Given $A = [1 \quad 2]$

i) what is A^+ (pseudoinverse)

ii) AA^+ and A^+A

iii) If x is in $N(A)$ what is A^+Ax?

iv) If x is in $C(A^T)$ what is A^+Ax?

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i) Using SVD

$$A = U \Sigma V^T$$

$$[1\times1 = 1\times1 \quad 1\times2 \quad 2\times2]$$

$$U = [1]$$

$$A^T A = V(\Sigma^T \Sigma) V^T$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} (1 \quad 2) = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

$$\lambda_1 = 0 \rightarrow \sigma = 0$$

$$\lambda = 5$$

$$\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} u = 0$$

$$u = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\lambda = 0$$

$$u = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$A = \left[1 \right] \left[\sqrt{5} \quad 0 \right] \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ \ldots & \ldots \end{bmatrix}^T$$

$$\cdots \quad \underset{\Sigma}{\uparrow} \qquad \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

$$\underset{V}{\uparrow}$$

<u>Note</u>:

$$A^+ = V \Sigma^{-1} U^T$$

$$= V \begin{pmatrix} 1/\sqrt{5} \\ 0 \end{pmatrix} U^T$$

$$= \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{5} \\ 0 \end{pmatrix}$$

$$A^+ = \frac{1}{5} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

ii) $AA^+ = (1 \quad 2) \; \frac{1}{\ } \; (1 \ \ 1$

$\frac{\ }{5}\begin{pmatrix} \\ 2 \end{pmatrix}$

$= 1$

$$A^+A = \frac{1}{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

iii) $N(A) = c\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

ie $x = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$$A^+ \underbrace{A\,x}_{= 0} = 0$$

iv) $C(A^T) = c\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\underbrace{\frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}}_{A^+A} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = x$$

$$\underline{\underline{A^+ A x = x}}$$