Exercises on Markov matrices; Fourier series

Problem 24.1: (6.4 #7. *Introduction to Linear Algebra:* Strang)

a) Find a symmetric matrix $\begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix}$ that has a negative eigenvalue.

b) How do you know it must have a negative pivot?

c) How do you know it can't have two negative eigenvalues?

Problem 24.2: (6.4 #23.) Which of these classes of matrices do A and B belong to: invertible, orthogonal, projection, permutation, diagonalizable, Markov?

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad B = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Which of these factorizations are possible for A and B: LU, QR, $S\Lambda S^{-1}$, or $Q\Lambda Q^T$?

Problem 24.3: (8.3 #11.) Complete A to a Markov matrix and find the steady state eigenvector. When A is a symmetric Markov matrix, why is $\mathbf{x}_1 = (1, \ldots, 1)$ its steady state?

$$A = \begin{bmatrix} .7 & .1 & .2 \\ .1 & .6 & .3 \\ \text{---} & \text{---} & \text{---} \end{bmatrix}.$$