CALCULUS.

calculus is the study of differentiation and integration.

calculus explains the changes in values, on a small and large scale, related to any function.

> Differential calculus is the rate of change of a variable or a quantity with respect to another variable/quantity represented by $f'(x) = dy/dx$.

Integral Calculus?

The process of evaluating the area under a curve or a function is called integral calculus.

Differentiation Formulas:

- $\frac{dk}{dx} = 0$; k is a constant.
- $\frac{d(x)}{dx} = 1$
- $\frac{d(kx)}{dx} = k$; k is a constant
- $\frac{d(x^n)}{dx} = nx^{n-1}$

Derivatives of Logarithmic and Exponential Functions.

$$\frac{d(e^x)}{dx} = e^x$$

$$\frac{d(\log(x))}{dx} = \frac{1}{x}$$

$$\frac{d(a^x)}{dx} = a^x \log a$$

$$\frac{d(x^x)}{dx} = x^x(1 + \log x)$$

$$\frac{d(\log_a x)}{dx} = \frac{1}{x} \times \frac{1}{\log a}$$

Derivatives of Trigonometric Functions.

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$$

Derivatives of Inverse Trigonometric Functions.

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cot^{-1} x)}{dx} = \frac{-1}{1+x^2}$$

Sum or difference Rule.

when the function is the sum or difference of two functions the derivative is the sum or difference of derivative of each function i.e,

$$f(x) = u(x) \pm v(x) \text{ then}$$

$$f'(x) = u'(x) \pm v'(x)$$

Product Rule:

when $f(x)$ is the sum of two $u(x)$ and $v(x)$ functions, it is the function derivative,

If $f(x) = u(x) \times v(x) \Rightarrow f'(x) = u'(x)v(x) + u(x)v'(x)$

Quotient Rule:

If the function $f(x)$ is in form of two functions $\frac{u(x)}{v(x)}$, the derivative of function can be expressed as;

If $f(x) = \frac{u(x)}{v(x)}$, then $f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$

Eg: calculate $f'(x)$; $f(x) = 2x^3 - 4x^2 + x - 33$.

$f'(x) = 6x^2 - 8x + 1$. $\left(\frac{d}{dx}(x^n) = nx^{n-1}\right)$

Eg. calculate $f(x) = e^x + x$.

$f'(x) = e^x + 1$.

Eg: $f(x) = 5x^3 - \tan x$

$f'(x) = 15x^2 - \sec^2 x$.

Eg: $f(x) = (e^x + 1)\tan x$.

$f'(x) = u'(x)v(x) + u(x)v'(x)$

$= e^x \tan x + (e^x + 1)\sec^2 x$

$= e^x \tan + (e^x + 1)\sec^2 x$.

Eg: $f(x) = \frac{\sin x}{x}$.

$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$

$= \frac{\cos x \cdot x - \sin x}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$

Integration.

Integration is the process of finding the antiderivative of a function. It is a similar way to add the slices to make it whole. The integration is the inverse of differentiation.

List of Integral Formulas.

* $\int dx = x + C$
* $\int a\,dx = ax + C$
* $\int x^n dx = \frac{x^{n+1}}{n+1} + C \; ; n \neq 1.$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \operatorname{cosec}^2 x\,dx = -\cot x + C.$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \operatorname{cosec} x \cdot \cot x \cdot dx = -\operatorname{cosec} x + C.$

$\int \frac{1}{x} dx = \ln|x| + C$

* $\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C \; ; a > 0 ; a \neq 1.$

* $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
* $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C.$

$\int \sin^n(x)dx = \frac{-1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$

$\int \cos^n(x)dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)dx$

$\int \tan^n(x)dx = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)dx$

$\int \sec^n(x)dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)dx$

$\int \operatorname{cosec}^n(x)dx = \frac{-1}{n-1}\operatorname{cosec}^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int \operatorname{cosec}^{n-2}(x)dx$

Eg: Find the integral of the function.

