Midterm 2 Part 1

Graded

Student

Krishna Nitin Pagrut

Total Points

43.5 / 45 pts

Question 1

MCQs

30 / 30 pts

- ✔ **+ 3 pts** Q1 is correct.
  Correct answer: *set 1:* **D**, *set 2:* **C**
- ✔ **+ 3 pts** Q2 is correct.
  Correct answer: *set 1:* **A**, *set 2:* **B**
- ✔ **+ 3 pts** Q3 is correct.
  Correct answer: *set 1:* **B**, *set 2:* **E**
- ✔ **+ 3 pts** Q4 is correct.
  Correct answer: *set 1:* **E**, *set 2:* **A**
- ✔ **+ 3 pts** Q5 is correct.
  Correct answer: *set 1:* **A**, *set 2:* **E**
- ✔ **+ 3 pts** Q6 is correct.
  Correct answer: *set 1:* **B**, *set 2:* **B**
- ✔ **+ 3 pts** Q7 is correct.
  Correct answer: *set 1:* **E**, *set 2:* **A**
- ✔ **+ 3 pts** Q8 is correct.
  Correct answer: *set 1:* **C**, *set 2:* **B**
- ✔ **+ 3 pts** Q9 is correct.
  Correct answer: *set 1:* **C**, *set 2:* **C**
- ✔ **+ 3 pts** Q10 is correct.
  Correct answer: *set 1:* **B**, *set 2:* **D**

Question 2

Max Flow Problem

Resolved **13.5** / 15 pts

✔ **+ 7 pts** The reduction to the Flow-Network problem is accurate, with a well-defined flow-network graph that includes clear explanations for the vertices, edges, and their capacities.

+ 6 pts Almost correct reduction to flow network.

+ 3.5 pts Significant progress has been made on the reduction. For example, vertices have been correctly added to represent the students and courses, along with the source and sink nodes.

+ 0 pts Incorrect reduction to flow-network problem.

✔ **+ 5 pts** The correctness proof is accurate, clearly explaining that the maximum flow is equivalent to the maximum assignment.

+ 2.5 pts Significant progress made towards a correctness proof.

+ 0 pts No correctness proof provided or totally wrong proof.

+ 3 pts The algorithm has polynomial time complexity, with complete calculation.

✔ **+ 1.5 pts** Non-polynomial time complexity provided, or partially correct calculation.

+ 0 pts Time complexity not provided.

+ 1.5 pts Mostly wrong or 10% option taken.

+ 0 pts Totally wrong or left blank.

Regrade Request

Submitted on: Dec 07

I think my time complexity is Polynomial, as the question requested. Can you explain how it is wrong?

To show that |f*| is polynomial, you need to assign a bound to it, which is missing. The total time complexity is not O(mn). It is O(m^2 n)

Reviewed on: Dec 09

CMPSC 465	Data Structures & Algorithms	
Fall 2024	Mingfu Shao and Debarati Das	Mid-Term 1 Part 1

Complete by: Monday, Nov. 18th, 2024, end of class
Set 1

Your Name: Krishna Pagrut

Your PSU Access ID: 955407380 (knp5451)

Your Recitation: 008R

INSTRUCTIONS:

1. Please clearly write your full name, your PSU Access User ID (i.e., xyz1234), and the recitation you are in (001–010) in the box above.
2. This exam contains 11 questions.
3. For questions 1–10 (i.e., multiple-choice questions) exactly one choice is correct.
4. Your answer to questions 1–10 MUST be recorded in the table on top of page 2.
5. For question 11, write your answers on the designated blank pages provided after the question.
6. You are not permitted to communicate with anyone during the exam time.
7. The exam is closed book, closed notes; no calculators, cell phones, or other electronic devices are allowed or are necessary.

