SECTION 2.1: AN INTRODUCTION TO LIMITS

LEARNING OBJECTIVES

- Understand the concept of (and notation for) a limit of a rational function at a point in its domain, and understand that "limits are local."
- Evaluate such limits.
- Distinguish between one-sided (left-hand and right-hand) limits and two-sided limits – and what it means for such limits to exist.
- Use numerical / tabular methods to guess at limit values.
- Distinguish between limit values and function values at a point.
- Understand the use of neighborhoods and punctured neighborhoods in the evaluation of one-sided and two-sided limits.
- Evaluate some limits involving piecewise-defined functions.

PART A: THE LIMIT OF A FUNCTION AT A POINT

Our study of calculus begins with an understanding of the expression $\lim_{x \to a} f(x)$, where a is a real number (in short, $a \in \mathbb{R}$) and f is a function. This is read as:

"the limit of $f(x)$ as x approaches a."

- **WARNING 1:** $\to$ means "approaches." Avoid using this symbol outside the context of limits.

- $\lim_{x \to a}$ is called a limit operator. Here, it is applied to the function f.

$\lim_{x \to a} f(x)$ is the real number that $f(x)$ approaches as x approaches a, **if such a number exists**. If $f(x)$ does, indeed, approach a real number, we denote that number by L (for limit value). We say the limit **exists**, and we write:

$$\lim_{x \to a} f(x) = L, \quad \text{or} \quad f(x) \to L \text{ as } x \to a.$$

These statements will be **rigorously defined** in Section 2.7.