

Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is $[2/7, 3/7, 6/7]$, and another is $[6/7, 2/7, -3/7]$. Let the third column be $[x, y, z]$. Since the length of the vector $[x, y, z]$ must be 1, there is a constraint that $x^2 + y^2 + z^2 = 1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

Ans:

$$2x + 3y + 6z = 0$$

$$6x + 2y - 3z = 0$$

$$14x + 7y = 0$$

$$Y = -2x$$

$$2x + 3(-2x) + 6z = 0$$

$$\Rightarrow -4x + 6z = 0$$

$$Z = 2/3x$$

Question 2: Find the eigenvalues and eigenvectors of the following matrix:

2	3
3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

Ans:

$$\text{Eigenvalue} = 11$$

$$\text{Eigen vector} = [1/\sqrt{10}, 3/\sqrt{10}]$$

Question 3: Suppose $[1, 3, 4, 5, 7]$ is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

Ans:

```
e = np.array([1,3,4,5,7])
e_unit = e / np.sqrt(np.dot(e,e))
e_unit

array([0.1, 0.3, 0.4, 0.5, 0.7])
```

Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

Ans:

$$M' * M = \begin{bmatrix} 14 & 17 \\ 17 & 21 \end{bmatrix}$$

14	17
17	21

Question 5: Consider the diagonal matrix M =

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.

Ans:

1/1	0	0
0	1/2	0
0	0	0

Question 6: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

Ans:

The four rows of a matrix have squared Frobenius norms of 14, 77, 194, 365, respectively. Thus, their respective probabilities are 0.02, 0.12, 0.29, and 0.56