8.02x - Physics II - Electricity & Magnetism

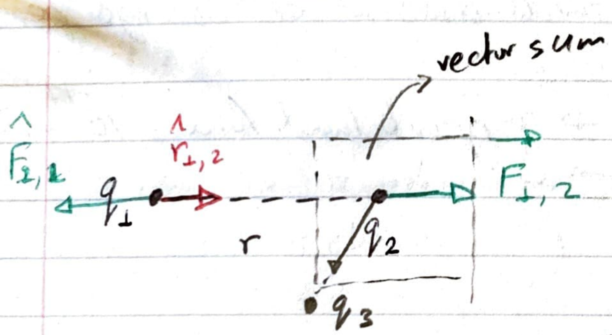
Lect 1:

- Electric Charges & Forces - Coulomb's Law - Polarization

- $m_p = m_n = 1.7 \times 10^{-27}$ kg.
- $m_e = 9.11 \times 10^{-31}$ kg.

* Characteristic of conductors → have free moving electrons
* —— // —— non-conductors → All electrons are fixed to atoms

2 charges (q_1, q_2)



$$\vec{F}_{1,2} = \frac{q_1 q_2}{r^2} k \, \hat{r}_{1,2}$$

unit vector $1 \to \text{to} \to 2$

↳ force of #2 due to 1

↳ (sign sensitive.)

$q_1 = q_2 = q_{p+} = q_{e-} \simeq 1.6 \times 10^{-19}$ C

$k = 9 \times 10^{9}$

Note: Colomb Force is similar (parallel) to how gravitational force equation.

$$k = \frac{1}{4\pi\varepsilon_0}$$

→ If we add a new charge then we use superposition principle

* Electric forces >> Gravitational forces

$$\frac{F_e}{F_g} \simeq 10^{36}$$

$P_1 \oplus \text{- - - -} \oplus P_2$, distance d

$$F_e = \frac{(1.6 \times 10^{-19})^2 (9 \times 10^9)}{d^2}$$

$$F_g = \frac{(1.7 \times 10^{-27})^2 (6.7 \times 10^{-11})}{d^2}$$

* $a_p = \frac{F_e}{m_p} \approx 10^{26}$ → than the gravitational force

(bring protons together) ↳ close to nucleus   * $d = 10^{-12}$ cm $= 10^{-14}$ m

∴ What holding the nucleus together?
↳ nuclear forces

* Since (d) cancells then why do we have different examples of forces (why do ~~planents~~ (planets) stay together from gravitational forces and not electrical force?

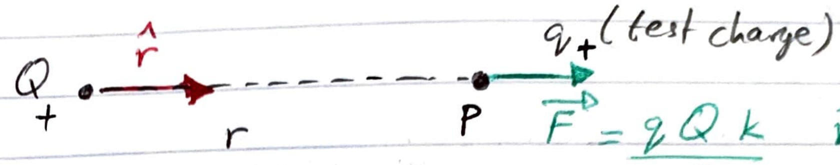
↳ Eventhough d cancells, most objects in the universe have a very small amount of charge per unit mass

eg Earth → 400×10^{3} C → } gravitational force $= 10^{17} \gg F_e$
Mars → 400×10^{3} C → }

Lect 2

- Electric Field Lines, Superposition, Inductive Charging, Induced Dipoles

* Electric Field.



$$\vec{F} = \frac{qQk}{r^2}\,\hat{r}$$

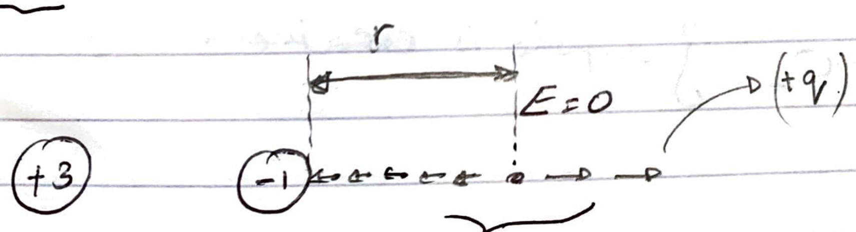
$$\vec{E_P} = \frac{\vec{F}}{q} = \frac{Qk}{r^2}\,\hat{r} \quad \left[\frac{N}{C}\right]$$

[Force experience by the test charge divided by the test charge ⇒ elliminate]

* Electric field is always in the direction that the Force is on a +ve test charge.

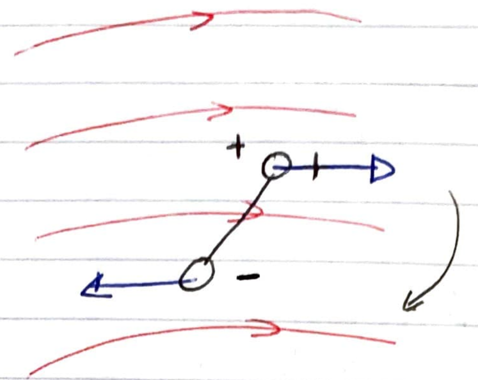
* Principle of superposition ⇒ $E_T = \sum_i \vec{E_i}$

Note:

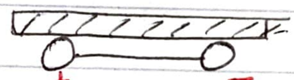


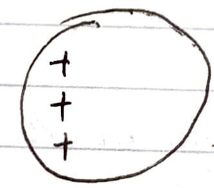
switches direction.

→ Finding distance r?



→ How do we charge the 2 circular contacts on the rod?





