POLITECNICO
MILANO 1863

Computing Infrastructures
-
System Dependability

Reliability Block Diagrams

The topics of the course: what are we going to see today?

<u>HW Infrastructures</u>:

System-level: Computing Infrastructures and Data Center Architectures, Rack/Structure;

Node-level: Server (computation, HW accelerators), Storage (Type, technology), Networking (architecture and technology);

Building-level: Cooling systems, power supply, failure recovery

<u>SW Infrastructures</u>:

Virtualization: Process/System VM, Virtualization Mechanisms (Hypervisor, Para/Full virtualization)

Computing Architectures: Cloud Computing (types, characteristics), Edge/Fog Computing, X-as-a service

<u>Methods</u>:

Reliability and availability of datacenters (definition, fundamental laws, **RBDs**)

Disk performance (Type, Performance, RAID)

Scalability and performance of datacenters (definitions, fundamental laws, queuing network theory)

Reliability Block Diagrams

An inductive model where a system is divided into blocks that represent distinct elements such as components or subsystems.

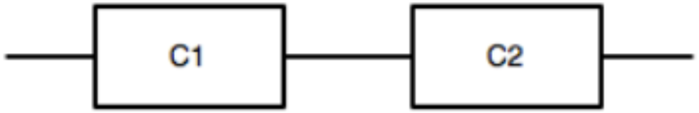
Every element in the RBD has its own reliability (previously calculated or modelled)

Blocks are then combined together to model all the possible *success paths*

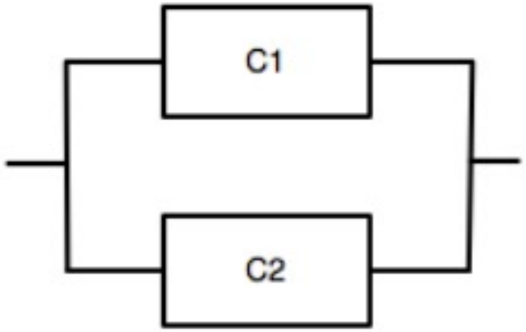
- **Model Topology can be different from the actual system topology**

Reliability Block Diagrams

RBDs are an approach to compute the reliability of a system starting from the reliability of its components



components in series



components in parallel

All components must be healthy for the system to work properly

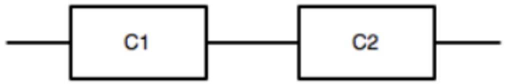
If one component is healthy the system works properly

Reliability Block Diagrams

- **Series**: System failure is determined by the failure of the *first* component

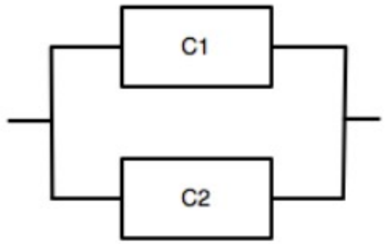
$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$

$$R_S(t) = R_{C1}(t) * R_{C2}(t)$$



- **Parallel:** System fails when the *last* component fails

$$R_S(t) = 1 - \prod \left(1 - R_i(t)\right)$$



$$R_S(t) = 1 - \left[(1 - R_{C1}(t)) * (1 - R_{C2}(t))\right]$$

$$R_S(t) = R_{C1}(t) + R_{C2}(t) - R_{C1}(t) * R_{C2}(t)$$

Reliability Block Diagrams

series

In general, if system S is composed by components with a reliability having an exponential distribution (very common case)

Failure in time

$$R_s(t) = e^{-\lambda_s t}$$

where

$$\lambda_s = \sum_{i=1}^{n} \lambda_i$$

Reliability Block Diagrams

series

In general, if system S is composed by components with a reliability having an exponential distribution (very common case)

Failure in time

$$R_s(t) = e^{-\lambda_s t} \quad \text{where} \quad \lambda_s = \sum_{i=1}^{n} \lambda_i$$

$$MTTF_S = \frac{1}{\lambda_S} = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} \frac{1}{MTTF_i}}$$

Reliability Block Diagrams

series

A special case: when all components are identical

$$R_s(t) = e^{-\lambda_s t}$$

$$R_S(t) = e^{-n\lambda t} = e^{-\frac{nt}{MTTF_1}} \qquad MTTF_S = \frac{MTTF_1}{n}$$

Reliability Block Diagrams

series

Availability:

$$A_S = \prod_{i=1}^{n} \frac{MTTF_i}{MTTF_i + MTTR_i}$$

When all components are the same:

$$A_S(t) = A_1(t)^n \qquad A = \left(\frac{MTTF_1}{MTTF_1 + MTTR_1} \right)^n$$

Reliability Block Diagrams

parallel

System P composed by *n* components

$$R_P(t) = 1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right)$$

Availability

$$A_P(t) = 1 - \prod_{i=1}^{n}\left(1 - A_i(t)\right)$$

$$A_P = 1 - \prod_{i=1}^{n}\left(1 - A_i\right) = 1 - \prod_{i=1}^{n}\frac{MTTR_i}{MTTF_i + MTTR_i}$$

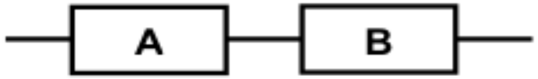
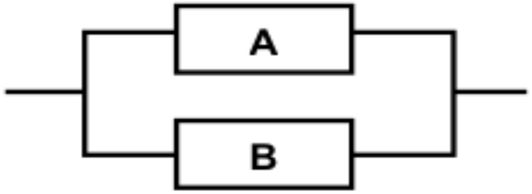
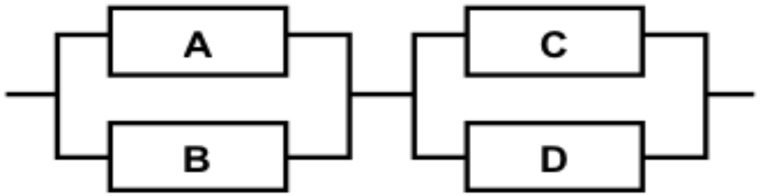
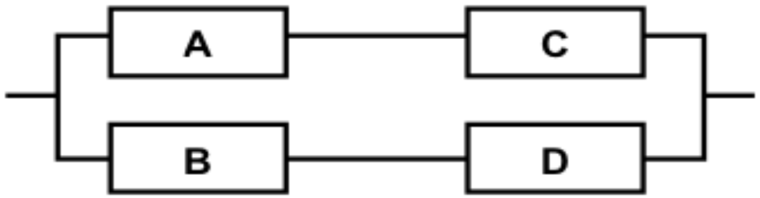
Reliability Block Diagrams (recap)

$$R_s = \prod_i^n R_i$$

$$R_s = 1 - \prod_i^n (1 - R_i)$$

Component redundancy

System redundancy

Type	Block Diagram Representation	System Reliability (R_S)
Series		$R_S = R_A R_B$ R_A = reliability, component A R_B = reliability, component B
Parallel		$R_S = 1-(1-R_A)(1-R_B)$
Series-Parallel		$R_S = [1-(1-R_A)(1-R_B)]^*$ $[1-(1-R_C)(1-R_D)]$ R_C = reliability, component C R_D = reliability, component D
Parallel-Series		$R_S = 1-(1-R_A R_C)^*$ $(1-R_B R_D)$

Example 1

What is the Reliability of the entire system knowing the Reliability of each component?

