Exercises on differential equations and e^{At}

Problem 23.1: (6.3 #14.a *Introduction to Linear Algebra:* Strang) The matrix in this question is skew-symmetric ($A^T = -A$) :

$$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \mathbf{u} \quad \text{or} \quad \begin{aligned} {u_1}' &= cu_2 - bu_3 \\ {u_2}' &= au_3 - cu_1 \\ {u_3}' &= bu_1 - au_2. \end{aligned}$$

Find the derivative of $||\mathbf{u}(t)||^2$ using the definition:

$$||\mathbf{u}(t)||^2 = {u_1}^2 + {u_2}^2 + {u_3}^2.$$

What does this tell you about the rate of change of the length of $\mathbf{u}$? What does this tell you about the range of values of $\mathbf{u}(t)$?

Problem 23.2: (6.3 #24.) Write $A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$ as $S\Lambda S^{-1}$. Multiply $Se^{\Lambda t}S^{-1}$ to find the matrix exponential e^{At}. Check your work by evaluating e^{At} and the derivative of e^{At} when $t = 0$.

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