

Monday, Oct 14, 2024

1. Let  $f^*$  be one maximum flow of network  $G = (V, E)$  with source  $s \in V$  and sink  $t \in V$ . Let  $T := \{v \in V \mid v \text{ can reach } t \text{ in } G(f^*)\}$ . Prove that  $(S := V \setminus T, T)$  is a minimum  $s$ - $t$  cut.
2. Recall that for a given network the mincut might not be unique. Given a minimum  $s$ - $t$  cut  $C$  for a network  $G = (V, E)$ , let  $S(C)$  denote the set of vertices that are on the same side of  $C$  as the source. Design a polynomial-time algorithm to compute  $\cap_{C \in \mathcal{C}} S(C)$  and  $\cup_{C \in \mathcal{C}} S(C)$ , where  $\mathcal{C}$  is the set of all minimum  $s$ - $t$  cuts of  $G$ .