$R_A = 0.95$

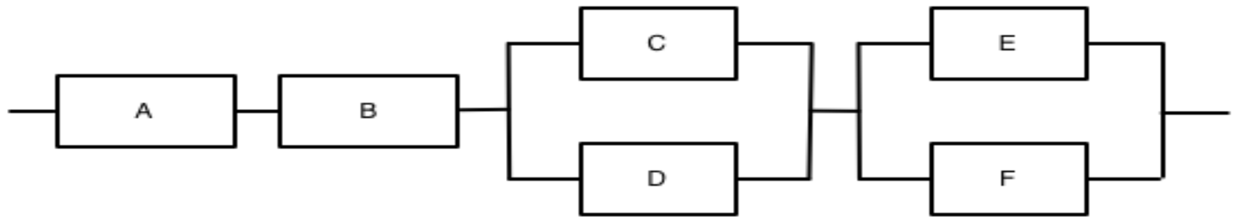
$R_B = 0.97$

$R_C = 0.99$

$R_D = 0.99$

$R_E = 0.92$

$R_F = 0.92$



Example 1

RBDs

What is the Reliability of the entire system knowing the Reliability of each component?

$R_A = 0.95$

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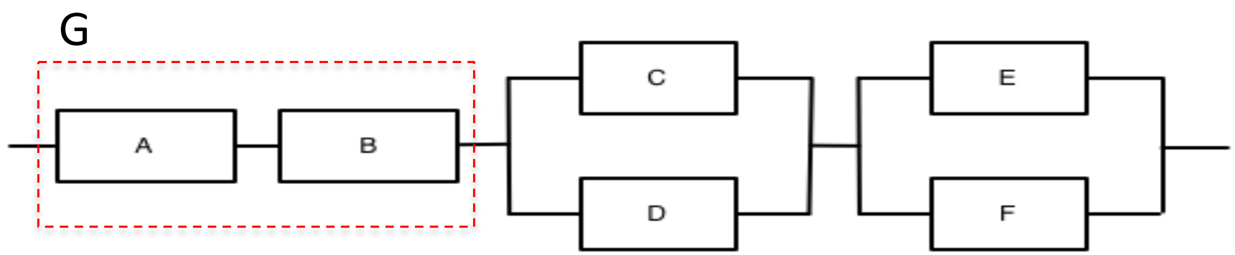
$R_F = 0.92$



$R_G = R_A * R_B$

$R_G = 0.9215$

Example 1

What is the Reliability of the entire system knowing the Reliability of each component?

$R_A = 0.95$

$R_B = 0.97$

$R_C = 0.99$

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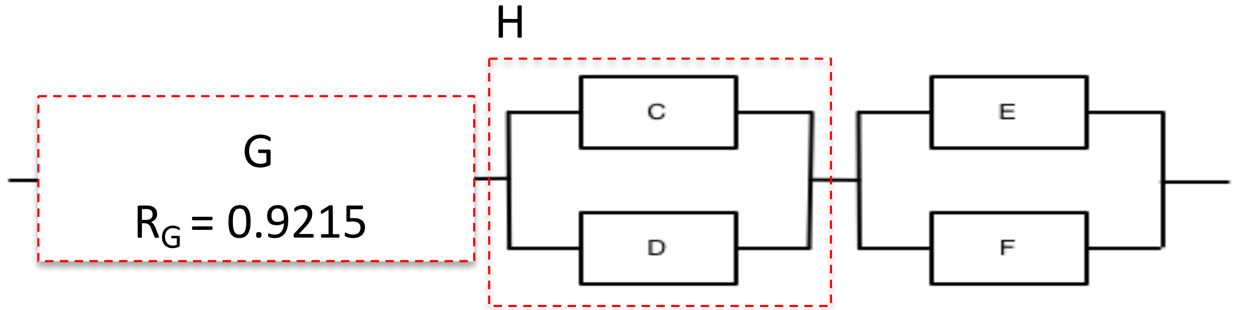

$R_H = 1-[(1-R_c)*(1-R_D)]$

$R_H = 0.9999$

Example 1

RBDs

What is the Reliability of the entire system knowing the Reliability of each component?

$R_A = 0.95$

$R_B = 0.97$

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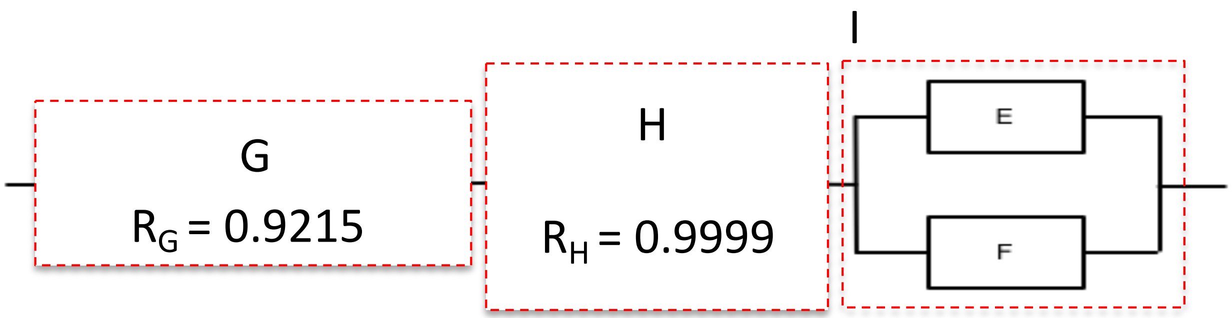
$R_D = 0.99$

$R_E = 0.92$

$R_F = 0.92$



$R_I = 1-[(1-R_E)*(1-R_F)]$

$R_I = 0.9936$

Example 1

What is the Reliability of the entire system knowing the Reliability of each component?

$R_A = 0.95$

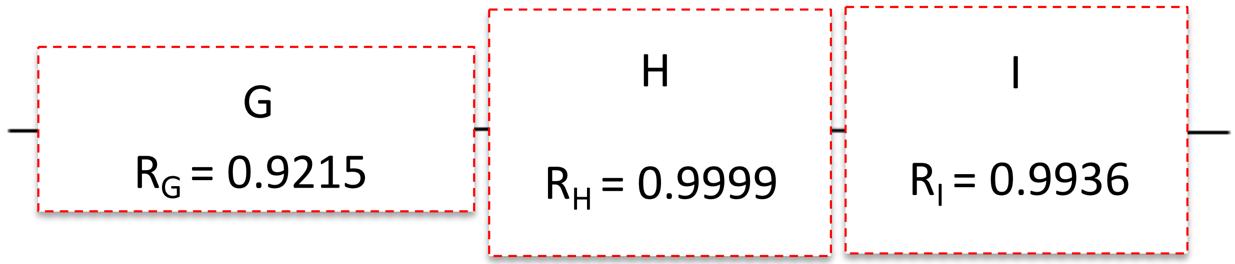
$R_B = 0.97$

$R_C = 0.99$

$R_D = 0.99$

$R_E = 0.92$

$R_F = 0.92$



$R_S = R_G * R_H * R_I = 0.9155$

Example 1

What is the Reliability of the entire system knowing the Reliability of each component?

