2

CHAPTER

PART 1

VALUE

# How to Calculate Present Values

**A corporation's shareholders** want maximum value and the maximum honest share price. To reach this goal, the company needs to invest in real assets that are worth more than they cost. In this chapter we take the first steps toward understanding how assets are valued and capital investments are made.

There are a few cases in which it is not that difficult to estimate asset values. In real estate, for example, you can hire a professional appraiser to do it for you. Suppose you own a warehouse. The odds are that your appraiser's estimate of its value will be within a few percent of what the building would actually sell for. After all, there is continuous activity in the real estate market, and the appraiser's stock-in-trade is knowledge of the prices at which similar properties have recently changed hands. Thus the problem of valuing real estate is simplified by the existence of an active market in which all kinds of properties are bought and sold.[1] No formal theory of value is needed. We can take the market's word for it.

But we need to go deeper than that. First, it is important to know how asset values are reached in an active market. Even if you can take the appraiser's word for it, it is important to understand *why* that warehouse is worth, say, $2 million and not a higher or lower figure. Second, the market for most corporate assets is pretty thin. Look in the classified advertisements in *The Wall Street Journal:* it is not often that you see a blast furnace for sale.

Companies are always searching for assets that are worth more to them than to others. That warehouse is worth more to you if you can manage it better than others can. But in that case, the price of similar buildings may not tell you what the warehouse is worth under your management. You need to know how asset values are determined.

In the first section of this chapter we work through a simple numerical example: Should you invest in a new office building in the hope of selling it at a profit next year? You should do so if net present value is positive, that is, if the new building's value today exceeds the investment that is required. A positive net present value implies that the rate of return on your investment is higher than your opportunity cost of capital, that is, higher than you could earn by investing in financial markets.

Next we introduce shortcut formulas for calculating present values. We show how to value an investment that delivers a steady stream of cash flows forever (a *perpetuity*) and one that produces a steady stream for a limited period (an *annuity*). We also look at investments that produce growing cash flows. We illustrate the formulas by applications to some personal financial decisions.

The term *interest rate* sounds straightforward enough, but rates can be quoted in various ways. We conclude the chapter by explaining the difference between the quoted rate and the true or effective interest rate.

By then you will deserve some payoff for the mental investment you have made in learning how to calculate present values. Therefore, in the next two chapters we try out these new tools on bonds and stocks. After that we tackle capital investment decisions at a practical level of detail.

For simplicity, every problem in this chapter is set out in dollars, but the concepts and calculations are identical in euros, yen, or any other currency.

[1] Needless to say, there are some properties that appraisers find nearly impossible to value—for example, nobody knows the potential selling price of the Taj Mahal, the Parthenon, or Windsor Castle.

## 2-1 Future Values and Present Values

### Calculating Future Values

Money can be invested to earn interest. So, if you are offered the choice between \$100 today and \$100 next year, you naturally take the money now to get a year's interest. Financial managers make the same point when they say that money has a *time value* or when they quote the most basic principle of finance: *a dollar today is worth more than a dollar tomorrow.*

Suppose you invest \$100 in a bank account that pays interest of $r = 7\%$ a year. In the first year you will earn interest of $.07 \times \$100 = \$7$ and the value of your investment will grow to \$107:

$$\text{Value of investment after 1 year} = \$100 \times (1 + r) = 100 \times 1.07 = \$107$$

By investing, you give up the opportunity to spend \$100 today and you gain the chance to spend \$107 next year.

If you leave your money in the bank for a second year, you earn interest of $.07 \times \$107 = \$7.49$ and your investment will grow to \$114.49:

$$\text{Value of investment after 2 years} = \$107 \times 1.07 = \$100 \times 1.07^2 = \$114.49$$

| Today | | Year 2 |
|---|---|---|
| \$100 | $\times\ 1.07^2$ | \$114.49 |

Notice that in the second year you earn interest on both your initial investment (\$100) and the previous year's interest (\$7). Thus your wealth grows at a *compound rate* and the interest that you earn is called **compound interest.**

If you invest your \$100 for $t$ years, your investment will continue to grow at a 7% compound rate to $\$100 \times (1.07)^t$. For any interest rate $r$, the future value of your \$100 investment will be

$$\text{Future value of } \$100 = \$100 \times (1 + r)^t$$

The higher the interest rate, the faster your savings will grow. Figure 2.1 shows that a few percentage points added to the interest rate can do wonders for your future wealth. For example, by the end of 20 years \$100 invested at 10% will grow to $\$100 \times (1.10)^{20} = \$672.75$. If it is invested at 5%, it will grow to only $\$100 \times (1.05)^{20} = \$265.33$.

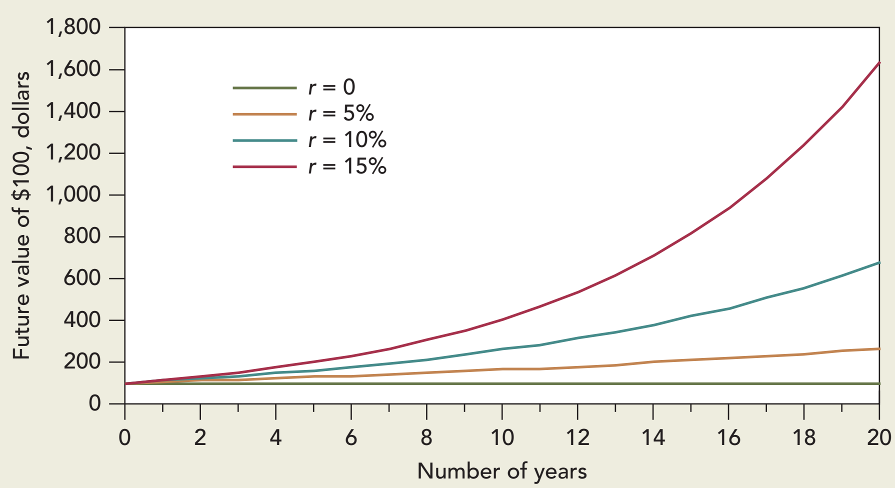



**FIGURE 2.1**

How an investment of \$100 grows with compound interest at different interest rates.

## Calculating Present Values

We have seen that \$100 invested for two years at 7% will grow to a future value of $100 \times 1.07^2 = \$114.49$. Let's turn this around and ask how much you need to invest *today* to produce \$114.49 at the end of the second year. In other words, what is the **present value (PV)** of the \$114.49 payoff?

