CMPSC 465 Data Structures & Algorithms
Fall 2024 Mingfu Shao and Debarati Das

HW 9

Due November 1st (Friday), 11:59 pm

Formatting: Each problem should begin on a new page. When submitting in Gradescope, try to assign pages to problems from the rubric as much as you can. Make sure you write all your group members' names. For the full policy on assignments, consult the syllabus.

1. (20 pts.)
   Let $G = (V, E)$ be an undirected graph with cost $c(e) \geq 0$ on the edges $e \in E$. Assume we are given an MST T in G. Now assume that a new edge $e = (u, v)$ is added to G and $c(e) = R$.

   1. Give an $O(|E|)$ time algorithm to test if T remains the MST for the modified graph (i.e., G with the new edge e). Analyze the running time. Can you do it in time $O(|V|)$?
   2. Suppose T is no longer the MST. Give an $O(|E|)$ time algorithm to update the tree T to the new MST.

2. (15 pts.)
   Let T be an MST of graph G. Given a connected subgraph H of G, show that $T \cap H$ is contained in some MST of H.

3. (15 pts.)
   Let T be a minimum spanning tree of a graph $G = (V, E)$ and V' be a subset of V. Let T' be a subgraph of T induced by V' (i.e., an edge $(u, v) \in T$ is present in T' iff both $u, v \in V'$) and G' be a subgraph of G induced by V'. Show that if T' is connected, then T' is a minimum spanning tree of G'.