Find the LU-decomposition of the matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{bmatrix}$$

when it exists.
For which real numbers a and b does it exist?

<u>Solution:</u>

(S1) Elimination
(S2) Keep track of Elimination matrices

$\therefore$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} (\times a) \qquad \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} (\times b)$$

$$\begin{bmatrix} \boxed{a} & a & a \\ b & b & a \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & a & 0 \\ \boxed{b} & b & a \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & \boxed{a} & 0 \\ 0 & \boxed{b} & a-b \end{bmatrix}$$

pivot

elimnate

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -b & 0 & 1 \end{bmatrix}$$

$\rightarrow$ $\boxed{\text{assume that } a \neq 0}$

*cannot do row exchange.

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & 0 & a-b \end{bmatrix}$$

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{b}{a} & 1 \end{pmatrix}$$

$\therefore$

$$\left[\begin{matrix} 1 & 0 & 1 \end{matrix} \right.$$

$$\boxed{U = \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & a-b \end{bmatrix}}$$

We started with

$$\underbrace{E_{32}E_{31}E_{21}}A = U$$

Need to move them to the other side to get L.

$$\Rightarrow A = \underbrace{\left(E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} \right)}_{L} U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & b/a & 1 \end{bmatrix}$$

$$\boxed{L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & b/a & 1 \end{bmatrix}}$$

$\boxed{\therefore \text{It exist when } a \neq 0}$ ②