You already know that the answer is \$100. But, if you didn't know or you forgot, you can just run the future value calculation in reverse and divide the future payoff by $(1.07)^2$:

$$\text{Present value} = \text{PV} = \frac{\$114.49}{(1.07)^2} = \$100$$

| Today | | Year 2 |
|---|---|---|
| | ← | |
| \$100 | $\div\ 1.07^2$ | \$114.49 |

In general, suppose that you will receive a cash flow of $C_t$ dollars at the end of year $t$. The present value of this future payment is

$$\text{Present value} = \text{PV} = \frac{C_t}{(1 + r)^t}$$

You sometimes see this present value formula written differently. Instead of *dividing* the future payment by $(1 + r)^t$, you can equally well *multiply* the payment by $1/(1 + r)^t$. The expression $1/(1 + r)^t$ is called the **discount factor.** It measures the present value of one dollar received in year $t$. For example, with an interest rate of 7% the two-year discount factor is

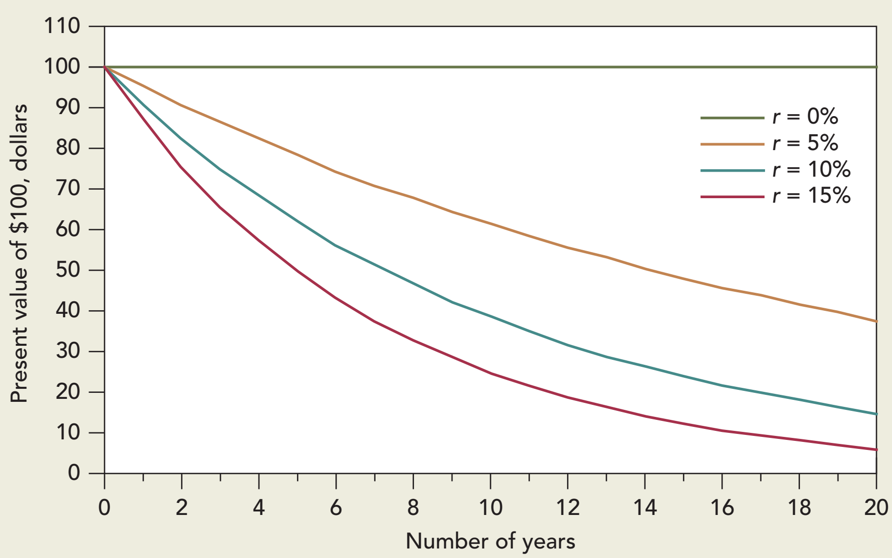
$$\text{DF}_2 = 1/(1.07)^2 = .8734$$

Investors are willing to pay \$.8734 today for delivery of \$1 at the end of two years. If each dollar received in year 2 is worth \$.8734 today, then the present value of your payment of \$114.49 in year 2 must be

$$\text{Present value} = \text{DF}_2 \times C_2 = .8734 \times 114.49 = \$100$$

**FIGURE 2.2**

Present value of a future cash flow of \$100. Notice that the longer you have to wait for your money, the less it is worth today.



The longer you have to wait for your money, the lower its present value. This is illustrated in Figure 2.2. Notice how small variations in the interest rate can have a powerful effect on the present value of distant cash flows. At an interest rate of 5%, a payment of \$100 in year 20 is worth \$37.69 today. If the interest rate increases to 10%, the value of the future payment falls by about 60% to \$14.86.

## Calculating the Present Value of an Investment Opportunity

How do you decide whether an investment opportunity is worth undertaking? Suppose you own a small company that is contemplating construction of an office block. The total cost of buying the land and constructing the building is \$370,000, but your real estate adviser forecasts a shortage of office space a year from now and predicts that you will be able sell the building for \$420,000. For simplicity, we will assume that this \$420,000 is a sure thing.

You should go ahead with the project if the present value (PV) of the cash inflows is greater than the \$370,000 investment. Suppose that the rate of interest on U.S. government securities is $r = 5\%$ per year. Then, the present value of your office building is:

$$\text{PV} = \frac{420{,}000}{1.05} = \$400{,}000$$

The rate of return $r$ is called the **discount rate, hurdle rate,** or **opportunity cost of capital.** It is an opportunity cost because it is the return that is foregone by investing in the project rather than investing in financial markets. In our example the opportunity cost is 5%, because you could earn a safe 5% by investing in U.S. government securities. Present value was found by *discounting* the future cash flows by this opportunity cost.

Suppose that as soon as you have bought the land and paid for the construction, you decide to sell your project. How much could you sell it for? That is an easy question. If the venture will return a surefire \$420,000, then your property ought to be worth its PV of \$400,000 today. That is what investors would need to pay to get the same future payoff. If you tried to sell it for more than \$400,000, there would be no takers, because the property would then offer an expected rate of return lower than the 5% available on government securities. Of course, you could always sell your property for less, but why sell for less than the market will bear? The \$400,000 present value is the only feasible price that satisfies both buyer and seller. Therefore, the present value of the property is also its market price.

## Net Present Value

The office building is worth \$400,000 today, but that does not mean you are \$400,000 better off. You invested \$370,000, so the **net present value (NPV)** is \$30,000. Net present value equals present value minus the required investment:

$$\text{NPV} = \text{PV} - \text{investment} = 400{,}000 - 370{,}000 = \$30{,}000$$

In other words, your office development is worth more than it costs. It makes a *net* contribution to value and increases your wealth. The formula for calculating the NPV of your project can be written as:

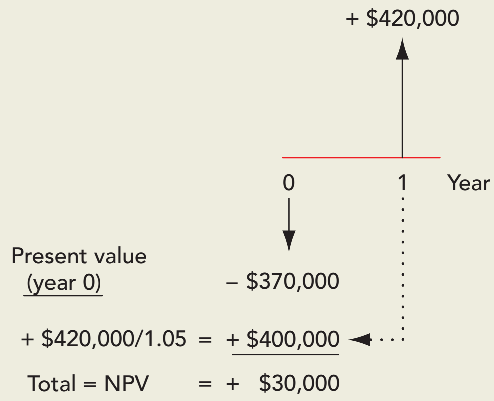
$$\text{NPV} = C_0 + C_1/(1 + r)$$

Remember that $C_0$, the cash flow at time 0 (that is, today) is usually a negative number. In other words, $C_0$ is an investment and therefore a cash outflow. In our example, $C_0 = -\$370{,}000$.

When cash flows occur at different points in time, it is often helpful to draw a time line showing the date and value of each cash flow. Figure 2.3 shows a time line for your office development. It sets out the present value calculations assuming that the discount rate $r$ is 5%.

**FIGURE 2.3**

Calculation showing the NPV of the office development.




## Risk and Present Value

We made one unrealistic assumption in our discussion of the office development: Your real estate adviser cannot be certain about the profitability of an office building. Those future cash flows represent the best forecast, but they are not a sure thing.

If the cash flows are uncertain, your calculation of NPV is wrong. Investors could achieve those cash flows with certainty by buying $400,000 worth of U.S. government securities, so they would not buy your building for that amount. You would have to cut your asking price to attract investors' interest.

Here we can invoke a second basic financial principle: *a safe dollar is worth more than a risky dollar.* Most investors avoid risk when they can do so without sacrificing return. However, the concepts of present value and the opportunity cost of capital still make sense for risky investments. It is still proper to discount the payoff by the rate of return offered by a risk-equivalent investment in financial markets. But we have to think of *expected* payoffs and the *expected* rates of return on other investments.[2]

Not all investments are equally risky. The office development is more risky than a government security but less risky than a start-up biotech venture. Suppose you believe the project is as risky as investment in the stock market and that stocks offer a 12% expected return. Then 12% is the opportunity cost of capital. That is what you are giving up by investing in the office building and *not* investing in equally risky securities.

Now recompute NPV with $r = .12$:

$$\text{PV} = \frac{420{,}000}{1.12} = \$375{,}000$$

$$\text{NPV} = \text{PV} - 370{,}000 = \$5{,}000$$

The office building still makes a net contribution to value, but the increase in your wealth is smaller than in our first calculation, which assumed that the cash flows from the project were risk-free.

The value of the office building depends, therefore, on the timing of the cash flows and their risk. The $420,000 payoff would be worth just that if you could get it today. If the office building is as risk-free as government securities, the delay in the cash flow reduces value by $20,000 to $400,000. If the building is as risky as investment in the stock market, then the risk further reduces value by $25,000 to $375,000.

