Using Bayesian Models to Approximate Human Risk Behavior in BART Task

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Abstract

In this project, we hope to study how people make decisions when faced with uncertainty, and whether human intuition reflects Bayesian inference. To do so, we built a game where participants have to learn the underlying distribution of the data and make decisions under certainty. We introduced a risk score as well as Bayesian model with varying horizon and decay rates to compare to human data. We performed 3 experiments and found that 1) there was an insignificant different in participants' scores when introducing loss aversion, 2) a horizon of 2 and discount rate of 0 in a Bayesian model was most consistent with participant game play, and 3) shifting a hypothesis space had an insignificant effect on future behavior. Future experiments would focus on improving the game interface and Bayesian models to further understand human behavior in risk taking. [1].

Keywords: Bayesian Decision Making; Human Decisions Under Risk; Loss Aversion;

Introduction

The fundamental phenomenon we hope to study is how people make decisions when faced with risk and uncertainty, as well as how they learn from new information to inform their decisions. People face risk and uncertainty everyday, ranging from small decisions like eating a tasty vs a healthy breakfast, to bigger decisions like choosing between career paths, buying insurance for belongings, personal investments and the greater exploration vs exploitation paradox applicable to many real life scenarios. Thus, making decisions under risk and uncertainty is an important cognitive skill. Understanding the behavior of cognitive science decision making under uncertainty can help understand and quantify how risk is incorporated in decision making processes, and potentially be utilized to build policy and tools that can aid better decision making.

In this study, we approximate risk in the framework of the Balloon Analogue Risk Task (BART) task, which is a game that measures the risk tendencies of individuals (C. W. Lejuez et al., 2002). The game presents individuals with a number of balloons that explode at certain points, and participants are given the task to maximize their score (money). At each point in the game, participants have 2 possible actions:

- Pumping air into the current balloon. If the balloon pops, they lose all the money they have on this balloon, and if it doesn't, their money increases by 1.
- Moving on to the next balloon and collecting the current amount of money they have.

Risk-taking behavior can be defined as that involve potential for harm while also providing an reward. Intuitive in this scenario, pumping air into the balloon is inherently risky. In (C. W. Lejuez et al., 2002), risk was approximated through the number of pumps participants added to balloons. In this work, we hope to expand on the foundations of BART presented, with the following goals

- Constructing a Bayesian model in the BART game to create a better approximation for risk
- Compare human performances in the BART game to Bayesian models to inform ourselves about the similarity and differences of human cognition in risk

Related Work

The Balloon Analogue Risk Task (BART) proposes a way to measure risk taking tendencies in individuals (C. W. Lejuez et al., 2002). Further research has demonstrated that it's an effective test to assess real world risk taking tendencies and smoking (C. W. Lejuez, Aklin, Zvolensky, & Pedulla, 2003) (C. Lejuez et al., 2003). Since the BART tests measure people's risk-taking tendencies, "the cognitive processes at work during the task are likely to be comparable to those used during real-world risky decision making." (Helfinstein, 2014), making this test a good model for us to study human decision making.

In addition, drawing on many of the concepts covered from this course, our goal was to expand on the BART game to study Bayesian inference in humans. Inspired by the Number Game as presented in lecture, we wanted to study how humans can infer probability distributions, especially in a game dealing with uncertainty. As far as we know, there have been no previous studies examining human Bayesian reasoning in the context of BART, thus motivating our study.

Methods

We created our version of the BART game and studied how participants infer probability distributions in a game dealing

[1] Project code can be found at https://github.com/Pen721/9.66-Final-Project

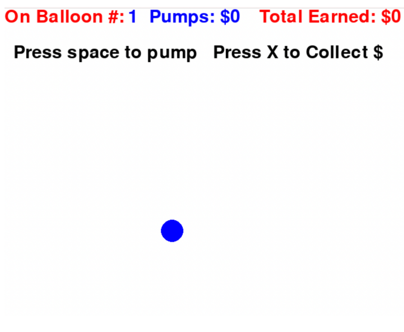

Figure 1: The interactive user interface for our BART game at the start of game play

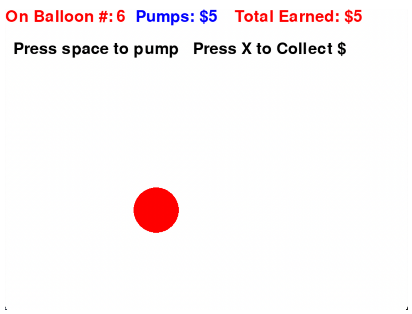

Figure 2: The interactive user interface for our BART game mid-game.

with uncertainty through various experiments across 30 experiment runs.

Game

We built our version balloon game using Python's PyGame package that imitates the original game interface presented in (C. W. Lejuez et al., 2002) shown in figure-2. We used keyboard controls for the popping and passing actions of the game, and introduced several varieties. As shown in Figure 1 and Figure 2 below, note that the balloon enlarges with the number of pumps. Additionally, the color of the balloon changes between even and odd balloons to denote a new balloon.

Probability Generation

To study how participants reacted to balloons generated by different probabilities, we built our own classes for generating balloons with the following distributions:

- Gaussian with integer mean ∈ [0, 10] and std ∈ [1, 3]
- Uniform distributions over [a, b] with a ¡ b and both integers between 1 and 10
- Limit distributions, a special case of uniform where all the balloons pop at a integer between 1 and 10
- Geometric distributions, which has a popping probability of p ∈ [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9] on each pump

Each balloon will pop after a maximum of 10 pumps (although for any individual balloon the number of pumps to pop it may be lower). Notably, participants will not be given this upper bound of 10 due to beta testing revealing that knowing this number anchored participants as compared to analyzing game data.