$R_A = 0.95$

$R_B = 0.97$

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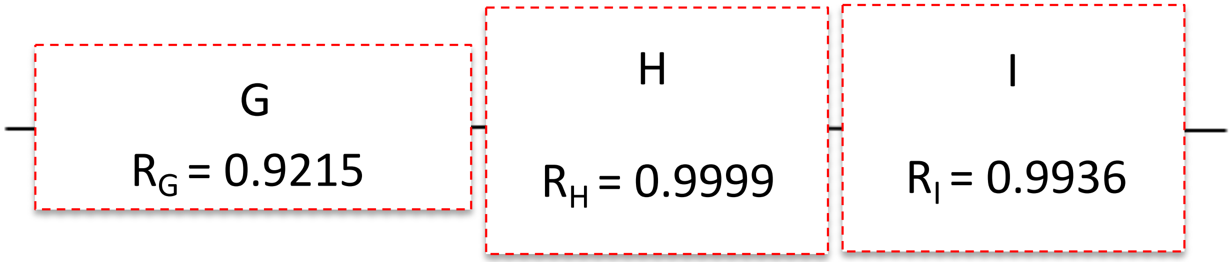


$R_S = R_G * R_H * R_I = 0.9155$

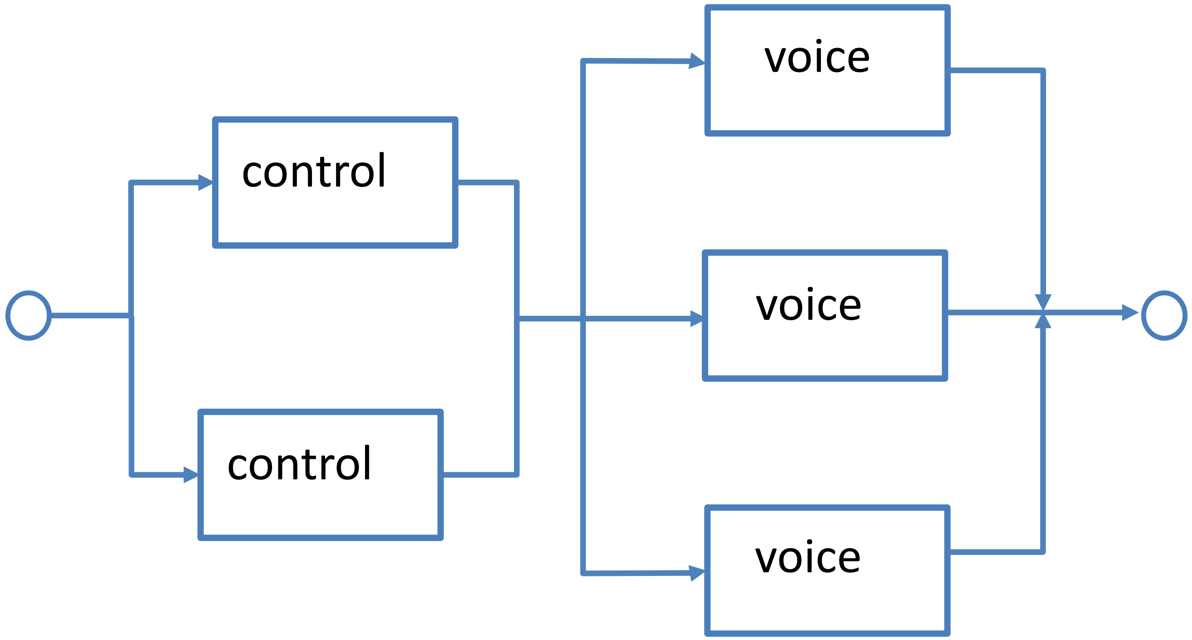
$R_s = R_A R_B [1-(1-R_C)(1-R_D)] [1-(1-R_E)(1-R_F)]$

$R_s = (0.95)(0.97)[1-(1-0.99)(1-0.99)] [1-(1-0.92)(1-0.92)]$

Example 2

2 control blocks and 3 voice channels:

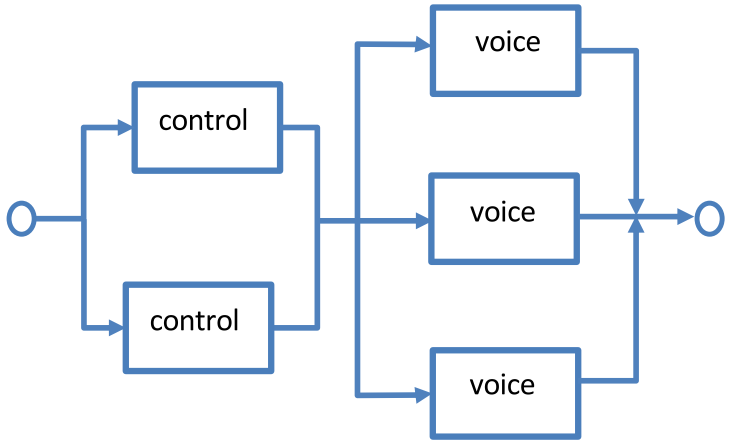
- system is up if at least 1 control channel and at least 1 voice channel are up



Example 2 – cont'd

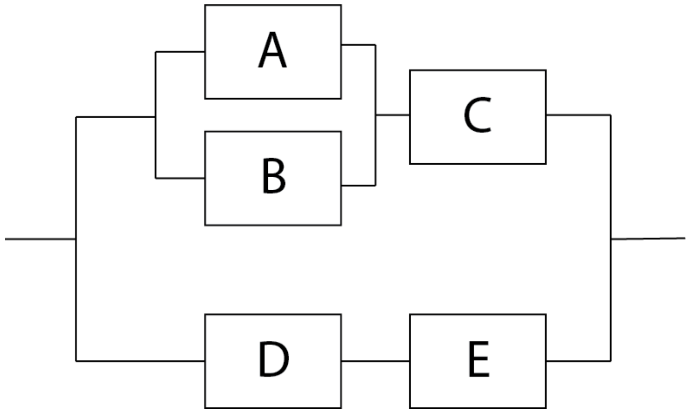
- Each control channel has reliability R_c
- Each voice channel has reliability R_v

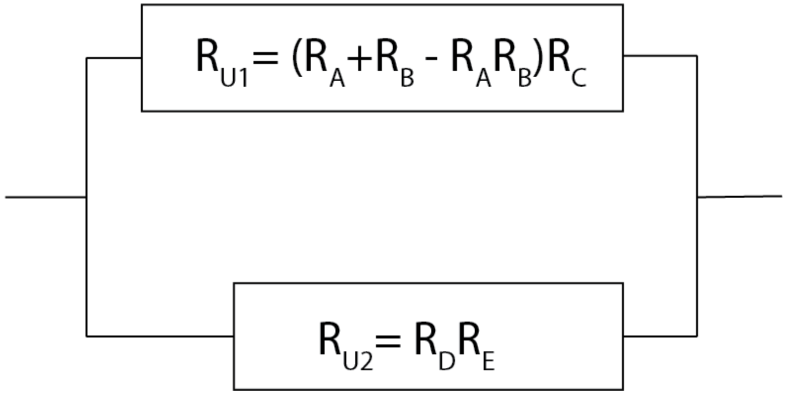
- Reliability:

