- Suppose A is a matrix, the complete solution to

$$A\mathbf{x} = \underbrace{\begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix}}_{= b} \text{ is}$$

all the solutions to Ax are given by

$$x = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}_{= x_p \text{ (particular sol.)}} + \underbrace{c \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}}_{= x_s \text{ (special sol)}}$$

What can you say about columns of A?

$\Rightarrow$ Solution

(size of A)

A should have 3 columns: why:

$$A = \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix}$$

Because we want to multiply A with an x with 3 entries.

c_1, c_2, c_3 are vectors in $\mathbb{R}^{4}$ - 4D

[take linear combination of results to A]

$A(\underline{\underline{x_p}} + c \cdot \underline{\underline{x_s}}) = \underline{\underline{b}}$ for any number c

$c = 0$ $\boxed{A \cdot x_p = b}$ * ①

$c = 1$ $A(x_p + x_s) = b$

$[Ax_p + Ax_s]$

$\downarrow$

b

$$\boxed{A \cdot x_s = 0}$$ *

Calculate c_1, c_2, c_3 ①

$A \quad x_p \quad = b$

$$\begin{array}{c} A \\ \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix} \end{array} \cdot \begin{array}{c} x_p \\ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\Rightarrow c_2 + c_3 = b \quad (1)}$$

$$A \cdot x_s = 0 \quad (2)$$

$$\begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = 0$$

$$\boxed{\Rightarrow 2c_2 + c_3 = 0 \quad (2)}$$

Solving simultaneous equations:

$$c_3 = -2c_2$$

$$c_2 - 2c_2 = b \Rightarrow \boxed{c_2 = -b}$$

$$\boxed{c_3 = 2 \cdot b}$$

$$\boxed{x = x_p + \underline{\underline{c}} \cdot x_s}$$

$$\underline{\underline{A \cdot x_s = 0}}$$

if we remove x_p

1 special solution

[dimension]

$$\dim(N(A)) = 1$$

$$\text{rank}(A) = 3 - 1 \quad (\text{columns} - \dim())$$

$$\underline{\underline{= 2}}$$

$:= \#$ linear independent columns.

$\Rightarrow c_1$ not a multiple of b

* So if the c_1 was also a multiple of b as the c_2, c_3 are

then the rank will be smaller than 2

↳ * This ~~cannot~~ happen