Midterm 2 Part 2

● Graded

Student

Krishna Nitin Pagrut

Total Points

29.5 / 35 pts

Question 1

Prim's and Kruskal's

20 / 20 pts

Prim

✔ **+ 10 pts** Steps for Prim's is correct.

Step 1: Vertex set: **{A}**; Cost: **0**
Step 2: Vertex set: **{A,B}**; Edge Set: **{AB}**; Cost: **1**
Step 3: Vertex set: **{A,B,C}**; Edge Set: **{AB,BC}**; Cost: **3**
Step 4: Vertex set: **{A,B,C,G}**; Edge Set: **{AB,BC,CG}**; Cost: **5**
Step 5: Vertex set: **{A,B,C,G,D}**; Edge Set: **{AB,BC,CG,GD}**; Cost: **6**
Step 6: Vertex set: **{A,B,C,G,D,F}**; Edge Set: **{AB,BC,CG,GD,GF}**; Cost: **7**
Step 7: Vertex set: **{A,B,C,G,D,F,H}**; Edge Set: **{AB,BC,CG,GD,GF,GH}**; Cost: **8**
Step 8: Vertex set: **{A,B,C,G,D,F,H,E}**; Edge Set: **{AB,BC,CG,GD,GF,GH,AE}**; Cost: **12**

+ 9 pts Correct, but the cost for Prim's is missing or incorrect.

+ 8 pts 1-2 steps for Prim's are incorrect.

+ 6 pts 3-4 steps for Prim's is incorrect.

+ 4 pts 5-6 steps for Prim's is incorrect.

+ 1 pt Mostly incorrect, or 10% option taken.

+ 0 pts Steps for Prim's is totally wrong or left blank.

Kruskal

✔ **+ 10 pts** Steps for Kruskal's is correct.

Step 1: Vertex set: **{A,B}**; Edge Set: **{AB}**; Cost: **1**
Step 2: Vertex set: **{A,B,F,G}**; Edge Set: **{AB,FG}**; Cost: **2**
Step 3: Vertex set: **{A,B,F,G,D}**; Edge Set: **{AB,FG,GD}**; Cost: **3**
Step 4: Vertex set: **{A,B,F,G,D,H}**; Edge Set: **{AB, FG,GD,GH}**; Cost: **4**
Step 5: Vertex set: **{A,B,F,G,D,H,C}**; Edge Set: **{AB, FG,GD,GH,BC}**; Cost: **6**
Step 6: Vertex set: **{A,B,F,G,D,H,C}**; Edge Set: **{AB, FG,GD,GH,BC,CG}**; Cost: **8**
Step 7: Vertex set: **{A,B,F,G,D,H,C,E}**; Edge Set: **{AB, FG,GD,GH,BC,CG,AE}**; Cost: **12**

+ 9 pts Correct, but the cost Kruskal's is missing or incorrect.

+ 8 pts 1-2 steps for Kruskal's are incorrect.

+ 6 pts 3-4 steps for Kruskal's is incorrect.

+ 4 pts 5-6 steps for Kruskal's is incorrect.

+ 1 pt Mostly incorrect, or 10% option taken.

+ 0 pts Steps for Kruskal's is totally wrong or left blank.

Question 2

Huffman Encoding

Resolved **9.5** / 15 pts

✔ **+ 1 pt** Correctly mentioned **n - 1** as the length of the longest codeword.

+ 7 pts Arguments for the longest length is correct.

✔ **+ 3.5 pts** Arguments are partially correct e.g., argued maximum length of a code is bounded by the depth of the tree; or each leaf corresponds to a code thus n leaf nodes imply depth is at most n-1.

+ 0 pts Wrongly mentioned length with incorrect arguments.

+ 8 pts Example is correct, with complete proof.

+ 6 pts Example correct with partially correct proof, e.g., the correct base case of correct induction hypothesis.

✔ **+ 4 pts** Only example is correct without the proof.

+ 2 pts Only example is partially correct without correct proof.

+ 0 pts Example and proof both are incorrect.

+ 1.5 pts Mostly wrong or 10% option taken.

+ 0 pts Totally wrong or left blank.

+ 15 pts (Makeup exam) Correct.

+ 10 pts (Makeup exam) Mostly correct; more details need to be shown.

+ 10 pts (Makeup exam) Mostly correct; needs to be generalized to bigger cases.

+ 5 pts (Makeup exam) Incomplete proof; one example is shown.

+ 5 pts (Makeup exam) Certain progress was made; but the wrong contradiction was stated.

+ 5 pts (Makeup exam) Certain progress was made; but the wrong statement was made.