Unfortunately, adjusting asset values for both time and risk is often more complicated than our example suggests. Therefore, we take the two effects separately. For the most part, we dodge the problem of risk in Chapters 2 through 6, either by treating all cash flows as if they were known with certainty or by talking about expected cash

[2] We define "expected" more carefully in Chapter 9. For now think of expected payoff as a realistic forecast, neither optimistic nor pessimistic. Forecasts of expected payoffs are correct on average.

flows and expected rates of return without worrying how risk is defined or measured. Then in Chapter 7 we turn to the problem of understanding how financial markets cope with risk.

## Present Values and Rates of Return

We have decided that constructing the office building is a smart thing to do, since it is worth more than it costs. To discover how much it is worth, we asked how much you would need to invest directly in securities to achieve the same payoff. That is why we discounted the project's future payoff by the rate of return offered by these equivalent-risk securities—the overall stock market in our example.

We can state our decision rule in another way: your real estate venture is worth undertaking because its rate of return exceeds the opportunity cost of capital. The rate of return is simply the profit as a proportion of the initial outlay:

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{420{,}000 - 370{,}000}{370{,}000} = .135, \text{ or } 13.5\%$$

The cost of capital is once again the return foregone by *not* investing in financial markets. If the office building is as risky as investing in the stock market, the return foregone is 12%. Since the 13.5% return on the office building exceeds the 12% opportunity cost, you should go ahead with the project.

Here, then, we have two equivalent decision rules for capital investment:[3]

- *Net present value rule.* Accept investments that have positive net present values.
- *Rate of return rule.* Accept investments that offer rates of return in excess of their opportunity costs of capital.[4]

## Calculating Present Values When There Are Multiple Cash Flows

One of the nice things about present values is that they are all expressed in current dollars—so you can add them up. In other words, the present value of cash flow (A + B) is equal to the present value of cash flow A plus the present value of cash flow B.

Suppose that you wish to value a stream of cash flows extending over a number of years. Our rule for adding present values tells us that the *total* present value is:

$$\text{PV} = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_T}{(1+r)^T}$$

This is called the **discounted cash flow** (or **DCF**) formula. A shorthand way to write it is

$$\text{PV} = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

where $\Sigma$ refers to the sum of the series. To find the *net* present value (NPV) we add the (usually negative) initial cash flow:

$$\text{NPV} = C_0 + \text{PV} = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

[3] You might check for yourself that these are equivalent rules. In other words, if the return of \$50,000/\$370,000 is greater than $r$, then the net present value $-$ \$370,000 + [\$420,000/(1 + $r$)] *must* be greater than 0.

[4] The two rules can conflict when there are cash flows at more than two dates. We address this problem in Chapter 5.

**EXAMPLE 2.1** ● Present Values with Multiple Cash Flows

Your real estate adviser has come back with some revised forecasts. He suggests that you rent out the building for two years at \$20,000 a year, and predicts that at the end of that time you will be able to sell the building for \$400,000. Thus there are now two future cash flows—a cash flow of $C_1 = \$20{,}000$ at the end of one year and a further cash flow of $C_2 = (20{,}000 + 400{,}000) = \$420{,}000$ at the end of the second year.

The present value of your property development is equal to the present value of $C_1$ plus the present value of $C_2$. Figure 2.4 shows that the value of the first year's cash flow is $C_1/(1 + r) = 20{,}000/1.12 = \$17{,}900$ and the value of the second year's flow is $C_2/(1 + r)^2 = 420{,}000/1.12^2 = \$334{,}800$. Therefore our rule for adding present values tells us that the *total* present value of your investment is

$$\text{PV} = \frac{C_1}{1 + r} + \frac{C_2}{(1 + r)^2} = \frac{20{,}000}{1.12} + \frac{420{,}000}{1.12^2} = 17{,}900 + 334{,}800 = \$352{,}700$$

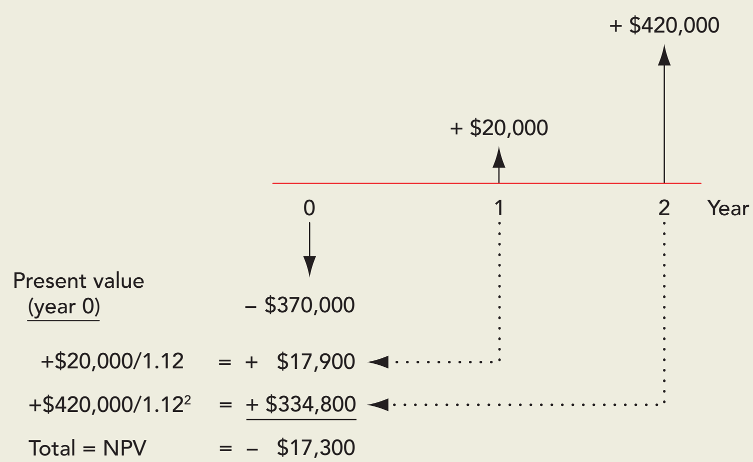
Sorry, but your office building is now worth less than it costs. NPV is negative:

$$\text{NPV} = \$352{,}700 - \$370{,}000 = -\$17{,}300$$

Perhaps you should revert to the original plan of selling in year 1.

**FIGURE 2.4**

Calculation showing the NPV of the revised office project.




Your two-period calculations in Example 2.1 required just a few keystrokes on a calculator. Real problems can be much more complicated, so financial managers usually turn to financial calculators especially programmed for present value calculations or to computer spreadsheet programs. A box near the end of the chapter introduces you to some useful Excel functions that can be used to solve discounting problems. In addition, the Web site for this book (**www.mhhe.com/bma**) contains appendixes to help get you started using financial calculators and Excel spreadsheets. It also includes tables that can be used for a variety of discounting problems.

### The Opportunity Cost of Capital

By investing in the office building you gave up the opportunity to earn an expected return of 12% in the stock market. The opportunity cost of capital is therefore 12%. When you discount the expected cash flows by the opportunity cost of capital, you are asking how much investors in the financial markets are prepared to pay for a security that produces a similar stream of future cash flows. Your calculations showed that investors would need to pay only \$352,700 for an investment that produces cash flows of \$20,000 at year 1 and \$420,000 at year 2. Therefore, they won't pay any more than that for your office building.

Confusion sometimes sneaks into discussions of the cost of capital. Suppose a banker approaches. "Your company is a fine and safe business with few debts," she says. "My bank will lend you the \$370,000 that you need for the office block at 8%." Does this mean that the cost of capital is 8%? If so, the project would be worth doing. At an 8% cost of capital, PV would be $20{,}000/1.08 + 420{,}000/1.08^2 = \$378{,}600$ and NPV $= \$378{,}600 - \$370{,}000 = +\ \$8{,}600$.

But that can't be right. First, the interest rate on the loan has nothing to do with the risk of the project: it reflects the good health of your existing business. Second, whether you take the loan or not, you still face the choice between the office building and an equally risky investment in the stock market. The stock market investment could generate the same expected payoff as your office building at a lower cost. A financial manager who borrows \$370,000 at 8% and invests in an office building is not smart, but stupid, if the company or its shareholders can borrow at 8% and invest the money at an even higher return. That is why the 12% expected return on the stock market is the opportunity cost of capital for your project.

