

Due December 6th (Friday), 11:59 pm

Formatting: Each problem should begin on a new page. When submitting in Gradescope, try to assign pages to problems from the rubric as much as you can. Make sure you write all your group members' names. For the full policy on assignments, consult the syllabus.

1. (20 pts.)

Consider a scenario where  $C$  is the set of clients and  $S$  is the set of servers. Each client  $i$  in set  $C$  can be served by server  $j$  in set  $S$  with a service cost  $c_{ij}$ . Because of the emerging issue of carbon emission and global warming at most  $k$  servers can stay functional at a time. Our objective is to find the optimal set of  $k$  servers so that each client can be served while minimizing the total service cost. Let OPT be the sum of the service costs over all clients for the optimal solution.

1. Design an LP representing the problem. [clearly describe in language the variables, describe the constraints and objective function]
  2. Write the dual LP.
2. (10 pts.) Convert the following LP into standard form form 1:

$$\begin{aligned} \text{maximize} \quad & 2x_1 + 7x_2 + x_3 \\ \text{subject to} \quad & x_1 - x_3 = 7 \\ & 3x_1 + x_2 \geq 24 \\ & x_2 \geq 0 \\ & x_3 \leq 0. \end{aligned}$$

3. (15 pts.)

A film producer wants to make a motion picture. For this she needs to choose among  $n$  available actors. Actor  $i$  demands a payment of  $s_i$  dollars to participate in the picture. The funding of the picture will come from  $m$  investors. The  $k_{th}$  investor will pay the producer  $p_k$  dollars, but only under the following condition. The investor has a list of actors  $L_k \subseteq \{1, \dots, n\}$ , and he will only invest iff all the actors on his list appear in the picture. The profit of the producer is the sum of payments from the investors that she agrees to take funding from, minus the sum of payments she makes to the actors that appear in the picture. The goal is to maximize the producer's profit. Give an integer linear programming to model this problem.