Final test - Part 1

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Abstract: In this work we use 10^4 realizations of a Monte Carlo simulation to compute the number of muons N, produced in cosmic-ray interactions in the atmosphere, that are detected during an observation of 10 s, by a detector with an efficiency $\epsilon = 0.87$. Using these values of N, we check that the interarrival times of the muons follow an exponential probability density function.

I. INTRODUCTION

We consider a detector placed on Earth surface that measures the arrival times of muons produced in cosmic-ray interactions in the atmosphere. The average rate of muons entering the detector is constant $r = 100$ muons/s, and the efficiency of the detector is $\epsilon = 0.87$. That is, 13% of the incident muons can go through it undetected. Then, the probability density function (PDF) of the number of muons detected during a time interval t is a Poisson distribution

$$p(N) = \frac{(r\epsilon t)^N}{N!} e^{-r\epsilon t}. \tag{1}$$

To compute the PDF of the time interval between muon detections, we need to compute the PDF of intervals between Poisson distributed events. We proceed as follows

1. We compute the probability of having no events in a time t
$$P(0) = e^{-r\epsilon t} \tag{2}$$
2. Considering that t is the interval between consecutive events, we compute
$$P(t > s) = e^{-r\epsilon s}. \tag{3}$$
Then the cumulative distribution is equal to
$$F(s) = 1 - P(t > s) = 1 - e^{-r\epsilon s}. \tag{4}$$
3. Finally, the PDF for the time interval between muon detections is
$$g(t) = \frac{dF(t)}{dt} = r\epsilon e^{-r\epsilon t}. \tag{5}$$

II. METHODOLOGY AND RESULTS

We run $NC = 10^4$ Monte Carlo simulations of 10 second observations with the detector described, where we get a number N of detected muons in each simulation. We sample the values of N from the PDF (1) and obtain the results showed in FIG.1.

Then, to compute the distribution of the time intervals between muon detections, we take the number of detections N, and assign an arrival time to each event using a uniform distribution. Then, we sort these values and compute the time difference Δt between them. We repeat this process for the NC values of N obtained with the Monte Carlo simulations. Using these values, one obtains the histogram showed in FIG.2, that fits the exponential distribution in (5) for a $t = 10s$ observation.

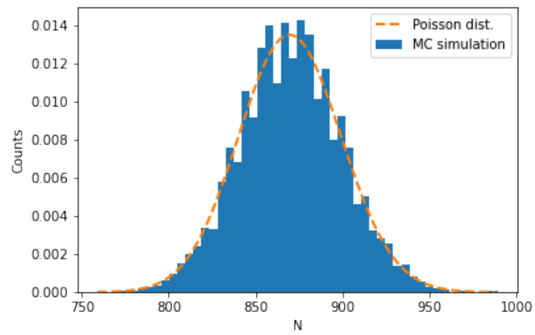

FIG. 1: Histogram of the values of N obtained in the NC Monte Carlo simulations and Poisson distribution showed in (1) for $t = 10s$ (orange).

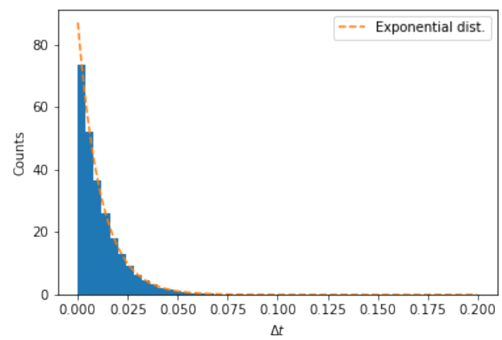

FIG. 2: Histogram of the values of Δt obtained using the values of N of the NC Monte Carlo simulations and exponential distribution showed in (5) for $t = 10s$ (orange).