$f(\theta)$: ~~ampl~~ amplitude to scatter into the counters when they are at $\measuredangle\,\theta$.

$\therefore\ |f(\theta)|^2$: experimentally determined probability.

⋉ Figure 3-7 (b) ⋉

Probability of some particle in $D_1 = |f(\theta)|^2 + |f(\pi-\theta)|^2$.

Note . Add probabilities NOT amplitudes ⋉

- Probability of an α-particle at $D_1 = |f(\theta)+f(\pi-\theta)|^2$