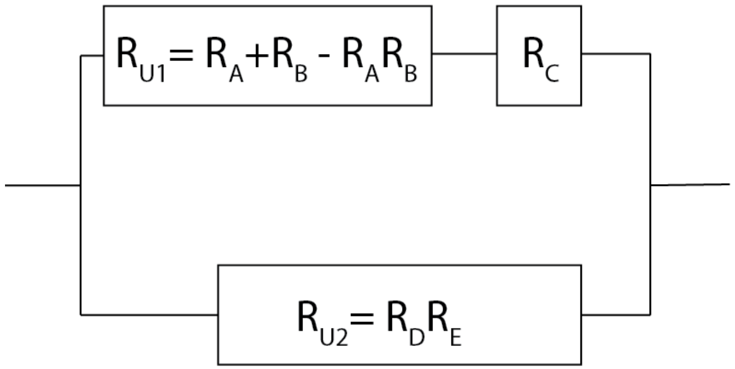


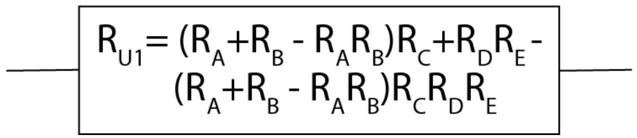
$$R = [1-(1-R_c)^2][1-(1-R_v)^3]$$

Example 3


A
B
C
D
E


$R_{U1} = (R_A + R_B - R_A R_B) R_C$
$R_{U2} = R_D R_E$


$R_{U1} = R_A + R_B - R_A R_B$
R_C
$R_{U2} = R_D R_E$

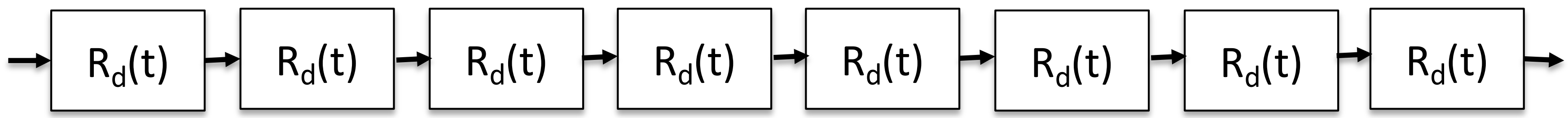

$R_{U1} = (R_A + R_B - R_A R_B) R_C + R_D R_E -$
$(R_A + R_B - R_A R_B) R_C R_D R_E$

RBD: used to model a system and calculate its reliability

We have a RAID-0 storage system composed of 8 parallel disks; each disk of the system may fail independently of the others;

If the reliability of each disk is $R_d(t)$, what is the overall reliability of the storage system?

How would you model the entire storage system using an RBD?



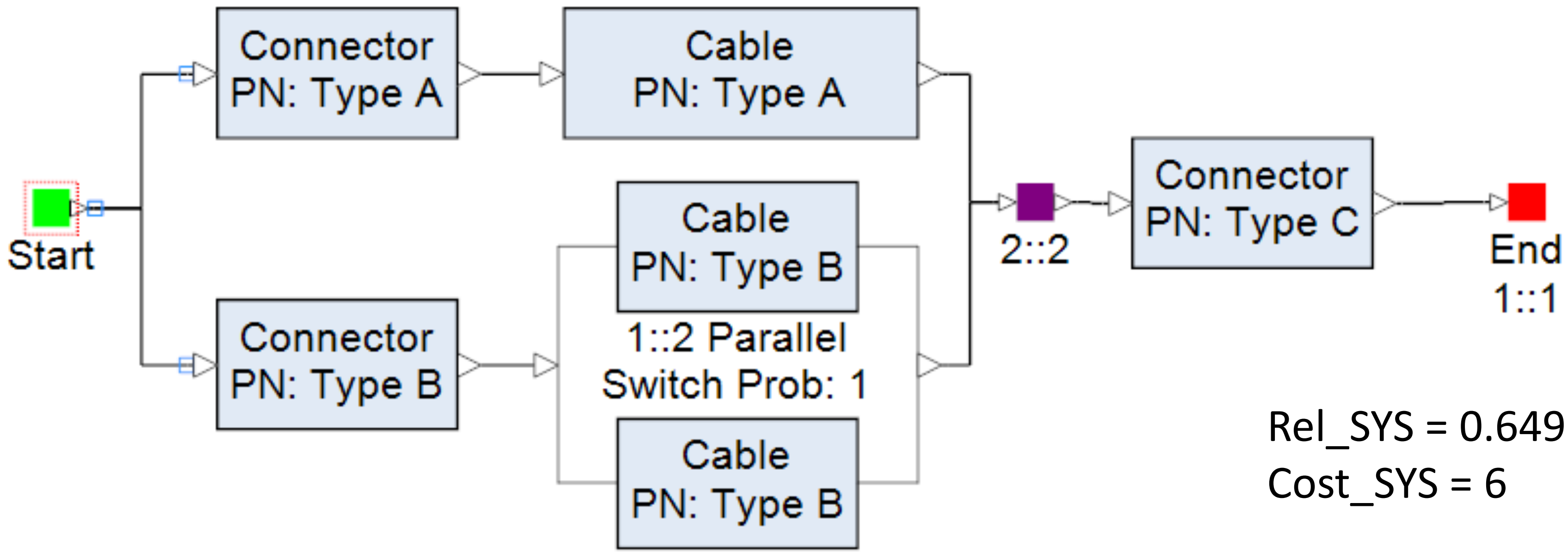
RBD: used to compare different alternatives

