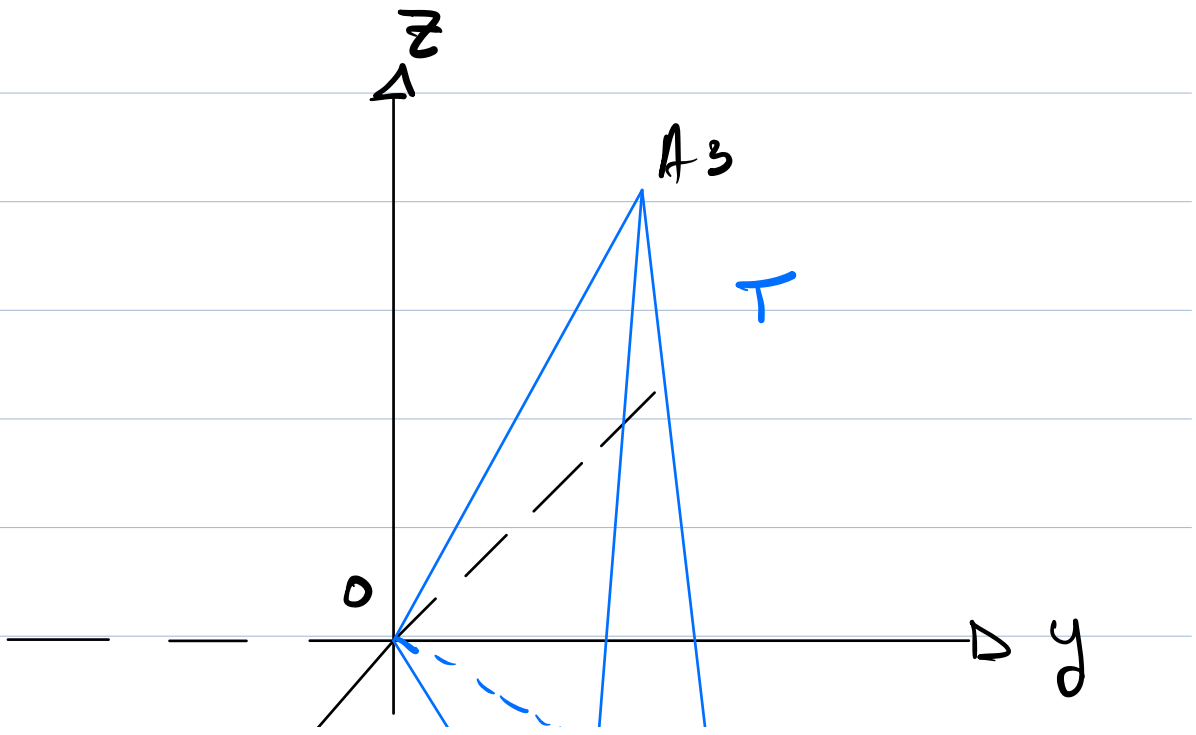
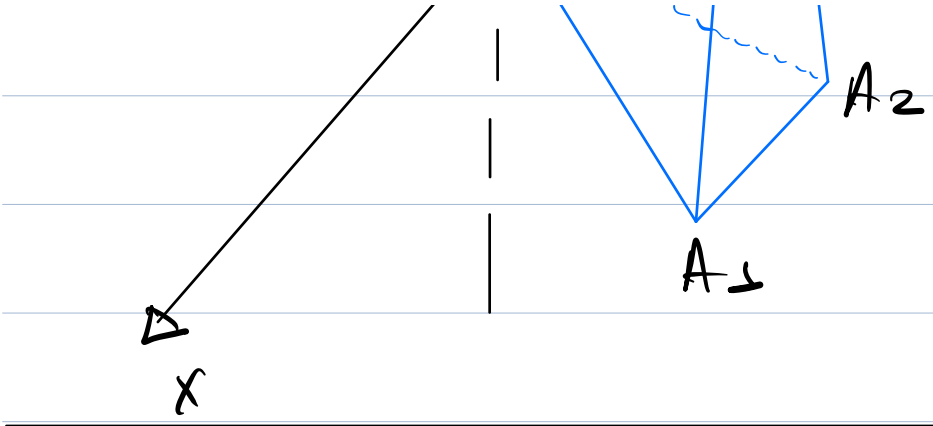
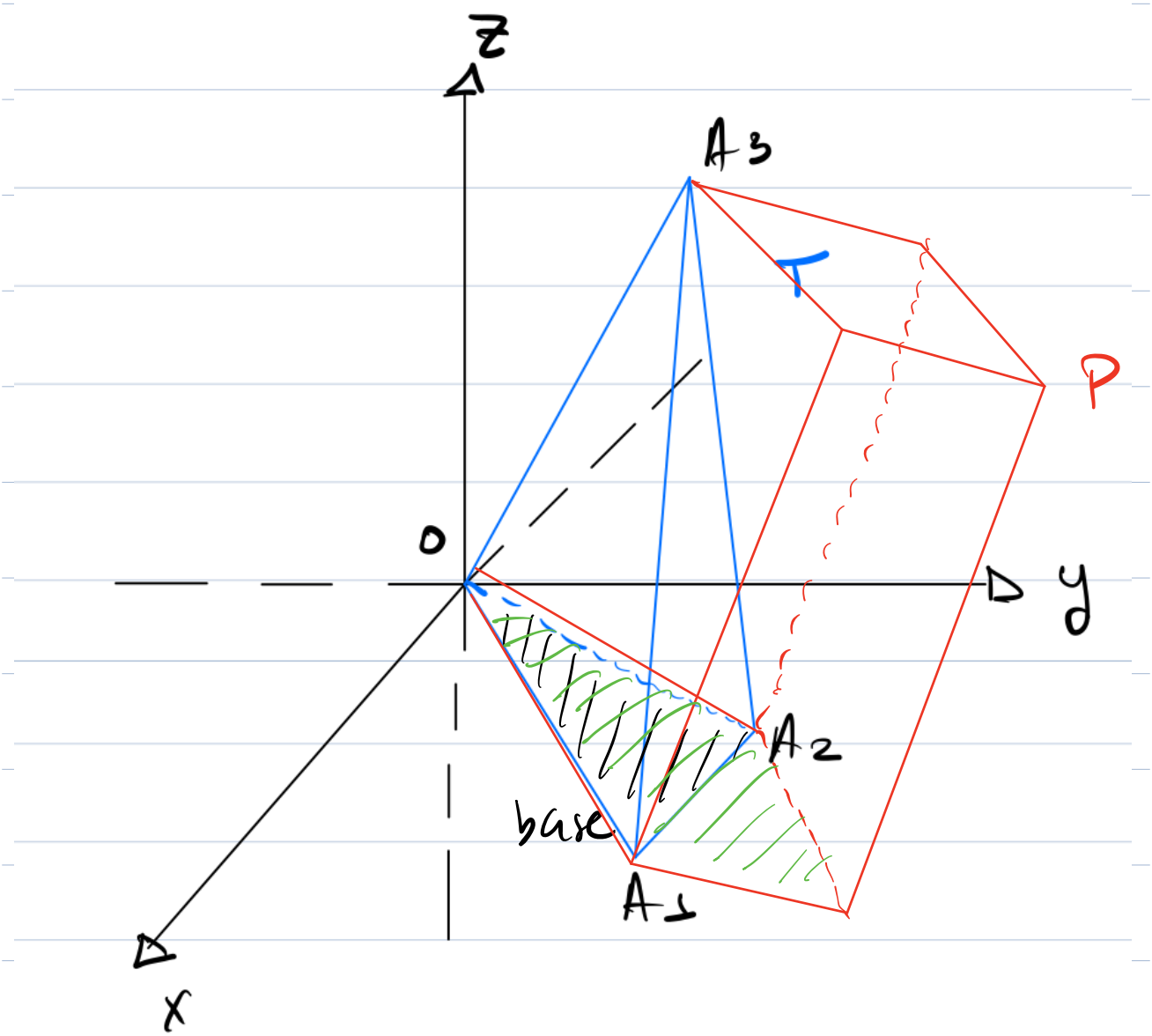
T is a tetrahedron with vertices $O(0,0,0)$, $A_1(2,2,-1)$, $A_2(1,3,0)$, $A_3(-1,1,4)$

1) Compute $Vol(T)$

2) If A_1, A_2, are fixed, but A_3 is moved to $A_3'(-201, -199, 104)$ Compute $Vol(T)$ again.




A2
A1
X


Z
A3
T
P
O
y
A2
base
A1
X

⇒ Volume of T

$$Vol(T) = \frac{1}{3} A(\Delta O A_1 A_2) \cdot h$$

⇒ Volume of P

$$Vol(P) = 2A(\Delta O A_1 A_2) \cdot h$$

∴

$$Vol(T) = \frac{1}{6} Vol(P)$$

⇒

$$Vol(P) = \left| \det \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & 0 \\ -1 & 1 & 4 \end{pmatrix} \right|$$

$= 12$

$\therefore \text{Vol}(T) = \frac{1}{6} \times 12 = \underline{2}$

2) $A_3 \to A_3'\ (-201, -199, 104)$

$$\text{Vol}(T)' = \frac{1}{6} \left| \det \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & 0 \\ -201 & -199 & 104 \end{pmatrix} \right|$$

$= 2$

$A_3' - A_3 = -100 A_1$