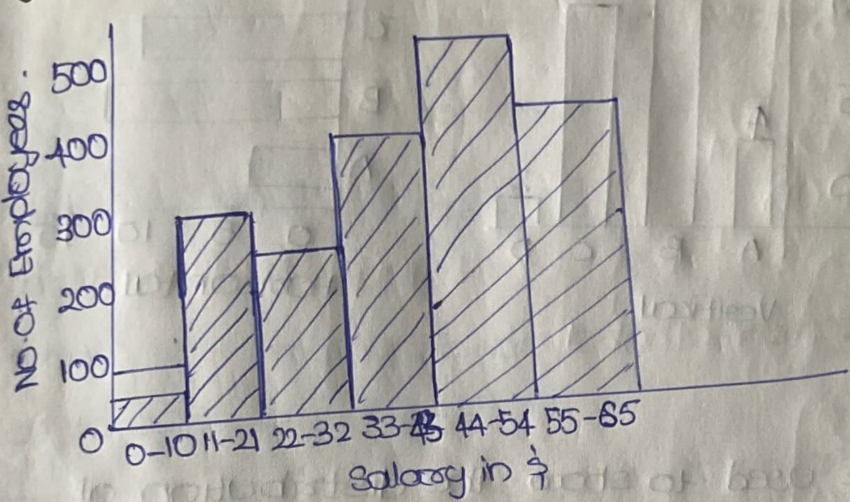
MATHEMATICS DISTRIBUTIONS.

Histogram.

> Histogram is a widely used graph to show the distribution of quantitative (numeric) data.

> It shows the frequency of values in the data, usually in intervals of values. Frequency is the amount of times that value appeared in the data.

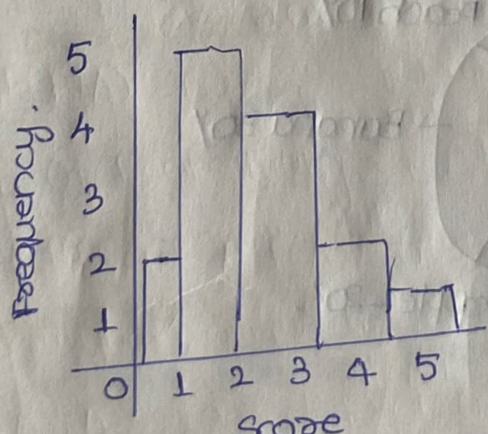
Eg: Distribution of salaries.



Frequency Histogram.

It is a special graph that uses vertical columns to show frequencies.

Scores : 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5.



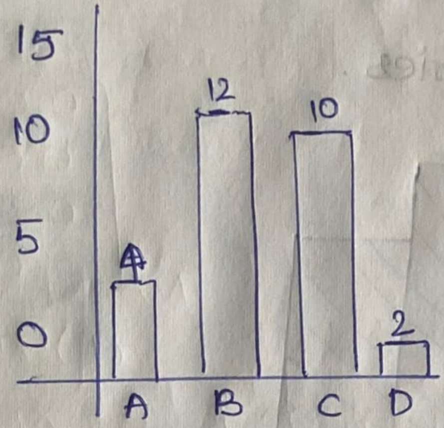
Bar Graph.

Bar Graph is a graphical display of data using bars of different heights.

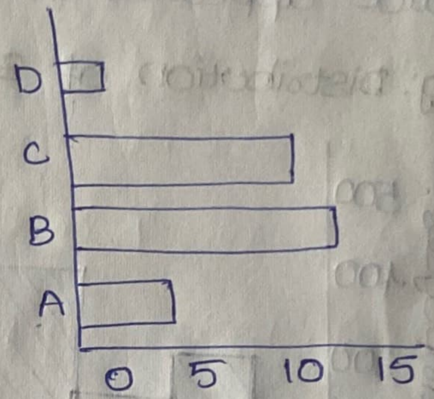
Bar graphs are used to show the distribution of qualitative data.

Eg:

Grade	A	B	C	D
students	4	12	10	2



Vertical.



