Image Denoising by Data-driven Orthonormal Basis

Mathematical Models and Methods for Image Processing

Edoardo Peretti

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Assignments

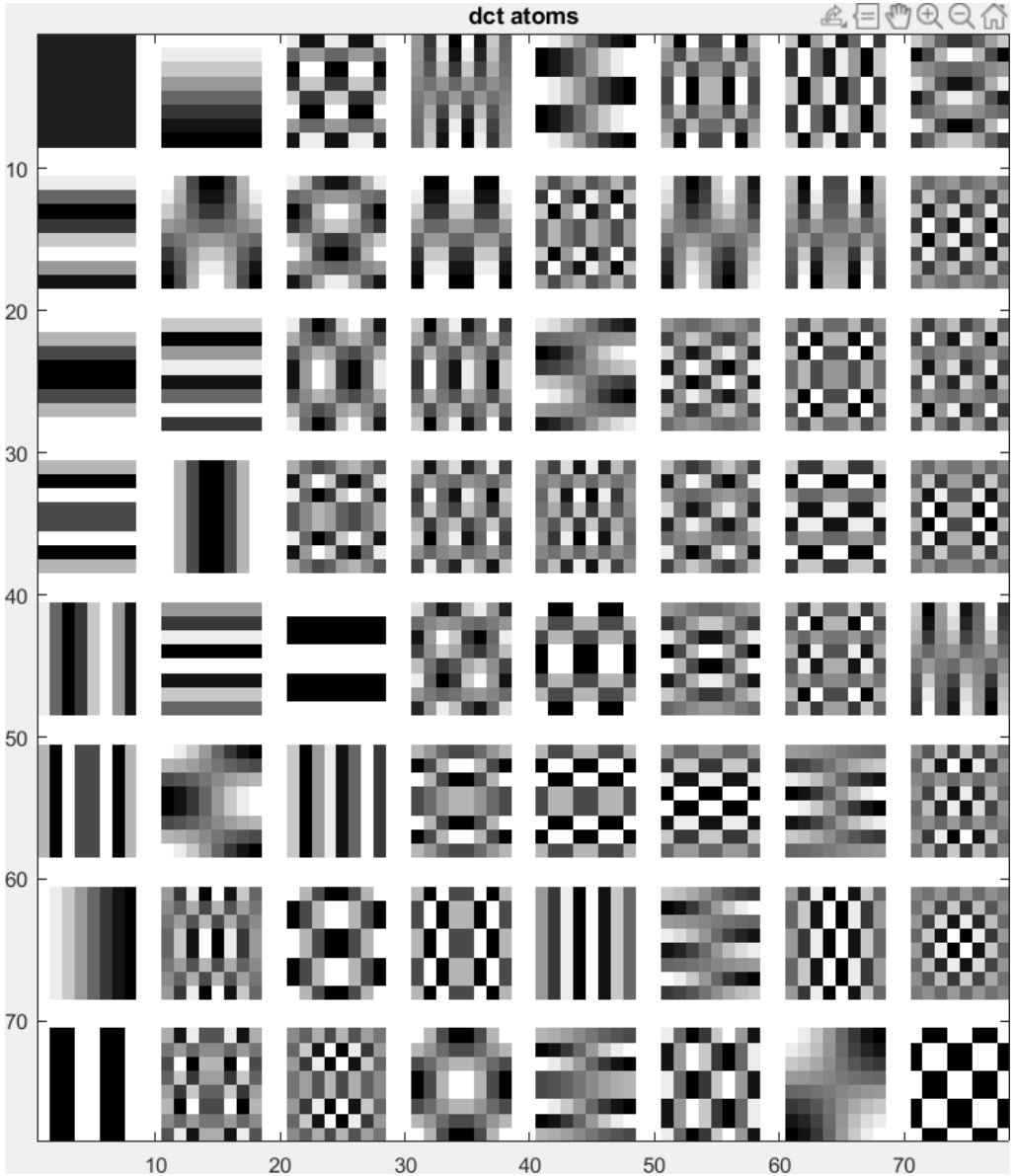
PCA-based denoising

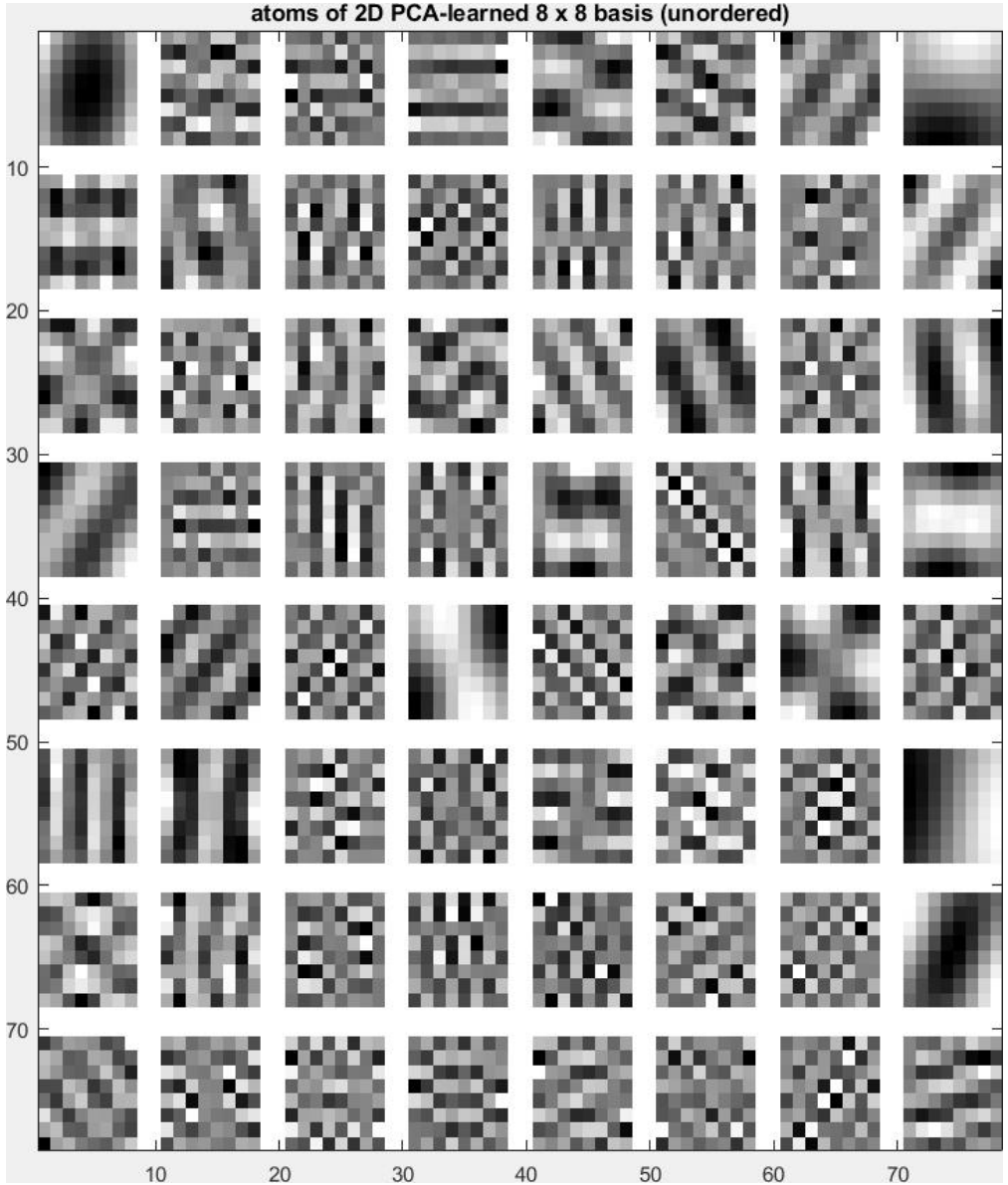
Assignment 1

Implement Global PCA-based denoising.

- Define the orthonormal basis by the PCA of all the image patches
- Visualize the atoms of the PC basis and compare these against DCT basis

DCT Atoms vs PC from Lena





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Assignment 1

Implement Global PCA-based denoising.