Bayesian Agent

Estimating Popping Probability We created a Bayesian Agent B that makes decisions using Bayesian logic to test against human decisions. The B has hypothesis space H with possible distributions $p(x)$ that generative the chance a balloon pops. Since this game is discrete, we note that a balloon has a popping point P if it pops once it reaches this size, and the probability of this balloon is

$$cdf(P-1,P) = \int_{P-1}^{P} p(x)\,dx$$

Similarly, the chance of observing a balloon that hasn't popped at size P is

$$cdf(P+1, maxsize) = \int_{P}^{maxsize} p(x)\,dx$$

At any given point in the game, the Bayesian agent has current balloon at size x_n past observations $obs = [(x_1, Popped), (x_2, Passed), (x_3, Popped)...(x_{n-1}, passed)]$, where x_i is the observed size of the ith balloon, and $Popped$ means the balloon popped at size x_i, and $Passed$ means the agent previously passed on this decision.

Based on the observations, the chance that a given hypothesis $h \in H = p(x)$ is the underlying distribution is therefore

$$P(h|obs) = \frac{P(obs|h) \cdot P(h)}{P(obs)}$$

$$P(h|obs) = \frac{\prod_{i=1}^{\infty} P(obs_i|h) \cdot \frac{1}{|H|}}{\sum_{h \in H} P(obs|h)}$$

where

$$P(obs_i|dist) = \begin{cases} cdf(x_i - 1, P), & obs_i popped \\ cdf(x_i, maxsize), & obs_i passed \end{cases}$$

Then, the Bayesian model estimates the chance of the current balloon popping on the next pump:

$$P(pop|x_n, h) = \frac{\int_{P-1}^{P} p(x)\,dx}{\int_{x_n}^{maxsize} p(x)\,dx}$$

And calculates the overall popping probability as

$$P(pop) = \sum_{h \in H} p(pop|x_n, h) \cdot p(h|obs)$$

Estimating Expected Value To calculate the expected value gained from popping the balloon, we used a dynamic programming approach to maximize an agent's policy over popping or passing at a given point. We defined a state $s = (balloonindex, size, score, horizon, obs)$. At each given state, the agent first approximates the chance of the balloon popping, and calculates the expected score S increase from pumping the balloon for horizon number of iterations. [2]

$$E[S, pump] = P(pop) \cdot -size + s(balloonindex + 1, size = 0, \\ score = 0, horizon - 1, obs + (size, pop)) + ((1 - P(pop) \\ \cdot s(balloonindex + 1, size + 1, score + 1, horizon - 1, obs))$$

and

$$E[S, pass] = 0 + s(balloonindex + 1, size = 0, score, \\ horizon - 1, obs + (size, pop))$$

at horizon 1, the agent simplify estimates the utility in the next pump:

$$max(E[S, pass] = 0, E[S, pump] = P(pop) \cdot -size + \\ (1 - P(pop)) \cdot 1)$$

At each evaluation, agent returns the action between pass and pump that returns the maximum expected utility, $max(E[S, pop], andE[s, pass])$ after evaluating across all future horizons. Note that a quick observation is that passing on a balloon gives 0 return, so it is always advantageous to pump on a balloon with size 0, since $-score$ starts off at 0, the expected utility from bumping at first is always positive.

Decay We introduce a decay variable d denoting how much information from subsequent horizons is discounted when we do our expected value calculation. For example, if $d = 1$, then it means values calculated from future horizons have no impact on our expected utility, where as $d = 0$ means we trust future value iterations just as much as we trust our current estimate, adding them together directly. Figure-5 shows the amount of points scored by a agent with Gaussian prior with various decay and horizons. Overall, we found that different

$$E[S, pop] = P(pop) \cdot -size + s(balloonindex + 1, size = 0, \\ score = 0, horizon - 1, obs + (size, pop)) + s((1 - P(pop) \\ (balloonindex + 1, size + 1, score + 1, horizon - 1, obs))$$

[2]note that horizon = 0 means agent would be making random moves between pass and pumping.

	Bayesian	Loss	Gain
Average Risk Point	0	5.3	6.8

Figure 3: Average risk points from experiment 1

Risk Taking Behavior

We used our model as a benchmark to measure risk taking behavior of individuals across a variety of situations. Instead of using the definition of risk presented in (C. Lejuez et al., 2003) as the total amount of pumps, we defend a system of "risk points". A player gains one risky point if the participant decided to pump the balloon when a Bayesian model passed, and they reduce one risky point if they passed before the Bayesian model did, and so, an individual is risk averse if they have a negative score, and risk loving if they have a positive score.

Results

We collected data points from 30 experiment game plays and listed the raw human data in appendix-figures-16 17 18 19 Before each experiment run, we will record the participant's gender, course, and age for demographic analysis. We ran 3 variations of experiments on each participant to test various hypotheses:

Loss Aversion

Our first experiment tested for the potential impact of loss aversion on participant's behavior in the game. To do this, we created two versions of the BART game. In the first version, participants were presented with a version of the game where the potential earnings from the current balloon (On the interface, Pumps:) are not included in the Total Earned amount. Please find the instructions shown to the participants included in the Appendix-10.

In the second version, the current potential earnings included in Total Earned such that having the balloon pop decreases the current total earnings on the screen. In theory, loss aversion means that individuals will value their score more and be less risky in the case. Please find the instructions shown to the participants included in the Appendix-11.

Our hypothesis with this first experiment is that when experience loss, individuals would be less risky since the cost of getting their points deducted is bigger than never receiving the points in the first place. However, our experimental data in table-3 suggests that the average risk point across the two groups weren't too different.

Standardized Risk Assessment

In our second experiment, we tested all participants against the same probability distribution to standardize the risk behavior of individuals. To measure risk, we compare the models individuals made and when they differed from Bayesian models significantly.

