CMPSC 465 Data Structures & Algorithms
Fall 2024 Mingfu Shao and Debarati Das **Recitation 1**

Sep 09, 2024

1. **Compare Growth Rates.** Order the following functions by asymptotic growth:

   (i) $f_1(n) = 3^n$
   (ii) $f_2(n) = n^{\frac{1}{3}}$
   (iii) $f_3(n) = 12$
   (iv) $f_4(n) = 2^{\log_2 n}$
   (v) $f_5(n) = \sqrt{n}$
   (vi) $f_6(n) = 2^n$
   (vii) $f_7(n) = \log_2 n$
   (viii) $f_8(n) = 2^{\sqrt{n}}$
   (ix) $f_9(n) = n^3$

   Solution $f_3, f_7, f_2, f_5, f_4, f_9, f_8, f_6, f_1$

2. **Prove Order of Growth.** Prove the following:

   (i) $\sum_{k=1}^{n} k^j = \Theta(n^{j+1})$ for any constant $j > 0$.
   (ii) $\sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$

   Solution

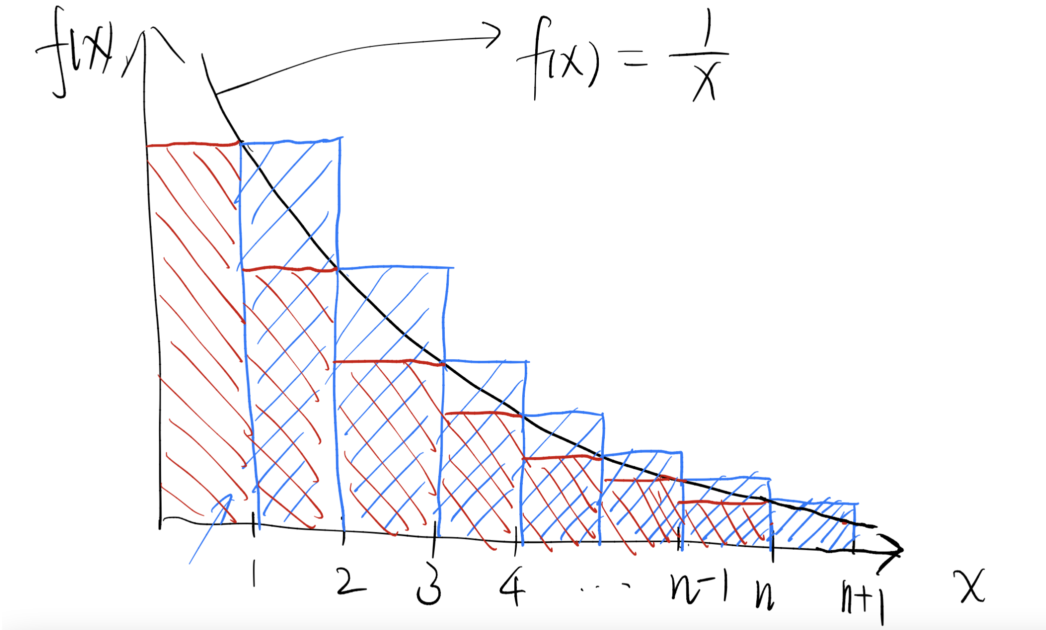
   (i) Since $k \leq n$ every term in the sum is at most n, so

$$\sum_{k=1}^{n} k^j = 1^j + \cdots + n^j \leq n^j + \cdots + n^j = \sum_{k=1}^{n} n^j = n^{j+1}.$$

   We do something similar for the lower bound. The only additional idea is that we only look at the second half of the sum. The smallest element in the second half of the sum correspond to $k = n/2$ (assuming without loss of generality that n is even). Then,

$$\sum_{k=1}^{n} k^j \geq \sum_{k=n/2}^{n} k^j \geq \sum_{k=n/2}^{n} (n/2)^j = 2^{-j-1} n^{j+1}.$$

   (ii) This can be proved using integration. We need to find both upper and lower bound for Θ.



Red area = Blue Area = $\sum_{k=1}^{n} \frac{1}{k}$

Red area is right shifted by one to obtain the blue area. Area under the graph of f(x) is greater than red area but less than blue area.

Area under the curve for 1 to n = $\int_1^n \frac{1}{x}dx$

As the actual value of f(x) goes to ∞ for 0, we can substitute it with 1 and red area will still be less than area under f(x).

Red Area (RHS) $\leq$ 1 + $\int_1^n \frac{1}{x}dx$ = 1 + $\log n$

Blue Area (RHS) $\geq \int_1^{n+1} \frac{1}{x}dx$ = $\log(n+1)$

The last two statements provide the upper and lower bound, thus $\sum_{k=1}^{n} \frac{1}{k} = \Theta(\log n)$