- Define the orthonormal basis by the PCA of all the image patches
- Visualize the atoms of the PC basis and compare these against DCT basis
- Compute projection and enforce sparsity w.r.t. this basis
- Implement the same aggregation scheme as for the DCT-based denoising

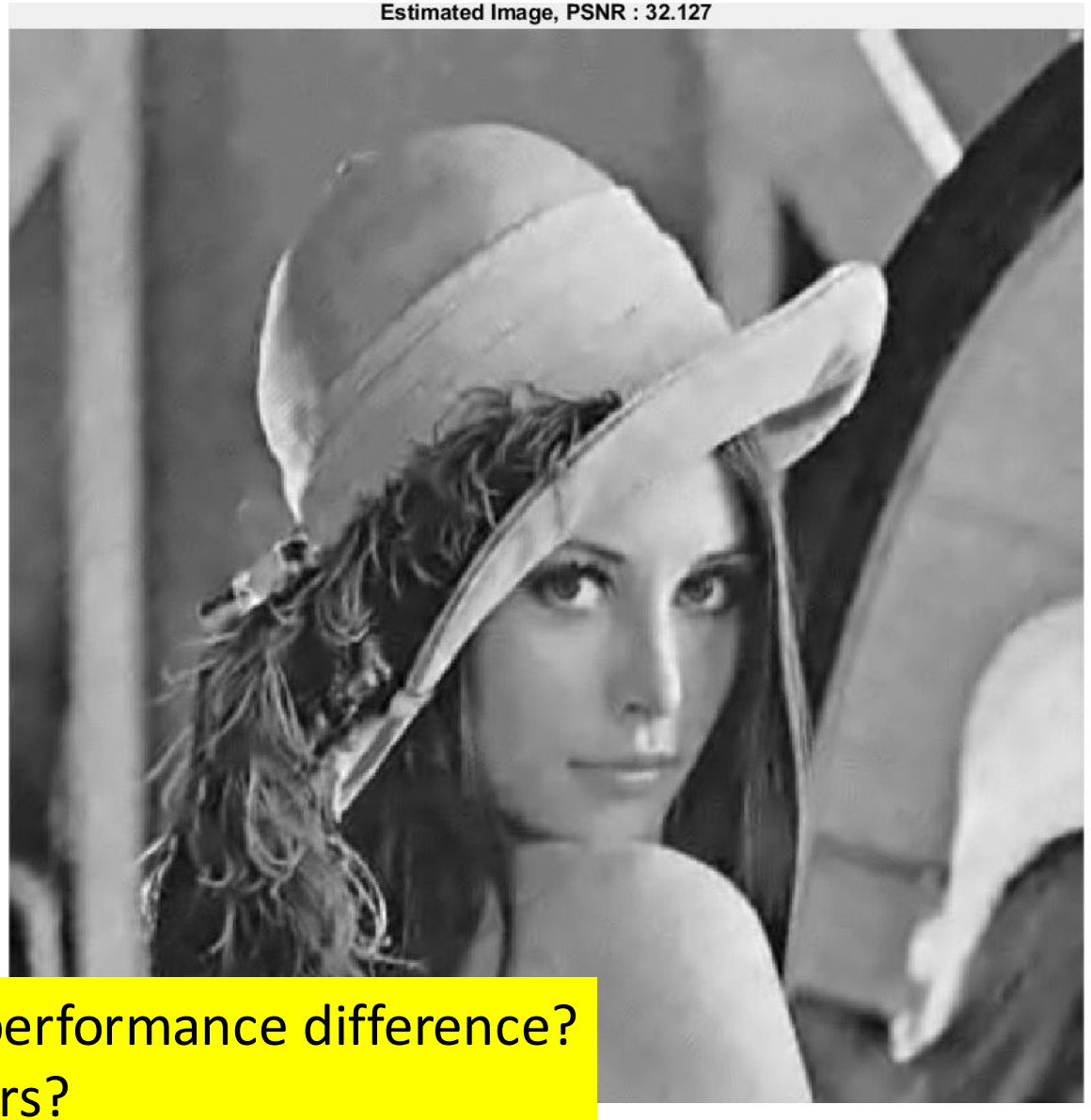
- Implement a variant where you project over the first k principal components

A reference paper

C.-A. Deledalle, J. Salmon, A. S. Dalalyan, *"Image denoising with patch based PCA: local versus global"* BMVC 2011, PDF.