→ ventograph

* Electron will get closer to the (+)
hence

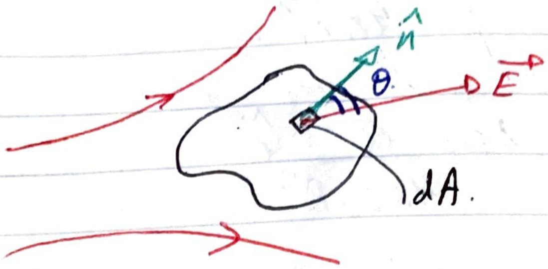
↳ Through induction → need to be connected through metal bar → in order for electrons to move

↳ When removing the bar

} dipole is created

Lect 3

- Electric Flux, Gauss' Law, Examples

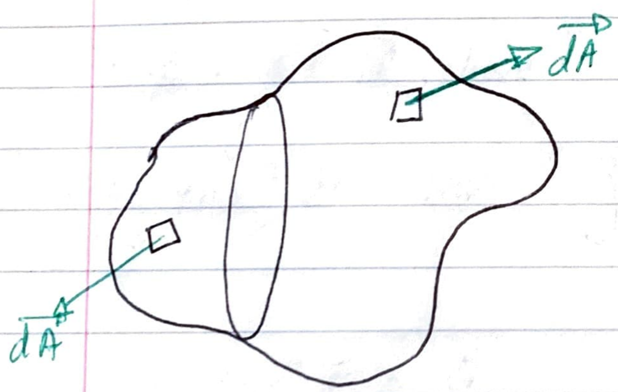


* open surface

$d\phi = \vec{E} \cdot \hat{n} dA$ [Electric Flux on surface dA]

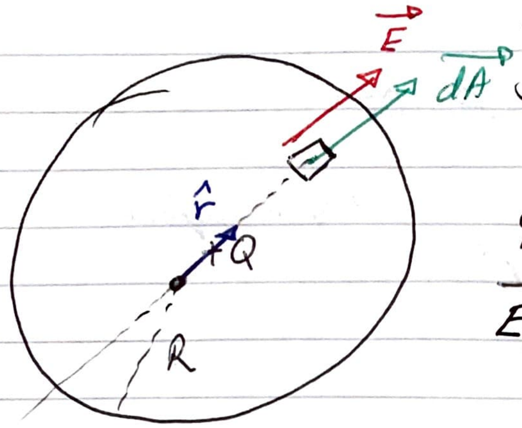
$= E dA \cos\theta$.

* Can calculate the flux over the whole area by doing an $\int$.



* close surface
* Normal ($\hat{n}$) always choosen from inside to outside.

$\phi = \oint \vec{E} \cdot \vec{dA}$



parallel $\therefore \cos\theta = 1$

$$\phi = 4\pi R^2 E$$

$$\vec{E}_R = \frac{Q}{4\pi \varepsilon_0 R^2}\,\hat{r}$$

$\therefore$

$$\Leftrightarrow \phi = \frac{Q}{\varepsilon_0}$$

* independent of distance R

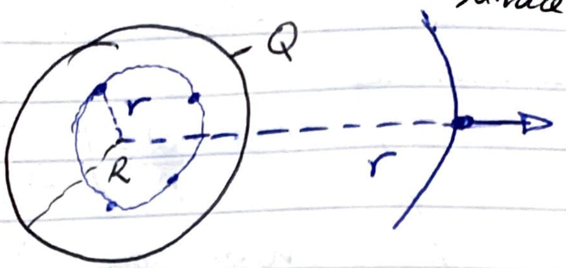
* Gauss's Law

- $\phi = \oint \vec{E} \cdot \vec{dA} = \frac{\sum Q_{ins}}{\varepsilon_0}$

closed surface

* Spherical symmetry

* thin surface. ⇒ charge Q uniformly distributed



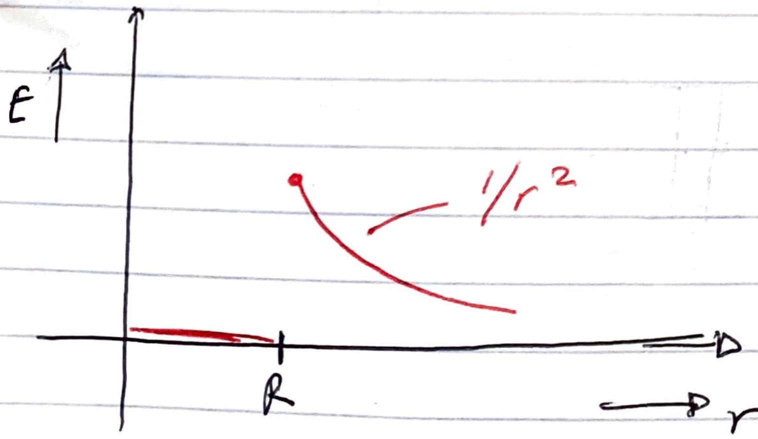
(S1) Need to choose Gauss surface.
↳ in this case sphere. • (electric field must be the same along Gauss surface) ① [symmetry argument]

② symmetry argument → (Electric field is either pointing radially inwards or outwards); since +ve charge → outwards