## 2-2 Looking for Shortcuts—Perpetuities and Annuities

### How to Value Perpetuities

Sometimes there are shortcuts that make it easy to calculate present values. Let us look at some examples.

On occasion, the British and the French have been known to disagree and sometimes even to fight wars. At the end of some of these wars the British consolidated the debt they had issued during the war. The securities issued in such cases were called consols. Consols are **perpetuities.** These are bonds that the government is under no obligation to repay but that offer a fixed income for each year to perpetuity. The British government is still paying interest on consols issued all those years ago. The annual rate of return on a perpetuity is equal to the promised annual payment divided by the present value:[5]

$$\text{Return} = \frac{\text{cash flow}}{\text{present value}}$$

$$r = \frac{C}{\text{PV}}$$

[5] You can check this by writing down the present value formula

$$\text{PV} = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

Now let $C/(1 + r) = a$ and $1/(1 + r) = x$. Then we have (1) $\text{PV} = a(1 + x + x^2 + \cdots)$.
Multiplying both sides by $x$, we have (2) $\text{PV}x = a(x + x^2 + \cdots)$.
Subtracting (2) from (1) gives us $\text{PV}(1 - x) = a$. Therefore, substituting for $a$ and $x$,

$$\text{PV}\left(1 - \frac{1}{1+r}\right) = \frac{C}{1+r}$$

Multiplying both sides by $(1 + r)$ and rearranging gives

$$\text{PV} = \frac{C}{r}$$

We can obviously twist this around and find the present value of a perpetuity given the discount rate $r$ and the cash payment $C$:

$$\text{PV} = \frac{C}{r}$$

The year is 2030. You have been fabulously successful and are now a billionaire many times over. It was fortunate indeed that you took that finance course all those years ago. You have decided to follow in the footsteps of two of your heroes, Bill Gates and Warren Buffet. Malaria is still a scourge and you want to help eradicate it and other infectious diseases by endowing a foundation to combat these diseases. You aim to provide \$1 billion a year in perpetuity, starting next year. So, if the interest rate is 10%, you are going to have to write a check today for

$$\text{Present value of perpetuity} = \frac{C}{r} = \frac{\$1 \text{ billion}}{.1} = \$10 \text{ billion}$$

Two warnings about the perpetuity formula. First, at a quick glance you can easily confuse the formula with the present value of a single payment. A payment of \$1 at the end of one year has a present value of $1/(1 + r)$. The perpetuity has a value of $1/r$. These are quite different.

Second, the perpetuity formula tells us the value of a regular stream of payments starting one period from now. Thus your \$10 billion endowment would provide the foundation with its first payment in one year's time. If you also want to provide an up-front sum, you will need to lay out an extra \$1 billion.

Sometimes you may need to calculate the value of a perpetuity that does not start to make payments for several years. For example, suppose that you decide to provide \$1 billion a year with the first payment four years from now. We know that in year 3 this endowment will be an ordinary perpetuity with payments starting in one year. So our perpetuity formula tells us that in year 3 the endowment will be worth $\$1/r = 1/.1 = \$10$ billion. But it is not worth that much now. To find *today's* value we need to multiply by the three-year discount factor $1/(1 + r)^3 = 1/(1.1)^3 = .751$. Thus, the "delayed" perpetuity is worth \$10 billion $\times$ .751 = \$7.51 billion. The full calculation is

$$\text{PV} = \$1 \text{ billion} \times \frac{1}{r} \times \frac{1}{(1+r)^3} = \$1 \text{ billion} \times \frac{1}{.10} \times \frac{1}{(1.10)^3} = \$7.51 \text{ billion}$$

## How to Value Annuities

An **annuity** is an asset that pays a fixed sum each year for a specified number of years. The equal-payment house mortgage or installment credit agreement are common examples of annuities. So are interest payments on most bonds, as we see in the next chapter.

Figure 2.5 illustrates a simple trick for valuing annuities. It shows the payments and values of three investments.

**Row 1** The investment in the first row provides a perpetual stream of \$1 starting at the end of the first year. We have already seen that this perpetuity has a present value of $1/r$.

**Row 2** Now look at the investment shown in the second row. It also provides a perpetual stream of \$1 payments, but these payments don't start until year 4. This investment is identical to the delayed perpetuity that we have just valued. In year 3, the investment will be an ordinary perpetuity with payments starting in one year and will be worth $1/r$ in year 3. Its value today is, therefore,

$$\text{PV} = \frac{1}{r(1+r)^3}$$

| | Cash flow Year: 1 | 2 | 3 | 4 | 5 | 6 . . . | Present value |
|---|---|---|---|---|---|---|---|
| **1. Perpetuity A** | $1 | $1 | $1 | $1 | $1 | $1 . . . | $\frac{1}{r}$ |
| **2. Perpetuity B** | | | | $1 | $1 | $1 . . . | $\frac{1}{r(1+r)^3}$ |
| **3. Three-year annuity (1 − 2)** | $1 | $1 | $1 | | | | $\frac{1}{r} - \frac{1}{r(1+r)^3}$ |

**FIGURE 2.5**

An annuity that makes payments in each of years 1 through 3 is equal to the difference between two perpetuities.

**Row 3** The perpetuities in rows 1 and 2 both provide a cash flow from year 4 onward. The only difference between the two investments is that the first one also provides a cash flow in each of years 1 through 3. In other words, the difference between the two perpetuities is an annuity of three years. Row 3 shows that the present value of this annuity is equal to the value of the row 1 perpetuity less the value of the delayed perpetuity in row 2:[6]

$$\text{PV of 3-year annuity} = \frac{1}{r} - \frac{1}{r(1+r)^3}$$

The general formula for the value of an annuity that pays \$1 a year for each of $t$ years starting in year 1 is:

$$\text{Present value of annuity} = \frac{1}{r} - \frac{1}{r(1+r)^t}$$

This expression is generally known as the $t$-year annuity factor.[7] Remembering formulas is about as difficult as remembering other people's birthdays. But as long as you bear in mind that an annuity is equivalent to the difference between an immediate and a delayed perpetuity, you shouldn't have any difficulty.

[6] Again we can work this out from first principles. We need to calculate the sum of the finite geometric series (1) $\text{PV} = a(1 + x + x^2 + \cdots + x^{t-1})$, where $a = C/(1 + r)$ and $x = 1/(1 + r)$.

Multiplying both sides by $x$, we have (2) $\text{PV}x = a(x + x^2 + \cdots + x^t)$.

Subtracting (2) from (1) gives us $\text{PV}(1 - x) = a(1 - x^t)$.