$\int_0^3 x^2 dx$

$= \left(\frac{x^3}{3}\right)_0^3 \Rightarrow \frac{3^3}{3} - \frac{0^3}{3} \Rightarrow 9$

$\int x^2 dx$

$\frac{x^3}{3} + C.$

Eg: $\int (x^2-1)(4+3x)dx.$

$\int (4x^2 + 3x^3 - 3x + 4)\, dx$

$4\frac{x^3}{3} + \frac{3x^4}{4} - \frac{3x^2}{2} + 4x + C.$

Eg: $\int (4x^2 - 2x)dx.$

$\int 4x^2 dx - \int 2x dx$

$\frac{4x^3}{3} - 2\frac{x^2}{2} + C$

$= \frac{4x^3}{3} - x^2 + C$

$= x^2\left(\frac{4x}{3} - 1\right) + C$

Integration By parts Formula.

$\int uv\, dx = u\int v dx - \int \left(u' \int v dx\right) dx$

ILATE RULE:

I: Inverse trigonometric functions

L: logarithmic functions.

A: Algebraic functions

T: Trigonometric functions

E: Exponential functions.

$\int x \cdot e^x dx$

$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\left(\int g(x)dx\right)dx$

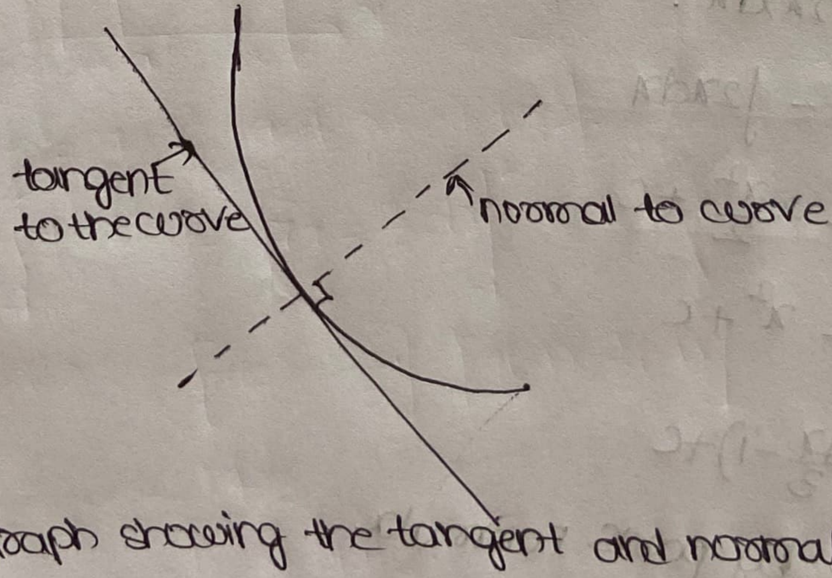
$= xe^x - e^x + C$

$= e^x(x-1) + C$

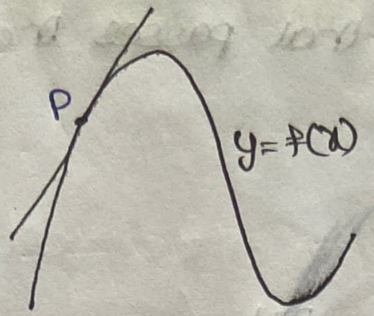
Tangents and Normals.

* Tangent to a curve is a line that touches the curve at one point and has the same slope as the curve at that point.

* A normal to a curve is a line perpendicular to a tangent to the curve.



graph showing the tangent and normal to a curve at a point.



