Which are subspaces of

$$\mathbb{R}^3 = \left\{ \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} \right\}$$

1) $b_1 + b_2 - b_3 = 0$

2) $b_1 b_2 - b_3 = 0$

3) $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

4) $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

1

$b_1 + b_2 - b_3 = 0 \rightarrow$ <u>linear</u>

$\updownarrow$

$$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = 0$$

✓ subspace

2

$b_1 b_2 - b_3 = 0 \rightarrow$ <u>not</u> linear.

$$\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{2} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$\therefore b_1 b_2 \neq b_3$
in the second $\therefore$

no subspace ✗

③

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

<u>linear combination</u>

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \underbrace{\frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}$$

$$\Rightarrow \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \quad + c_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = (c_1 + 1/2) \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + (c_2 + 1/2) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(4)

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$c_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

No ✗

Always have a 1 in the second entry