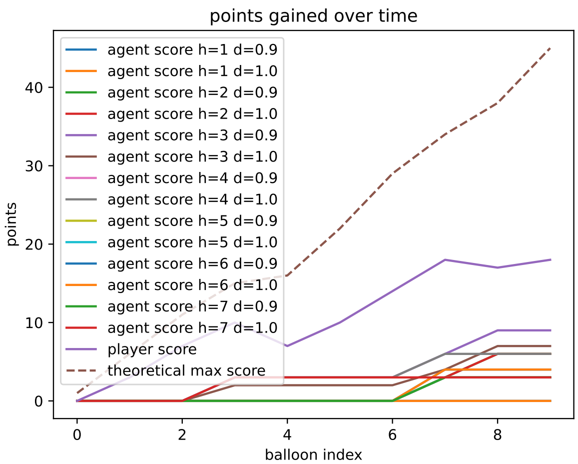

Figure 4: Human data compared with model output in standard distribution tested on all participants

Please find the instructions shown to the participants included in the Appendix-12. We utilized results from this experiment to fit parameters on the Bayesian model, with figure-5

Hypothesis Space Differences Finally, in our third experiment we will test participants playing the game with and without sample probability distributions for the maximum balloon size. The goal of this experiment is to test whether participants seeing these distributions affects their game behavior. Please find the instructions shown to the participants included in the Appendix-13. Through our experiments, we found that individuals did perform more similarly to the Bayesian model after seeing the graphs qualitatively. In our experiments, we observed that showing participants graphs made them behave more similarly to the Bayesian model performance, and the average risk score difference between the two categories was around 1.

Bayesian Model Parameters

Decay and Horizon Values We tried to fit a decay value to our human data, figure-5 is an example illustrating horizon and decay on the final Bayesian model effects the model prediction compared to human data in a Gaussian distribution balloon sample. In figure-6, we compare the performance of a human player against models with a variety of horizons and decay rates.

After evaluation of human data over experiments, we found that a horizon of $h = 2$ and decay rate of $d = 0$ or $d = 0.1$ fit what most closely resembled human decisions, which meant the model heavily valued future information with very little discount rate.

Varying Horizon We initially expected a Bayesian model with a large horizon to perform more similarly to individuals, as a large horizon would be able to internalize the value of

Figure 5: Amount of points model gained with varying horizon and decay, on the observation = [1, 5, 5, 4, 1, 6, 7, 5, 4, 7] generated by underlying function $\mathcal{N}(4,4)$

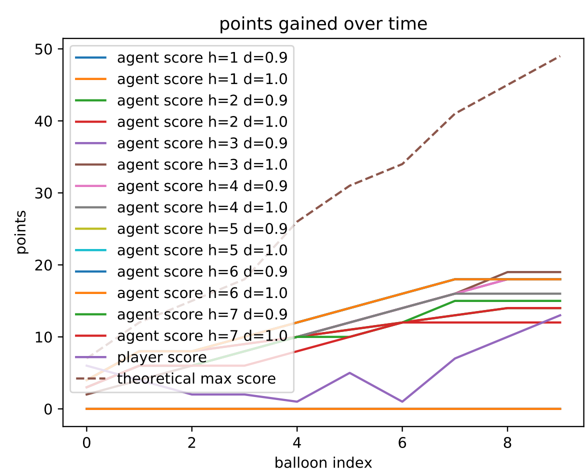

Figure 6: Player score compared to agent scores over the course of the game in experiment 3

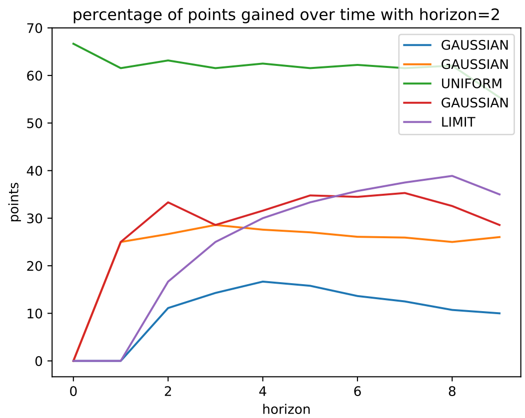

Figure 7: Average percentage of total points gained over game play, horizon = 2

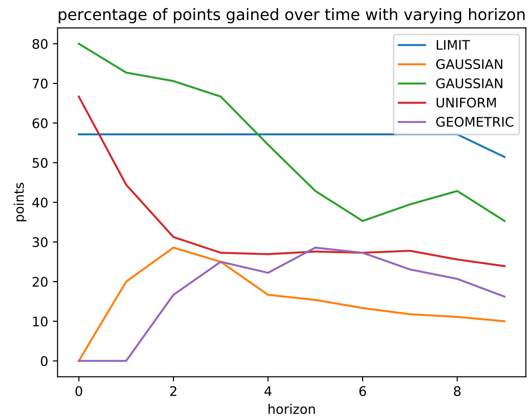

Figure 8: Percentage of points gained over game play, Bayesian model with varying horizon 7 before index 3, 2 afterwards)

	Human	Varying Horizon	Horizon=2
Score Percentage	36.3 %	32.5 %	30.7 %
Percentage On Gaussian Distribution	36.8 %	33.3 %	35.3 %

Figure 9: Percentage of points gained across different models

exploration in expected utility calculations. However, in table-15, we found that models with large horizons often learned the wrong distribution.

Motivated by the fact that many individuals first explore then exploit (for example, popping the first 3 balloons for the sake of learning the distribution and optimizing afterwards), we implemented a Bayesian model with varying horizon. This model starts off with a large expected utility horizon (n=7) for the sake of exploration, and changed to a small horizon (n=2) once it has acquired sufficient information. In figure-7, we illustrate the learning outcome of a Bayesian model with constant horizon $h = 2$ of various distributions, and in figure-8 we illustrate the learning outcome of a Bayesian model with varying horizon.

To quantitatively compare the models, we illustrated the average performance of models compared in humans across all data, and found that both models compared similarly in both average score percentage and score percentage when dealing with a Gaussian distribution [3].

In addition, please refer to the Appendix for 15, which shows the Model Inference of Distributions Seeing Balloons Generated by N(4,4).