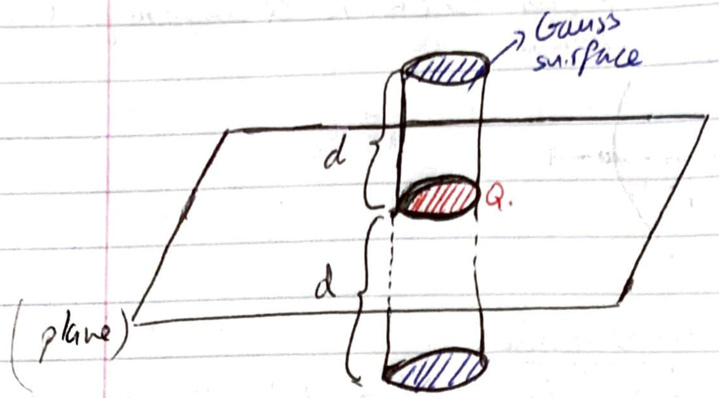
Surface Area of sphere

*> $\phi = 4\pi r^2 \cancel{E} = \frac{Q_{ins}}{\varepsilon_0}$ } No ~~q~~ Q inside ∴ $\underline{E = 0}$ $r < R$

*> $\phi = 4\pi r^2 E = \frac{Q_{ins}}{\varepsilon_0}$ ∴ No $Q_{ins} \neq 0$ ∴ $E = \frac{Q_{ins}}{4\pi r^2 \varepsilon_0}$ $r > R$



* Flat Horizontal plane symmetry

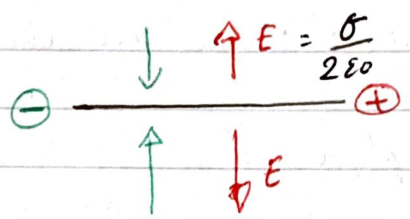


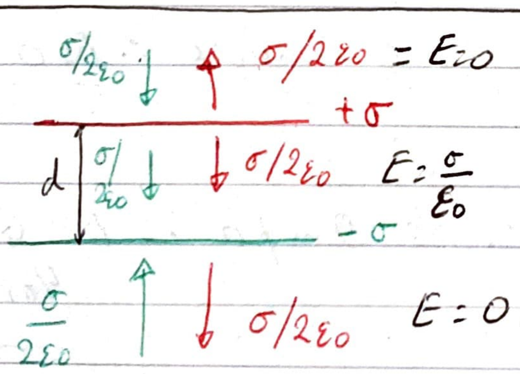
* infinite large plane
* $\sigma = \frac{Q}{A} \left[\frac{C}{m^2}\right]$

(charge density).

* Top & Bottom planes of Gauss surface <u>must</u> be parallel to plane
* Vertical walls of Gauss surface are perpendicular to plane
* 2 (d) must be the same

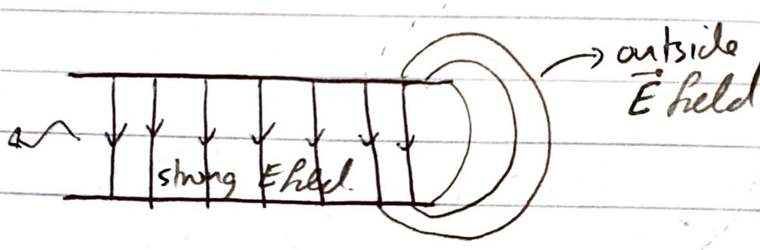
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$





What is the electric field anywhere in space?

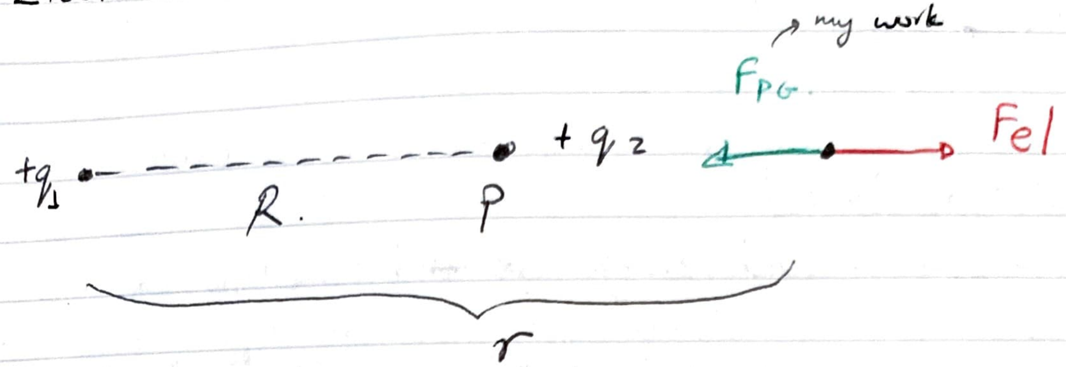
Electric Field.



Lect 4

- Electrostatic Potential, Electric Energy, Equipotential Surface

- Electrostatic Potential Energy (U)
- Electric Potential (V)



$$\left[-\frac{1}{r}\right]_R^{\infty}$$

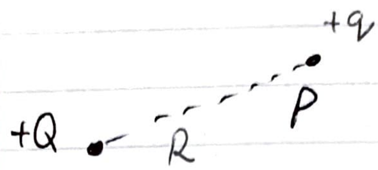
- $$U = W_{PG} = \underbrace{\int_{\infty}^{R} \vec{F}_{PG} \cdot \vec{dr} = \int_{R}^{\infty} F_{el} \cdot \vec{dr}}_{\text{forces are equal but opposite direction}} = \frac{q_1 q_2}{4\pi \varepsilon_0} \int_{R}^{\infty} \frac{dr}{r^2}$$

$\therefore\ U = \dfrac{q_1 q_2}{4\pi \varepsilon_0 R}$ [J] ⇒ scalar
(sign sensitive if different polarity)

* Conservative forces ⇒ work that has to be done from 1 point to another is independent of the path.