Projection over k PC

Sparsity w.r.t U



- Can you explain such a performance difference?
- Are these linear operators?

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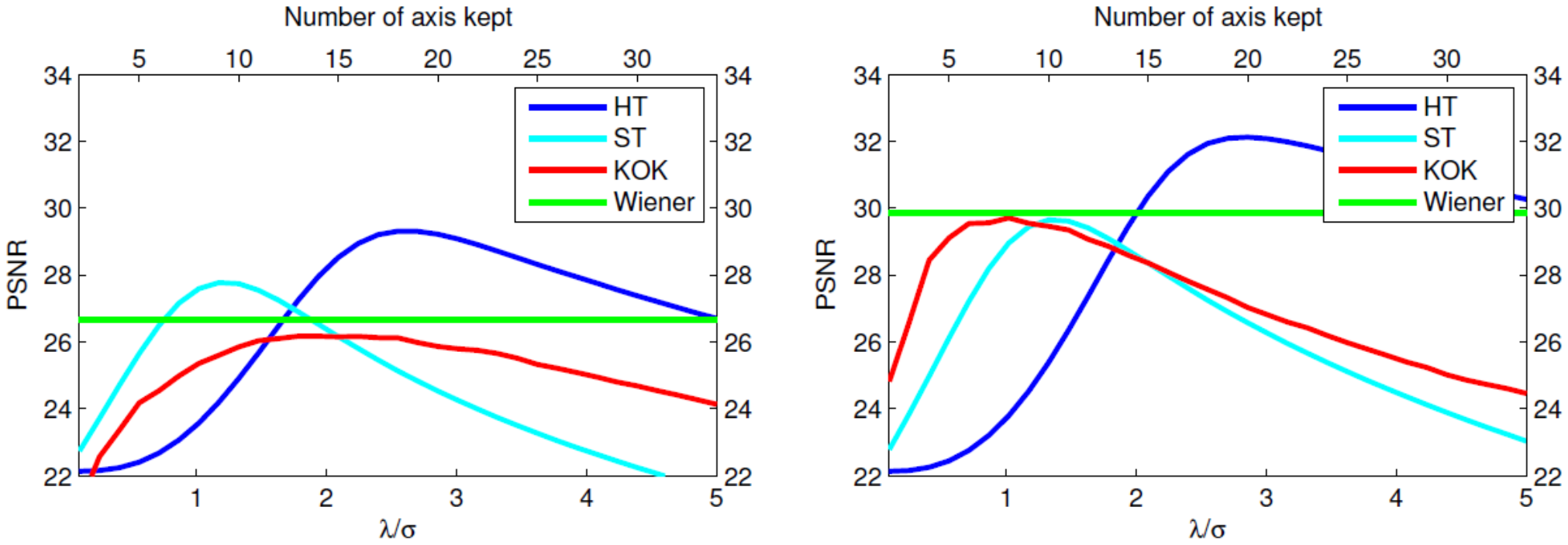
See also paper results



(a) Cameraman (b) House

Figure 3: Comparing various strategies of reconstruction from the projections onto the basis provided by PCA, for House and Cameraman ($\sigma = 20$): Hard Thresholding, Soft Thresholding, "Keep or Kill" and Wiener Filtering. The x axes are different. The number of coefficients kept for the "Keep or Kill" strategy (top) and the threshold ratio λ/σ (bottom).

C.-A. Deledalle, J. Salmon, A. S. Dalalyan, *"Image denoising with patch based PCA: local versus global"* BMVC 2011, PDF.