Discussion

In this project, we built a version of the BART game and compared human performance to Bayesian model decisions to measure risk and learn about the underlying decision process when dealing with risk. We conducted the 3 following experiments and gathered data from participants to analyze:
Firstly, we conducted an experiment aiming to study the effects of risk aversion in BART, and the findings was that although introducing a loss did decrease the relative risk score of participants compared with those who just lost potential points, the difference wasn't significant to form a conclusion. Possible reasons for this could be that since we still showed the balloon size on screen for participants playing the version where they didn't lose points explicitly still tracked their balloon size and implicitly counted it as a loss, or because we didn't align enough incentives to create an environment where they would value the points enough to experience loss aversion. A possible future step could be conducting the same experiment but making UI changes to the game as well as providing physical prizes for participants to better study loss aversion under stress and stakes.
In the second experiment, we tested the same distribution against all participants to understand which parameters in Bayesian models most corresponded to our participants and found that a horizon of 2 and discount rate of 0 was most consistent with what we observed across participants, as well as tested the hypothesis of a varying horizon. In the future, we'd like to improve on the varying horizon method to inform Bayesian models to adjust the planning horizon as a parameter of their observations to potentially create a way to improve model performance.
In the third experiment, we tested the idea of shifting a hypothesis space and showing participants two sets of instructions to see what perturbation it would cause to participants. However, we found that it didn't cause a huge difference in participant behavior. This could be due to the fact that our sample of distributions wasn't a significant portion of the hy-

[3] We choose to highlight Gaussian here as it is a entropy-maximizing distribution

pothesis space (we showed them 4 out of 150 possible distributions), or that the participants already initially had a similar hypothesis space to what we constructed. An interesting observation we had in our experiments was that many participants would construct a possible distribution and then shift it based on new information, which may be fundamentally different from how our model works, which is systematically evaluating across all possibilities. As a possible future step, we'd like to investigate how humans cognitively update the parameters of distributions and make efficient computations across horizons, and how different sampling algorithms like MCMC could be applied to improve our model.

Author Contributions

Penny: Mainly responsible for designing and coding the Bayesian model framework including the normal Bayesian model, varieties of the model with varying horizon, non loss averse Bayesian model (not introduced in this project since it needed a large amount of computations that wasn't doable with CPU), various tests for evaluating the model, experiment design (experiment, finding people, writing instructions), data storage (storing /retrieving), data analysis / processing methods like risk score and running experiments, model fitting, made graphs / figures, and collected data from participants and wrote manuscript along with Caroline.

Caroline: Mainly responsible for building and testing the BART game. Game included both user interface design as well as participant data storage (in collaboration with Penny). Along with Penny, I interviewed participants, worked on experiment design, wrote participant Experiment instructions, did a literature review on BART as a metric for risk aversion, and wrote manuscript.

Note that our data collection method was tough because we had to make participants play the game on our computer and write our own processing code to store and retrieve it.

Acknowledgments

Thank you to the Course Staff for a great semester!

References

Helfinstein, S. T. C. E. K. K. H. M. J. A. S. F. W. C. T. D. L. E. D. B. R. M. . P. R. A., S. M. (2014). Predicting risky choices from brain activity patterns. *Proceedings of the National Academy of Sciences*, *111*(7), 2470-2475.

Lejuez, C., Aklin, W. M., Jones, H. A., Richards, J. B., Strong, D. R., Kahler, C. W., & Read, J. P. (2003). The balloon analogue risk task (bart) differentiates smokers and nonsmokers. *Experimental and clinical psychopharmacology*, *11*(1), 26.

Lejuez, C. W., Aklin, W. M., Zvolensky, M. J., & Pedulla, C. M. (2003). Evaluation of the balloon analogue risk task (bart) as a predictor of adolescent real-world risk-taking behaviours. *Journal of adolescence*, *26*(4), 475–479.

Lejuez, C. W., Read, J. P., Kahler, C. W., Richards, J. B., Ramsey, S. E., Stuart, G. L., … Brown, R. A. (2002). Evaluation of a behavioral measure of risk taking: the balloon analogue risk task (bart). *Journal of Experimental Psychology: Applied*, *8*(2), 75.

EXP1.1
You have 10 balloons in front of you. Each balloon will pop after a defined (but unknown to you!) amount of pumps and a max size (by which all balloons will pop). The points in which the balloons pop might be generated by some sort of underlying distribution that's unknown, your goal is to maximize the total amount of money you collect by the end!

At each point in the game, you have the following options:

1. You can press space ( ) to pump, in which case if the balloon doesn't pop, your score increases by 1!
   a. If the balloon pops, you **lose** all the money you have gathered from the current balloon, and transition to the next balloon
2. You can press X to to collect all the money at this given point, in which case you collect all the points you earned

Best of luck!

Figure 10: Instructions for Experiment 1 first run.

EXP1.2
You have 10 balloons in front of you. Each balloon will pop after a defined (but unknown to you!) amount of pumps and a max size (by which all balloons will pop). The points in which the balloons pop might be generated by some sort of underlying distribution that's unknown, your goal is to maximize the total amount of money you collect by the end!

At each point in the game, you have the following options:

3) You can press space ( ) to pump, in which case your balloon increases in size by 1
4) You can press up to to collect all the money at this given point, in which case you collect money equal to the size of the balloon
   a) if the balloon size is 3, you earn 3 for passing before the balloon pops!

Best of luck!

Figure 11: Instructions for Experiment 1 second run.

Appendix

Experiment 1 human data from experiment with loss aversion 16 and without loss aversion. 17
Experiment 2 human data, where all participants tested against the same observation. 18

EXP2
You have 10 balloons in front of you. Each balloon will pop after a defined (but unknown to you!) amount of pumps and a max size (by which all balloons will pop). The points in which the balloons pop might be generated by some sort of underlying distribution that's unknown, your goal is to maximize the total amount of money you collect by the end!

At each point in the game, you have the following options:

3. You can press space ( ) to pump, in which case if the balloon doesn't pop, your score increases by 1!
   a. If the balloon pops, you **lose** all the money you have gathered from the current balloon, and transition to the next balloon
4. You can press up to to collect all the money at this given point, in which case you collect all the points you earned

Best of luck!

Figure 12: Instructions for Experiment 2.

EXP3.2
You have 10 balloons in front of you. Each balloon will pop after a defined (but unknown to you!) amount of pumps and a max size (by which all balloons will pop). The points in which the balloons pop might be generated by some sort of underlying distribution that's unknown but we have **attached some examples here**, your goal is to maximize the total amount of money you collect by the end!

At each point in the game, you have the following options:

7. You can press space ( ) to pump, in which case if the balloon doesn't pop, your score increases by 1!
   a. If the balloon pops, you **lose** all the money you have gathered from the current balloon, and transition to the next balloon
8. You can press up to to collect all the money at this given point, in which case you collect all the points you earned

Best of luck!