+ 0 pts (Makeup exam) Totally wrong or left blank.

💬 **+ 1 pt** Point adjustment

Regrade Request **Submitted on: Dec 09**

Hello. My solution of n-1 is correct. Can you explain how it is incorrect? I am a little confused.

You already received the point for correctly mentioning (n-1), even though earlier in your answer you state that "the longest codeword becomes greater than nlogn", which is incorrect. I can give one more point for your correct example, but that's it. Regraded.

Reviewed on: Dec 10

CMPSC 465 Data Structures & Algorithms
Fall 2024 Mingfu Shao and Debarati Das

Mid-Term 2 Part 2

Complete by: Wednesday, Nov. 20th, 2024, end of class

Your Name: Krishna Pagrut

Your PSU Access ID: Knp5451 (955407280)

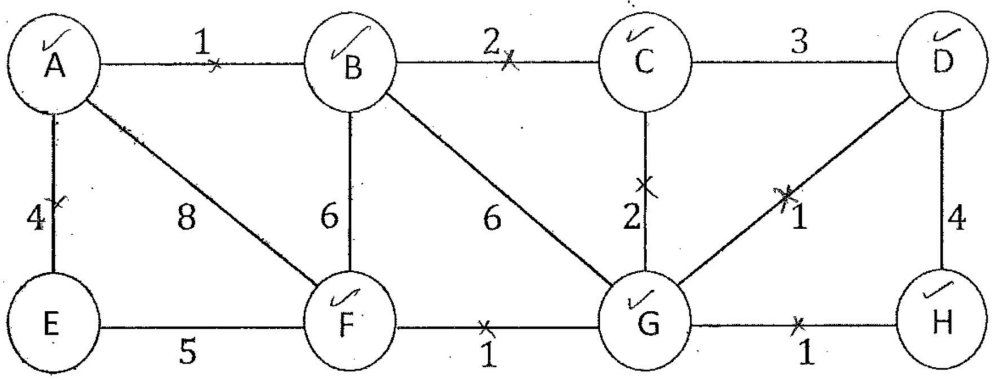
Your Recitation: 008R

INSTRUCTIONS:

1. Please clearly write your full name, your PSU Access User ID (i.e., xyz1234), and the recitation you are in (001–010) in the box above.
2. This exam consists of 2 questions. Write your answers on the designated blank pages provided after each question.
3. You are not permitted to communicate with anyone during the exam time.
4. The exam is closed book, closed notes; no calculators, cell phones, or other electronic devices are allowed or are necessary.

MST

1. (10+10 pts.) Suppose we want to find a minimum spanning tree for the following graph.



Run Prim's Algorithm; whenever there is a choice of nodes, always use alphabetic ordering (eg. start from node A). Mention all intermediate steps, corresponding spanning tree and associated cost.

Run Kruskal's Algorithm; whenever there is a choice of edges, pick an arbitrary one. Mention all intermediate steps, corresponding spanning tree and associated cost.

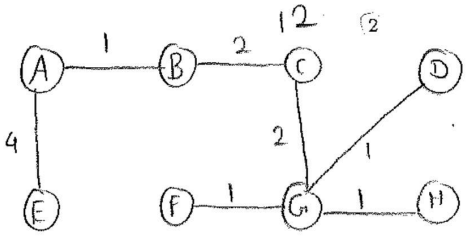
Prim's ⇒ Let $S := \phi$ be the set that will contain the MST

Make a priority Queue H with Key → cost(v) = min we e:(u,v)|u∈S

Every node has a prev()

S	A	B	C	D	E	F	G	H	cost	
{}	0/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil	0	(1)
{A}		1/A	∞/nil	∞/nil	4/A	8/A	∞/nil	∞/nil	0	(2)
{A,B}			2/B	∞/nil	4/A	6/B	6/B	∞/nil	1	(3)
{A,B,C}				3/C	4/A	6/B	2/C	∞/nil	3	(4)
{A,B,C,G}				1/G	4/A	1/G		1/G	5	(5)
{A,B,C,G,D}					4/A	1/G		1/G	6	(6)
{A,B,C,G,D,F}					4/A			1/G	7	(7)
{A,B,C,G,D,F,H}					4/A				8	(8)
{A,B,C,G,D,F,H,E}									12	(9)

