Problem 1.1

$$x_1 w_1 + x_2 w_2 + x_3 w_3$$

$$w_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

to give $\emptyset$ vector :

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} &① \; x_1 + 4x_2 + 7x_3 = 0 \\ &② \; 2x_1 + 5x_2 + 8x_3 = 0 \\ &③ \; 3x_1 + 6x_2 + 9x_3 = 0 \end{aligned}\right\} \text{simultaneous equations.}$$

1. $2x_1 + 5x_2 + 8x_3 = 0 \quad \times 1$

$x_1 + 4x_2 + 7x_3 = 0 \quad \times 2$

$$\Rightarrow \begin{array}{r} \cancel{2x_1} + 5x_2 + 8x_3 = 0 \\ \cancel{2x_1} + 8x_2 + 14x_3 = 0 \\ \hline -3x_2 - 6x_3 = 0 \end{array} \quad -$$

$-3x_2 = 6x_3$

$x_2 = -2x_3$

Take 2 & 3

$x_1 + 4x_2 + 7x_3 = 0 \quad \times 3$

$3x_1 + 6x_2 + 9x_3 = 0 \quad \times 1$

$$\begin{array}{r} \cancel{3x_1} + 12x_2 + 21x_3 = 0 \\ \cancel{3x_1} + 6x_2 + 9x_3 = 0 \\ \hline \end{array} \quad -$$

$6x_2 + 12x_3 = 0 \rightsquigarrow -6x_2 - 12x_3 = 0$

$\therefore$ <u>Note</u> :

$$-3x_2 - 6x_3 = 0$$
$$-6x_2 - 12x_3 = 0$$

$\therefore$ we can guess that $x_2 = -2$, $x_3 = 1$ & $x_1 = 1$

$\Rightarrow x_1 = 1$, $x_2 = -2$, $x_3 = 1$
& $w_1 + 2w_2 + w_3 = 0$

$\Rightarrow$ Vectors are <u>dependent</u> because there is a combination of the vectors that gives the zero vector

The 3 vectors lie in a plane

The matrix W with those column
is not invertible

Problem 1.2

Multiply:

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 9 \\ 11 \end{bmatrix}$$

Problem 1.3

A 3×2 matrix A times a 2×3 matrix B $\Rightarrow$ equals to a 3×3 AB?

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \times \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \underline{\text{True}}$$

$3 \times 2 \qquad 2 \times 3$

In order to multiply 2 matrices, the <u>number of columns</u> of A must be equal to the number of <u>rows of B</u>

The product AB will have the same number of rows as the first matrix & the same number of column as the second.

A(m by n) times B(n by p) equals to AB (m × p)