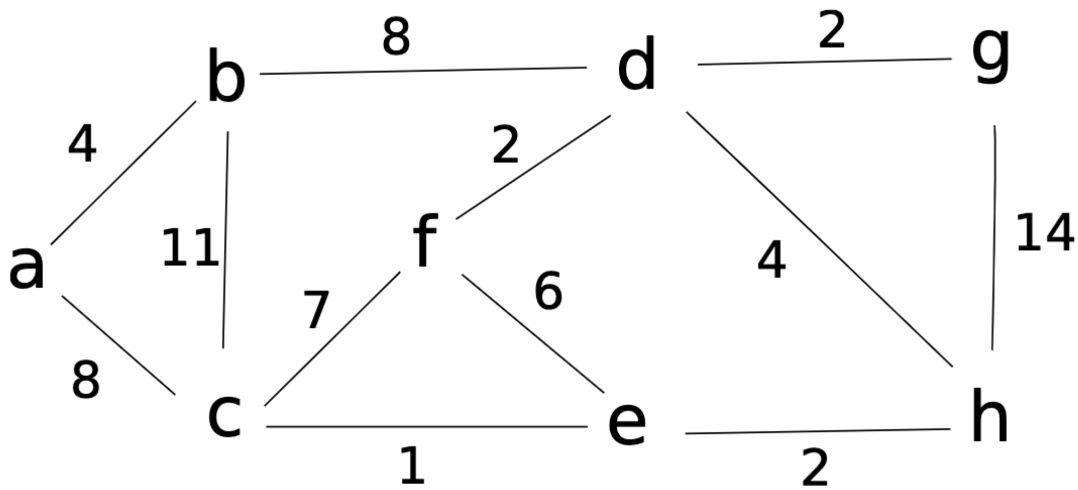
CMPSC 465	Data Structures & Algorithms	
Fall 2024	Mingfu Shao and Debarati Das	Worksheet 8

October 28, 2024

1. (pts.) Prim's

   Suppose Prim's algorithm is run on the following graph, starting at node f.



   a) Draw a table showing the intermediate *cost* and *prev* values of all the nodes at each iteration of the algorithm.
   b) Show the final MST.
   c) Run Kruskal's algorithm to find the MST of this graph. Give the edges in the order in which they are added to the MST. Use alphabetical ordering to avoid ambiguities.
   d) How would you find the *maximum spanning tree*?

2. (pts.) MST: True or False

Consider an undirected and connected graph $G = (V, E)$. Do not assume that edge weights are distinct unless this is specifically stated. For the following statements, give a short explanation for why it is true or give a counterexample if it is false.

(a) If G has more than $|V| - 1$ edges, and there is a unique heaviest edge e, then e can cannot be part of an MST.

(b) If G has a cycle with a unique heaviest edge e, then e cannot be part of any MST.

(c) Let e be any edge of minimum weight in G. Then e must be part of some MST.

(d) If the lightest edge in a graph is unique, then it must be part of every MST.

(e) If e is part of some MST of G, then it must be a lightest edge across some cut of G.

(f) If G has a cycle with a unique lightest edge e, then e must be part of every MST.

(g) The shortest-path tree computed by Dijkstra's algorithm is necessarily an MST.

(h) The shortest path between two nodes is necessarily part of some MST.

(i) Prim's algorithm works correctly when there are negative edges.