Assignment 2 (Optional)

- Implement a function to estimate the U matrix (and the preprocessing, the number of PC to preserve) within an image region

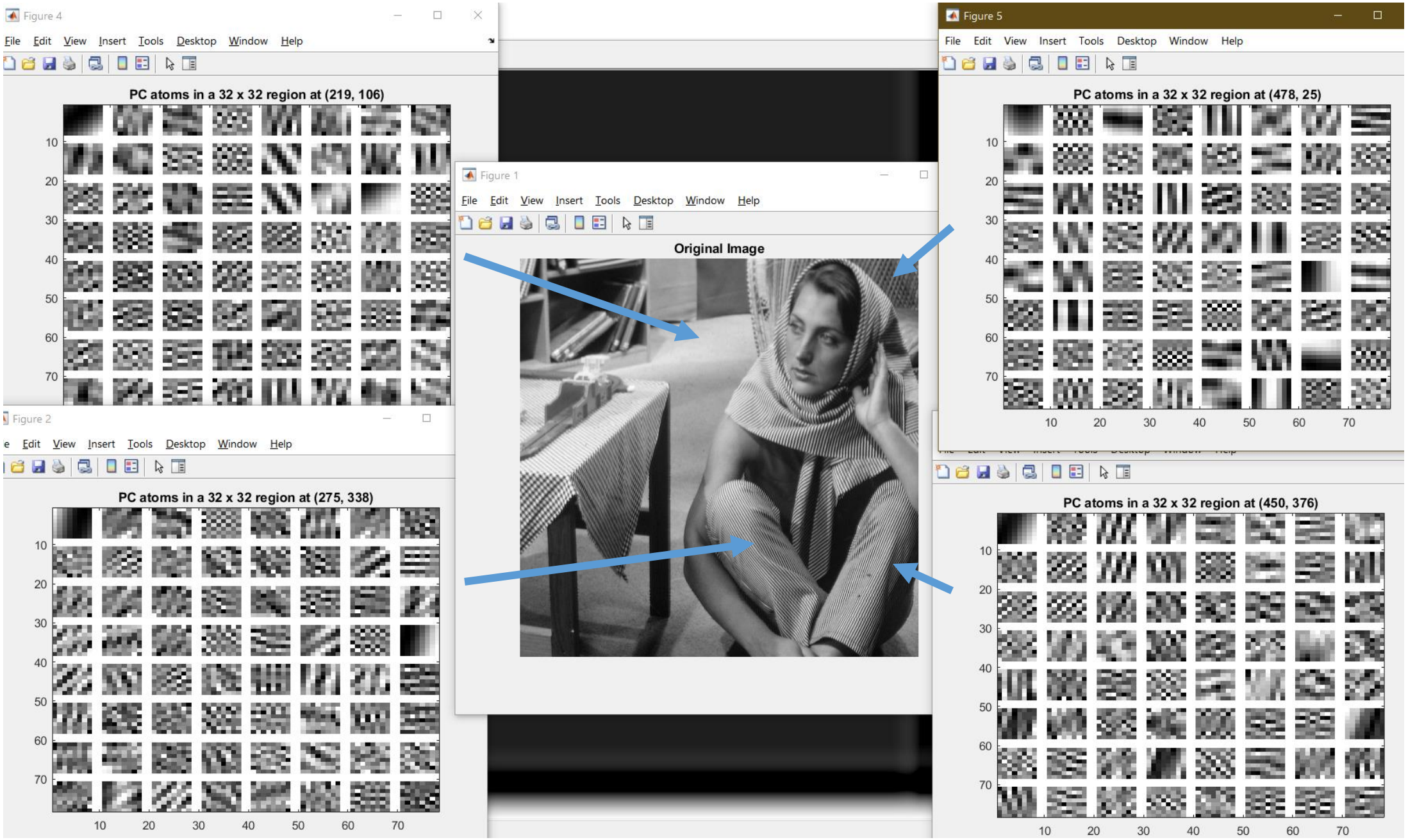


Figure 4
File Edit View Insert Tools Desktop Window Help
PC atoms in a 32 x 32 region at (219, 106)
Figure 5
File Edit View Insert Tools Desktop Window Help
PC atoms in a 32 x 32 region at (478, 25)
Figure 1
File Edit View Insert Tools Desktop Window Help
Original Image
Figure 2
e Edit View Insert Tools Desktop Window Help
PC atoms in a 32 x 32 region at (275, 338)
PC atoms in a 32 x 32 region at (450, 376)
10 20 30 40 50 60 70