Prim's MST ⇒



Cost(MST) = 12

Trees on pg 54

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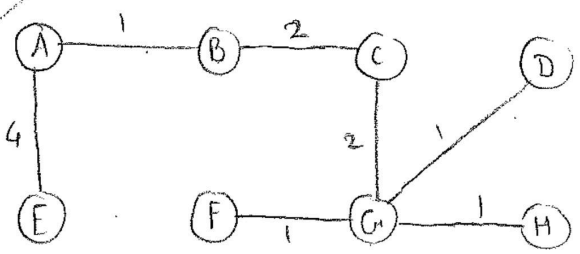
Kruskal's MST ⇒

① Using Directed Tree Data structure make trees of all nodes
② Sort E and pick smallest edge
③ Pick edge (F,G) & merge sets {F, G}
④ Pick edge (A, B) & merge sets {A, B}
⑤ Pick edge (G, D) & merge sets {G, D}
⑥ Pick edge (G, H) & {G, D, H}
⑦ Pick edge (B, C) & {A, B, C}
⑧ Pick edge (C, G) &

{A, B, C, G, D, F, D}

Kruskal's MST



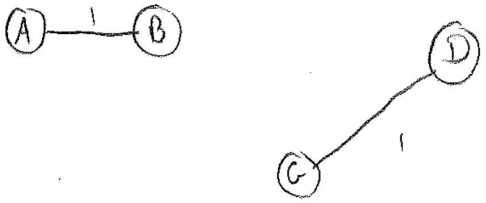
Kruskals Sets merged.	(without cycle) Edge Picked	Current cost	Step
{A, B}	A, B	1	①
{G, D}	G, D	2	②
{G, D, H}	G, H	3	③
{G, D, H, F}	G, F	4	④
{A, B, C}	B, C	6	⑤
{A, B, C, G, D, H, F}	C, G	8	⑥
{A, B, C, G, D, H, F, E}	E, A	12	⑦

Cost = 12

Trees on pg 5 ⇒

(1) 

1

(2) 

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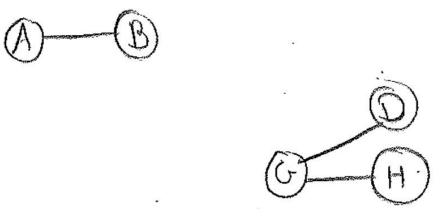
(3) 

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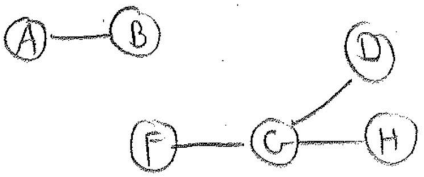
(4) 

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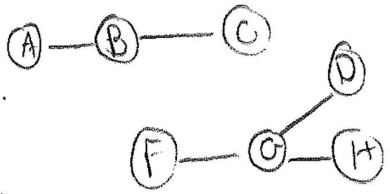
(5) 

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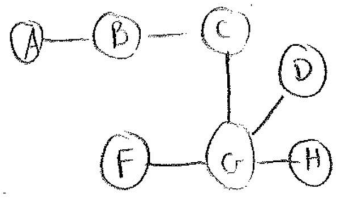
(6) 

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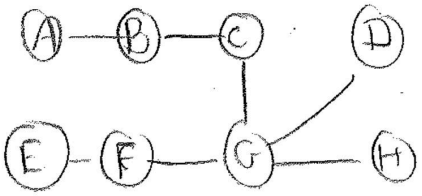
(7) 