Therefore, substituting for $a$ and $x$,

$$\text{PV}\left(1 - \frac{1}{1+r}\right) = C\left[\frac{1}{1+r} - \frac{1}{(1+r)^{t+1}}\right]$$

Multiplying both sides by $(1 + r)$ and rearranging gives

$$\text{PV} = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

[7] Some people find the following equivalent formula more intuitive:

$$\text{Present value of annuity} = \underbrace{\frac{1}{r}}_{\substack{\text{perpetuity}\\ \text{formula}}} \left[\underbrace{1}_{\substack{\$1\\ \text{starting}\\ \text{next year}}} - \underbrace{\frac{1}{(1+r)^t}}_{\substack{\text{minus \$1}\\ \text{starting at}\\ t+1}}\right]$$

## EXAMPLE 2.2 • Costing an Installment Plan

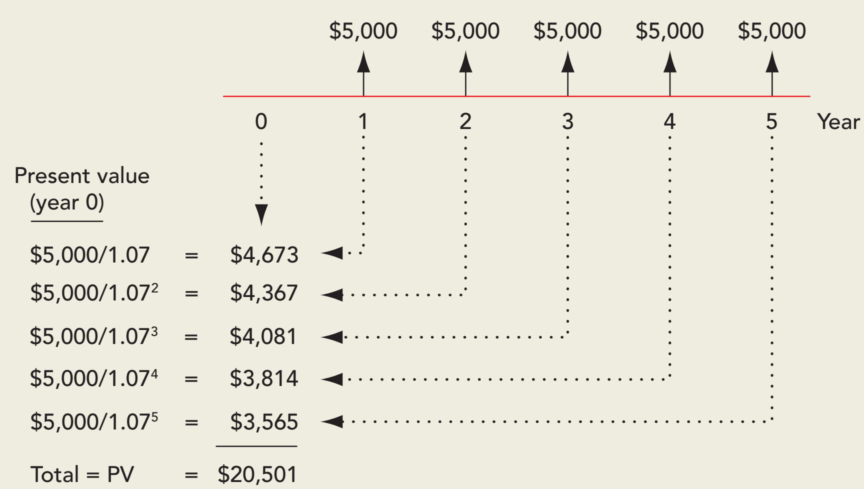
Most installment plans call for level streams of payments. Suppose that Tiburon Autos offers an "easy payment" scheme on a new Toyota of \$5,000 a year, paid at the end of each of the next five years, with no cash down. What is the car really costing you?

First let us do the calculations the slow way, to show that, if the interest rate is 7%, the present value of these payments is \$20,501. The time line in Figure 2.6 shows the value of each cash flow and the total present value. The annuity formula, however, is generally quicker:

$$\text{PV} = 5{,}000\left[\frac{1}{.07} - \frac{1}{.07(1.07)^5}\right] = 5{,}000 \times 4.100 = \$20{,}501$$

**FIGURE 2.6**

Calculations showing the year-by-year present value of the installment payments.




## EXAMPLE 2.3 • Winning Big at the Lottery

When 13 lucky machinists from Ohio pooled their money to buy Powerball lottery tickets, they won a record \$295.7 million. (A fourteenth member of the group pulled out at the last minute to put in his own numbers.) We suspect that the winners received unsolicited congratulations, good wishes, and requests for money from dozens of more or less worthy charities. In response, they could fairly point out that the prize wasn't really worth \$295.7 million. That sum was to be repaid in 25 annual installments of \$11.828 million each. Assuming that the first payment occurred at the end of one year, what was the present value of the prize? The interest rate at the time was 5.9%.

These payments constitute a 25-year annuity. To value this annuity we simply multiply \$11.828 million by the 25-year annuity factor:

$$\begin{aligned}\text{PV} &= 11.828 \times \text{25-year annuity factor}\\ &= 11.828 \times \left[\frac{1}{r} - \frac{1}{r(1+r)^{25}}\right]\end{aligned}$$

At an interest rate of 5.9%, the annuity factor is

$$\left[\frac{1}{.059} - \frac{1}{.059(1.059)^{25}}\right] = 12.9057$$

The present value of the cash payments is $\$11.828 \times 12.9057 = \$152.6$ million, much below the well-trumpeted prize, but still not a bad day's haul.

Lottery operators generally make arrangements for winners with big spending plans to take an equivalent lump sum. In our example the winners could either take the \$295.7 million spread over 25 years or receive \$152.6 million up front. Both arrangements had the same present value.

## PV Annuities Due

When we used the annuity formula to value the Powerball lottery prize in Example 2.3, we presupposed that the first payment was made at the end of one year. In fact, the first of the 25 yearly payments was made immediately. How does this change the value of the prize?

If we discount each cash flow by one less year, the present value is increased by the multiple $(1 + r)$. In the case of the lottery prize the value becomes $152.6 \times (1 + r) = 152.6 \times 1.059 = \$161.6$ million.

A level stream of payments starting immediately is called an **annuity due.** An annuity due is worth $(1 + r)$ times the value of an ordinary annuity.

## Calculating Annual Payments

Annuity problems can be confusing on first acquaintance, but you will find that with practice they are generally straightforward. In Example 2.4, you will need to use the annuity formula to find the amount of the payment given the present value.

### EXAMPLE 2.4 ● Finding Mortgage Payments

Suppose that you take out a \$250,000 house mortgage from your local savings bank. The bank requires you to repay the mortgage in equal annual installments over the next 30 years. It must therefore set the annual payments so that they have a present value of \$250,000. Thus,

$$\text{PV} = \text{mortgage payment} \times \text{30-year annuity factor} = \$250{,}000$$
$$\text{Mortgage payment} = \$250{,}000/\text{30-year annuity factor}$$

Suppose that the interest rate is 12% a year. Then

$$\text{30-year annuity factor} = \left[\frac{1}{.12} - \frac{1}{.12(1.12)^{30}}\right] = 8.055$$

and

$$\text{Mortgage payment} = 250{,}000/8.055 = \$31{,}037$$

The mortgage loan is an example of an *amortizing loan*. "Amortizing" means that part of the regular payment is used to pay interest on the loan and part is used to reduce the amount of the loan.

Table 2.1 illustrates another amortizing loan. This time it is a four-year loan of $1,000 with an interest rate of 10% and annual payments. The annual payment needed to repay the loan is $315.47. In other words, $1,000 divided by the four-year annuity factor is $315.47. At the end of the first year, the interest charge is 10% of $1,000, or $100. So $100 of the first payment is absorbed by interest, and the remaining $215.47 is used to reduce (or "amortize") the loan balance to $784.53.

Next year, the outstanding balance is lower, so the interest charge is only $78.45. Therefore $315.47 − 78.45 = $237.02 can be applied to amortization. Because the loan is progressively paid off, the fraction of each payment devoted to interest steadily falls over time, while the fraction used to reduce the loan increases. By the end of year 4 the amortization is just enough to reduce the balance of the loan to zero.

| Year | Beginning-of-Year Balance | Year-end Interest on Balance | Total Year-end Payment | Amortization of Loan | End-of-Year Balance |
|---|---|---|---|---|---|
| 1 | $1,000.00 | $100.00 | $315.47 | $215.47 | $784.53 |
| 2 | 784.53 | 78.45 | 315.47 | 237.02 | 547.51 |
| 3 | 547.51 | 54.75 | 315.47 | 260.72 | 286.79 |
| 4 | 286.79 | 28.68 | 315.47 | 286.79 | 0 |

**TABLE 2.1** An example of an amortizing loan. If you borrow $1,000 at an interest rate of 10%, you would need to make an annual payment of $315.47 over four years to repay that loan with interest.

## Future Value of an Annuity

Sometimes you need to calculate the *future* value of a level stream of payments.

**EXAMPLE 2.5** ● Saving to Buy a Sailboat

Perhaps your ambition is to buy a sailboat; something like a 40-foot Beneteau would fit the bill very well. But that means some serious saving. You estimate that, once you start work, you could save $20,000 a year out of your income and earn a return of 8% on these savings. How much will you be able to spend after five years?

We are looking here at a level stream of cash flows—an annuity. We have seen that there is a shortcut formula to calculate the *present* value of an annuity. So there ought to be a similar formula for calculating the *future value* of a level stream of cash flows.

