Objecties jergenvalues
- Deferminants (det A)
- Properties 1,2,3, 4-10

† Sighs-

- Dekrminant

Loss a number associated with any square matrix.

[det A or [AI]

Sthe dekrimmant encodes a lot of inhormation about the Matrix

Logest her invertability

when the determinant is non-zero

· We already know that

[ab] = ad-bc (determnent)

[cd]

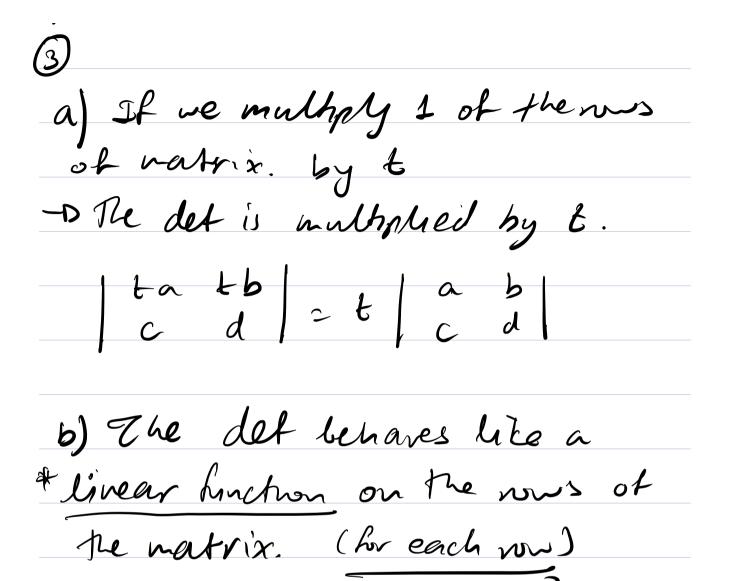
- Properties

O det I = 1

2) Exchange rows: Pererse tre sign of the determinant.

(eg)

fran Prop: 1



4) It has equal rows -sdet=0 Los because properly (2), exchange whe

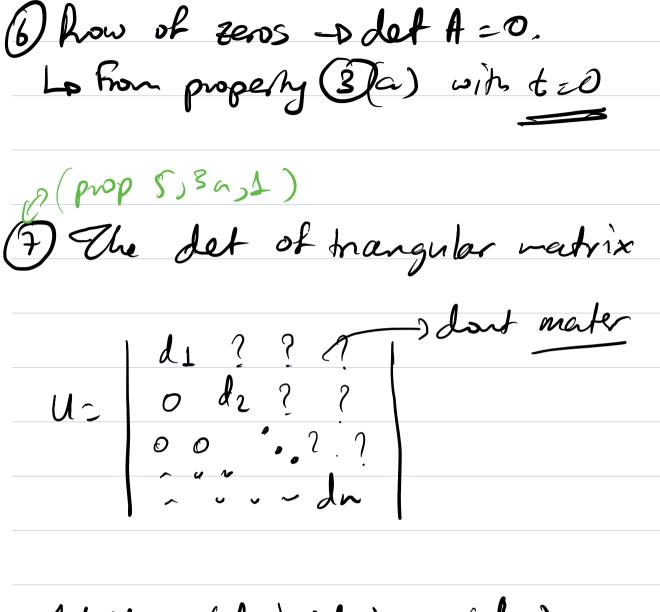
## seve matrix

Form elementers For Subhact (Lx rowi) hom (rowk)

Det does NOT change. &

2 a b | -la - lb |

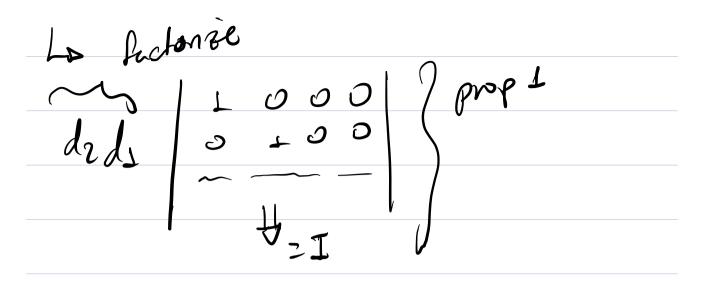
 $= \begin{vmatrix} a & b \\ c & d \end{vmatrix} - l \begin{vmatrix} a & b \\ a & b \end{vmatrix} = p$ 



det u = (d<sub>1</sub>) (d<sub>2</sub>) - · · (d<sub>n</sub>)

-s product of prots

Note that we cannot use elimination to get a diagonal matrix if one of the  $d_i$  is zero. In that case elimination will give us a row of zeros and property 6 gives us the conclusion we want.



8) det A exactly when A is singular

Dise elemination to get a now of zeros

Lo prop 6 HD det =0

Delimination produces a hill set of private de, de, de, de.

Lodet \$0

& Mote: If A is sincular A-1 does

not exist and det A-1 is underved.

det A<sup>2</sup> = (det A)<sup>2</sup>

det 2A = 2<sup>n</sup> det A.

? => factorriy Prictor 2

out of every ow.

(all rows -...)

Lo Spunder to releve.

If we doubte the length, width and height of 3D box, we increase its volume by multiplie of  $2^3$ -8

(10) det A = det A

1 a b | 1 a c |

## | c d | 2 | b d |

We have one loose end to worry about. Rule 2 told us that a row exchange changes the sign of the determinant. If it's possible to do seven row exchanges and get the same matrix you would by doing ten row exchanges, then we could prove that the determinant equals its negative. To complete the proof that the determinant is well defined by properties 1, 2 and 3 we'd need to show that the result of an odd number of row exchanges (odd permutation) can never be the same as the result of an even number of row exchanges (even permutation).