Figure 13: Instructions for Experiment 3.

Ballon size(x) and probabilities(y):

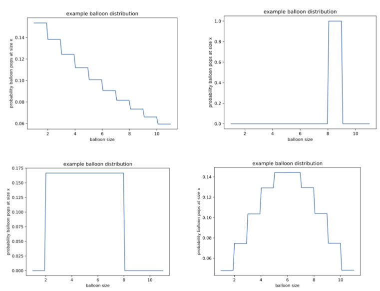

Figure 14: Distributions shown to participants second trial of Experiment 3

Decay	1	2	3	4	5	6	7
0.1	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$
0.2	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$
0.3	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$
0.4	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$
0.5	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$
0.6	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$	$\mathcal{N}(4,4)$
0.7	$\mathcal{N}(4,4)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$
0.8	$\mathcal{N}(4,4)$	$\mathcal{N}(3,9)$	$\mathcal{N}(2,9)$	$\mathcal{N}(2,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,9)$
0.9	$\mathcal{N}(4,4)$	$\mathcal{N}(2,9)$	$\mathcal{N}(1,9)$	$\mathcal{N}(1,9)$	$\mathcal{N}(3,4)$	$\mathcal{N}(3,9)$	$\mathcal{N}(3,4)$
1.0	$\mathcal{N}(4,4)$	$\mathcal{N}(1,9)$	$\mathcal{N}(1,9)$	$\mathcal{N}(1,9)$	$\mathcal{N}(2,9)$	$\mathcal{N}(3,4)$	$\mathcal{N}(2,9)$

Figure 15: Model Inference of Distributions Seeing Balloons Generated by N(4,4)

