

Monday, Oct 14, 2024

1. Let f^* be one maximum flow of network $G = (V, E)$ with source $s \in V$ and sink $t \in V$. Let $T := \{v \in V \mid v \text{ can reach } t \text{ in } G(f^*)\}$. Prove that $(S := V \setminus T, T)$ is a minimum s - t cut.
2. Recall that for a given network the mincut might not be unique. Given a minimum s - t cut C for a network $G = (V, E)$, let $S(C)$ denote the set of vertices that are on the same side of C as the source. Design a polynomial-time algorithm to compute $\cap_{C \in \mathcal{C}} S(C)$ and $\cup_{C \in \mathcal{C}} S(C)$, where \mathcal{C} is the set of all minimum s - t cuts of G .