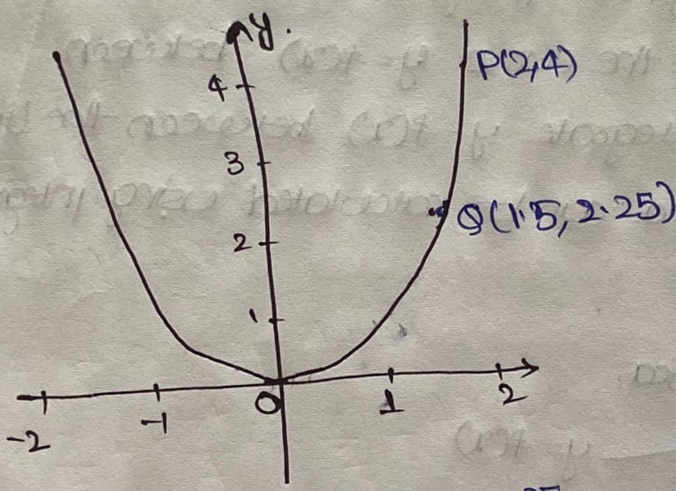
slope of the tangent at P.

slope of a curve $y = f(x)$ at the point P means the slope of the tangent at the point P.

By definition, the slope is given by:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

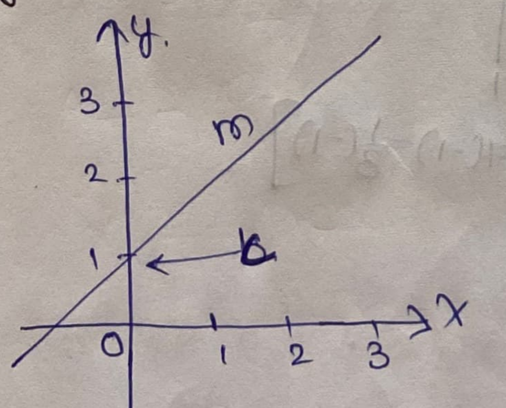
Eg:-



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2.25}{2 - 1.5} = 3.5.$$

Equation of a straight line.

$y = mx + b$



Eg:- Find slope of a line that passes through the points $(2,-1)$ and $(-5,3)$

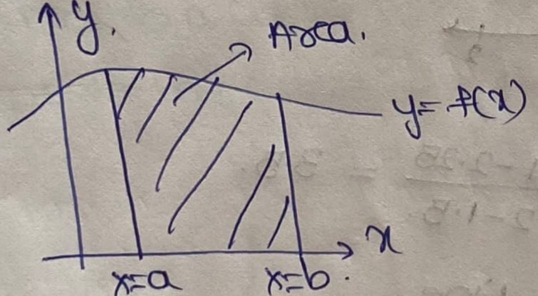
$$m, \text{ slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$\left[\begin{array}{c} = \cancel{\dfrac{-5-2}{3+1}} \\ \cancel{-7}/4 \end{array}\right] = \frac{3+1}{-5-2} = \frac{-4}{7}.$$

Application of Integration:

Area under the curve Formula.

The area under a curve between two points is found out by doing a definite integral between the two points. To Find the area of the curve $y=f(x)$ between $x=a$ and $x=b$, integrate $y=f(x)$ between the limits of a and b. This area can be calculated using integration with given limits.



* $f(x) = 7-x^2$ and $x=-1$ to 2.

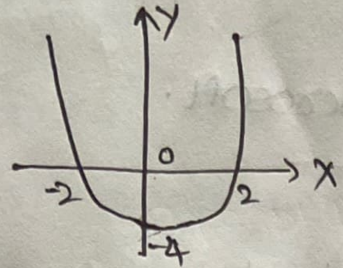
$$\int_{-1}^{2} 7-x^2 = \int 7x - \frac{x^3}{3}\,dx \Big|_{-1}^{2}$$

$$= \left[7(2) - \frac{(2)^3}{3}\right] - \left[7(-1) - \frac{1}{3}(-1)\right]$$

$$= 54/3.$$

$$= 18 \text{ sq. units.}$$

what is the area between the curve $y = x^2 - 4$ and the x-axis?



$$\int_{-2}^{2} x^2 - 4\,dx = \left.\frac{x^3}{3} - 4x\right]_{-2}^{2} = \left[\frac{8}{3} - 8\right] - \left[\frac{-8}{3} + 8\right]$$

$$= \frac{16}{3} - 16$$

$$= -10.67.$$ (Area is negitive bec. it is below x-axis)