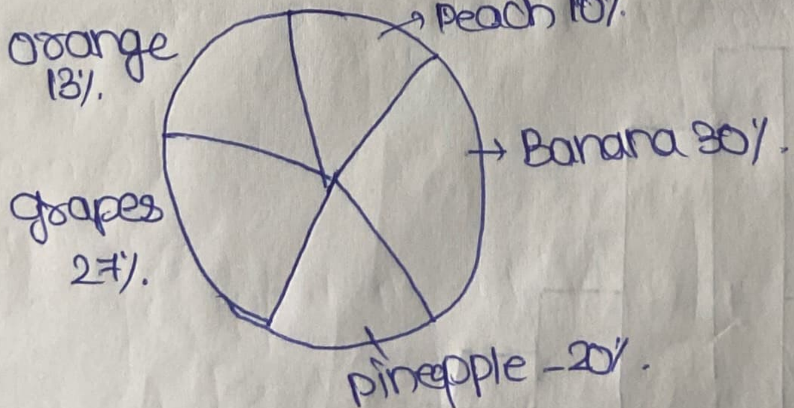
Horizontal.

Pie chart:

Pie graphs are used to show the distribution of qualitative data.

Each category is represented with a slice in the 'pie'.

The size of each slice represents the frequency of values from that category in data.



LINEAR ALGEBRA

matrix.

A matrix represents a collection of numbers arranged in an order of rows and columns. It is necessary to enclose the elements of a matrix in parentheses or brackets.

Eg:
$$m = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

This matrix [m] has 3 rows and 3 columns. Each element of matrix [m] can be referred to by its row and column number. For eg. $a_{23} = 6$.

order of a matrix.

order of a matrix is defined in terms of its number of rows and columns.

order of matrix = No. of rows x No. of columns.

Trace of a matrix: tr(A): which is used only for square matrix and equals the sum of diagonal elements of matrix.

$$tr(A) = 1+5+9 = 15.$$

matrix Dimensions

$$C = \begin{bmatrix} 2 & 5 & 3 \end{bmatrix}_{(1\times 3)}$$

$$C = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix}_{2\times 3.}$$

square matrices.

square matrix has the same number of rows and columns.

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3.}$$

Diagonal matrices.

Diagonal matrix has values on the diagonal entries, and zero on the rest.

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Scalar matrices.

scalar matrix has equal diagonal entries and zero on the rest.

$$C = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}_{4 \times 4}$$

Identity matrix (I)

Identity matrix has 1 on the diagonal and 0 on the rest

This matrix equivalent of 1.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

If you multiply any matrix with identity matrix, the result equals the original.

Zero matrix

zero matrix has only zeros (null matrix)

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Equal matrices.

matrices are Equal if each element correspond

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix}$$

Negative matrices.

$$-\begin{bmatrix} -2 & 5 & 3 \\ -4 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 & -3 \\ 4 & -7 & -1 \end{bmatrix}$$

Adding matrices.

If two matrices have the same dimension, we can add.

$$\underset{2\times3}{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix}} + \underset{2\times3}{\begin{bmatrix} 4 & 7 & 1 \\ 2 & 5 & 3 \end{bmatrix}} = \underset{2\times3}{\begin{bmatrix} 6 & 12 & 4 \\ 6 & 12 & 4 \end{bmatrix}}$$

Subtracting matrices.

If two matrices have the same dimension, we can subtract

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 7 & 1 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 2 \\ 2 & 2 & -2 \end{bmatrix}$$

Scalar multiplication.

while numbers in rows and columns are called matrices, single numbers are called scalars.

It is easy to multiply a matrix with a scalar.

Just multiply each number in the matrix with the scalar.

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 4 & 10 & 6 \\ 8 & 14 & 1 \end{bmatrix}$$

Transpose a matrix.

To transpose a matrix, means to replace rows with columns.

when you swap rows and columns, you rotate the matrix around it's diagonal.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Matrix multiplication.

If A and B are two matrix

$m \times n \times n \times p = m \times p$.

Eg:- 1×3 by $3 \times 1 \rightarrow 1 \times 1$ as result

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\times4 + 2\times5 + 3\times6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}$$

Eg: 3×1 by $1 \times 3 \rightarrow 3 \times 3$ as result

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4\times1 & 4\times2 & 4\times3 \\ 5\times1 & 5\times2 & 5\times3 \\ 6\times1 & 6\times2 & 6\times3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$$

Eg: $$\underset{2\times3}{\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}} \underset{3\times2}{\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}} \rightarrow \begin{bmatrix} & \\ & \end{bmatrix}_{2\times2}$$

$$\begin{bmatrix} 1\times2 + (-2)3 + 1\times1 & 1\times1 + (-2)(2) + 1\times1 \\ 2\times2 + 1(3) + (3)(1) & 2\times1 + 1(2) + 3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

Eg:- $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

Determinant.