RBDs

Cable Bundle

Each block has R = 0.9

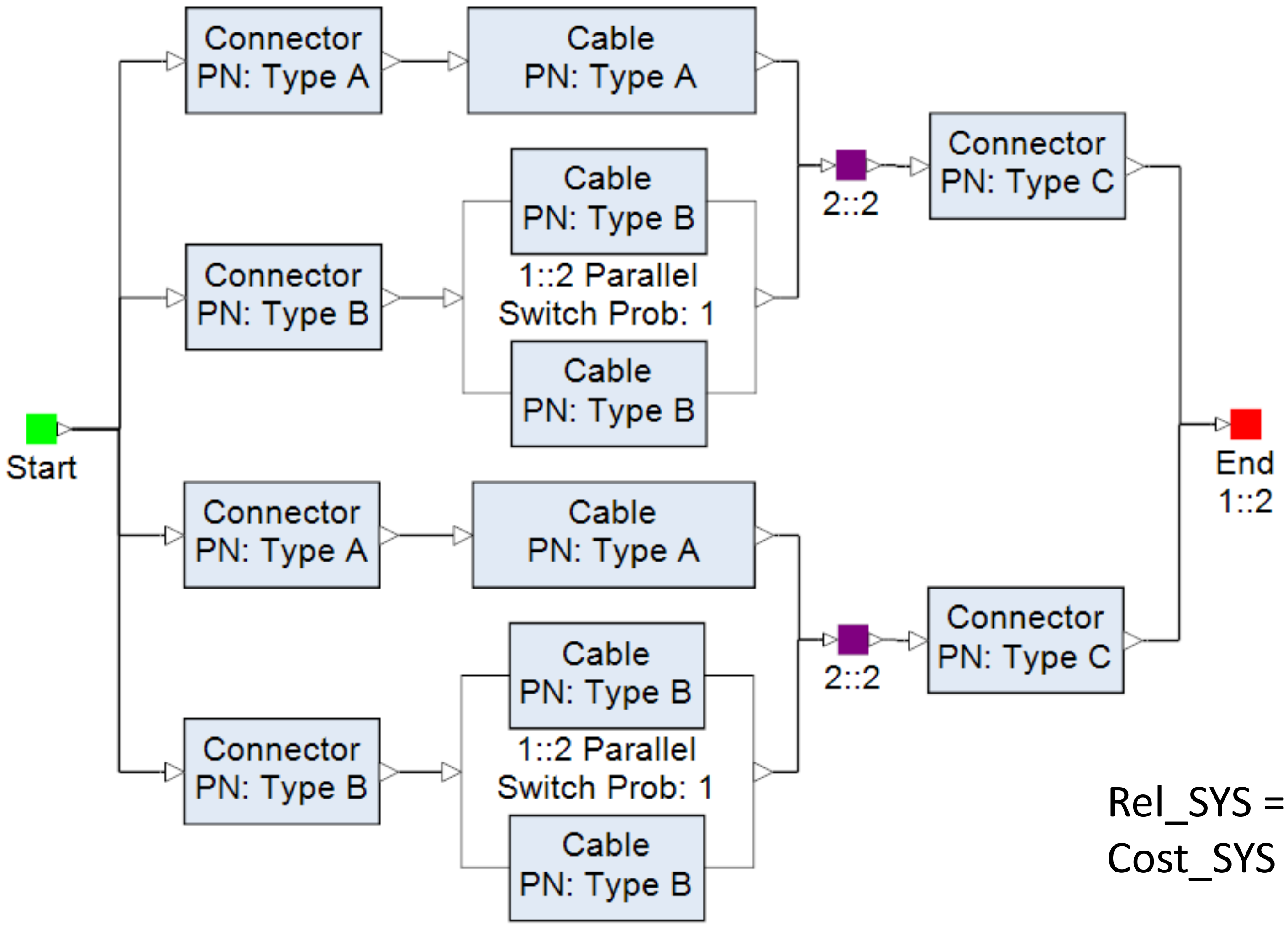
Each block costs 1



Rel_SYS = 0.649539
Cost_SYS = 6

Alternative 1

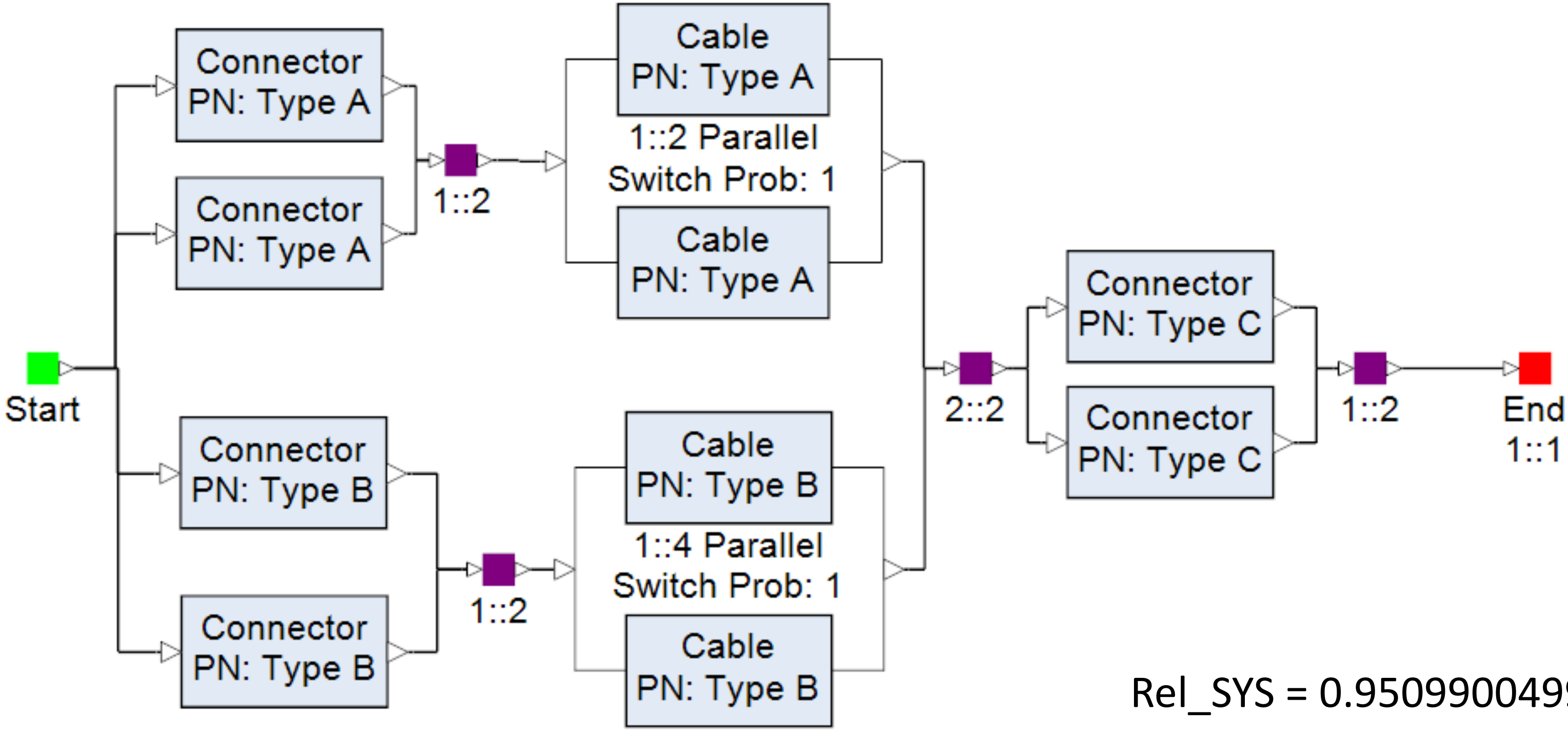
RBDs



Rel_SYS = 0.877177

Cost_SYS = 12

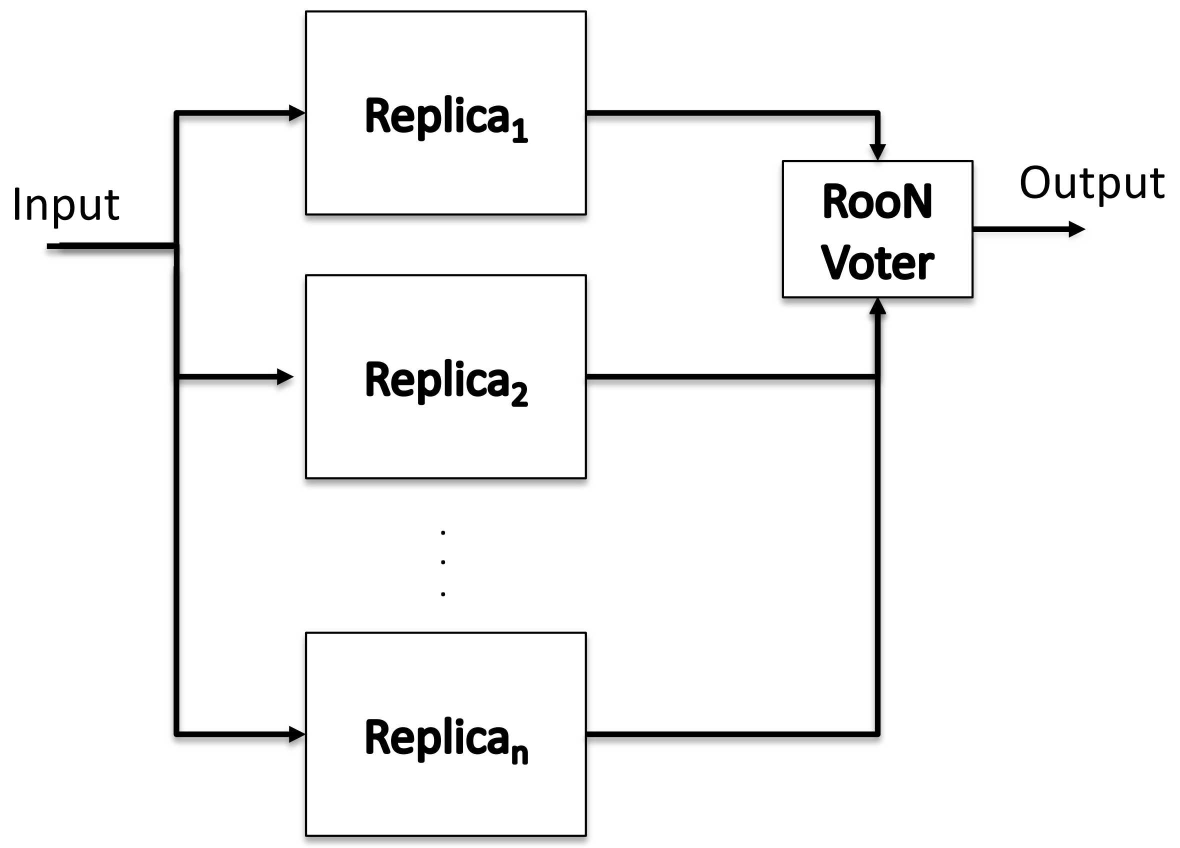
Alternative 2



Rel_SYS = 0.9509900499
Cost_SYS = 10

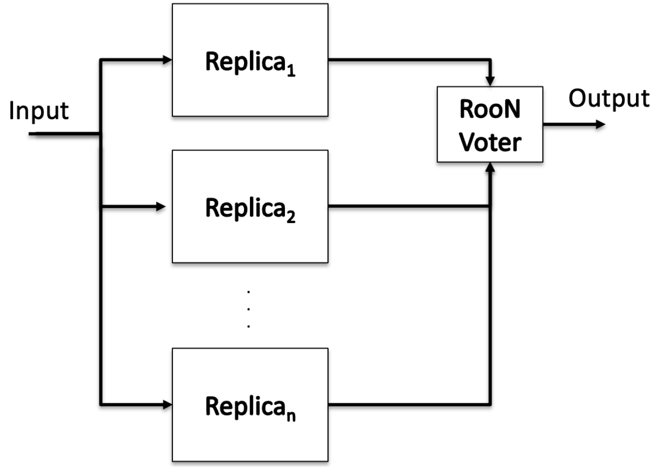
r out of *n* redundancy (RooN)

A system composed of *n* identical replicas where at least *r* replicas have to work fine for the entire system to work fine



r out of *n* redundancy (RooN)

RBDs

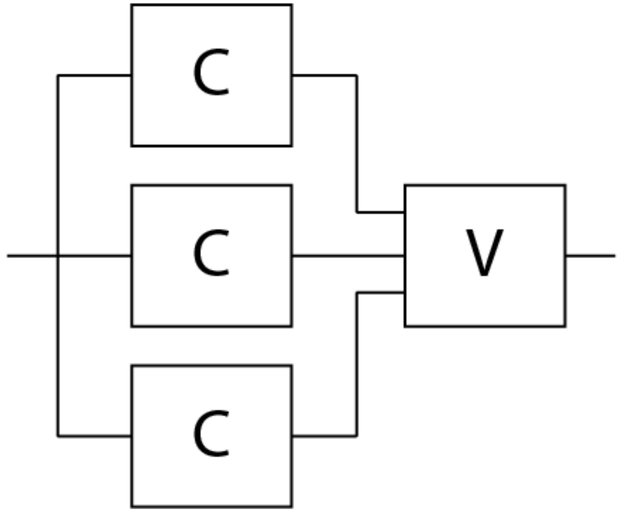


R_s = System reliability
R_c = Component reliability
R_V= Voter Reliability
n = Number of components
r = Minimum number of components which must survive

$$R_S(t) = RV \sum_{i=r}^{n} R_c^i (1 - R_C)^{n-i} \underbrace{\frac{n!}{i!\,(n-i)!}}$$

Binomial coefficient
