Exercises on symmetric matrices and positive definiteness

Problem 25.1: (6.4 #10. *Introduction to Linear Algebra:* Strang) Here is a quick "proof" that the eigenvalues of all real matrices are real:

$$\textbf{False Proof: } A\mathbf{x} = \lambda\mathbf{x} \text{ gives } \mathbf{x}^T A\mathbf{x} = \lambda\mathbf{x}^T\mathbf{x} \text{ so } \lambda = \frac{\mathbf{x}^T A\mathbf{x}}{\mathbf{x}^T\mathbf{x}} \text{ is real.}$$

There is a hidden assumption in this proof which is not justified. Find the flaw by testing each step on the 90° rotation matrix:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

with $\lambda = i$ and $\mathbf{x} = (i, 1)$.

Problem 25.2: (6.5 #32.) A *group* of nonsingular matrices includes AB and A^{-1} if it includes A and B. "Products and inverses stay in the group." Which of these are groups?

a) Positive definite symmetric matrices A.

b) Orthogonal matrices Q.

c) All exponentials e^{tA} of a fixed matrix A.

d) Matrices D with determinant 1.

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