Overview of linear algebra

- Linear algebra progresses from vectors to matrices to subspaces

Vectors:

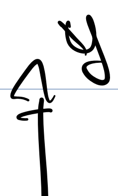
- Take linear combination
- Can subtract, add, multiply by scalar

eg $\quad x_1 u + x_2 v_2 + x_3 w = b$

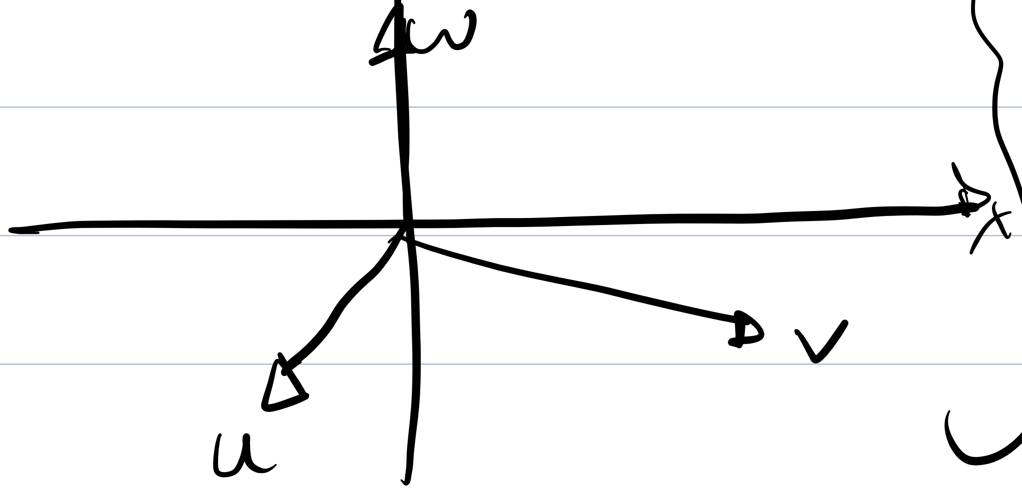
scalar

eg $\quad \mathbb{R}^3$ is 3 dimensions.

$$u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Suppose we take all



possible linear comb.

$\Downarrow$

get a plane [if added]

\# independent do not lie on the same plane.

→ How do I take combinations?

- Always put the columns of vectors into matrix

$$Ax = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(row • column.)

$$= \begin{bmatrix} x_1 \end{bmatrix}$$

Combination of

$$\begin{bmatrix} \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix}$$

columns.

difference matrix [first] because it takes differences].

eg

$$A \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

difference of squares results to odd numbers

What if we take b first?

→ Solve 3 equations.

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$AD \begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 + b_1 \\ b_3 + b_2 + b_1 \end{bmatrix}$$

- What is the matrix multiplied that will result to b

$$x = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{Inverse matrix.}} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

→ If $Ax = b$

$x = A^{-1}b$ is if invertible

* multiplying A in other words it called transform.

Since A can be called difference transform.

∴ A^{-1} can be called sum transform.

Example #2

line of solutions.

$$C = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} c \\ c \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Change from the above example:

$\{ G' x \}$ okay only if

$$\rightarrow \begin{bmatrix} x_1 - x_3 \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

↳ can we solve it?

satisfied

If we add them:

$$0 = b_1 + b_2 + b_3$$

solution

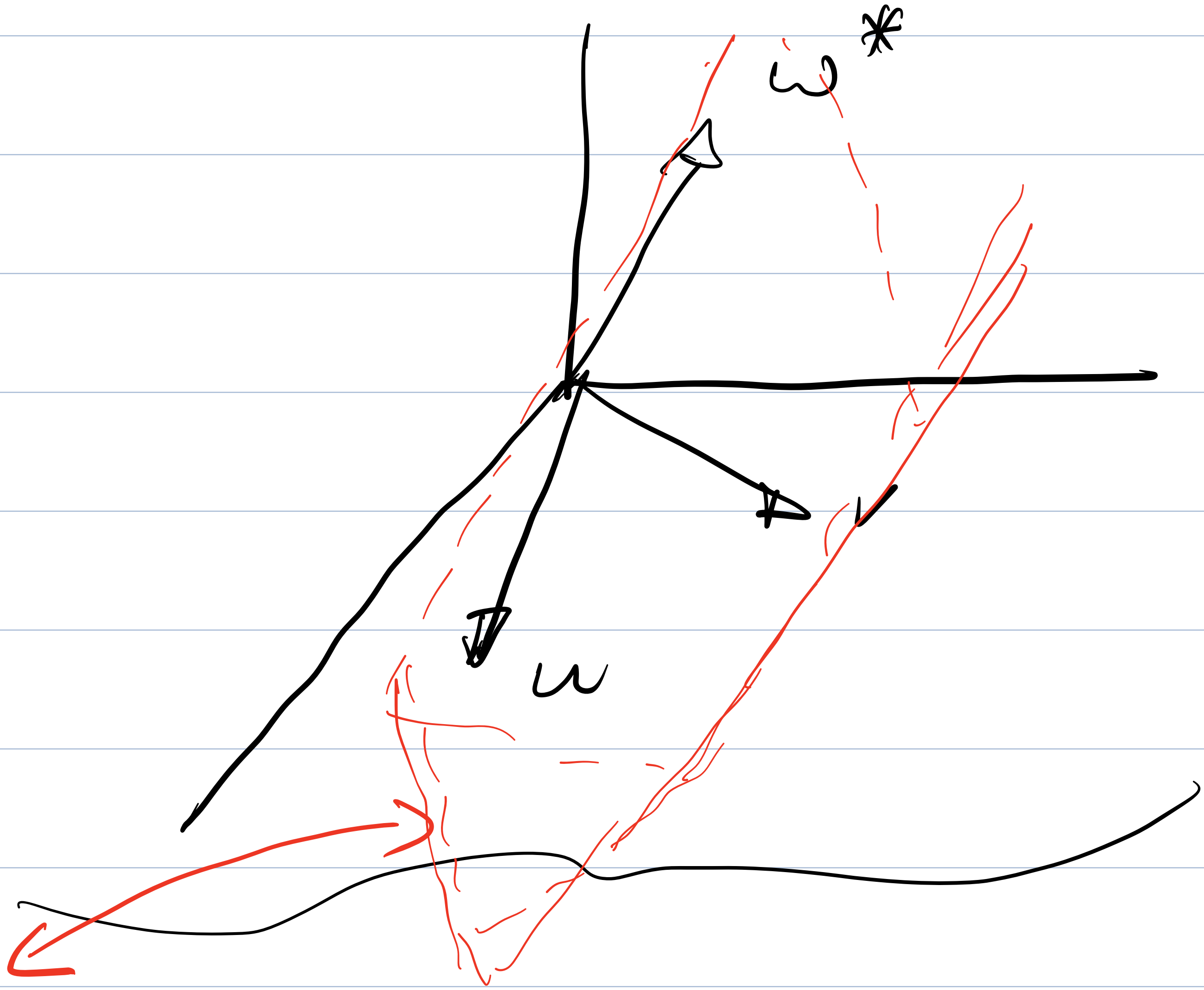
Problem

* No G inverse can return it back [because it results to $\underline{0}$]

∴ invertible.

Problem

<u>Geometrically</u>



→ All of them lie on the same plane

* Taking all combination of

u, v, w → results to the
whole subspace.

Basis : • meaning vectors independent
• result to the whole subspace
• Matrix will be invertible

(↳ Plane is a subspace)

Note :

- all combs of u, v, w*
=
all vectors $C_1 x$

$\hookrightarrow \begin{bmatrix} u & v & w^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underline{\text{subspace}}$

- What $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ do we get?

$\hookrightarrow \underline{\underline{0 = b_1 + b_2 + b_3}}$

If we look at the components of C

$$\underline{\underline{C = \begin{bmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} & \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{bmatrix}}}$$

$=0 \quad =0 \quad =0$

forming a $\underline{\underline{\text{plane}}}$

- A subspace → plane

↳ Vector space : bunch of vectors
∴ take all combinations.

→ See lecture #2 notes *

- The subspace of $\mathbb{R}^3$ are:

- the origin,
- a line through the origin,
- a plane through the origin,
- all of $\mathbb{R}^3$.