$\binom{n}{i}$

Triple Modular Redundancy – TMR



System works properly if

- 2 out of 3 components work properly AND the voter works properly

$$R_{TMR} = R_v[\sum_{i=2}^{3}\binom{3}{i}R_m^i(1-R_m)^{3-i}] = R_v[R_m^3 + 3R_m^2(1-R_m)] = R_v(3R_m^2 - 2R_m^3)$$

$$MTTF_{TMR} = \int_0^\infty R_{TMR}dt = \int_0^\infty R_v(3R_m^2 - 2R_m^3)dt = \int_0^\infty e^{-\lambda_v t}(3e^{-2\lambda_m t} - 2e^{-3\lambda_m t})dt$$

$$= \frac{3}{2\lambda_m + \lambda_v} - \frac{2}{3\lambda_m + \lambda_v} \cong \frac{3}{2\lambda_m} - \frac{2}{3\lambda_m} = \left(\frac{5}{6}\right)\left(\frac{1}{\lambda_m}\right) = \frac{5}{6}MTTF_{simplex}$$

Triple Modular Redundancy – TMR – GOOD or BAD?

RBDs

- $MTTF_{TMR}$ is shorter than $MTTF_{symplex}$
- Can tolerate transient faults and permanent faults
- Higher reliability (for shorter missions)

When do we have the same reliability?