Determinant is a scalar value that can be calculated from the elements of a square matrix

matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad$ $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Finding the Determinant of a three by three matrix

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$\det(A) = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Adjoint of a matrix.

Adjoint of a matrix A is the transpose of cofactor matrix o

A.

Denoted by Adj A.

To find cofactor.

$$C_{ij} = (-1)^{i+j} \det(M_{ij})$$

Eg:- Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$ Adj A = ?

$A_{11} = \begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix} = -7$

$A_{12} = 1$

$A_{13} = 1$

$A_{21} = 6$

$A_{22} = 0$

$A_{23} = -2$

$A_{31} = -1$

$A_{32} = -1$

$A_{33} = 1$

$$= \begin{bmatrix} -7 & 1 & 1 \\ 6 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$$

Adj A = transpose of cofactor matrix

$$= \begin{vmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix}$$

Inverse of matrix.

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A) \quad \text{where } |A| \neq 0.$$

Eg:- $\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{4\times6 - 7\times2} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$

$A_{11} = 6$
$A_{12} = -2$
$A_{21} = -7$
$A_{22} = 4$

$= \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$

$\text{Adj}A = \begin{bmatrix} 6 & -2 \\ -7 & 4 \end{bmatrix}^T$

$= \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$, $A^{-1} = ?$ $\quad A^{-1} = \frac{\text{Adj}A}{|A|}$

cofactor matrix of $A = \begin{bmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & -8 & 4 \end{bmatrix}^T$

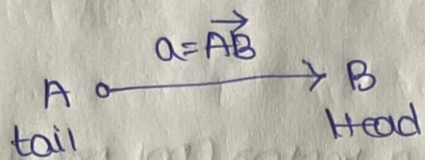
$\text{Adj}A = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix} = 4(-7) - (-6)(5) - 3(-6) = 28 + 30 + 18$
$= 20.$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{20} \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$$

VECTORS.

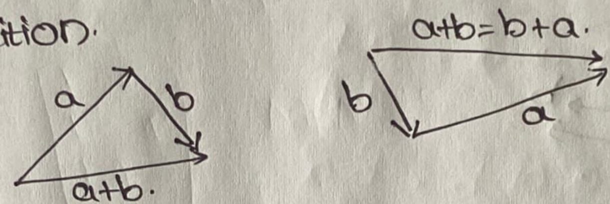
image is a vector.
length shows magnitude
Arrow shows Direction.



Force & Velocity:

Force is a vector with a magnitude and Direction
velocity is a vector with a magnitude and Direction.

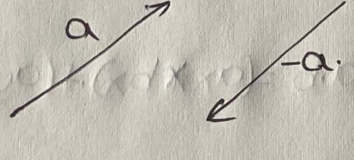
vector Addition.



line from tail of a to head of b is vector a+b. and it doesn't matter which order we add them.

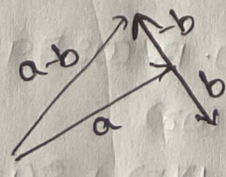
vector subtraction.

vector -a is the opposite of +a.
vector a and vector -a has the same magnitude in opposite directions.



subtracting.

First we reverse the direction of vector we want to subtract, then add them as usual.

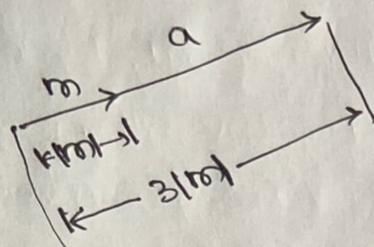


magnitude of a vector

magnitude of a vector is shown by two vertical bars on either side of a vector

$$|a| = \sqrt{(x^2+y^2)}$$

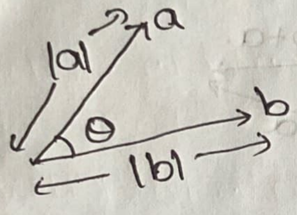
multiplying a vector by a scalar.



multiplying a vector by a vector (Dot product and cross product)

Dot product (The result is a scalar)

vector (cross product)



Dot product:- (a·b) = Dot product of a and b.