Electric Potential (V) → Work / unit charge

$$U = \frac{qQ}{4\pi\varepsilon_0 R} \quad [J]$$

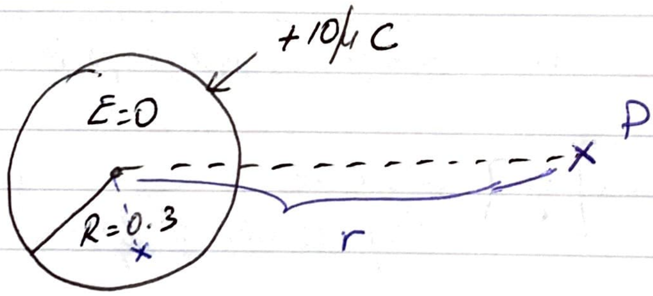


Electric Potential
↳ Work per unit charge $\infty \to P$

$\therefore$

$$V_P = \frac{Q}{4\pi\varepsilon_0 R} \quad [J/C] = [V]\text{olt} \quad (\text{can be both +ve, -ve})$$

$V_\infty = 0.$



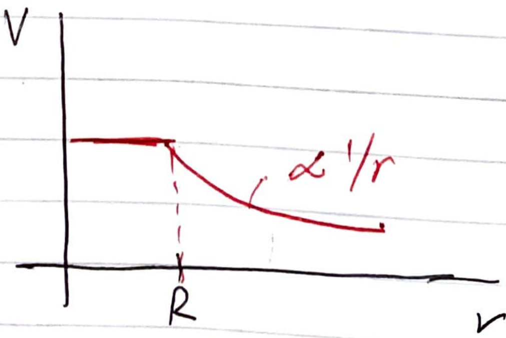
- $V_P = \int_r^\infty \frac{F_{el}}{q} \cdot d\vec{r} = \int_r^\infty \vec{E} \cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0 r} \quad r > R$

- $r = R = 3 \times 10^5\ V$
- $r = 0.6 = 150\ kV$
- $r = 3 = 30\ kV$

$\Big\}$ inversely proportional to r

$\Rightarrow r \to \infty = V = \phi$

- $W_{P\infty} = q \cdot V$



$+Q_1$

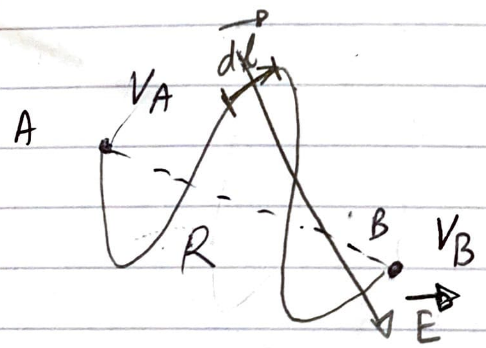
$\bullet$ P

$$V_{P_T} = V_{P\ Q_1 \text{ alone}} + V_{P\ Q_2 \text{ alone}}$$

$-Q_2$

* Equipotential's of different values can never intersect.

- +ve charges will move from high potential to low potential
- -ve charges will move from low potential to high potential.



$$V_A = \int_A^{\infty} \vec{E} \cdot \vec{dr}$$

$$V_B = \int_B^{\infty} \vec{E} \cdot \vec{dr}$$

$\therefore$ $V_A - V_B = \int_A^B \vec{E} \cdot \vec{dr}$ or $V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dr}$

Note: We can change $dr \to dl$ $\therefore$ ✓

eg $A = 150V$ $B = 50V$

$\therefore$

PG $+q$ pocket bobby $\to B$ $\to$ Do work $= W_{PG} = q \cdot V_B$
bobby $\to A$. $= 50 \times q$

$qV_A = 150q$

Δ Potential Energy ; $+q$ "$A \to B$" $\Rightarrow q(V_A - V_B) = \underbrace{K_B - K_A}_{\text{kinetic energy}}$.

• Since

$q(V_A - V_B) = K_B - K_A$

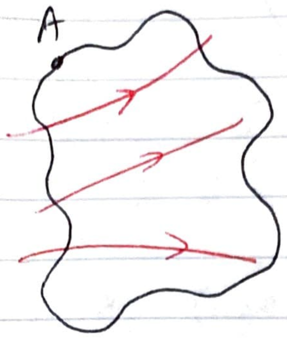
$(1.6 \times 10^{-19})(100) = \frac{1}{2} m_e v_A^2$

$(150-50)$

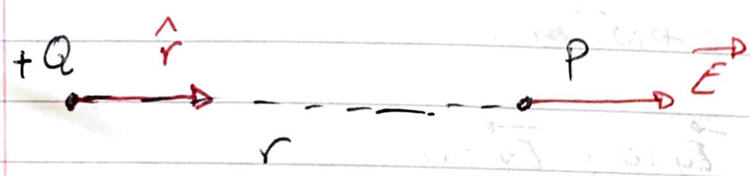
Lect 5

- $E = -\text{grad } V$, Conductors, Electrostatic Shielding (Faraday cage)

> $V_A - V_A = 0 = \oint_A^A \vec{E} \cdot d\vec{l}$ → Close line



(1) $\oint_{\text{closed line}} E \cdot dl = 0$



- $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

- $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

$\Rightarrow \dfrac{dV}{dr}\hat{r} = -\dfrac{Q}{4\pi\varepsilon_0 r^2}$ $\times\hat{r}$ → Turn them into vectors
↳ Because $\vec{E}$ vector

$\therefore \boxed{\vec{E} = -\dfrac{dV}{dr}\hat{r}}$

Note:

- $|E_x| = \left|\dfrac{\Delta V}{\Delta x}\right|_{yz}$ $\left[\dfrac{V}{m}\right]$ *Make a side step into 1 direction & measure potential across it.

