Problem 22.1 (6.2 #6)

$$A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$$

- To find eigenvectors we need to first find eigenvalues

$\therefore$

$$\det(A - \lambda I) = \begin{bmatrix} 4-\lambda & 0 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (4-\lambda)(2-\lambda) - 0 = 0$$

$$\lambda_1 = 4 \qquad \lambda_2 = 2$$

$\therefore$

Eigenvectors: Solve $(A - \lambda I)x = 0$

λ_1

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \Rightarrow y = 2z.$$

$\therefore$ eigenvector is any multiple of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

λ_2

$$(A - \lambda_2 I)x = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \Rightarrow y = 0$$

z = free variable

$\therefore$ eigenvector is any multiple of

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore$ Therefore the columns of the matrices S that diagonalize A, are non zero multiples of $(2,1)$ & $(1,0)$

Since $A^{-1} = S\Lambda^{-1}S^{-1}$,
the same S will diagonalize A^{-1}

Problem 22.2 *
(6.2 # 16).

$$A = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix}$$

each of the columns sum to 1
$\therefore$ Markov matrix.

$\therefore$ eigenvalue $\Rightarrow \lambda_1 = 1$.

Trace $= .7$ so other eigenvalue is

$\lambda_2 = .7 - 1 = -.3$

$\Rightarrow$ Corresponding eigenvectors

$$(A - \lambda_1 I)x_1 = \begin{bmatrix} -.4 & .9 \\ .4 & -.9 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = (9, 4)$

$$(A - \lambda_2 I)x_2 = \begin{bmatrix} .9 & .9 \\ .4 & .4 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow y = -z \Rightarrow x_2 = (1, -1)$

Putting together:

$S = \begin{bmatrix} 9 & 1 \\ 4 & -1 \end{bmatrix}$ and $\Lambda = \begin{bmatrix} 1 & \\ & -3 \end{bmatrix}$

As $k \to \infty$, $\Lambda^k \to \begin{bmatrix} 1 & \\ & 0 \end{bmatrix}$

so

$$S \Lambda^{\infty} S^{-1} \to \begin{bmatrix} 9 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & \\ & 0 \end{bmatrix} (1/13) \begin{bmatrix} 1 & 1 \\ 4 & -9 \end{bmatrix} = \frac{1}{13} \underbrace{\begin{bmatrix} 9 & 9 \\ 4 & 4 \end{bmatrix}}_{\text{steady state vector.}}$$