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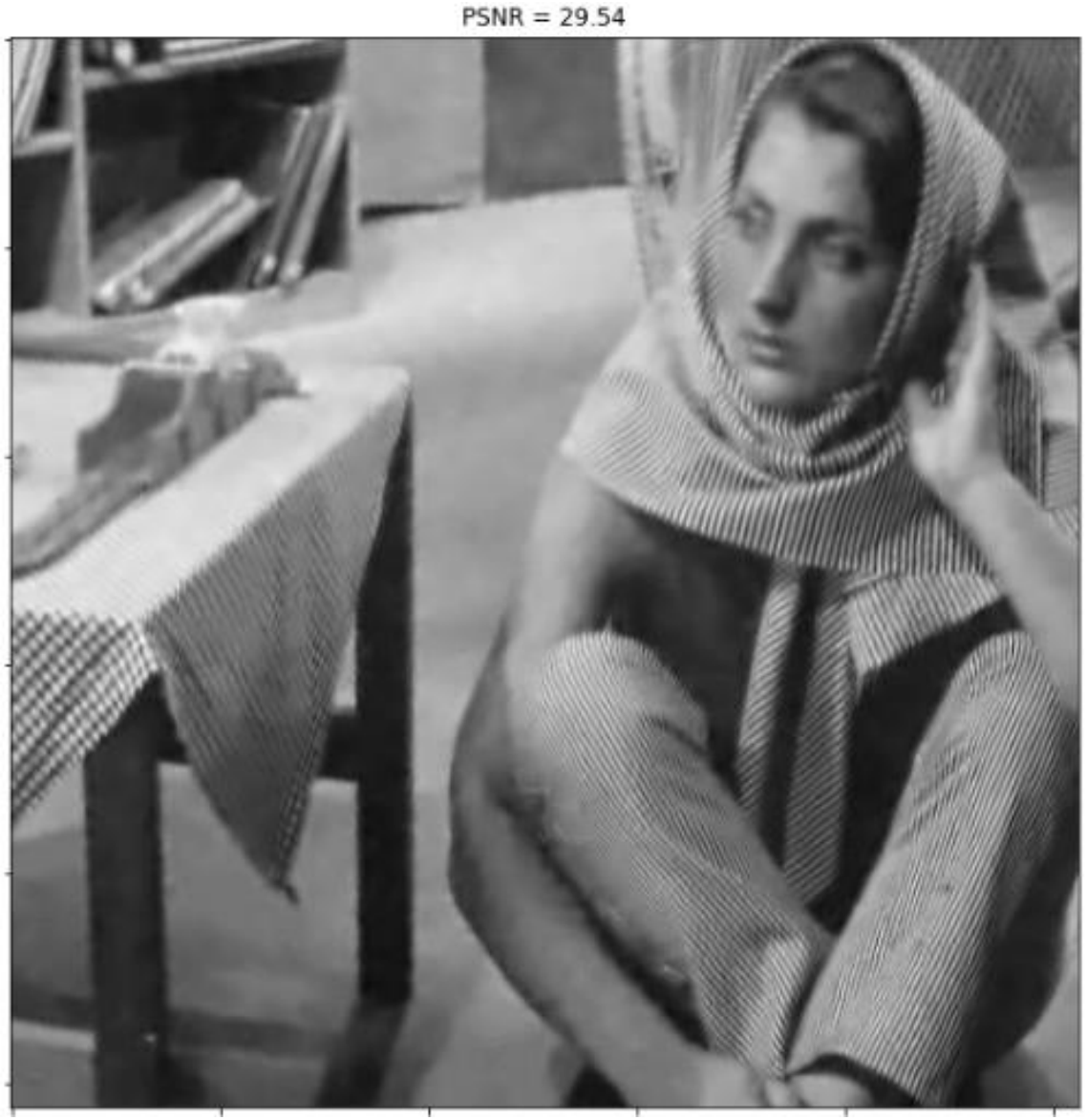
Assignment 2 (Optional)

- Implement a function to estimate the U matrix (and the preprocessing, the number of PC to preserve) within an image region