- $E_y = \left|\dfrac{\Delta V}{\Delta y}\right|_{xz}$

In Cartesian coordinates

$$-\vec{E} = -\left(\underbrace{\frac{\partial V}{\partial x}}_{E_x}\hat{x} + \underbrace{\frac{\partial V}{\partial y}}_{E_y}\hat{y} + \underbrace{\frac{\partial V}{\partial z}}_{E_z}\hat{z}\right)$$

$\therefore$

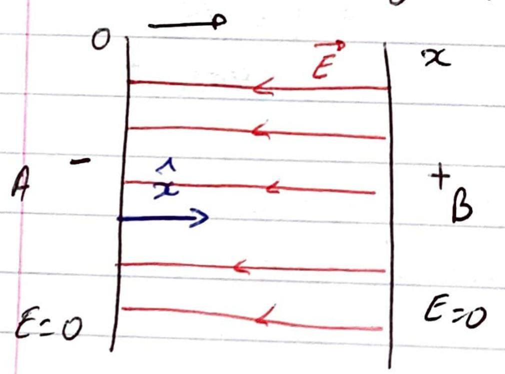
$\vec{E} = -$ gradient V

eg

$V = 10^5 x \qquad x = 0 \to 10^{-2}\,m$

$\therefore$

$\vec{E} = -10^5 \hat{x}\ ;\ \vec{E_y} = 0\ ;\ \vec{E_z} = 0$



* opposite direction of $\hat{x}$ because $(-)$

$|E| = \frac{\sigma}{\varepsilon_0} = 10^5$

$$\therefore V_A - V_B = \int_A^B \vec{E} \cdot d\vec{x}$$

$$= -10^5 \int_A^B \hat{x} \cdot d\vec{x}$$

$$= -10^5 \int_A^B dx = -10^5 \left[x_B - x_A\right]$$

$$\Rightarrow = -1000\,V \quad \left[-10^5 \times 10^{-2}\right]$$

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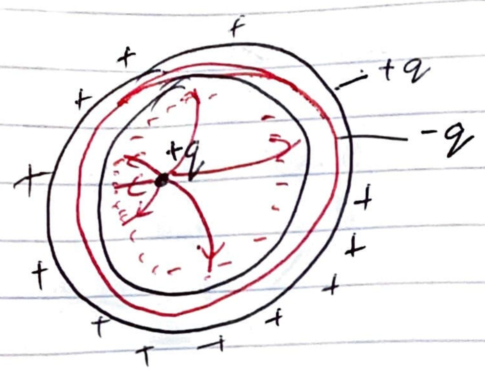


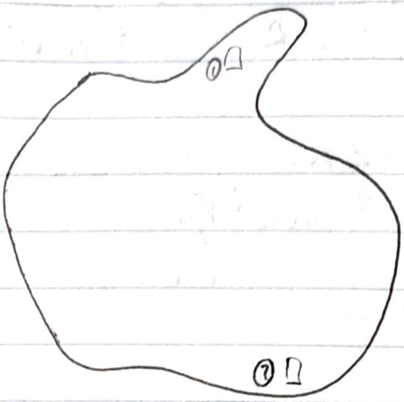
$\therefore$ the whole thing is equipotential
$\therefore E = \phi$ inside.

* Faradays Cage (Electrostatic Shielding)
eg car radios.

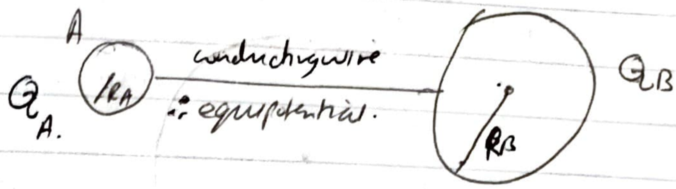
Lect 6

- High voltage Breakdown, Lightning, Sparks, St-Elmo's Fire

* Different σ. At ① & ②
$\therefore E \uparrow$ ①



* solid conductor

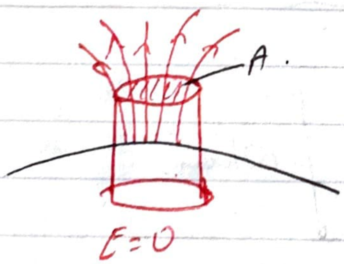


$$V_A = \frac{Q_A}{4\pi\varepsilon_0 R_A} = V_B = \frac{Q_B}{4\pi\varepsilon_0 R_B}$$

$\therefore$

$$\frac{Q_A}{R_A} = \frac{Q_B}{R_B}$$ $\therefore$ eg: $R_B = 5R_A$
$\therefore$ more charge.

$\Rightarrow \sigma = \frac{Q}{4\pi R^2}$; $Q_B = 5Q_A$; $\sigma_B = \frac{1}{5}\sigma_A$.



$AE = \frac{\sigma A}{\varepsilon_0}$ $\therefore$ $\boxed{E = \frac{\sigma}{\varepsilon_0}}$ conductor.

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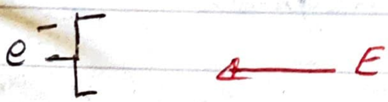
*



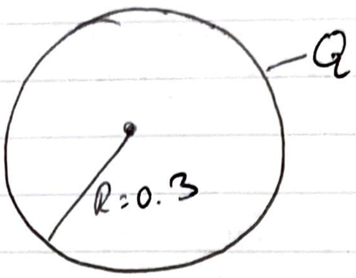
- electron will start to accelerate → (direction)
- The electron will start to collide with hydrogen & ~~nitrogen~~ nitrogen molecules in the air.

