Problem 21.1

(6.1 # 19.)

eigen values $\lambda = 0$, λ $\to$ singular.

$\lambda = 1$, $(\lambda - 1)$

$\lambda = 2$, $(\lambda - 2)$ $\therefore$ rank $= 2$

a) Rank: since 3×3 rank can be at most $n - 1 = 2$.

b) det $|B^T B|$

since B is singular ($\lambda = 0$) $\therefore$

$\det(B^T) \det(B) = 0$.

c) There is not enough information to find eigenvalues of $B^T B$.

d) $(B^2+I)^{-1}$?

If $p(t)$ is a polynomial and if x is an eigenvector A, with eigenvalue λ, then

$$p(A)x = p(\lambda)\, x$$

We also know that if λ is an eigenvalue of A the $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

$$\Rightarrow (B^2+I)^2 = \frac{1}{0^2+1}, \frac{1}{1^2+1}, \frac{1}{2^2+1}$$

$$1 \qquad 1/2 \qquad 1/5$$

Problem 21.2

eigen values

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

Triangular matrix.

For B

$$\det(B - \lambda I) = (-\lambda)(2-\lambda)(-\lambda) - 3(2-\lambda)$$
$$= (\lambda^2 - 3)(2-\lambda)$$

$\therefore$ Eigenvalues are $\pm\sqrt{3}$ & 2

$$\det(C-\lambda I) = (2-\lambda)\left[(2-\lambda)^2-4\right]$$
$$-2\left[2(2-\lambda)-4\right]+2\left[4-2(2-\lambda)\right]$$

$$= \lambda^3-6\lambda^2 = \lambda^2(\lambda-6)$$

$$\lambda = 0 \qquad \lambda = 6$$