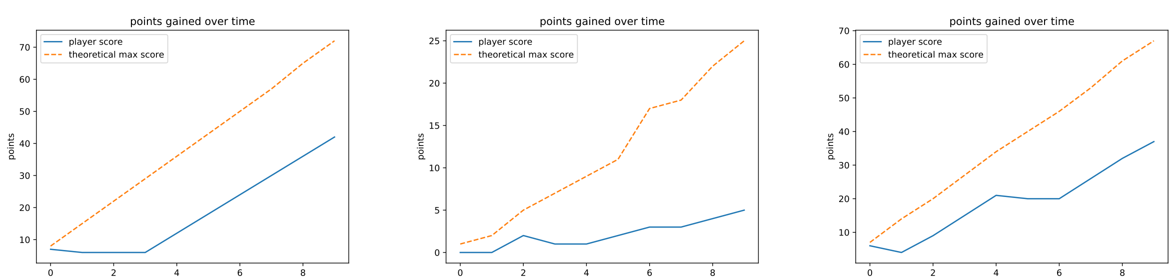

Figure 16: Human data from experiment 1 with lossAversion

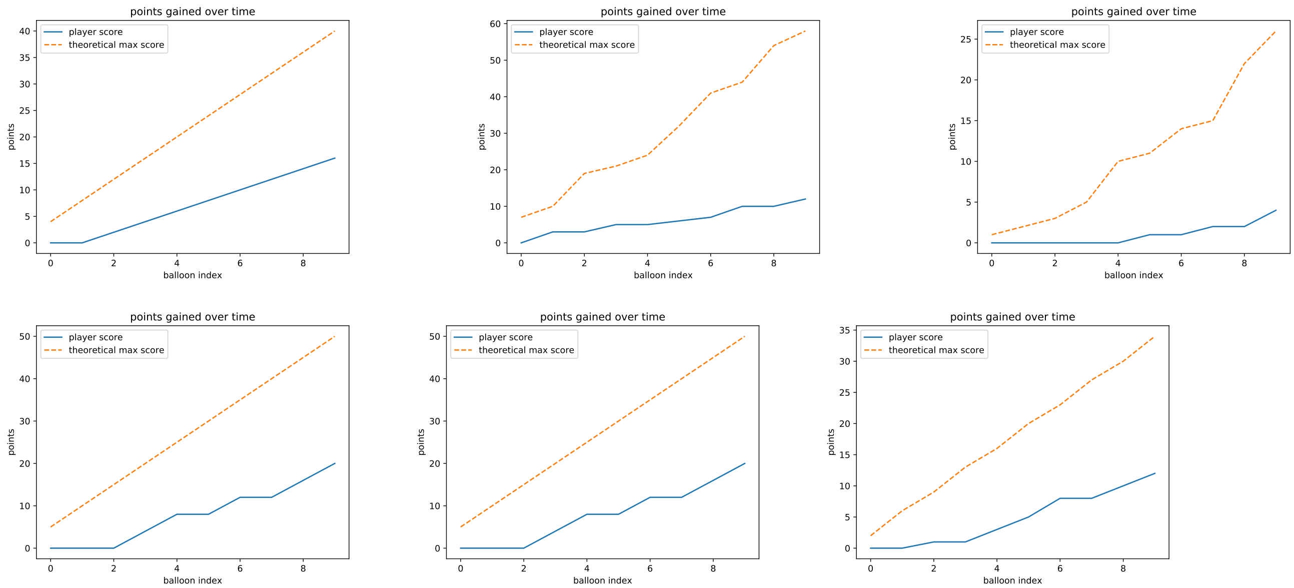

Figure 17: Human data from experiment 1 with no lossAversion