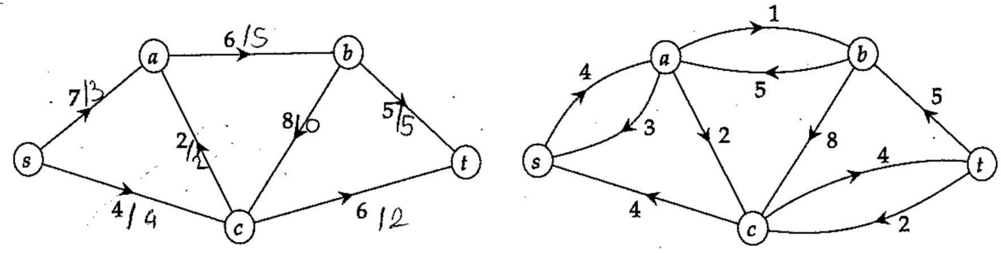
Your answer to questions 1–10:

Question	1	2	3	4	5	6	7	8	9	10
Your Answer	D	A	B	E	A	B	E	C	C	B

1. (3 pts.) The left figure shows a network G and the right figure shows the residual graph G_f with respect to a certain flow f. (The nubmers next to edges are capacities for both graphs.) What is the value of this flow f?

   A. 3
   B. 4
   C. 5
   D. 7

$|f| = 7$



network G residual graph G_f

2. (3 pts.) If the weight of each edge of a graph is increased by 50, then the minimum spanning tree stays the same.

   A. True
   B. False

3. (3 pts.) Let C be one minimum s-t cut of a network and let $e \in C$ be one edge in it. Suppose that we increase the capacity of e by 1, then the value of the maximum flow will increase by 1.

   A. True for all networks.
   B. True for some networks but not all.
   C. False for all networks.

4. (3 pts.) Given a graph, Prim's and Krusakl's algorithms output the same minimum spanning tree.

   A. Always
   B. Never
   C. If the edge weights are unique

D. If the minimum spanning tree is unique

E. Both C and D

$|f^*| = c(S^*, T^*)$

$|M| = |V_1|$ only when Bipartite

5. (3 pts.) Statement A: for a network the value of its maximum flow is always equal to the capacity of its minimum s-t cut. Statement B: for a bipartite graph the number of vertices in its minimum vertex-cover is always equal to the number of edges in its maximum matching.

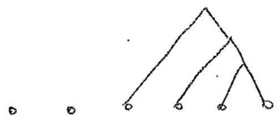
A. A is true and B is true.

B. A is true and B is false.

C. A is false and B is true.

D. A is false and B is false.

6. (3 pts.) (111, 000, 010, 011, 100, 001) is a



Not huffmann

A. Huffman code

B. Prefix-free code

C. Both prefix-free and Huffman code

D. None of the above

7. (3 pts.) Identify the valid Horn formulas

A. $\bar{x} \vee \bar{y} \vee z, \Rightarrow w$

B. $\bar{x} \vee \bar{y} \vee \bar{z} \vee \bar{w},\ x \vee y \Rightarrow z$

C. $\bar{x} \vee \bar{y} \vee \bar{z} \vee \bar{w},\ x \wedge \bar{y} \Rightarrow z$

D. All of the above

E. None of the above

8. (3 pts.) In the greedy set cover in every iteration the algorithm selects the subset that contains

A. maximum number of items

B. minimum number of items

C. maximum number of uncovered items

D. maximum number of covered items

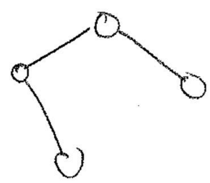
9. (3 pts.) Which one of the following is NOT true for the undirected graph G given below?

A. Maximum matching of G contains 3 edges.

B. Minimum vertex-cover of G contains 4 vertices.

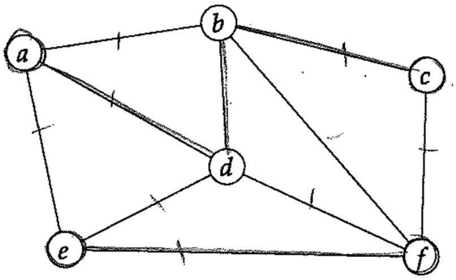
C. $\{a, c, e, f\}$ is a vertex-cover of G. bd left

D. The maximum matching of G is not unique.



$|M| < |V_1|$

$V_1 = \{d, b, e, c\}$



10. (3 pts.) In Kruskal's algorithm, any intermediate solution is always a single connected component of the MST.