Think first how much your savings are worth today. You will set aside $20,000 in each of the next five years. The present value of this five-year annuity is therefore equal to

$$\begin{aligned} \text{PV} &= \$20{,}000 \times \text{5-year annuity factor} \\ &= \$20{,}000 \times \left[\frac{1}{.08} - \frac{1}{.08(1.08)^5}\right] = \$79{,}854 \end{aligned}$$

Now think how much you would have after five years if you invested \$79,854 today. Simple! Just multiply by $(1.08)^5$:

$$\text{Value at end of year 5} = \$79{,}854 \times 1.08^5 = \$117{,}332$$

You should be able to buy yourself a nice boat for \$117,000.

In Example 2.5 we calculated the future value of an annuity by first calculating its present value and then multiplying by $(1 + r)^t$. The general formula for the future value of a level stream of cash flows of \$1 a year for $t$ years is, therefore,

$$\begin{aligned}\text{Future value of annuity} &= \text{present value of annuity of \$1 a year} \times (1 + r)^t \\ &= \left[\frac{1}{r} - \frac{1}{r(1 + r)^t}\right] \times (1 + r)^t = \frac{(1 + r)^t - 1}{r}\end{aligned}$$

## 2-3 More Shortcuts—Growing Perpetuities and Annuities

### Growing Perpetuities

You now know how to value level streams of cash flows, but you often need to value a stream of cash flows that grows at a constant rate. For example, think back to your plans to donate \$10 billion to fight malaria and other infectious diseases. Unfortunately, you made no allowance for the growth in salaries and other costs, which will probably average about 4% a year starting in year 1. Therefore, instead of providing \$1 billion a year in perpetuity, you must provide \$1 billion in year 1, 1.04 × \$1 billion in year 2, and so on. If we call the growth rate in costs $g$, we can write down the present value of this stream of cash flows as follows:

$$\begin{aligned}PV &= \frac{C_1}{1 + r} + \frac{C_2}{(1 + r)^2} + \frac{C_3}{(1 + r)^3} + \cdots \\ &= \frac{C_1}{1 + r} + \frac{C_1(1 + g)}{(1 + r)^2} + \frac{C_1(1 + g)^2}{(1 + r)^3} + \cdots\end{aligned}$$

Fortunately, there is a simple formula for the sum of this geometric series.[8] If we assume that $r$ is greater than $g$, our clumsy-looking calculation simplifies to

$$\text{Present value of growing perpetuity} = \frac{C_1}{r - g}$$

Therefore, if you want to provide a perpetual stream of income that keeps pace with the growth rate in costs, the amount that you must set aside today is

$$PV = \frac{C_1}{r - g} = \frac{\$1 \text{ billion}}{.10 - .04} = \$16.667 \text{ billion}$$

You will meet this perpetual-growth formula again in Chapter 4, where we use it to value the stock of mature, slowly growing companies.

[8] We need to calculate the sum of an infinite geometric series $PV = a(1 + x + x^2 + \cdots)$ where $a = C_1/(1 + r)$ and $x = (1 + g)/(1 + r)$. In footnote 5 we showed that the sum of such a series is $a/(1 - x)$. Substituting for $a$ and $x$ in this formula,

$$PV = \frac{C_1}{r - g}$$

## Growing Annuities

You are contemplating membership in the St. Swithin's and Ancient Golf Club. The annual membership dues for the coming year are \$5,000, but you can make a single payment of \$12,750, which will provide you with membership for the next three years. In each case no payments are due until the end of the first year. Which is the better deal? The answer depends on how rapidly membership fees are likely to increase over the three-year period. For example, suppose that fees are payable at the end of each year and are expected to increase by 6% per annum. The discount rate is 10%.

The problem is to calculate the value of a three-year stream of cash flows that grows at the rate of $g = .06$ each year. Of course, you could calculate each year's cash flow and discount it at 10%. The alternative is to employ the same trick that we used to find the formula for an ordinary annuity. This is illustrated in Figure 2.7. The first row shows the value of a perpetuity that produces a cash flow of \$1 in year 1, $\$1 \times (1 + g)$ in year 2, and so on. It has a present value of

$$\text{PV} = \frac{\$1}{(r - g)}$$

The second row shows a similar growing perpetuity that produces its first cash flow of $\$1 \times (1 + g)^3$ in year 4. It *will* have a present value of $\$1 \times (1 + g)^3/(r - g)$ in year 3 and therefore has a value today of

$$\text{PV} = \frac{\$1}{(r - g)} \times \frac{(1 + g)^3}{(1 + r)^3}$$

The third row in the figure shows that the difference between the two sets of cash flows consists of a three-year stream of cash flows beginning with \$1 in year 1 and growing each year at the rate of $g$. Its value is equal to the difference between our two growing perpetuities:

$$\text{PV} = \frac{\$1}{(r - g)} - \frac{\$1}{(r - g)} \times \frac{(1 + g)^3}{(1 + r)^3}$$

In our golf club example, the present value of the three annual membership dues would be:

$$\begin{aligned} \text{PV} &= [1/(.10 - .06) - (1.06)^3/(.10 - .06)(1.10)^3] \times \$5{,}000 \\ &= 2.629 \times \$5{,}000 = \$13{,}146 \end{aligned}$$

| | Cash flow | | | | | | |
|---|---|---|---|---|---|---|---|
| Year: | 1 | 2 | 3 | 4 | 5 | 6 . . . | Present value |
| 1. Growing perpetuity A | \$1 | $\$1 \times (1 + g)$ | $\$1 \times (1 + g)^2$ | $\$1 \times (1 + g)^3$ | $\$1 \times (1 + g)^4$ | $\$1 \times (1 + g)^5$ . . . | $\frac{1}{r - g}$ |
| 2. Growing perpetuity B | | | | $\$1 \times (1 + g)^3$ | $\$1 \times (1 + g)^4$ | $\$1 \times (1 + g)^5$ . . . | $\frac{(1 + g)^3}{(r - g)(1 + r)^3}$ |
| 3. Growing 3-year annuity (1 − 2) | \$1 | $\$1 \times (1 + g)$ | $\$1 \times (1 + g)^2$ | | | | $\frac{1}{r - g} - \frac{(1 + g)^3}{(r - g)(1 + r)^3}$ |

**FIGURE 2.7**

A three-year stream of cash flows that grows at the rate *g* is equal to the difference between two growing perpetuities.

| Cash Flow, $ | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Year:** | **0** | **1** | **2 . . .** | **. . . t − 1** | **t** | **t + 1 . . .** | **Present Value** |
| Perpetuity | | 1 | 1 . . . | 1 | 1 | 1 . . . | $\frac{1}{r}$ |
| t-period annuity | | 1 | 1 . . . | 1 | 1 | | $\frac{1}{r} - \frac{1}{r(1+r)^t}$ |
| t-period annuity due | 1 | 1 | 1 . . . | 1 | | | $(1+r)\left(\frac{1}{r} - \frac{1}{r(1+r)^t}\right)$ |
| Growing perpetuity | | 1 | $1 \times (1+g)$ . . . | $1 \times (1+g)^{t-2}$ | $1 \times (1+g)^{t-1}$ | $1 \times (1+g)^t$ . . . | $\frac{1}{r-g}$ |
| t-period growing annuity | | 1 | $1 \times (1+g)$ . . . | $1 \times (1+g)^{t-2}$ | $1 \times (1+g)^{t-1}$ | | $\frac{1}{r-g} - \frac{1}{r-g} \times \frac{(1+g)^t}{(1+r)^t}$ |

**TABLE 2.2** Some useful shortcut formulas.

If you can find the cash, you would be better off paying now for a three-year membership.

