Find the determinants of:

$$A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$ called Vendermont matrix.

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

→ lets do elimination.

$$|A| = \left.\begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}\right\} = \text{equal rows}$$

$$= 0$$

$$|B| = \begin{vmatrix} 1 & a & a^2 \\ 0 & \underline{b-a} & b^2-a^2 = \underline{(b-a)}(b+a) \\ 0 & \underline{c-a} & c^2-a^2 = \underline{(c-a)}(c+a) \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} & & \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

can see that it's almost upper triangular

$\Rightarrow$ Do another elimination

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-b \end{vmatrix}$$

$$= (b-a)(c-a)(c-b)$$

③

rank 1

$$C = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix} \right\} \text{ matrix}$$

$$|C| = 0$$

4

$$D = \left\{ \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} \right\} \text{ skew symmetric matrix.}$$

factor out
each row

$$|D| = |D^T| = |-D| \overset{?}{=} \neq -|D|$$

$$= (-1)^3 |D|$$

$$= -|D|$$

$$\therefore |D| = 0$$

Note:

Is it true that all skew symmetric matrix have a det = 0?

↳ No, In our case we had

$a(-1)^{③}_{1} |D|$

④ even number

$1 \times |D| =$

$\therefore$

we have $|D| = |D|$

can be any number & not necessarly ϕ