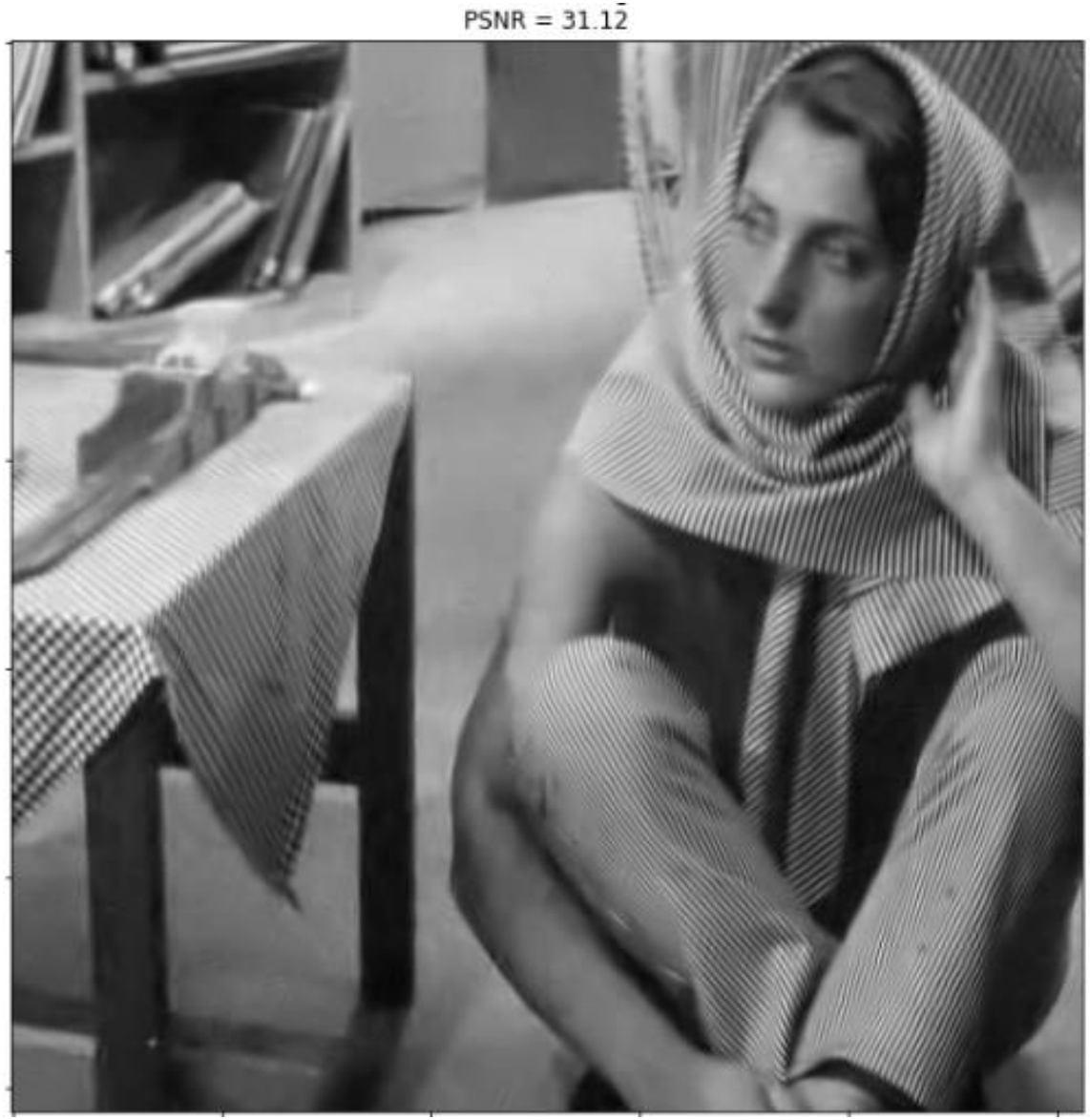
Implement denoising by Local PCA basis

- First, define image regions where to compute PCA as tiles over an image (no overlap)
  - Hint: generate a collection of basis beforehand, then use the right basis in each pixel (depending on the location)
- Then define overlapping regions, to obtain for each patch, possibly estimates from different basis. Aggregate always by averaging
  - Hint: a simpler strategy than expanding the above approach is to define overlapping regions over an image. In each region extract PC and then perform denoising. Aggregate all the results at the end

Global PCA



Local PCA



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