Find the singular value decomposition of the matrix:

$$G = \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$$

→ we want:

$$G = U \Sigma V^T$$

- $C^T C = V \Sigma^T \Sigma V^T$
- $CV = U\Sigma$

$$\therefore C^T C = \begin{bmatrix} 5 & -1 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 26 & 18 \\ 18 & 74 \end{bmatrix}$$

$$\det(C^TC-\lambda I) = \det\begin{pmatrix} 26-\lambda & 18 \\ 18 & 74-\lambda \end{pmatrix}$$

$$= \lambda^2 - 100\lambda + 1600$$

$$= (\lambda-20)(\lambda-80)$$

eigenvectors:

$$C^TC - 20I = \begin{pmatrix} 6 & 18 \\ 18 & 54 \end{pmatrix}$$

$$v_1 = \begin{pmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{pmatrix}$$

$$C^TC - 80I = \begin{pmatrix} -54 & 18 \\ 18 & -6 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{pmatrix}$$

$\Leftrightarrow$

$$V = \begin{pmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 2\sqrt{5} & 0 \\ 0 & 4\sqrt{5} \end{pmatrix}$$

- $CV = U\Sigma$

$$\begin{pmatrix} 5 & 5 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{pmatrix}$$

$$= \begin{pmatrix} -\sqrt{10} & 2\sqrt{10} \\ \sqrt{10} & 2\sqrt{10} \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}}_{\underline{\underline{U}}} \begin{pmatrix} 2\sqrt{5} & \\ & 4\sqrt{5} \end{pmatrix}$$