Objectives:

- Four fundamental subspaces (for matrix A)

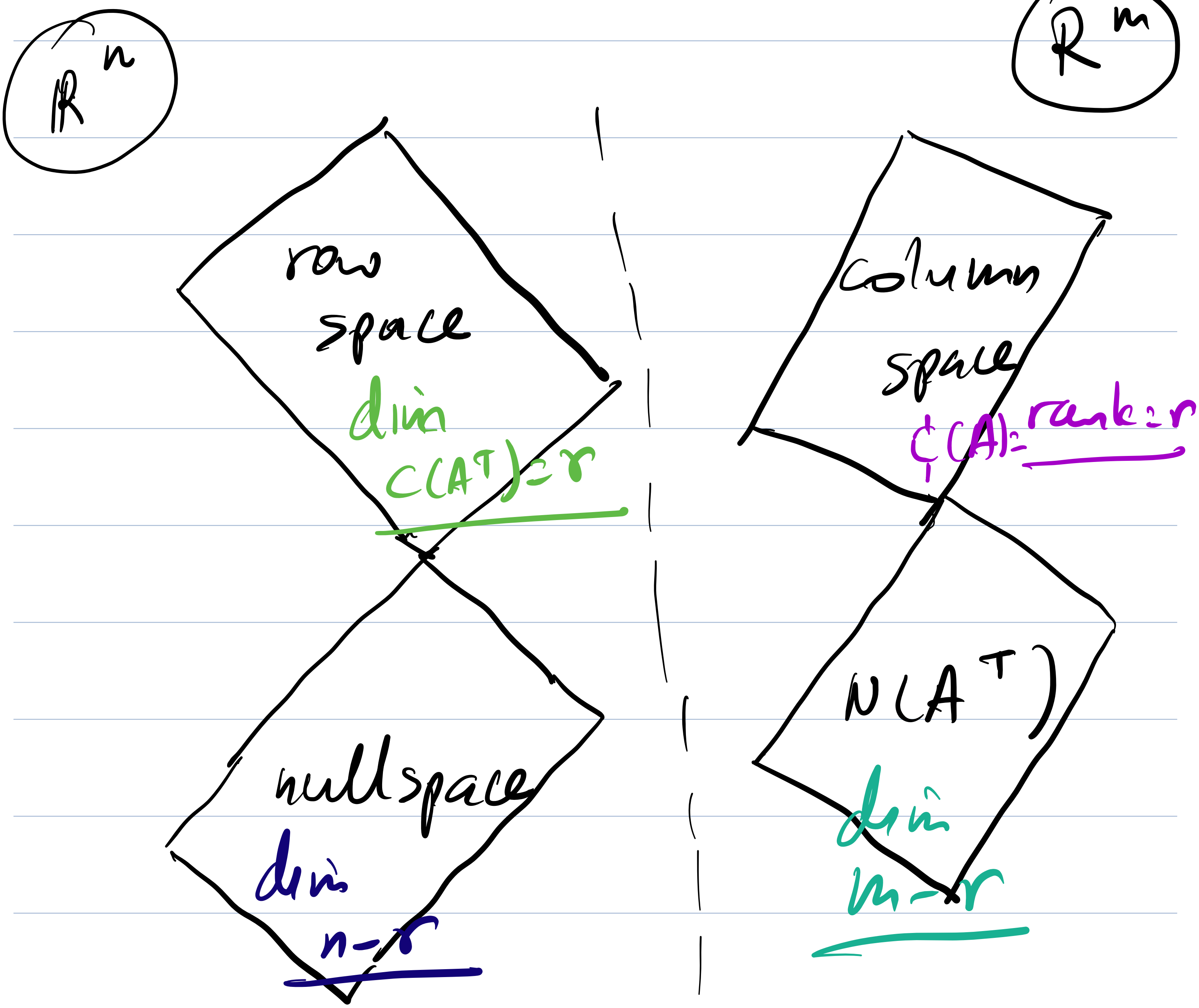
4 subspaces

- Column space $C(A)$ in $\mathbb{R}^m$
- Null space $N(A)$ in $\mathbb{R}^n$
- Row space: all combinations of rows.

  = all combs of the columns of A^T

  $\Rightarrow C(A^T)$ in $\mathbb{R}^n$
- Null space of A^T: $N(A^T) \subseteq \mathbb{R}^m$

[left null space of A]



	$C(A)$	$C(A^T)$	$N(A)$
basis?	pivot col		special solut.
dimension?	r	r	$n-r$

eg

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \boxed{1} & 2 & 3 & 1 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{array}{cc} \quad I & \qquad F \\ \multicolumn{2}{c}{\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \end{array} = R$$

basis

$C(R) \neq C(A)$ * different column spaces

* same row space.

* Basis for row space:
First r rows of R.

4^{th} space: $N(A^T)$

$\to A.$

$A^T y = 0$ $\qquad y^T A^{TT} = 0^T$

$$\begin{bmatrix} & \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$[y^T][A] = [0]$

left.

To find a basis for the left nullspace we reduce an augmented version of A:

$$\left[\begin{array}{cc} A_{m\times n} & I_{m\times n} \end{array} \right] \longrightarrow \left[\begin{array}{cc} R_{m\times n} & E_{m\times n} \end{array} \right].$$

From this we get the matrix E for which $EA = R$. (If A is a square, invertible matrix then $E = A^{-1}$.) In our example,

$$EA = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R.$$

New vector space (M)

All 3×3 matrices! $\Big\}$ $A+B$, cA

(not interested in AB)

Subspaces of M:

- all upper triangular matrices
- all symmetric matrices
- D, all diagonal matrices.

D is the intersection of the first two spaces. Its dimension is 3; one basis for D is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}.$$