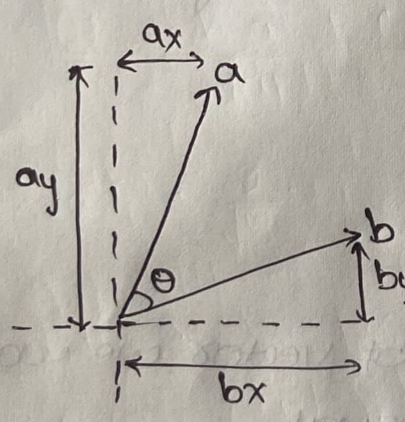
a·b = |a| × |b| × cos(θ)

|a| = magnitude of vector a

|b| = magnitude of vector b

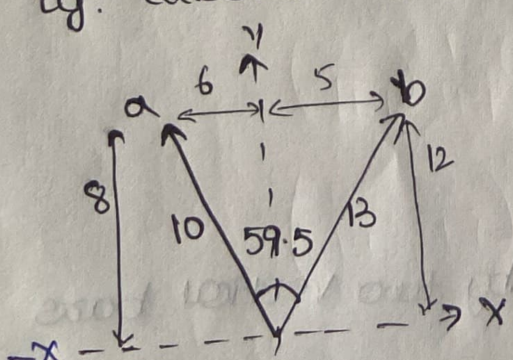
θ is angle between a and b.

(or)



a·b = (ax * bx) + (ay × by)

Eg: calculate a·b.



a·b = 10· 13· cos 59.5 ≅ 66.

(or)

a·b = (-6)(5) + 8(12)

= 66.

Eg:- $A = 2i - 3j + 7k$ and

$B = -4i + 2j - 4k$. Find $A \cdot B$.

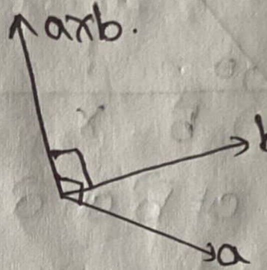
$$A \cdot B = (2)(-4) + (-3)(2) + (7)(-4)$$

$$= -8 - 6 - 28$$

$$= -42$$

Cross product

Cross product $a \times b$ of two vectors is another vector that is at right angles to both.



Cross product matrix

$A = ai + bj + ck$

$B = xi + yj + zk$.

$$A \times B = \begin{vmatrix} i & j & k \\ a & b & c \\ x & y & z \end{vmatrix}$$

$$= (bz - cy)i - (az - cx)j + (ay - bx)k.$$

$$= (bz - cy)i + (cx - az)j + (ay - bx)k.$$

Eg:- $\vec{x} = 5\vec{i} + 6\vec{j} + 2\vec{k}$ $\vec{x} \times \vec{y} = ?$

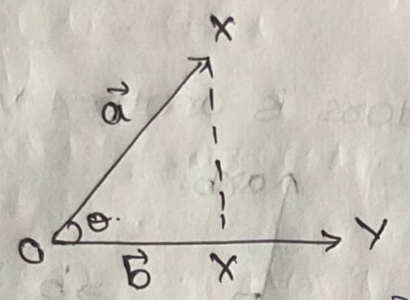
$\vec{y} = \vec{i} + \vec{j} + \vec{k}$

$$\vec{x} \times \vec{y} = \begin{vmatrix} i & j & k \\ 5 & 6 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 4\vec{i} - 3\vec{j} - \vec{k}.$$

Vector Projection Formula.

Vector projection is of two types ∵ Scalar projection tells about the magnitude of vector projection and the other is the vector projection which says about itself and represents the unit vector.



projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

projection of $\vec{a}$ on $\vec{b}$ = $\dfrac{\vec{b}\cdot\vec{a}}{|\vec{b}|}$

Eg: Find the scalar projection of $b = (-4,1)$ onto $a = (1,2)$

projection of b onto a.

$$P_a(b) = |b|\cos(\theta)$$

$$= \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{(-4)(1) + (1)(2)}{\sqrt{1^2+2^2}}$$

$$P_a(b) = \frac{-2}{\sqrt{5}}.$$

Eg: Find the vector projection of $a = (1,2)$ onto $b = (-4,1)$

$$P_b(a) = \left(\frac{a\cdot b}{|b|}\right)\cdot\frac{b}{|b|}$$

$$= \left(\frac{-2}{\sqrt{17}}\right)\frac{1}{\sqrt{17}}(-4,1)$$

$$= \left(\frac{8}{17}, \frac{-2}{17}\right)$$