Elimination with Matrices

Note: Elimination is commonly used to computer systems.

Objectives

Elimination → Sucess
Elimination → Failure

Back-Substitutions.

Elimination matrices

Matrix multiplication

Lets start:

$$\left.\begin{array}{l} x + 2y + z = 2 \\ 3x + 8y + z = 12 \end{array}\right\} \text{forming in a matrix}$$

$4y + z = 2$

$[Ax = b]$

1착 pivot

$$A = \begin{bmatrix} \boxed{1} & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix}$$

- So what does elimination do?

(S1) Purpose: remove x part of eq. 2 [eliminate x].

(2,1)

2^{nd} pivot.

$$\Rightarrow \begin{matrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 4 & 1 \end{matrix} \qquad \begin{matrix} \text{(pivot row)} \\ (\times 3) \\ \\ \end{matrix}$$

[forward elimination].

(S2) Wipe out (3, 2)

$$\begin{matrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 0 & \boxed{5} \end{matrix} \quad u$$

* Purpose $\Rightarrow$ $A \to u$

pivots <u>cannot be</u> 0

~ How could this fail (fail to come up with 3 pivots)

$\hookrightarrow$

(eg) If 1^{st} pivot $= 0$

(Incase if there is a 0 at the pivot position $\to$ exchange rows)

(3) Last night 1 became a 5 to −4 for example will result to 0 pivot.

∴ matrix not invertible.

Back-Substitution

$$
\begin{array}{cccc|c}
 & & & & b \\
\boxed{1} & 2 & 1 & 2 \\
3 & 8 & 1 & 12 \\
0 & 4 & 1 & 2
\end{array}
\Rightarrow
\begin{array}{c}
\times 3 \\
2 \\
6 \\
2
\end{array}
$$

Last step

$$\begin{array}{c} 2 \\ 6 \end{array}$$

c is what happens to b

$$-10$$

$$\begin{aligned} x + 2y + z &= 2 &\leadsto\ & x = 2 \\ 2y - 2z &= 6 &\leadsto\ & y = 1 \\ 5z &= -10 &\leadsto\ & z = -2 \end{aligned}$$

back subst.

- Elimination Matrices

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

Note:
matrix x column
= column

$\leadsto$ Row operations

$$\underbrace{\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}_{(1\times 3)} \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times 3}}$$

$\Rightarrow$ $1 \times$ row 1

$2 \times$ row 2

$7 \times$ row 3

$\therefore$ Matrices: subtract $3 \times$ row 1 from row 2

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 4 & 1 \end{bmatrix}$$

Elementary Matrix

$(2,1)$

(S2) : Subtract 2 x row 2 from row 3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

E

$(3,2)$

We start with

$$E_{32}(E_{21}A) = u$$

$$\underbrace{(E_{32}E_{21})}A = u$$

Associative law

$\Rightarrow$ Other type of Elementary Matrix
(Matrix that exchanges 2 rows)

↳ permutation Matrix

eg Exchange rows 1 and 2

$$\begin{bmatrix} 0 & 1 \\ \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$

P

Exchange columns

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

Note: You cannot change the order of Matrices

[commutative Law is FALSE]

Inverses

eg

$$E^{-1} \qquad E \qquad = I$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

*Note: The step was subtract $3 \times$ row_1 from row_2

(Undoes elimination)