Problem 9.1 (3.5 #2. Introduction to Linear Algebra: Strang).

Find the largest possible number of independent vectors among:

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Solution:

We can see that

- $v_4 = v_2 - v_1$
- $v_5 = v_3 - v_1$

$\therefore v_{4,5,6} \Rightarrow$ dependent on vectors $v_{1,2,3}$

- $v_6 = v_3 - v_2$

CED

$$\begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{array} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \to \begin{bmatrix} \boxed{1} & 1 & 1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{bmatrix}$$

$\hookrightarrow$ 3 pivots $\Rightarrow$ independent

rank = 3

Problem 9.2 (3.5 # 20)

Find a basis for the plane $x-2y+3z=0$ in $\mathbb{R}^3$

Then find a basis for the intersection of plane with the xy plane.

Then find a basis for all vectors perpendicular to the plane.

Solution

$\Rightarrow$ Since the plane $=0$, then its part of the <u>null space</u>

$\therefore$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

∴ The special solutions are $Ax=0$

$$\underbrace{v_1 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}}_{\text{basis for the null space of } A \text{ and thus for the plane.}}$$

→ Intersection of this plane with xy plane contains v_1 and does <u>not</u> contain v_2 → Line

Need to be $\perp$ to the plane

∴

$$\begin{bmatrix} 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$