Exercises on column space & nullspa

Problem 6.1

(3.1 #30. Introduction to Linear Algebra)

Suppose S and T are 2 subspaces of vector space V

a) Definition:

The sum $S+T$ contains all $s+t$ of a vector s in S and a vector t in T.

Show that $S+T$ satisfies the requirements (addition and scalar multiplication) for a vector space.

$\Rightarrow$ Let s, s' be vectors in S

let s, t' be vectors in T

$(s+t) + (s'+t') = (s+s') + (t+t')$

$\therefore c(s+t) = cs + ct$

$\therefore$ Scaler multiplication ✓

(b) If S & T are lines in $\mathbb{R}^m$, what is the difference between $S+T$ and $S \cup T$?

That union contains all vectors from S & T or both.

Explain this statement:

The <u>span</u> of $S \cup T$ is $S+T$

$\Rightarrow$ If S & T are distinct lines then $S+T$ is a plane whereas $S \cup T$ is only the 2 lines.

→ The span of $S \cup T$ is the set of all combinations of vectors in this union, of 2 lines. In particular it contains all sums $s+t$ of a vector s in S and vector t in T, and these sums form $S+T$.

Since $S+T$ contains both S & T it contains $S \cup T$

Further $S+T$ is a vector space. So it contains all combinations of vectors in itself; in particular it contains the span of $S \cup T$

Thus the span of $S \cup T$ is $S+T$

Problem 6-2: (3.2 #18)

The plane $x-3y-z=12$ is parallel to the plane $x-3y-x=0$.
One particular point on this plane is $(12,0,0)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ 0 \\ 0 \end{bmatrix} + y\begin{bmatrix} \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow$ equation $x = 12+3y+z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 + 3y + z \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Problem 6.3: (3.2 #36)

How is the nullspace $N(C)$ related to the subspace $N(A)$ & $N(B)$ if $C = \begin{bmatrix} A \\ B \end{bmatrix}$

$\Rightarrow N(C) = N(A) \cap N(B)$

contains both subspaces

$$Cx = \begin{pmatrix} Ax \\ Bx \end{pmatrix} = 0$$

if Ax & $Bx = 0$.