↳ If electron has enough KE to ionised that molecule.
$1e^- \to 2e^-$ (original + electrons from ion) → ∴ avalanche effect.
(⇒ get a spark. (when electron becomes neutral → emit light)

$O_2 = 12.5eV$; $N_2 = 15eV$ } required energy to ionised.

* $\text{current} = \frac{\text{charge}}{\text{time}} = \frac{C}{sec}$ [A]

* Ventograph



$V = \frac{Q}{4\pi\varepsilon_0 R}$; $E = \frac{Q}{4\pi\varepsilon_0 R^2}$

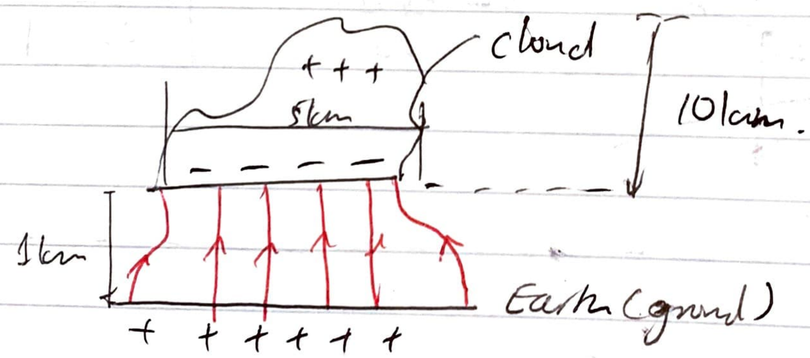
$\therefore V = ER.$

$\uparrow 3\times10^6$ V/m.

R	V
3mm	10 kV
3cm	100kV
30cm	1MV

*cannot exceed (breakdown)

$Q_{max} = 10\mu C$; $V_{max} = 3\times10^5 V.$

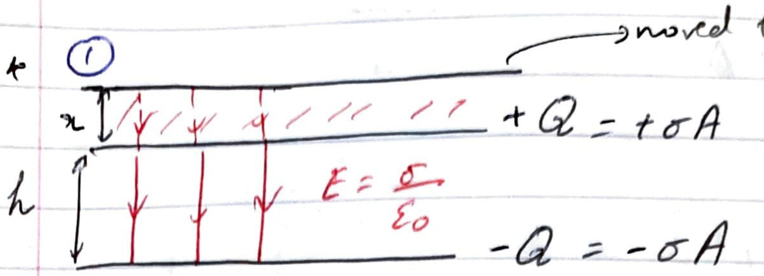


$\Delta V = E\cdot d. = 3\cdot10^6 \times 10^3 \simeq 3\times10^9 V.$

* Corona discharge
* Marsh air → Malhería story)
Napoleon

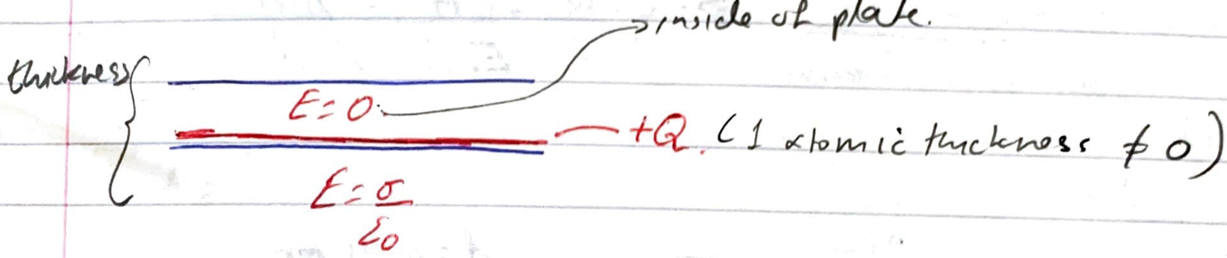
Lect 7 -

Capacitance, Electric Field Energy



$\therefore W_{PG} = F_{PG} \cdot x$

* Enlargement of upper plate ①



$\therefore$ Electric field is the average of those 2 in charge Q

$\therefore$ force $\left(\frac{1}{2}\right)$

$$\Rightarrow W_{PG} = \frac{1}{2} Q E \cdot x = \frac{1}{2} \sigma A E \cdot x \frac{\varepsilon_0}{\varepsilon_0} = \boxed{\frac{1}{2} \varepsilon_0 E^2 \underbrace{A x}_{\text{Volume}}}$$

$$\Rightarrow \frac{W_{PG}}{\text{volume}} = \frac{1}{2} \varepsilon_0 E^2 \; [J/m^3] \rightarrow \text{field energy density.}$$

- $U = \int_{\text{all space}} \frac{1}{2} \varepsilon_0 E^2 \, \underbrace{dV}_{\text{volume}}$

> $U = \int \frac{1}{2} \varepsilon_0 E^2 dV = \frac{1}{2} \varepsilon_0 \left(\frac{\sigma}{\varepsilon_0}\right)^2 Ah = \underline{\frac{1}{2} QV}$

$Q = A \cdot \sigma$ $\qquad = \underline{\frac{1}{2} CV^2}$

$V = E \cdot h$ → potential. Δ

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* $\underline{\text{Capacitance}} = \frac{Q}{V}$ [F]

- $C = \frac{\text{charge}}{\text{potential difference}}$ $\Rightarrow C = \frac{Q}{V} = \frac{\sigma A}{E \cdot d} = \frac{\cancel{\sigma} A \varepsilon_0}{\cancel{\sigma} d} = \underline{\frac{A \varepsilon_0}{d}}$