- $R_{TMR}(t) = R_C(t)$

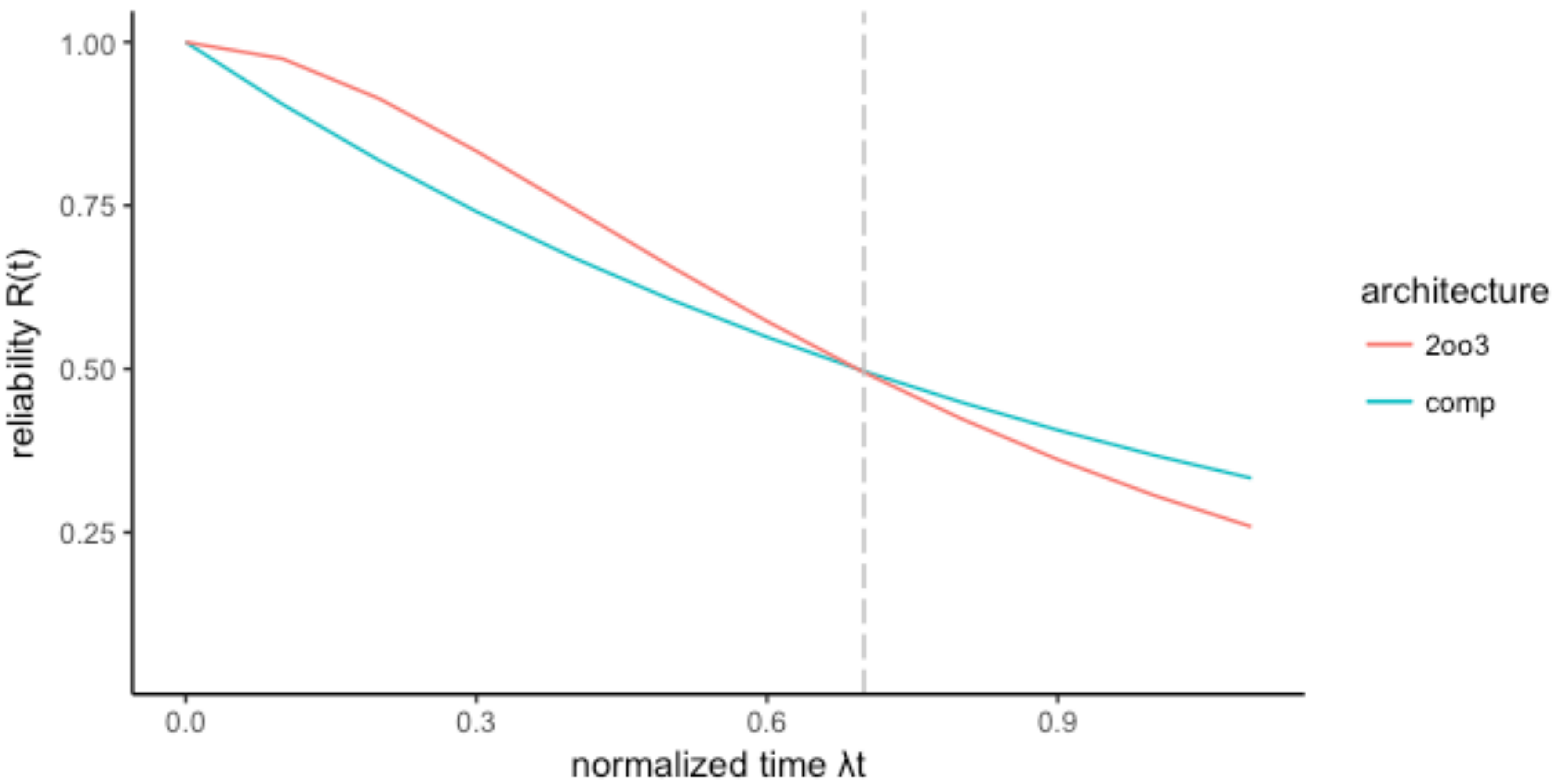
$$3e^{-2\lambda_m t} - 2e^{-3\lambda_m t} = e^{-\lambda_m t}$$

$$t = \frac{\ln 2}{\lambda_m} \cong 0.7\ MTTF_C$$

$R_{TMR}(t) > R_C(t)$ when the mission time is shorter than 70% of $MTTF_C$

TMR

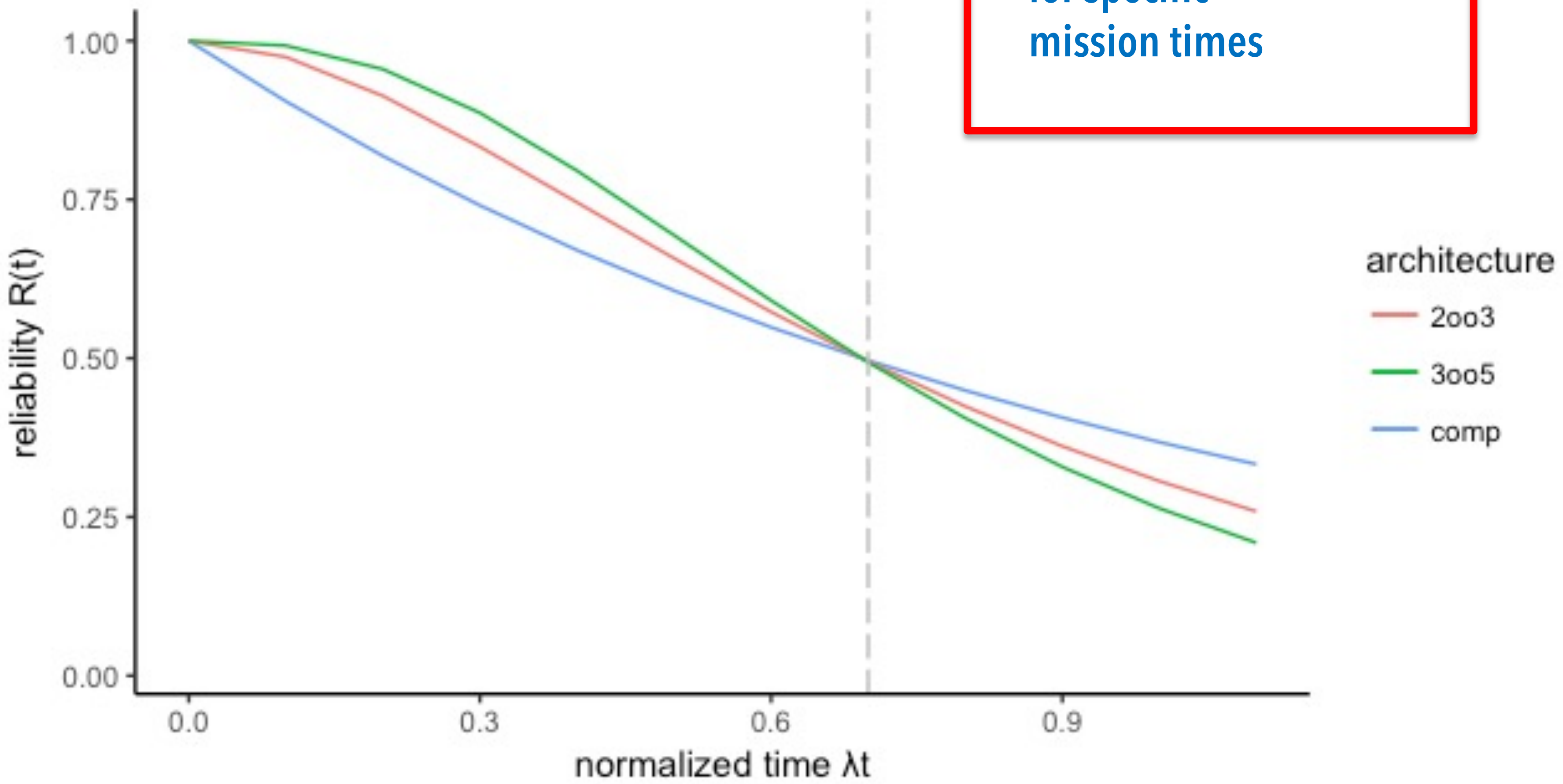
TMR: 2 out of 3 components (voter is a 'perfect' element)



