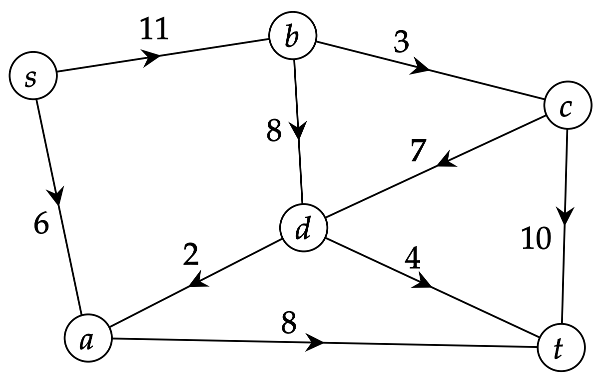
CMPSC 465 Data Structures & Algorithms
Fall 2024 Mingfu Shao and Debarati Das

HW 7

Due October 18th (Friday), 11:59 pm

Formatting: Each problem should begin on a new page. When submitting in Gradescope, try to assign pages to problems from the rubric as much as you can. Make sure you write all your group members' names. For the full policy on assignments, consult the syllabus.

1. (6 + 3 + 3 pts.) Consider the following network $G = (V, E)$ with source s and sink t:



   (a) Run the Ford-Fulkson algorithm on this instance. For each iteration, illustrate the flow f (i.e., $f(e)$ for every $e \in E$), the augmenting path p, the bottleneck capacity $x(p)$, and the value of the flow at the end of the iteration.

   (b) Let f^* to be the maximum-flow found in (a). Give the s-t cut (S^*, T^*), where $S^* := \{v \in V \mid s \text{ can reach } v \text{ in } G_{f^*}\}$, and $T^* := V \backslash S^*$. Show the cut-edges and the capacity of this s-t cut.

   (c) Let f^* to be the maximum-flow found in (a). Give the s-t cut (S', T'), where $T' := \{v \in V \mid v \text{ can reach } t \text{ in } G_{f^*}\}$, and $S' := V \backslash T'$. Show the cut-edges and the capacity of this s-t cut.

2. (12 pts.) You are given a directed graph $G = (V, E)$ with positive integral capacity $c(e)$ on $e \in E$, a source $s \in V$, and a sink $t \in V$. You are also given an integral maximum s-t flow f of G. Now we pick one edge $e^* \in E$ and reduce its capacity by 1. Design an algorithm to find a maximum flow in the resulting network in $O(|E| + |V|)$ time.

3. (12 pts.) There are n guests, w different kinds of wines in one party. Each guest has a preference list that specifies the wines they like. A guest is well-served if they are served a wine from their preference list. Each kind of wine can be served to at most b guests. Describe a polynomial-time algorithm to compute the maximum number of guests that can be well-served. Hint: reduce this problem to a network flow problem.