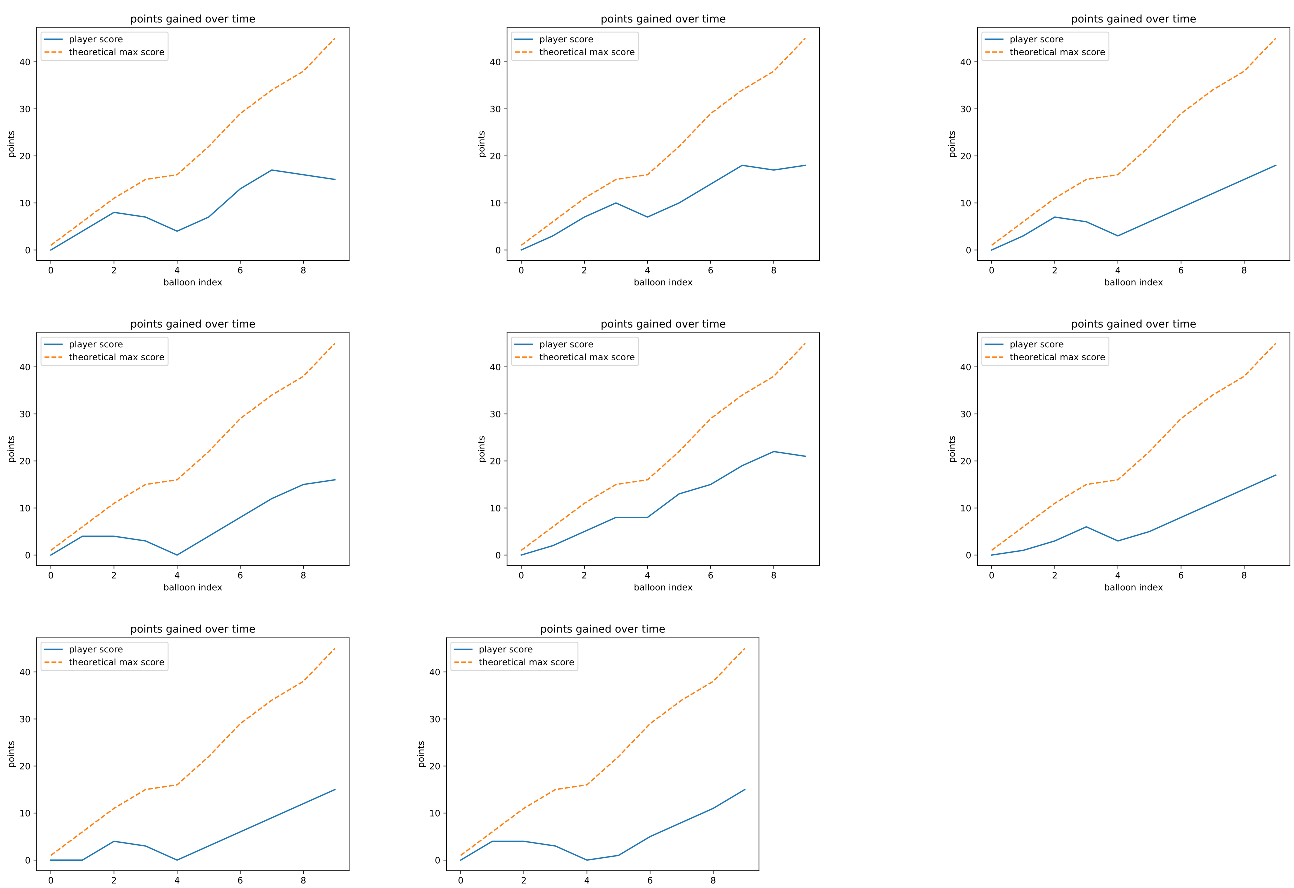

Figure 18: Human data from experiment 2

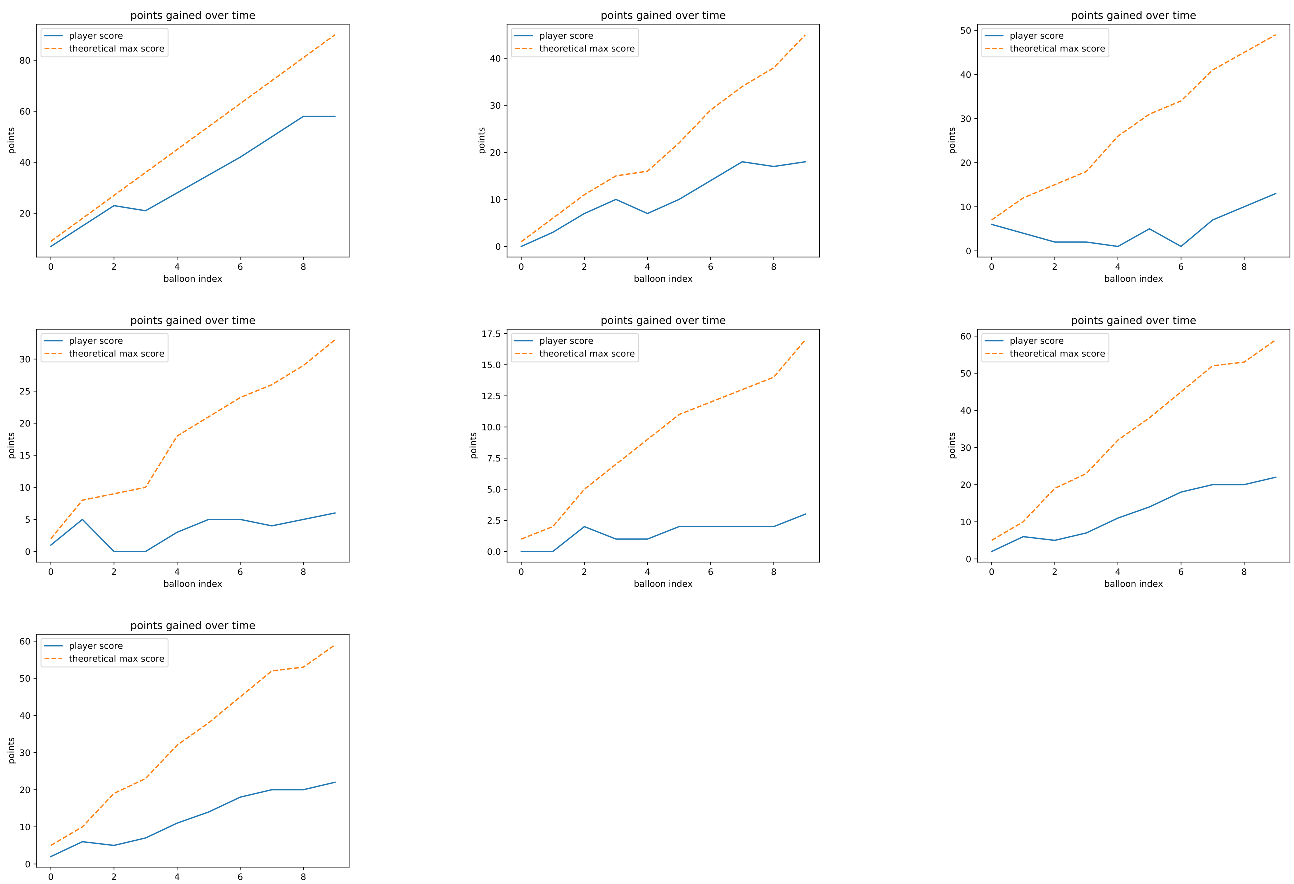

Figure 19: Human data from experiment 3

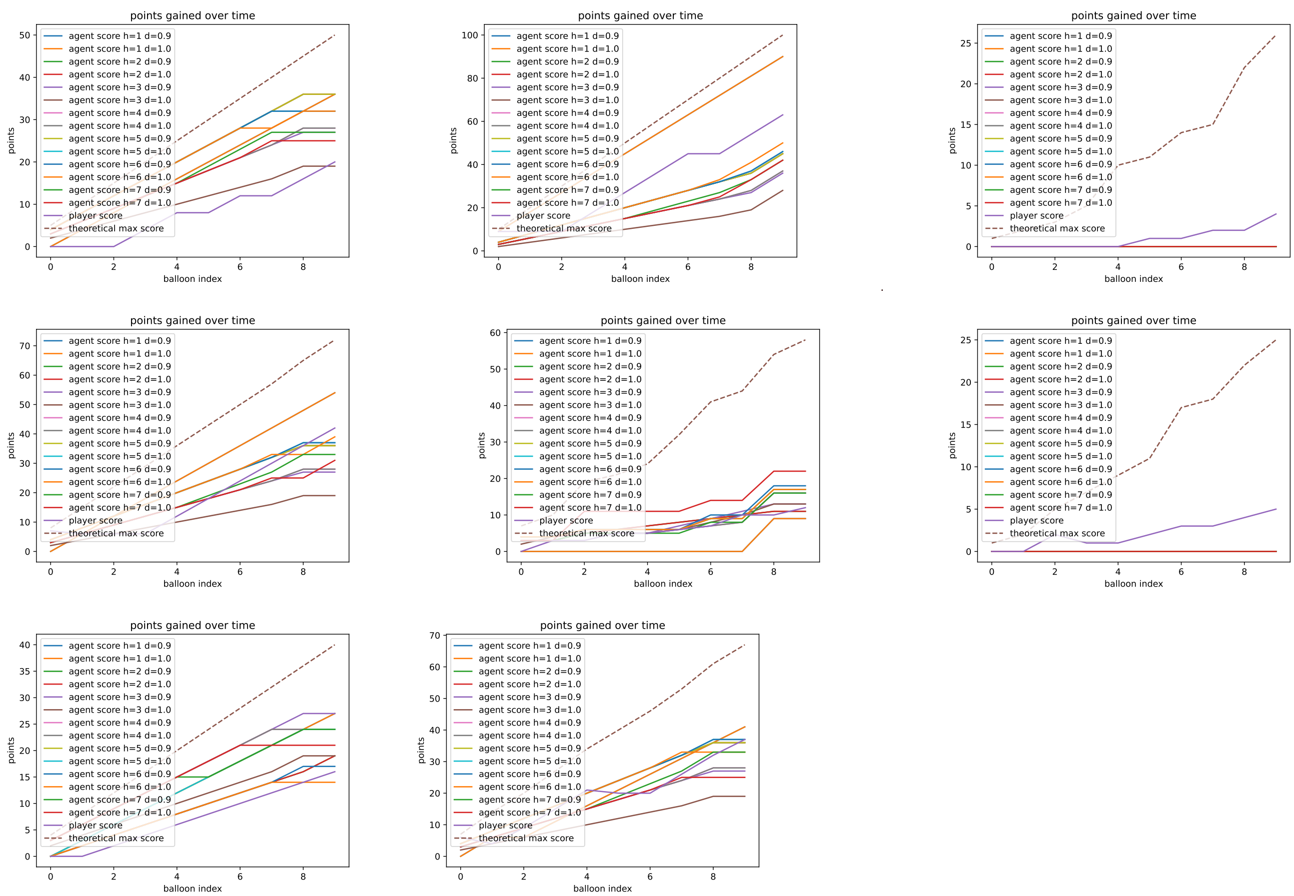

