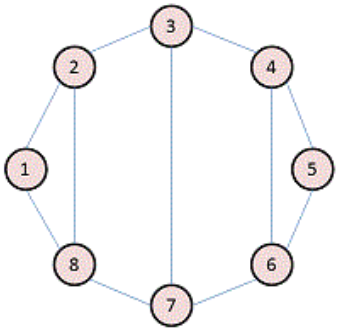
Communities

Question 1:

For the following graph:



Write the adjacency matrix A, the degree matrix D, and the Laplacian matrix L. For each, find the sum of all entries and the number of nonzero entries.

Ans:

Adjacency matrix A

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	**0**	1	0	0	0	0	0	1
2	**1**	0	1	0	0	0	0	1
3	**0**	1	0	1	0	0	1	0
4	**0**	0	1	0	1	1	0	0
5	**0**	0	0	1	0	1	0	0
6	**0**	0	0	1	1	0	1	0
7	**0**	0	1	0	0	1	0	1
8	**1**	1	0	0	0	0	1	0

Sum of all entries: 22

Number of nonzero entries: 22

Degree Matrix D

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	**2**	0	0	0	0	0	0	0
2	**0**	3	0	0	0	0	0	0
3	**0**	0	3	0	0	0	0	0
4	**0**	0	0	3	0	0	0	0
5	**0**	0	0	0	2	0	0	0
6	**0**	0	0	0	0	3	0	0
7	**0**	0	0	0	0	0	3	0
8	**0**	0	0	0	0	0	0	3

Sum of all entries: 22

Number of nonzero entries: 8

Laplacian Matrix L

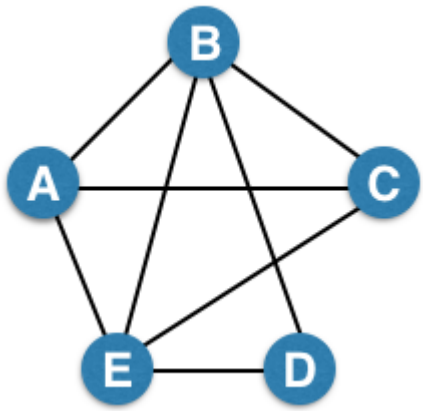
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	**2**	-1	0	0	0	0	0	-1
2	**-1**	3	-1	0	0	0	0	-1
3	**0**	-1	3	-1	0	0	-1	0
4	**0**	0	-1	3	-1	-1	0	0
5	**0**	0	0	-1	2	-1	0	0
6	**0**	0	0	-1	-1	3	-1	0
7	**0**	0	-1	0	0	-1	3	-1
8	**-1**	-1	0	0	0	0	-1	3

Sum of all entries: 0

Number of nonzero entries: 30

Question 2:

Consider the following undirected graph (i.e., edges may be considered bidirectional):



Run the "trawling" algorithm for finding dense communities on this graph and find all complete bipartite subgraphs of types $K_{3,2}$ and $K_{2,2}$. Note: In the case of $K_{2,2}$, we consider {{W, X}, {Y, Z}} and {{Y, Z}, {W, X}} to be identical.

Ans:

First step: get the adjacent edges

A = {B,C,E}

B = {A,C,D,E}

C = {A,B,E}

D = {B,E}

E = {A,B,C,D}

Type $K_{3,2}$

{{A,C,D}, {B,E}}

Type $K_{2,2}$

{{A,E}, {B,C}}

{{A,B}, {C,E}}

{{B,E},{A,C}}

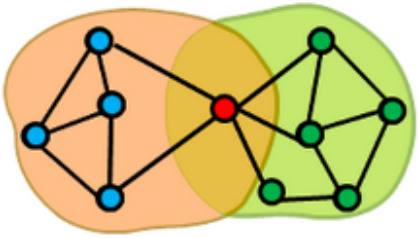
{{B,E}, {A,D}}

{{B,C},{A,E}} **not included as it is identical to {{A,E}, {B,C}}**

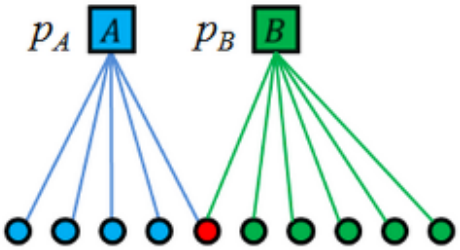
{{B,E},{C,D}}

Question 3:

We fit AGM to the network on the left, and found the parameters on the right:



Network



Learned AGM parameters

Find the optimal values for p_A and p_B

Ans:

$p_{A = 0.7}$

$p_{B = 0.6}$