The vector space of all polynomials in x of degree ≤ 2 has a basis $1, x, x^2$.

Let w_1, w_2, w_3 be a different basis of polynomials whose values at $x = -1, 0, 1$ are given by:

x	w_1	w_2	w_3	y
-1	1	0	0	6
0	0	1	0	5
1	0	0	1	4

a) Express $y(x) = -x+5$ in the basis!

b) Find the change of basis matrices

$(1, x, x^2) \rightleftarrows (w_1, w_2, w_3)$

c) Find the matrix of "taking

"derivatives" in both bases.

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a) $y(x) = \alpha \cdot w_1(x) + \beta \cdot w_2(x) + \gamma \cdot w_3(x)$ / $x=-1$, $x=0$, $x=1$

$y(-1) = \alpha \cdot w_1(-1) + \beta w_2(-1) + \gamma w_3(-1)$

$\ldots(0) = \ldots$

$\ldots(1) = \ldots$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ b \\ \gamma \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

$y = 6w_1 + 5w_2 + 4w_3.$

b) $1 = w_1 + w_2 + w_3$

$x = -w_1 + w_3$

$x^2 = w_1 + w_3$

$$A = \begin{array}{c c} & \begin{array}{ccc} 1 & x & x^2 \end{array} \\ & \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{array}{c} w_1 \\ w_2 \\ w_3 \end{array} \end{array}$$

$$(1, x, x^2) \to (w_1, w_2, w_3)$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}$$

$$(w_1, w_2, w_3) \to (1, x, x^2)$$

$$\left(1', \ x', \ x^{2}{}' \right.$$

(c) $$D_x = \begin{array}{cc} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ x \\ x^2 \end{array} \end{array}$$

$$D_\omega = A \cdot D \cdot A^{-1}$$

$$= \begin{bmatrix} -3/2 & 2 & -1/2 \\ -1/2 & 0 & 1/2 \\ 1/2 & -2 & 3/2 \end{bmatrix}$$