Section: _____ Name:_____

Read the following directions carefully. DO NOT turn to the next page until the exam has started.

Write your name and section number at the top right of this page:

Class	**Section Number**
Bret 8:50	001
Sahifa 1:20	003
Miranda 9:55	004
Sahifa 3:30	005
Sahifa 8:50	006
Cameron 1:20	007

As you complete the exam, write your initials at the top right of each other page.

When the exam start time is called, you may turn the page and begin your exam.
If you need more room, there is a blank page at the end of the exam, or we can give you some scratch paper.

Some multiple choice questions are "Select ONE" while others are "Select ALL that apply". Pay attention to the question type and only mark one option if it says "Select ONE". Fill in the circles completely.

If you finish early, you can hand your exam to your instructor or TA and leave early.

Otherwise, stop writing and hand your exam to your instructor or TA when the exam stop time is called.

1. The number of credits that a UW-Madison course is worth follows this distribution, with some numbers removed (Not real data).

x	1	2	3	4
$P(X = x)$	?	?	0.4	0.4

(a) Suppose $P(X = 1)$ and $P(X = 2)$ are equal. What are $P(X = 1)$ and $P(X = 2)$?
Probabilities must sum to 100%. 1 - 0.4 - 0.4 = 0.2, both are equal so they are both 0.1.

(b) Write out a mathematical expression for $E[X]$, no need to evaluate or simplify.

$$(1 \times 0.1) + (2 \times 0.1) + (3 \times 0.4) + (4 \times 0.4)$$

(c) Write out a mathematical expression for $Var[X]$. You may write $E[X]$ instead of the full expression above where needed.

$$[(1 - E[X])^2 \times 0.1] + [(2 - E[X])^2 \times 0.1] + [(3 - E[X])^2 \times 0.4] + [(4 - E[X])^2 \times 0.4]$$

2. The PACER test is a fitness test used to assess aerobic fitness. It involves running back and forth between two lines which are twenty meters apart. An audio series of "beeps" is played, and you must make it to the opposite line before the next beep sounds. If you do so, you have completed "a lap". If you fail to do so, the test is over. The beeps get closer together over time.

   Consider one person running the PACER test, and let X be the number of laps they successfully complete (e.g. make it to the other side before the beep). Does X follow a binomial distribution? Briefly comment on each of the individual assumptions and why it is or is not met.

   - (B)inary trials passes, each lap is completed or failed.
   - (I)ndependent trials fails; once you fail a lap, the test is over and you cannot pass any further laps.
   - (N)umber of trials fails; The number of laps is not fixed, you run until you fail.
   - (S)ame probability fails: The beeps get closer together, making later laps more difficult than earlier laps.

3. A spinner at a carnival will either land on "Lose" or "Win" when spun. Assume spins are independent and each one has the same probability of landing on "Win".

   (a) (3 points) The carnival claims the spinner has a win probability of $p = 0.4$. Assuming this is correct, which lines of R code below can be used to find the probability of winning exactly twice in 3 spins? **Select ALL that apply**.

   - ○ `0.6*0.6`
   - ○ `0.6*0.6*0.4`
   - ● `3*0.6*0.6*0.4`
   - ● `dbinom(2, 3, 0.4)`
   - ○ `dbinom(2, 3, 0.6)`
   - ● `dbinom(1, 3, 0.6)`
   - ○ `pbinom(2, 3, 0.4)`
   - ○ `pbinom(2, 3, 0.6)`
   - ○ `pbinom(1, 3, 0.4)`

   (b) (3 points) You suspect that the true win probability p is actually less than 0.4. Write a null and alternative hypothesis to test the value of p.

   We are specifically looking for evidence that p is less than 0.4, so that must be our alternative.

   $$H_0 : p = 0.4 \quad \text{versus} \quad H_A : p < 0.4$$

   or

   $$H_0 : p \geq 0.4 \quad \text{versus} \quad H_A : p < 0.4$$

   (c) (2 points) You decide to complete the test with significance level $\alpha = 0.05$. You spin the spinner 10 times and win 2 times, giving a p-value of $P\Big(Binom(10, 0.4) \leq 2\Big) = 0.167$. What is the most accurate conclusion of the test? **Select ONE**.

   - ● Because our p-value is greater than α, we do not have evidence of $p < 0.4$.
   - ○ Because our p-value is greater than α, we have evidence of $p < 0.4$.
   - ○ We have evidence that the true $p = 0.4$.
   - ○ We have evidence that the true $p = 0.2$.

4. You are interested in comparing the performance of Student A at University A with Student B at University B.

   Grade point averages at University A are normally distributed with mean 2.8 and standard deviation 0.4. Grade point averages at University B are normally distributed with mean 2.5 and standard deviation 0.5.

   (a) Student A has a GPA of 3.2. What would Student B's grade point average be if they were at the same percentile at University B as Student A is at University A?

   Student A is one Univ. A SD above the Univ A. mean. So student B must be one Univ. B SD above the Univ. B mean, e.g. 2.5 + 0.5 = 3.0.

   (b) Consider taking a random sample of five students from University A. What is the distribution of $\bar{X}$, the average of their GPA's? **Select ONE**.

   - ○ $\bar{X} \sim N(2.8, 0.4)$
   - ○ $\bar{X} \sim N(2.8, 0.4 \cdot \sqrt{5})$
   - ● $\bar{X} \sim N\left(2.8, \frac{0.4}{\sqrt{5}}\right)$
   - ○ We do not have enough information to determine the distribution.

5. We are interested in the true percentage, p, of UW-Madison students who know that Bucky Badger's full first name is Buckingham. We ask 100 students, and construct a 90% confidence interval for p as (0.50, 0.70).

   (a) Which of the following correctly calculates the critical value for a 90% CI? **Select ONE**.

   - ○ `qnorm(0.9)`
   - ● `qnorm(0.95)`
   - ○ `qnorm(0.975)`
   - ○ `dbinom(2, 3, 0.4)`

   (b) Which of the following statements are TRUE about this interval? **Select ALL that apply**.

   - ● A 95% confidence interval would have a larger margin of error than this one.
   - ○ A 95% confidence interval would have the same margin of error as this one.
   - ● If we had asked 500 students AND decreased confidence level to 80%, the resulting interval would be narrower than this one.
   - ○ If we had asked 500 students AND decreased confidence level to 80%, we do not know if the resulting interval would be narrower or wider than this one.

   (c) Which of the following conclusions can be made from this confidence interval? **Select ONE**.

   - ○ At least half of all UW-Madison students know this fact.
   - ○ p is below 0.8.
   - ● 60% of the students sampled knew this fact.
   - ○ We are 95% confident that 60% of all UW students know this fact.