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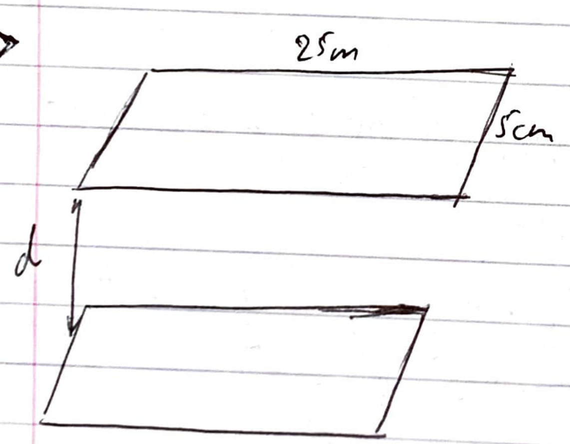


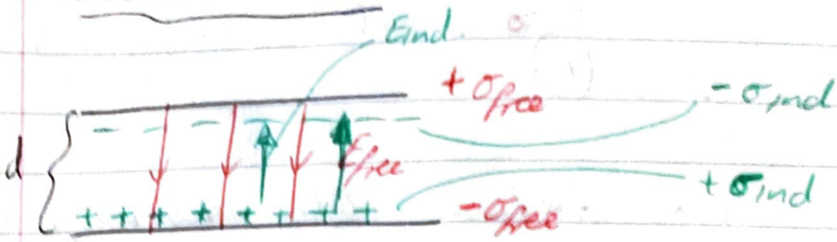
$d = 0.01$ mm.

$\therefore C = 1 \mu F$

Lect 8

- Polarization, Dielectrics, Van de Graaff Generator, Capacitors

* Dielectrics → Polarization



$E_{free} = \frac{\sigma_{free}}{\varepsilon_0}$; $E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}$

$\Rightarrow \vec{E}_{net} = \vec{E}_{free} + \vec{E}_{ind}$ $\qquad \sigma_{ind} = b\,\sigma_{free}$ $\therefore$ $\overset{\text{constant}}{b} < 1$

$\therefore$

$E_{net} = E_{free} - E_{ind}$ $\qquad E_i = b E_f$.

$E_{net} = E_{free}\,\underbrace{(1-b)}_{\to \frac{1}{K}}$ | K = dielectric constant

$$\boxed{\vec{E} = \frac{\vec{E}_{free}}{K}}$$

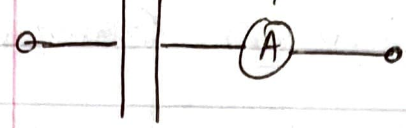
* Gauss's Law: $\oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} \sum Q_{ins,net}$

$Q_{net} = \overset{+}{Q_{free}} + \overset{-}{Q_{ind}}$

$(\Rightarrow) \oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} \frac{\sum Q_{free}}{K}$

(I) $d = 1mm$
$V = 1500V$

d
Ameter



* once charge → disconnect PS
* Ammeter show a short surge of current

** $d \to 7mm$. Qfree → no change cause trapped.

$V = 10kV$

E → no change
V↑ → ×7
[$V = E \cdot d$.

II : $V = 10kV$ $d = 7mm$.
* insert dielectric (K)

Qfree → no change (A) → nothing.
E ↓ K.
V ↓ K

↑ $C = \frac{Q_{free}}{V\downarrow}$ → by a factor of K

$\left[C = \frac{A \cdot \varepsilon_0}{d} K \right]$ → dielectric constant between the 2 plates

* $E = \frac{\sigma_{free}}{\varepsilon_0 K}$ (1)

* $V = E \cdot d$ (2)

* $C = \frac{Q_{free}}{V} = \frac{A \cdot \varepsilon_0}{d} K$. (3)

III

* Do not disconnect PS

* $V = 1500V$ / $d = 1mm$ (A)

$d = 7mm$

↳ V fixed ⇒ $E \downarrow (\times 7)$

$C \downarrow (\times 7)$

Q_{free} $(\times 7) \downarrow$ (A)

IV $V = 1500V$; $d = 7mm$

$K \Rightarrow 14$

(E) NOT CHANGE

$(V = E \cdot d)$

V no change → connected with PS (A)

$Q_{free} \uparrow K$

$C \uparrow K$

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* (Van De Graff.)

Lect 9

- Electric Currents, Resistivity, Conductivity, Ohm's Law

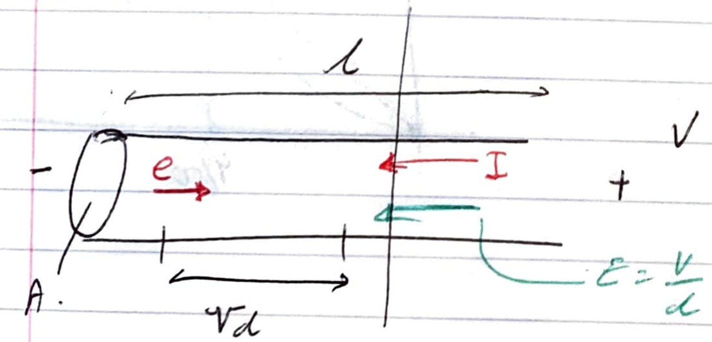
* Conductor (Cu) $\to$ 300 K ; $v_e > \approx 10^6$ m/sec.
time between collisions $\tau \approx 3\times10^{-14}$ sec
# of free electrons / m^3 $n \approx 10^{29}$.