Too many formulas are bad for the digestion. So we will stop at this point and spare you any more of them. The formulas discussed so far appear in Table 2.2.

## 2-4 How Interest Is Paid and Quoted

In our examples we have assumed that cash flows occur only at the end of each year. This is sometimes the case. For example, in France and Germany the government pays interest on its bonds annually. However, in the United States and Britain government bonds pay interest semiannually. So if the interest rate on a U.S. government bond is quoted as 10%, the investor in practice receives interest of 5% every six months.

If the first interest payment is made at the end of six months, you can earn an additional six months' interest on this payment. For example, if you invest $100 in a bond that pays interest of 10% compounded semiannually, your wealth will grow to $1.05 \times \$100 = \$105$ by the end of six months and to $1.05 \times \$105 = \$110.25$ by the end of the year. In other words, an interest rate of 10% compounded semiannually is equivalent to 10.25% compounded annually. The *effective annual interest rate* on the bond is 10.25%.

Let's take another example. Suppose a bank offers you an automobile loan at an **annual percentage rate,** or **APR,** of 12% with interest to be paid monthly. This means that each month you need to pay one-twelfth of the annual rate, that is, $12/12 = 1\%$ a month. Thus the bank is *quoting* a rate of 12%, but the effective annual interest rate on your loan is $1.01^{12} - 1 = .1268$, or 12.68%.[9]

Our examples illustrate that you need to distinguish between the *quoted* annual interest rate and the *effective* annual rate. The quoted annual rate is usually calculated as the total

[9] In the U.S., truth-in-lending laws oblige the company to quote an APR that is calculated by multiplying the payment each period by the number of payments in the year. APRs are calculated differently in other countries. For example, in the European Union APRs must be expressed as annually compounded rates, so consumers know the effective interest rate that they are paying.

annual payment divided by the number of payments in the year. When interest is paid once a year, the quoted and effective rates are the same. When interest is paid more frequently, the effective interest rate is higher than the quoted rate.

In general, if you invest $1 at a rate of $r$ per year compounded $m$ times a year, your investment at the end of the year will be worth $[1 + (r/m)]^m$ and the effective interest rate is $[1 + (r/m)]^m - 1$. In our automobile loan example $r = .12$ and $m = 12$. So the effective annual interest rate was $[1 + .12/12]^{12} - 1 = .1268$, or 12.68%.

## Continuous Compounding

Instead of compounding interest monthly or semiannually, the rate could be compounded weekly ($m = 52$) or daily ($m = 365$). In fact there is no limit to how frequently interest could be paid. One can imagine a situation where the payments are spread evenly and continuously throughout the year, so the interest rate is continuously compounded.[10] In this case $m$ is infinite.

It turns out that there are many occasions in finance when continuous compounding is useful. For example, one important application is in option pricing models, such as the Black–Scholes model that we introduce in Chapter 21. These are continuous time models. So you will find that most computer programs for calculating option values ask for the continuously compounded interest rate.

It may seem that a lot of calculations would be needed to find a continuously compounded interest rate. However, think back to your high school algebra. You may recall that as $m$ approaches infinity $[1 + (r/m)]^m$ approaches $(2.718)^r$. The figure 2.718—or $e$, as it is called—is the base for natural logarithms. Therefore, $1 invested at a continuously compounded rate of $r$ will grow to $e^r = (2.718)^r$ by the end of the first year. By the end of $t$ years it will grow to $e^{rt} = (2.718)^{rt}$.

**Example 1** Suppose you invest $1 at a continuously compounded rate of 11% ($r = .11$) for one year ($t = 1$). The end-year value is $e^{.11}$, or $1.116. In other words, investing at 11% a year *continuously* compounded is exactly the same as investing at 11.6% a year *annually* compounded.

**Example 2** Suppose you invest $1 at a continuously compounded rate of 11% ($r = .11$) for two years ($t = 2$). The final value of the investment is $e^{rt} = e^{.22}$, or $1.246.

Sometimes it may be more reasonable to assume that the cash flows from a project are spread evenly over the year rather than occurring at the year's end. It is easy to adapt our previous formulas to handle this. For example, suppose that we wish to compute the present value of a perpetuity of $C$ dollars a year. We already know that if the payment is made at the end of the year, we divide the payment by the *annually* compounded rate of $r$:

$$\mathrm{PV} = \frac{C}{r}$$

If the same total payment is made in an even stream throughout the year, we use the same formula but substitute the *continuously* compounded rate.

**Example 3** Suppose the annually compounded rate is 18.5%. The present value of a $100 perpetuity, with each cash flow received at the end of the year, is 100/.185 = $540.54. If

[10] When we talk about *continuous* payments, we are pretending that money can be dispensed in a continuous stream like water out of a faucet. One can never quite do this. For example, instead of paying out $1 billion every year to combat malaria, you could pay out about $1 million every 8¾ hours or $10,000 every 5¼ minutes or $10 every 3⅙ seconds but you could not pay it out *continuously*. Financial managers *pretend* that payments are continuous rather than hourly, daily, or weekly because (1) it simplifies the calculations and (2) it gives a very close approximation to the NPV of frequent payments.

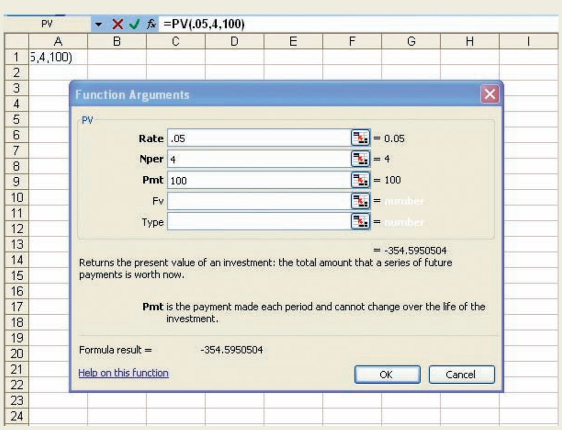
# USEFUL SPREADSHEET FUNCTIONS

## Discounting Cash Flows

Spreadsheet programs such as Excel provide built-in functions to solve discounted-cash-flow (DCF) problems. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel asks you for the inputs that it needs. At the bottom left of the function box there is a Help facility with an example of how the function is used.

Here is a list of useful functions for DCF problems and some points to remember when entering data:

- **FV:** Future value of single investment or annuity.
- **PV:** Present value of single future cash flow or annuity.
- **RATE:** Interest rate (or rate of return) needed to produce given future value or annuity.
- **NPER:** Number of periods (e.g., years) that it takes an investment to reach a given future value or series of future cash flows.
- **PMT:** Amount of annuity payment with a given present or future value.
- **NPV:** Calculates the value of a stream of negative and positive cash flows. (When using this function, note the warning below.)
- **XNPV:** Calculates the net present value of a series of unequal cash flows at the date of the first cash flow.
- **EFFECT:** The effective annual interest rate, given the quoted rate (APR) and number of interest payments in a year.




- **NOMINAL:** The quoted interest rate (APR) given the effective annual interest rate.

