

- Find q_1, q_2, q_3 (orthonormal)
from a, b, c (columns of A).
- Then write A as QR
(Q orthonormal, R upper triangular)

$$A = \begin{matrix} & \begin{matrix} a & b & c \end{matrix} \\ \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix} \end{matrix}$$

→ Take q_1 (Start with a and make
it orthonormal to q_1)

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$$\Rightarrow q_1 = \frac{a}{\|a\|} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad a = \|a\| q_1$$

→ q_2 : (Need to subtract projection

of b onto q_1

$$q_2 = b - \frac{(b \cdot q_1) q_1}{\|q_1\|^2}$$

$$q_2' = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

• Why $q_2' \Rightarrow D$ because even though

$$\text{dot product of } \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \text{ \& } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = [0]$$

length is not $= 1$

$$\therefore q_2 = \frac{q_2'}{3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$q_3' = c - (c \cdot q_1) q_1 - (c \cdot q_2) q_2$$

$$= \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - 6 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

$$q_3 = \frac{q_3'}{\|q_3'\|} = \frac{q_3'}{5} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

②

$$A = QR$$

upper
triangular

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$a \quad b \quad c \qquad q_1 \quad q_2 \quad q_3$

$$\begin{aligned} -a &= 1q_1 + 0q_2 + 0q_3 \\ -b &= 2q_1 + 3q_2 + 0q_3 \\ -c &= 4q_1 + 6q_2 + 5q_3 \end{aligned} \quad \left. \vphantom{\begin{aligned} -a \\ -b \\ -c \end{aligned}} \right\} \text{see orange}$$