

Suppose

$$B = \begin{pmatrix} 1 & & \\ 2 & 1 & \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Find a basis for and compute the dimension of each of the 4 fundamental spaces.

Solution:

• column space: $\text{rank}(B) = 2$ (pivots)

A basis for $\text{Col}(B) =$

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

- Null space: $\dim N(B) = 1$ ($3 - 2$)
A basis for $N(B)$

$$\begin{pmatrix} -3/5 \\ -1 \\ 1 \end{pmatrix}$$

- Row space: $\dim C(B^T) = 2$
A basis for $C(B^T)$

$$\begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

- Left null space: $\dim N(B^T) = 1$

$$E_2 \begin{pmatrix} 1 & & & \\ -2 & 1 & & \\ \hline 1 & 0 & 1 & \end{pmatrix} \quad B = \begin{pmatrix} 5 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

A basis for $N(B^T)$ is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$



