Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7, 2/7, -3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that $x^2+y^2+z^2 = 1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

Ans:

2x + 3y + 6z = 0

6x + 2y - 3z = 0

14x + 7y = 0

Y = -2x

2x + 3(-2x) + 6z = 0

=> -4x + 6z = 0

Z = 2/3x

Question 2: Find the eigenvalues and eigenvectors of the following matrix:

2	3
3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

Ans:

2-λ	3
3	10-λ

$$(2-\lambda)(10-\lambda)-9=0$$

$$20-2\lambda-10\lambda+\lambda^2-9$$

$$\lambda^2-12\lambda+11$$

$\lambda = 11, \lambda = 1$

$2x + 3y = 11x; 3y = 9x; y = 3x$

$3x + 10y = 11y; 3x = y, y = 3x$

$[1,3] => [1/\sqrt{10}, 3/\sqrt{10}]$

Therefore, eigenvalue = 11, and eigen vector = $[1/\sqrt{10}, 3/\sqrt{10}]$

Question 3: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

Ans:

```
e = np.array([1,3,4,5,7])
e_unit = e / np.sqrt(np.dot(e,e))
e_unit
```

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array([0.1, 0.3, 0.4, 0.5, 0.7])
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Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

Ans:

$M^T * M$ = [[14 ,17], [17, 21]]

14	17
17	21

Question 5: Consider the diagonal matrix M =

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.

***Ans*:**

1/1	0	0
0	1/2	0
0	0	0

Question 6: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

***Ans*:**

The four rows of a matrix have squared Frobenius norms of 14, 77, 194, 365, respectively. Thus, their respective probabilities are 0.02 (14/650), 0.12 (77/650), 0.29 (194/650), and 0.56 (365/650)