Given the invertible matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

Find the eigenvalues & eigenvectors of A^2, A^{-1} $-I$

$\Rightarrow Av = \lambda v$

$\cdot\ A^2 v = A(Av) = A(\lambda v) = \lambda(Av)$

$= \lambda^2 v$

$\Rightarrow A^{-1} v = A^{-1}\left(\dfrac{Av}{\lambda}\right) = \underbrace{A^{-1}A}_{I} \dfrac{v}{\lambda} = \dfrac{1}{\lambda} v.$

$(\lambda \neq 0)$

$\hookrightarrow (A^{-1} - I)v = (\lambda^{-1} - 1)v$

$\det(A - \lambda I) = 0$

$$\Rightarrow \det\begin{pmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & -2 \\ 0 & 1 & 4-\lambda \end{pmatrix}$$

$$= (1-\lambda)\det\begin{pmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{pmatrix}$$

$$= (1-\lambda)(\lambda^2 - 5\lambda + 6)$$

$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

$\Rightarrow \lambda = 1, 2, 3.$

i.e.

$-\lambda = 1$

$$0 = (A - I)v = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix} v$$

$$\Rightarrow v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\vdots$	A	A^2	$A^{-1} - I$
eigen value	λ	λ^2	$\lambda^{-1} - 1$
eigen vectors	– " –	– " –	– " –