All the inputs in these functions can be entered directly as numbers or as the addresses of cells that contain the numbers. Three warnings:

1. PV is the amount that needs to be invested today to produce a given future value. It should therefore be entered as a negative number. Entering both PV and FV with the same sign when solving for RATE results in an error message.
2. Always enter the interest or discount rate as a decimal value.
3. Use the NPV function with care. It gives the value of the cash flows one period *before* the first cash flow and not the value at the date of the first cash flow.

### SPREADSHEET QUESTIONS

The following questions provide opportunities to practice each of the Excel functions.

**2.1** (FV) In 1880 five aboriginal trackers were each promised the equivalent of 100 Australian dollars for helping to capture the notorious outlaw Ned Kelly. One hundred and thirteen years later the granddaughters of two of the trackers claimed that this reward had not been paid. If the interest rate over this period averaged about 4.5%, how much would the A$100 have accumulated to?

**2.2** (PV) Your company can lease a truck for $10,000 a year (paid at the end of the year) for six years, or it can buy the truck today for $50,000. At the end of the six years the truck will be worthless. If the interest rate is 6%, what is the present value of the lease payments? Is the lease worthwhile?

**2.3** (RATE) Ford Motor stock was one of the victims of the 2008 credit crisis. In June 2007, Ford stock price stood at $9.42. Eighteen months later it was $2.72. What was the annual rate of return over this period to an investor in Ford stock?

**2.4** (NPER) An investment adviser has promised to double your money. If the interest rate is 7% a year, how many years will she take to do so?

**2.5** (PMT) You need to take out a home mortgage for \$200,000. If payments are made annually over 30 years and the interest rate is 8%, what is the amount of the annual payment?

**2.6** (XNPV) Your office building requires an initial cash outlay of \$370,000. Suppose that you plan to rent it out for three years at \$20,000 a year and then sell it for \$400,000. If the cost of capital is 12%, what is its net present value?

**2.7** (EFFECT) First National Bank pays 6.2% interest compounded annually. Second National Bank pays 6% interest compounded monthly. Which bank offers the higher effective annual interest rate?

**2.8** (NOMINAL) What monthly compounded interest rate would Second National Bank need to pay on savings deposits to provide an effective rate of 6.2%?

the cash flow is received continuously, we must divide \$100 by 17%, because 17% continuously compounded is equivalent to 18.5% annually compounded ($e^{.17} = 1.185$). The present value of the continuous cash flow stream is 100/.17 = \$588.24. Investors are prepared to pay more for the continuous cash payments because the cash starts to flow in immediately.

For any other continuous payments, we can always use our formula for valuing annuities. For instance, suppose that you have thought again about your donation and have decided to fund a vaccination program in emerging countries, which will cost \$1 billion a year, starting immediately, and spread evenly over 20 years. Previously, we used the annually compounded rate of 10%; now we must use the continuously compounded rate of $r = 9.53\%$ ($e^{.0953} = 1.10$). To cover such an expenditure, then, you need to set aside the following sum:[11]

$$\text{PV} = C\left(\frac{1}{r} - \frac{1}{r} \times \frac{1}{e^{rt}}\right)$$

$$= \$1 \text{ billion}\left(\frac{1}{.0953} - \frac{1}{.0953} \times \frac{1}{6.727}\right) = \$1 \text{ billion} \times 8.932 = \$8.932 \text{ billion}$$

If you look back at our earlier discussion of annuities, you will notice that the present value of \$1 billion paid at the *end* of each of the 20 years was \$8.514 billion. Therefore, it costs you \$418 million—or 5%—more to provide a continuous payment stream.

Often in finance we need only a ballpark estimate of present value. An error of 5% in a present value calculation may be perfectly acceptable. In such cases it doesn't usually matter whether we assume that cash flows occur at the end of the year or in a continuous stream. At other times precision matters, and we do need to worry about the exact frequency of the cash flows.

[11] Remember that an annuity is simply the difference between a perpetuity received today and a perpetuity received in year *t*. A continuous stream of *C* dollars a year in perpetuity is worth *C/r*, where *r* is the continuously compounded rate. Our annuity, then, is worth

$$\text{PV} = \frac{C}{r} - \text{present value of } \frac{C}{r} \text{ received in year } t$$

Since *r* is the continuously compounded rate, *C/r* received in year *t* is worth $(C/r) \times (1/e^{rt})$ today. Our annuity formula is therefore

$$\text{PV} = \frac{C}{r} - \frac{C}{r} \times \frac{1}{e^{rt}}$$

sometimes written as

$$\frac{C}{r}(1 - e^{-rt})$$

## SUMMARY

Firms can best help their shareholders by accepting all projects that are worth more than they cost. In other words, they need to seek out projects with positive net present values. To find net present value we first calculate present value. Just discount future cash flows by an appropriate rate $r$, usually called the *discount rate, hurdle rate,* or *opportunity cost of capital:*

$$\text{Present value (PV)} = \frac{C_1}{(1 + r)} + \frac{C_2}{(1 + r)^2} + \frac{C_3}{(1 + r)^3} + \cdots$$

Net present value is present value plus any immediate cash flow:

$$\text{Net present value (NPV)} = C_0 + \text{PV}$$

Remember that $C_0$ is negative if the immediate cash flow is an investment, that is, if it is a cash outflow.

The discount rate $r$ is determined by rates of return prevailing in capital markets. If the future cash flow is absolutely safe, then the discount rate is the interest rate on safe securities such as U.S. government debt. If the future cash flow is uncertain, then the expected cash flow should be discounted at the expected rate of return offered by equivalent-risk securities. (We talk more about risk and the cost of capital in Chapters 7 to 9.)

Cash flows are discounted for two simple reasons: because (1) a dollar today is worth more than a dollar tomorrow and (2) a safe dollar is worth more than a risky one. Formulas for PV and NPV are numerical expressions of these ideas.

Financial markets, including the bond and stock markets, are the markets where safe and risky future cash flows are traded and valued. That is why we look to rates of return prevailing in the financial markets to determine how much to discount for time and risk. By calculating the present value of an asset, we are estimating how much people will pay for it if they have the alternative of investing in the capital markets.

You can always work out any present value using the basic formula, but shortcut formulas can reduce the tedium. We showed how to value an investment that makes a level stream of cash flows forever (a *perpetuity*) and one that produces a level stream for a limited period (an *annuity*). We also showed how to value investments that produce growing streams of cash flows.

When someone offers to lend you a dollar at a quoted interest rate, you should always check how frequently the interest is to be paid. For example, suppose that a \$100 loan requires six-month payments of \$3. The total yearly interest payment is \$6 and the interest will be quoted as a rate of 6% compounded semiannually. The equivalent *annually compounded rate* is $(1.03)^2 - 1 = .061$, or 6.1%. Sometimes it is convenient to assume that interest is paid evenly over the year, so that interest is quoted as a continuously compounded rate.



## PROBLEM SETS

### BASIC

**1.** At an interest rate of 12%, the six-year discount factor is .507. How many dollars is \$.507 worth in six years if invested at 12%?

**2.** If the PV of \$139 is \$125, what is the discount factor?

**3.** If the cost of capital is 9%, what is the PV of \$374 paid in year 9?

**4.** A project produces a cash flow of \$432 in year 1, \$137 in year 2, and \$797 in year 3. If the cost of capital is 15%, what is the project's PV?

**5.** If you invest \$100 at an interest rate of 15%, how much will you have at the end of eight years?