# **Dimensionality Reduction**

**Question 1**: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7,2/7,-3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that  $x^2+y^2+z^2=1$ . However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

### Ans:

$$2x + 3y + 6z = 0$$

$$6x + 2y - 3z = 0$$

$$14x + 7y = 0$$

$$Y = -2x$$

$$2x + 3(-2x) + 6z = 0$$

$$=> -4x + 6z = 0$$

$$Z = 2/3x$$

**Question 2**: Find the eigenvalues and eigenvectors of the following matrix:



You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

## Ans:

Eigenvalue = 11

Eigen vector =  $[1/\sqrt{10}, 3/\sqrt{10}]$ 

**Question 3**: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

Ans:

```
e = np.array([1,3,4,5,7])
e_unit = e / np.sqrt(np.dot(e,e))
e_unit
array([0.1, 0.3, 0.4, 0.5, 0.7])
```

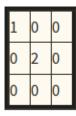
**Question 4**: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

#### Ans:

$$M' * M = [[14, 17], [17, 21]]$$

14	17
17	21

Question 5: Consider the diagonal matrix M =



Compute its Moore-Penrose pseudoinverse.

# Ans:

1/1	0	0
0	1/2	0
0	0	0

**Question 6**: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

# Ans:

The four rows of a matrix have squared Frobenius norms of 14, 77, 194, 365, respectively. Thus, their respective probabilities are 0.02, 0.12, 0.29, and 0.56