$F = eE \to$ acceleration $= a = \frac{F}{m_e}$; $v_d = a\tau \Rightarrow \frac{eE}{m_e}\tau$. (drift velocity)

eg

Cu $l = 10m$, $\Delta V = 10V$ $\Big\}$ $E = 1\ \frac{V}{m}$

* $\therefore v_d = \frac{1.6\times10^{-19} \cdot 3\times10^{-14}}{10^{-30}} \approx 5\times10^{-3}$ m/s.



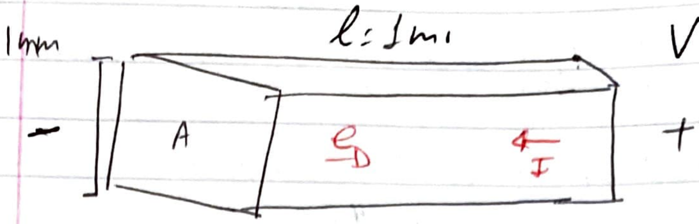
~~Volume~~ = $\boxed{v_d \cdot A n e = I}$

* $I = \frac{e^2 n\tau}{m_e} A\cdot E$. $\to \sigma$ = conductivity eg $\sigma_{Cu}(300K) \simeq 10^8$

$I = \sigma A \frac{V}{l}$; $\boxed{V = \frac{l}{\sigma A} I}$ [Ohms Law]

$\frac{l}{\sigma A} \to R$

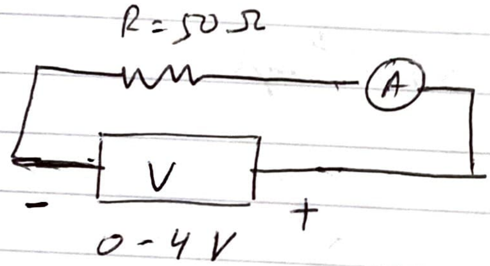
$\rho = \frac{1}{\sigma}$ (resistivity) $V = IR$ $R = \frac{l}{\sigma A} = \frac{l\rho}{A}$. *

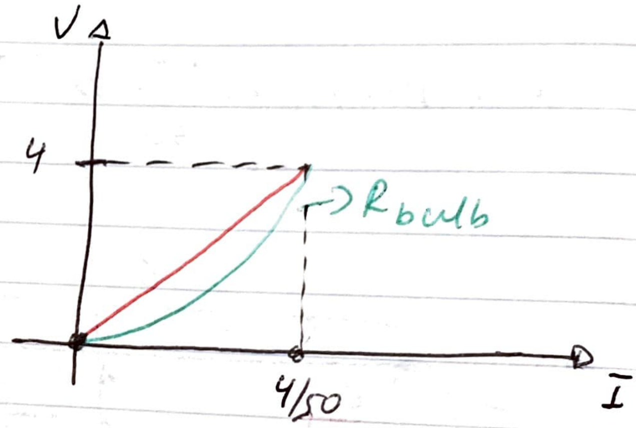


* good conductors σ
eg Au / Ag / Cu ≈ 10^8
∴ $\rho = 10^{-8}$
($R = 10^{-2}\,\Omega$) $I = 100A$

$A = 10^{-6}$
$R = 10^{6}\rho$

* good insulators σ
eg glass ≈ $10^{-12} - 10^{-16}$
$\rho = 10^{12} - 10^{16}$
($R = 10^{20}\,\Omega$) $I = 10^{-20}A$

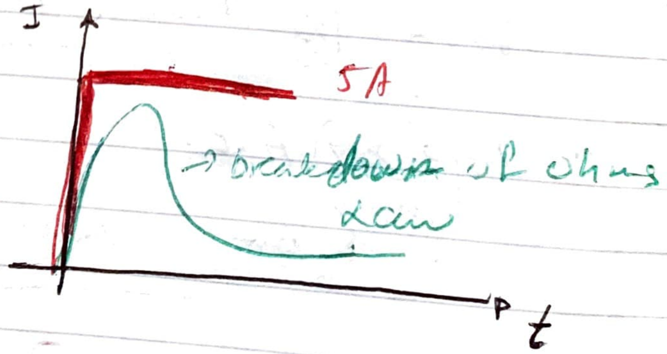


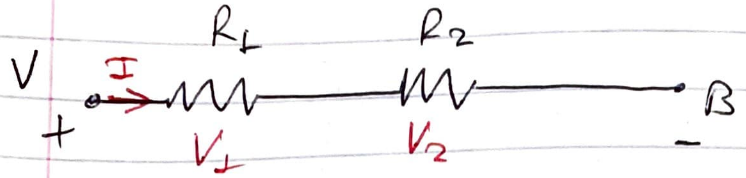


$V = IR(T)$

$\frac{V}{I} \neq C(I)$

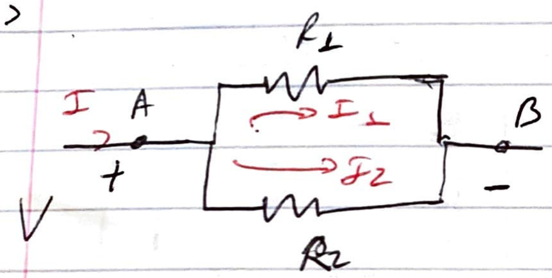
> $V = 125V$; $R_{cold} = 25\Omega$
$R_{hot} = 250\Omega$





$V = I(R_1 + R_2)$

$V_1 = IR_1$; $V_2 = IR_2$



$V = I_1 R_1$

$V = I_2 R_2$

$I = I_1 + I_2$