Figure 20: Experiment 1 Agent vs Player Graphs

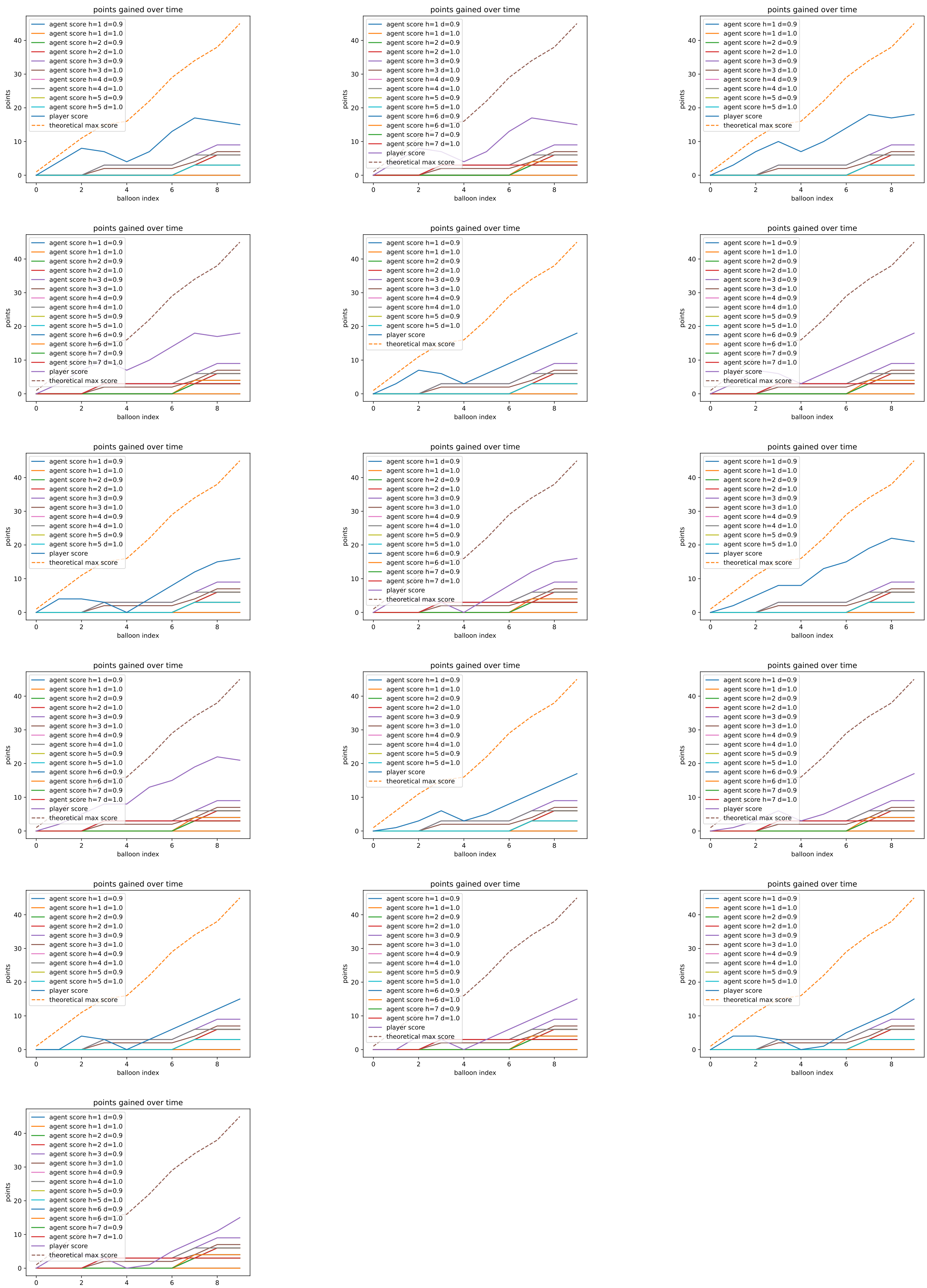

Figure 21: Experiment 2 Agent vs Player Graphs

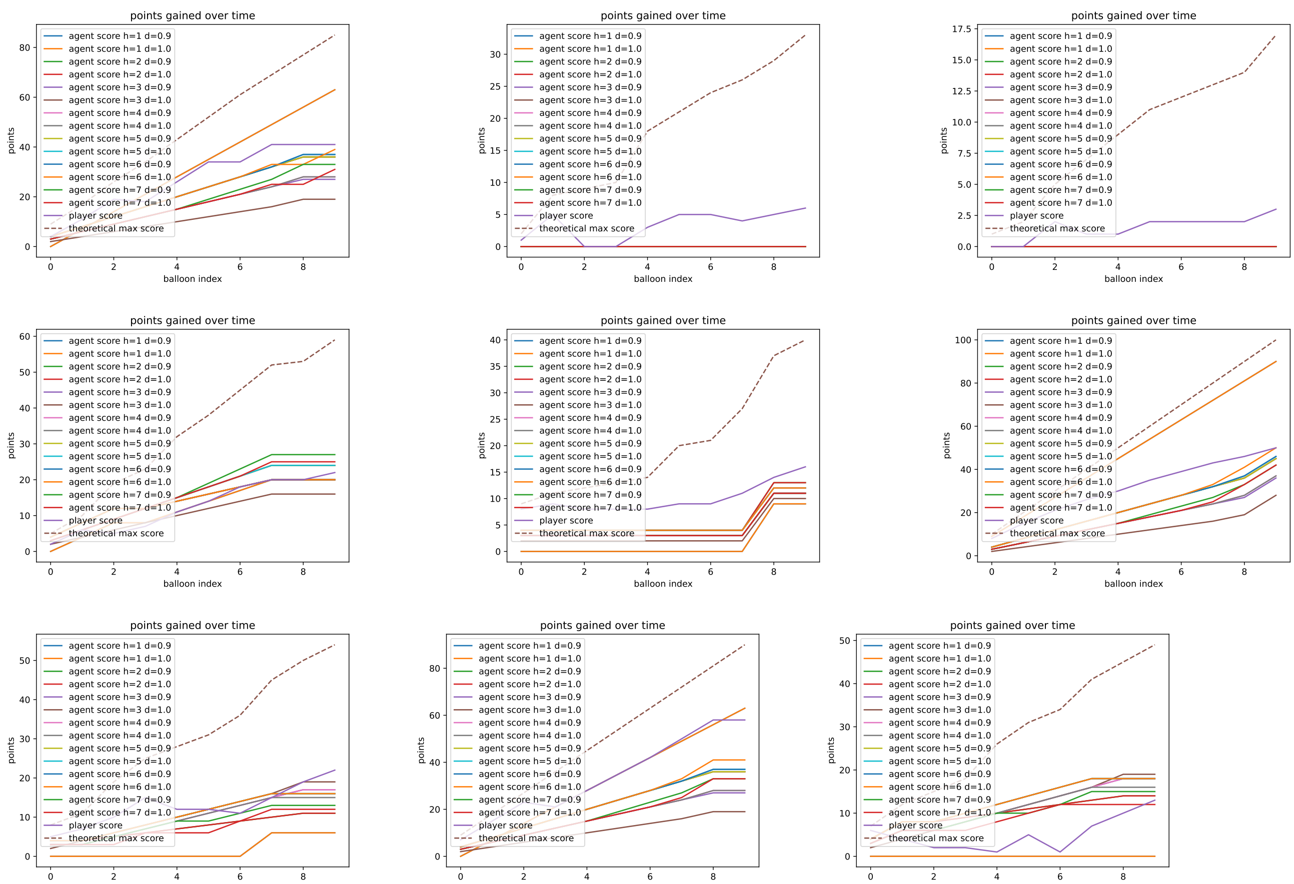

Figure 22: Experiment 3 Agent vs Player Graphs