12

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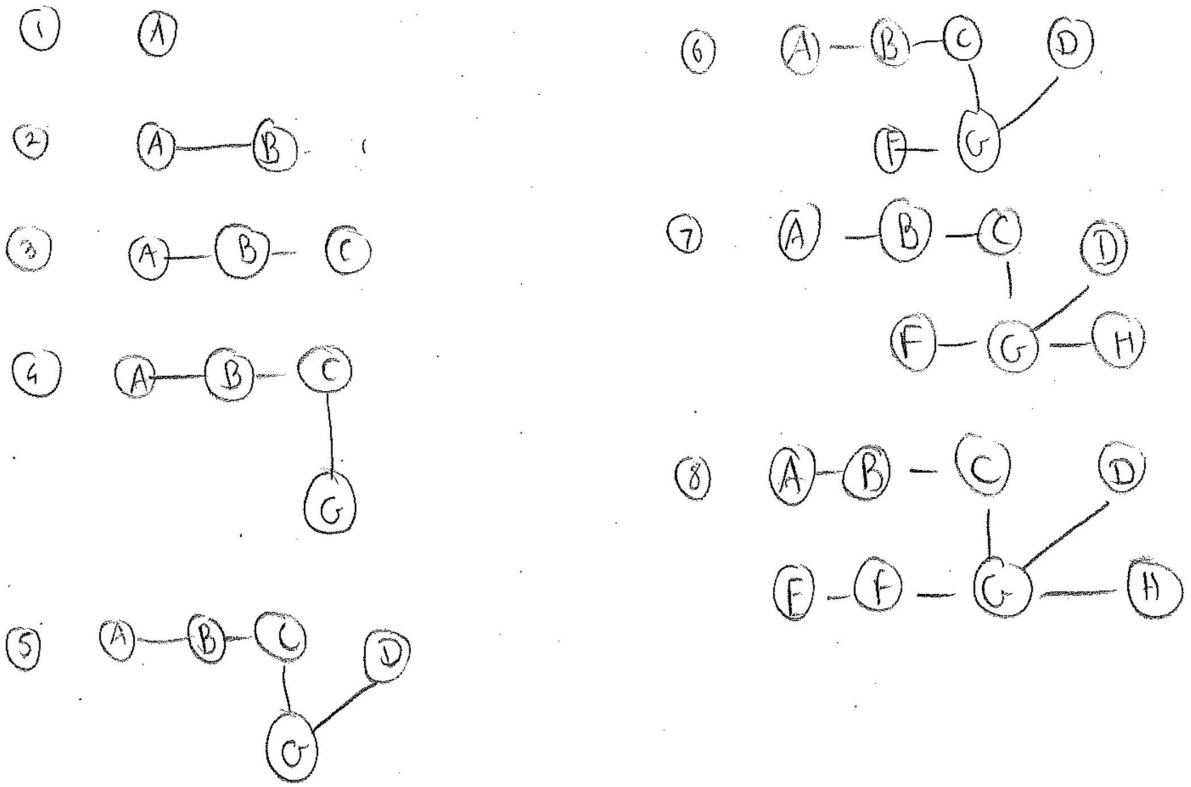
Kruskal's setup



(7+8 pts.) Under a Huffman encoding of n symbols with frequencies $f_1, f_2, \ldots, f_n$, what is the longest a codeword could possibly be and why? Give an example set of frequencies that would produce this case, and argue that it is the longest possible.

Symbols character set $C = \{x_1, x_2 \ldots x_n\}$,

$freq(x_i) = f_i$, $1 \leq i \leq n$.

(a) For any encoding the longest code word every symbol must have the longest possible encoding. Consider the case where $f_1 \neq f_2 \neq \cdots \neq f_n$ all frequencies are unique

In this case every symbol will have an encoding of length atleast $\lceil \log(n) \rceil$ except for the node with max freq.

i.e in a tree with 'n' leaves the rank(root) $\geq \lceil \log n \rceil$

which means that the depth(leaf) $\geq \lceil \log n \rceil$.

In the case when all frequencies are unique the depth(leaf) $\lceil \log n \rceil$

Thus, the longest code word becomes greater than $n \log n$

Reason for Rank(root) $\geq \lceil \log n \rceil$ because in a Full binary tree with n leaves there can be at most 2^n nodes in the tree

~~Max length will be~~

max length is $(n-1)$

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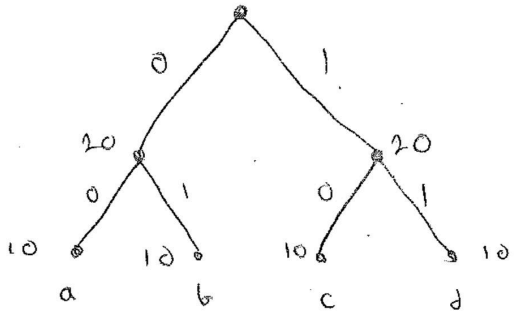
(b) Example ⇒

$C = \{a, b, c, d\}$ $f_a = 10,\ f_b = 10,\ f_c = 10,\ f_d = 10$

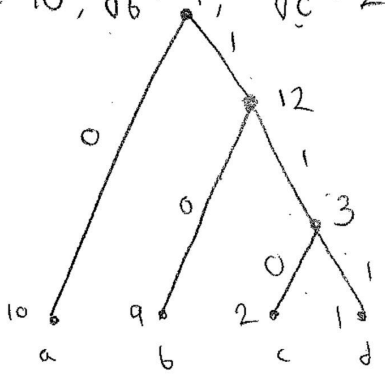


a = 00
b = 01
c = 10
d = 11

Length of 'abcd' = $4 \cdot \lceil \log 4 \rceil = 8$ 00 01 10 11

Now to show above is correct change all of the frequencies

$f_a = 10,\ f_b = 9,\ f_c = 2,\ f_d = 1$



a = 0
b = 10
c = 110
d = 111

Length of 'abcd' = 9

978

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