Section: ___________ Name:___________________________________

Read the following directions carefully. DO NOT turn to the next page until the exam has started.

Write your name and section number at the top right of this page:

Class	**Section Number**
Bret 8:50	001
Sahifa 1:20	003
Miranda 9:55	004
Sahifa 3:30	005
Sahifa 8:50	006
Cameron 1:20	007

As you complete the exam, write your initials at the top right of each other page.

When the exam start time is called, you may turn the page and begin your exam.
If you need more room, there is a blank page at the end of the exam, or we can give you some scratch paper.

Some multiple choice questions are "Select ONE" while others are "Select ALL that apply". Pay attention to the question type and only mark one option if it says "Select ONE". Fill in the circles completely.

If you finish early, you can hand your exam to your instructor or TA and leave early.

Otherwise, stop writing and hand your exam to your instructor or TA when the exam stop time is called.

1. Researchers have planted wildflowers in several plots in an attempt to study local insect wildlife. Each day, the number of monarch butterflies in each plot is observed. Supopose the number of monarch butterflies per plot is given by random variable X with the following partial probability distribution:

x	0	1	2	3
$P(X = x)$	?	?	0.2	0.1

The average number of monarch butterflies per plot is $E(X) = 0.8$.

(a) (4 points) Find $P(X = 0)$ and $P(X = 1)$.
From the definition of expected value,

$$\begin{aligned} E(X) &= \sum x \cdot P(x) \\ 0.8 &= 0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot 0.2 + 3 \cdot 0.1 \\ 0.8 &= P(1) + 0.4 + 0.3 \\ 0.1 &= P(1) \end{aligned}$$

Since the probabilities must sum to 1, we have $P(0) = 1 - (0.1 + 0.2 + 0.1) = 0.6$.

(b) (4 points) Write an expression for $\text{Var}(X)$. It is sufficient to write the numerical expression without doing the arithmetic to simplify your response to a single number.

$$\begin{aligned} V(X) &= \sum (x - E(X))^2 \cdot P(x) \\ &= (0 - 0.8)^2 \cdot 0.6 \ + \ (1 - 0.8)^2 \cdot 0.1 \ + \ (2 - 0.8)^2 \cdot 0.2 \ + \ (3 - 0.8)^2 \cdot 0.1 \end{aligned}$$

2. 32 men competed in the 2024 Paris Qualification Round for shot put. If an athlete threw the ball at least 21.35 meters, they qualified for the final. It is NOT valid to assume all athletes have equal skill.

   (a) (4 points) Let X be the number of athletes who qualify for the final. Determine whether X meets the assumptions of a binomial random variable by briefly explaining why each individual assumption is or is not met.

   B: The trials are binary because each athlete either qualifies or does not qualify.

   I: The trials are independent because the result of one athlete does not affect the performance of the other athletes.

   N: There are a fixed number of trials ($n = 32$).

   S: The trials do not have the same probability, because the athletes have different skill levels and do not have the same probability of qualifying.

   Because the same probability assumption is not met, X is not binomial.

   (b) (6 points) Regardless of your answer to the previous question, assume that X, the number of athletes who qualify for the final, follows a binomial distribution, with some number of trials n and some probability of success p.

   Which lines of R code below can be used to calculate the probability that 10 or more athletes qualify for the final? **Select ALL that apply**.

   - ○ `pbinom(9, n, p)`
   - ● `1 - pbinom(9, n, p)`
   - ● `pbinom(9, n, p, lower.tail = F)`
   - ○ `1 - pbinom(9, n, p, lower.tail = F)`
   - ○ `1 - pbinom(10, n, p)`
   - ○ `pbinom(10, n, p)`

3. At a certain hospital, the number of liters of blood donated per week is given by normal variable D with $\mu_D = 52, \sigma_D = 7$. The number of liters of blood required for transfusions per week is given by normal variable R with $\mu_R = 56, \sigma_R = 4$. You can assume R and D are independent.

   (a) (4 points) The number of liters required this week is $r = 48$. What number of donated liters has the same percentile as $r = 48$? In other words, if $P(R \leq 48) = p$, find d such that $P(D \leq d) = p$.
   The value $r = 48$ is exactly 2 standard deviations below the mean of $\mu_R = 56$, since $56 - 2(4) = 48$.

   The corresponding value of d will be two standard deviations below the mean $\mu_D = 52$. This is $d = 52 - 2(7) = 38$.

   (b) (5 points) Identify the sampling distribution for $\bar{D}_{52}$, the average liters of blood donated per week over a 52-week year.

$$\bar{D}_{52} \quad \sim \quad N\ (\ 52\ , \quad \frac{7}{\sqrt{52}}\ )$$

   The sampling distribution of the sample mean, in general, is given by

$$\bar{X} \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right)$$

4. A 95% confidence interval for the true proportion of left-handed Americans is reported as (0.086, 0.112).

   (a) (3 points) Which line of R code correctly calculates the critical value / quantile score for the margin of a 95% CI? **Select ONE**.

   - ○ `qnorm(0.05)`
   - ○ `qnorm(0.95)`
   - ○ `qnorm(0.9)`
   - ● `qnorm(0.975)`

   (b) (4 points) Which of the following statements are true about the CI? **Select ALL that apply**.

   - ○ A 98% CI on the same data would be narrower than the 95% CI.
   - ● A 98% CI on the same data would be wider than the 95% CI.
   - ● A 98% CI would have the same point estimate as the 95% CI.
   - ○ A 98% CI would have a different point estimate as the 95% CI.

   (c) (3 points) A hypothesis test of $H_0 : p = 0.1$ versus $H_A : p \neq 0.1$ (where p is the true proportion of left-handed Americans) is performed. The test resulted in a p-value smaller than the chosen significance level $\alpha = 0.05$. Which of the following statements is the best conclusion of this test? **Select ONE**.

   - ○ The true proportion of left-handed Americans is 0.1.
   - ○ The true proportion of left-handed Americans is not 0.1.
   - ○ We do not have evidence that the true proportion of left-handed Americans is different from 0.1.
   - ● We have evidence that the true proportion of left-handed Americans is different from 0.1