(Linear space & Linear subspace)

Problem:

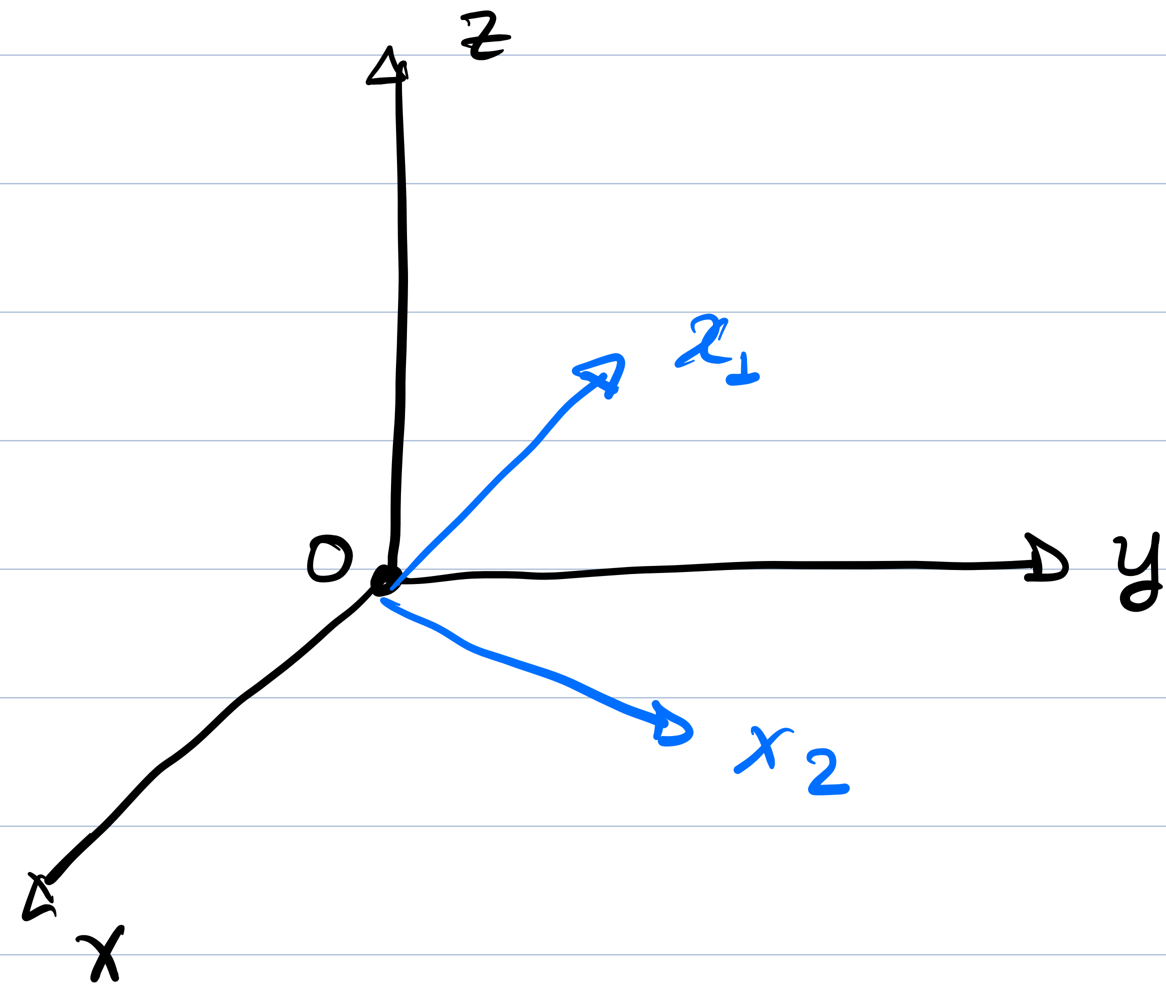
Reminder:

Conditions of a set to belong to linear space?

① You take the sum of any 2 elements from that set → needs to be still in the same set

② Any multiple of any element from the set → still needs to be in the same set

$$x_1 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$



1. Find V_1 = subspace generated by x_1

V_2 = subspace generated by x_2

looking for the smallest

Describe $V_1 \cap V_2$

subspace that contains x_1 / x_2

2. Find V_3 = subspace generated by $\{x_1, x_2\}$

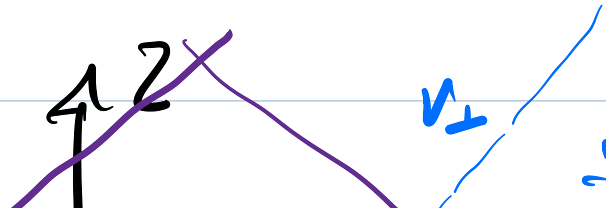
Is V_3 equal to $V_1 \cup V_2$?