    A. True

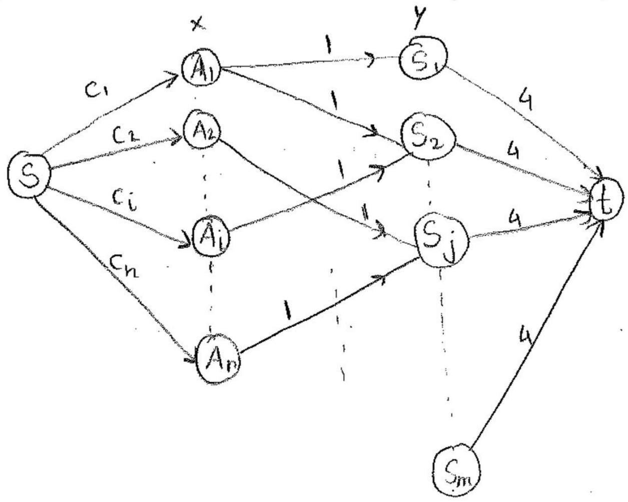
    ✓B. False

11. (7+5+3 pts.) In a semester the computer science department offers n courses to m students. Due to prerequisite requirements, each student has a specific set of courses the student is eligible to take. The i-th course has a maximum capacity of c_i students, $1 \le i \le n$. Each student can enroll in up to 4 courses. Design a polynomial-time algorithm that assigns courses to student so that the total number of course enrollments across all courses is maximized. Describe your algorithm, prove its correctness, and analyze its time complexity.

Courses A_i has max capacity c_i, $1 \leq i \leq n$

Student S_j $(1 \leq j \leq m)$ can enroll in upto 4 courses

$\boxed{\text{Algorithm}}$: Consider the following Bipartite graph



Graph $G = (V, E, s, t, c(\cdot))$ construction $\Rightarrow$

Construct a graph G where

$V = \{\text{Set of all cources}\} \cup \{\text{Set of all students}\} \cup \{s, t\} = X \cup Y \cup \{s,t\}$

$V = \{A_i \mid 1 \leq i \leq n\} \cup \{S_j \mid 1 \leq j \leq m\} \cup \{s, t\}$

→ We set the edges (E) in the graph as follows ⇒ $[\{(s,A_i),(A_i,S_j),(S_j,t)\}]$

- Source 's' is connected to every A_i, $1 \leq i \leq n$ with capacity $c(s,A_i) = c_i$, $1 \leq i \leq n$

- All ~~studens~~ are connected to sink 't' with capacity 4 $c(S_j,t) = 4$, $1 \leq j \leq m$

- An edge (A_i,S_j) exists only if Student S_j has to take that class i.e 'A_i' is a part of the prereq set of student S_j, $1 \leq i \leq n$, $1 \leq j \leq m$.

→ Now on this graph we perform "Ford-Fulkerson" Algorithm to get the max flow $|f^*|$ = max matching $|M|$ this $|f^*|$ will be the total no. of enrollments that will occure in that semester.

~~→ Now we define a min cut (S^*, T^*) where $S^* = \{$All vertices reachable from Source 's' in $G_{f^*}\}$, $T^* = V \setminus S^*$~~

→ After this process the edges from A_i to S_j that are saturated will define the assignment. A saturated edge (A_i,S_j) means student S_j is assigned course A_i.

$\boxed{\text{Time Complexity}}$ ⇒ Building the graph takes $O(mn)$ time

The running of Ford-Fulkerson takes

$$O(|f^*|[(m+n+2) + (mn+n+m)]) = O(|f^*| \cdot mn)$$

We Know that here the capacities are constant & flow is updated by one unit to the total of the max assignment. i.e it is constant.

So time complexity is $O(mn)$ which is polynomial Time.

Proof of Correctness $\Rightarrow$

We need to show that the $|f^*|$ is indeed the max course enrollments.

Since every edge in (A_i, S_j) has a capacity of 1 and the capacities of the edges from 's' and to 't' represent the appropriate restrictions we can say that there is a one-to-one correspondence between the max flow and the max assignments.

If $|f^*|$ is the max flow then $|f^*|$ is the max assignments is proved via the one-to-one correspondence.

If K is the max assignments then $K = |f^*| \Rightarrow$

We can show this by contradiction. Lets assume for the sake of proof by contradiction that $K \neq |f^*|$

Case 1: $K > |f^*|$: This provides a contradiction as it states that more flow can be pushed from s to t which contradicts the correctness of Ford Fulkson.

Case 2: $K < |f^*|$: This means that there exists at least one student s' whose node is violating the capacity constraint of our network flow graph.

Thus, our algorithm is correct

$\square$