nMR

RBDs

TMR: 2oo3 and nMR: 3oo5

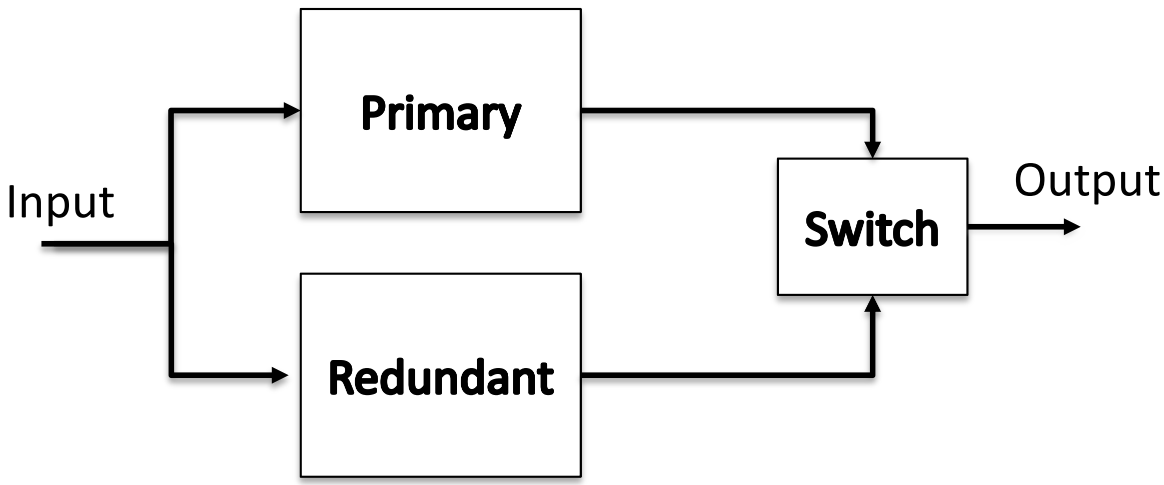


Redundancy is useful for specific mission times

Standby redundancy

A system may be composed of two parallel replicas:

- The primary replica working all time, and
- The redundant replica (generally disable) that is activated when the primary replica fails



Standby redundancy

RBDs

A system may be composed of two parallel replicas:

- The primary replica working all time, and
- The redundant replica (generally disable) that is activated when the primary replica fails

- **What do we need for such a redundancy to be operational?**

Obviously, we need:

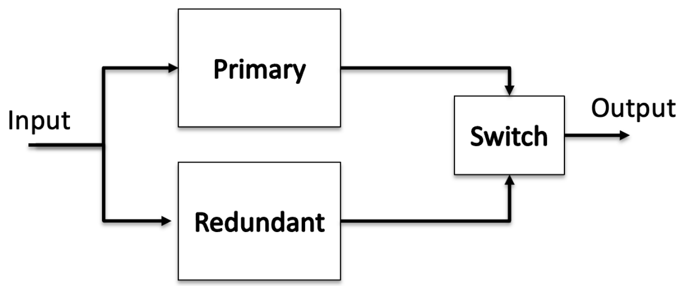
- A mechanism to determine whether the primary replica is working properly or not (on-line self check)
- A dynamic switching mechanism to disable the primary replica and activate the redundant one

Standby redundancy – Quick Formulas

Standby Parallel Model	System Reliability
Equal failure rates, perfect switching	$R_s = e^{-\lambda t}(1+\lambda t)$
Unequal failure rates, perfect switching	$R_s = e^{-\lambda_1 t} + \lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) / (\lambda_2 - \lambda_1)$
Equal failure rates, imperfect switching	$R_s = e^{-\lambda t}(1+ \mathrm{R_{switch}} \lambda t)$
Unequal failure rates, imperfect switching	$R_s = e^{-\lambda_1 t} + \mathrm{R_{switch}} \lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) / (\lambda_2 - \lambda_1)$

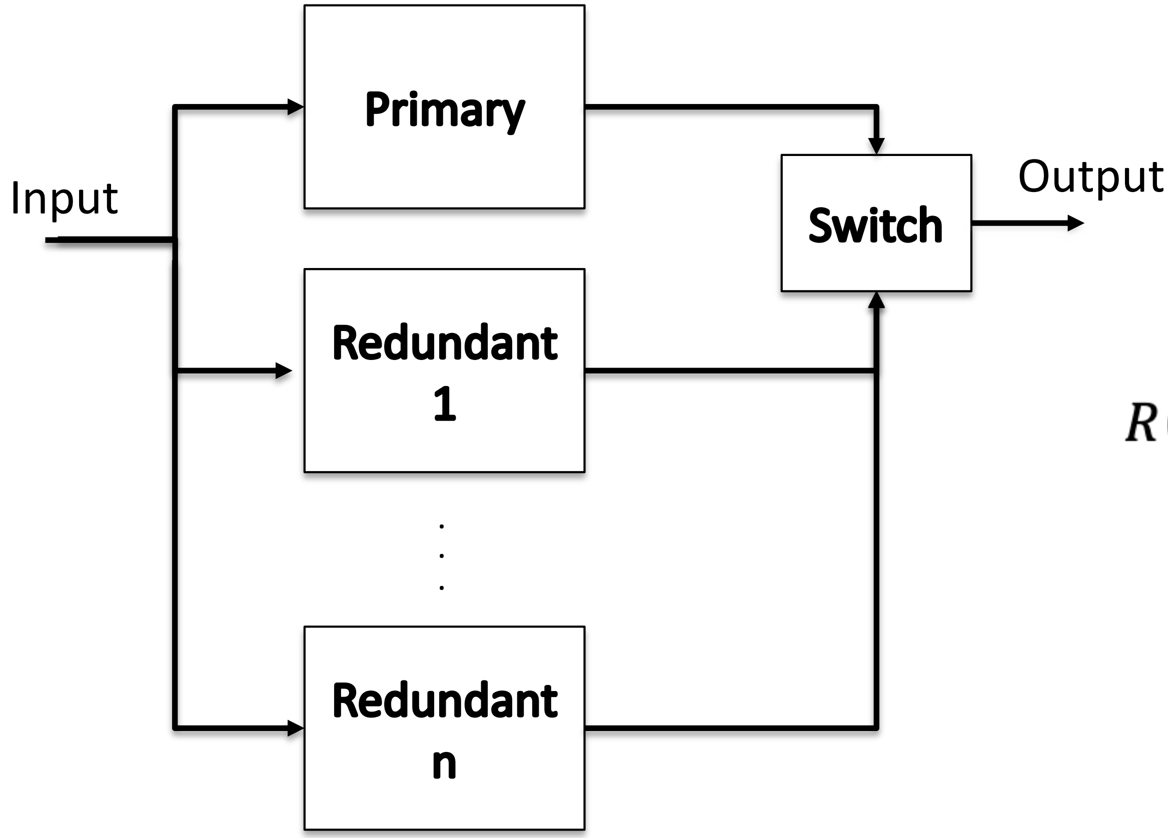
where

R_s = System reliability
λ = Failure rate
t = Operating time
R_{switch} = Switching reliability



Standby redundancy – Quick Formulas

More in general, a system having one primary replica and *n* redundant replicas (with identical replicas and perfect switching)



$$R(t) = e^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!}$$