Find a subspace S of V_3 s.t. $x_1 \notin S$, $x_2 \notin S$

3. What is $V_3 \cap \{x\text{-}y \text{ plane}\}$?

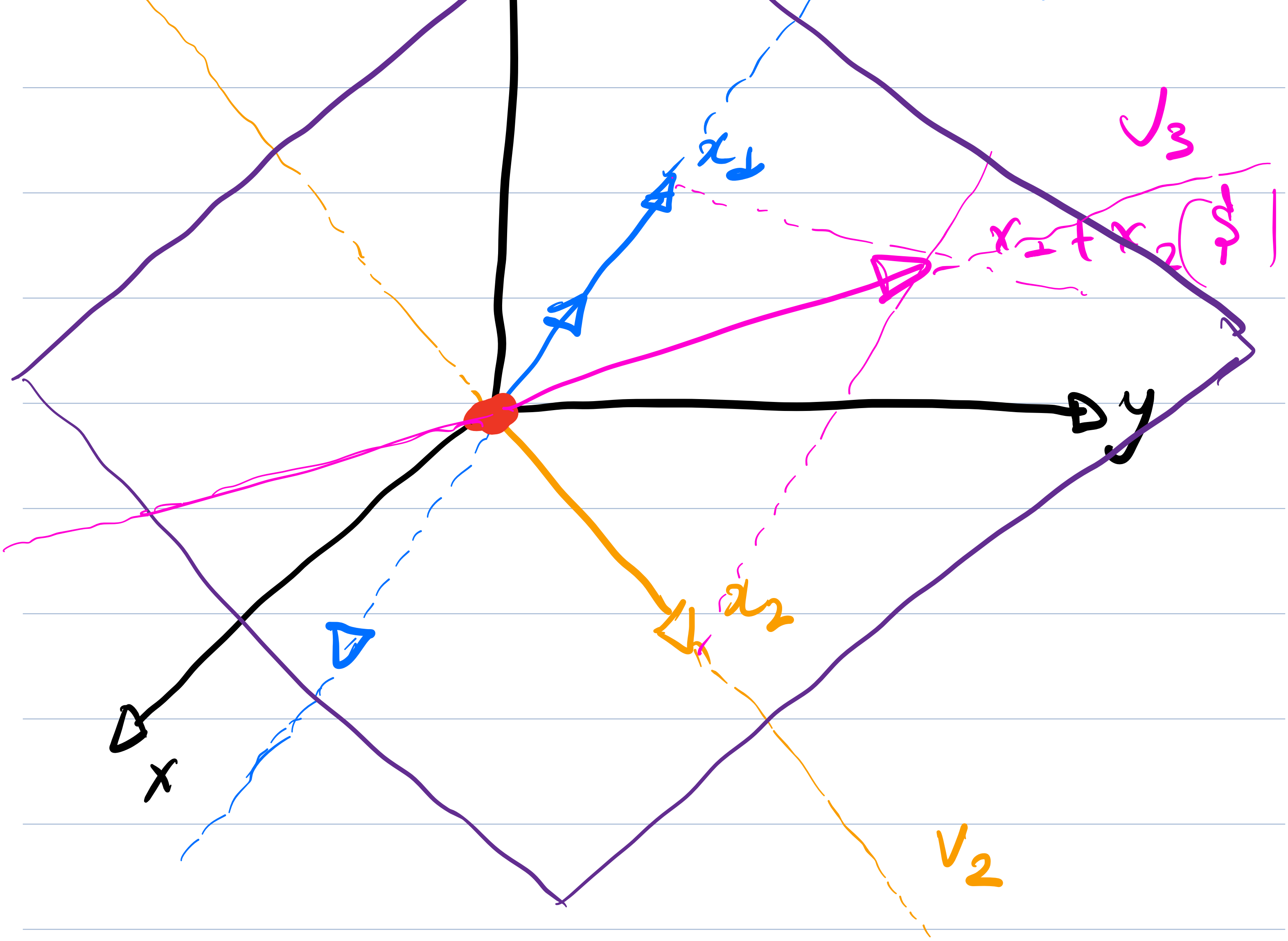
Solution:

①



* Satisfy both conditions

$\therefore V_1$



$\Rightarrow V_1 \cap V_2 = \{ 0 \}$ // a subspace of $\mathbb{R}^3$

2.

$x_1 + x_2 = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \Bigg\} \notin V_1 \cup V_2$

∴ Not possible that is in the subspace.

3. $V_3 \to$ plane ∴ intersection of 2 planes in R^3 (x-y & $V3$)

∴ $\to$ straight line.

$Z = \{0\}$

$\Rightarrow$ $V_3 \cap \{x\text{-}y \text{ plane}\} = V_2$