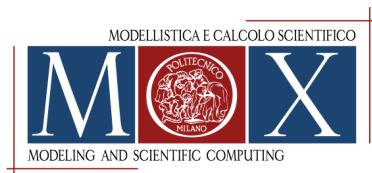


# NUMERICAL LINEAR ALGEBRA

Prof. Paola Antonietti

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Politecnico di Milano  
<https://antonietti.faculty.polimi.it>

TA: Dr. Michele Botti



POLITECNICO  
MILANO 1863

DEPARTMENT  
OF MATHEMATICS

## P0: Introduction

# Course information

## Lectures/TA:

Prof. Paola F. Antonietti ([paolaantonietti@polimi.it](mailto:paolaantonietti@polimi.it))

Dr. Michele Botti ([michele.botti@polimi.it](mailto:michele.botti@polimi.it))

**Timetable:** The calendar is available on WeBeep [[Link](#)]. Please check it regularly as there might be changes.

**Office hours:** Upon request. Please send an email to [paolaantonietti@polimi.it](mailto:paolaantonietti@polimi.it)

**Recorded lectures:** Links to the recorded lectures will be progressively made available on the WeBeep page course [[Link](#)].

**Slides:** Slides will be progressively made available on the WeBeep page

# Exam

The exam consists of two components: course hands-on challenges (completed during the course) and a written test.

An optional oral exam may be taken upon the student's request.

		Assigned on	Submission deadline
<b>Course Hands-on Challenge I</b>	10% of the final grade	<b>September 23, 2024</b>	<b>October 7, 2024 (h: 23:59)</b>
<b>Course Hands-on Challenge II</b>	10% of the final grade	<b>October 14, 2014</b>	<b>October 28, 2024 (h: 23:59)</b>
<b>Written examination</b>	80% of the final grade	Any of the exam session	Any of the exam session

Read carefully the exam guidelines  
on WeBeep!!

# Exam - Written test

The written exam will be held in the computer room. It covers all theoretical and practical topics discussed in the lectures and lab sessions. Some of the questions and problems will require numerical solutions, including the implementation and programming of numerical algorithms. The exam focuses on the definition and application of key lemmas, theorems, and important examples. Light calculations may be required. The use of course materials, including books, notes, or any form of preparation aids, is not permitted.

**First written examination session: November 9, 2024 (TBC)**

# Exam - Course Hands-on Challenges

Two hands-on exercises are conducted during the course.

Course Hands-on Challenges are designed to apply both theoretical and computational knowledge to practical scenarios. The exercises are completed in small groups (3 students per group) during the course and consist of simple tasks on practical topics. Students are required to collaborate and submit their assignments (via a Microsoft MS form) according to the following schedule:

- **Course Hands-on Challenge I: Assigned on September 23, 2024. Submission deadline: October 7, 2024 (23:59)**
- **Course Hands-on Challenge II: Assigned on October 14, 2024. Submission deadline: October 28, 2024 (23:59)**

# Groups - please adhere to the following guidelines

- Group formation:
  - Each group must consist of three students. All group members should collaborate to submit a single answer for the challenges assigned.
- Deadline
  - Groups must be formed by **September 22, 2024, at 23:59**.
- To register your group, please fill out the dedicated Group Registration Form by the deadline.
- Submission process:
  - The form will require you to list the **names** and student **person code** of all group members.
  - Ensure all group members agree on the composition before submission, as no changes will be allowed after the deadline.
  - Please organise yourselves and complete the form on time to avoid any delays.

If you encounter any issues or have any questions, do not hesitate to reach out for assistance.

# Groups - please fill in the form

Deadline: **September 22, 2024, at 23:59**

NUMERICAL LINEAR ALGEBRA -  
GROUPS FOR CHALLENGES



<https://forms.office.com/e/7eXmxcE8AC>

# Modelling and Scientific Computing

# Modelling and Scientific Computing

- Scientific computing uses computational methods and algorithms to solve complex scientific problems, integrating mathematics, computer science, and domain-specific knowledge.
- The field fosters collaboration among applied mathematicians, engineers, and domain-specific scientists, facilitating physics-based decision-making that supports sustainable development.



# **Key Enabling technologies in Scientific Computing**

## **Mathematical Models and Numerical Algorithms**

Enable the simulation of complex system interactions, with a rigorous physics-inspired perspective

## **High-Performance Computing (HPC)**

HPC systems enable the processing of complex simulations, allowing researchers to model complex systems, optimize processes and predict impacts with unprecedented accuracy

## **Machine Learning Algorithms**

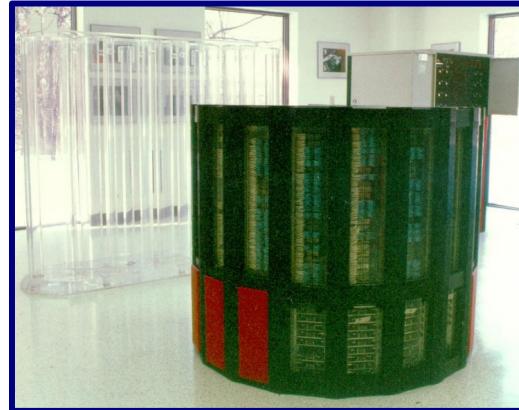
Analyze large volumes of data to identify patterns and trends, inform physics-based models facilitating predictive analytics and enhance decision-making

# Superpowers of supercomputers

1985: CRAY-2

4 processors

1.95 GigaFLOPS (peak)



2024: FRONTIER - HPE CRAY EX235A, Oak Ridge National Laboratory (US).

8.699.904 Cores

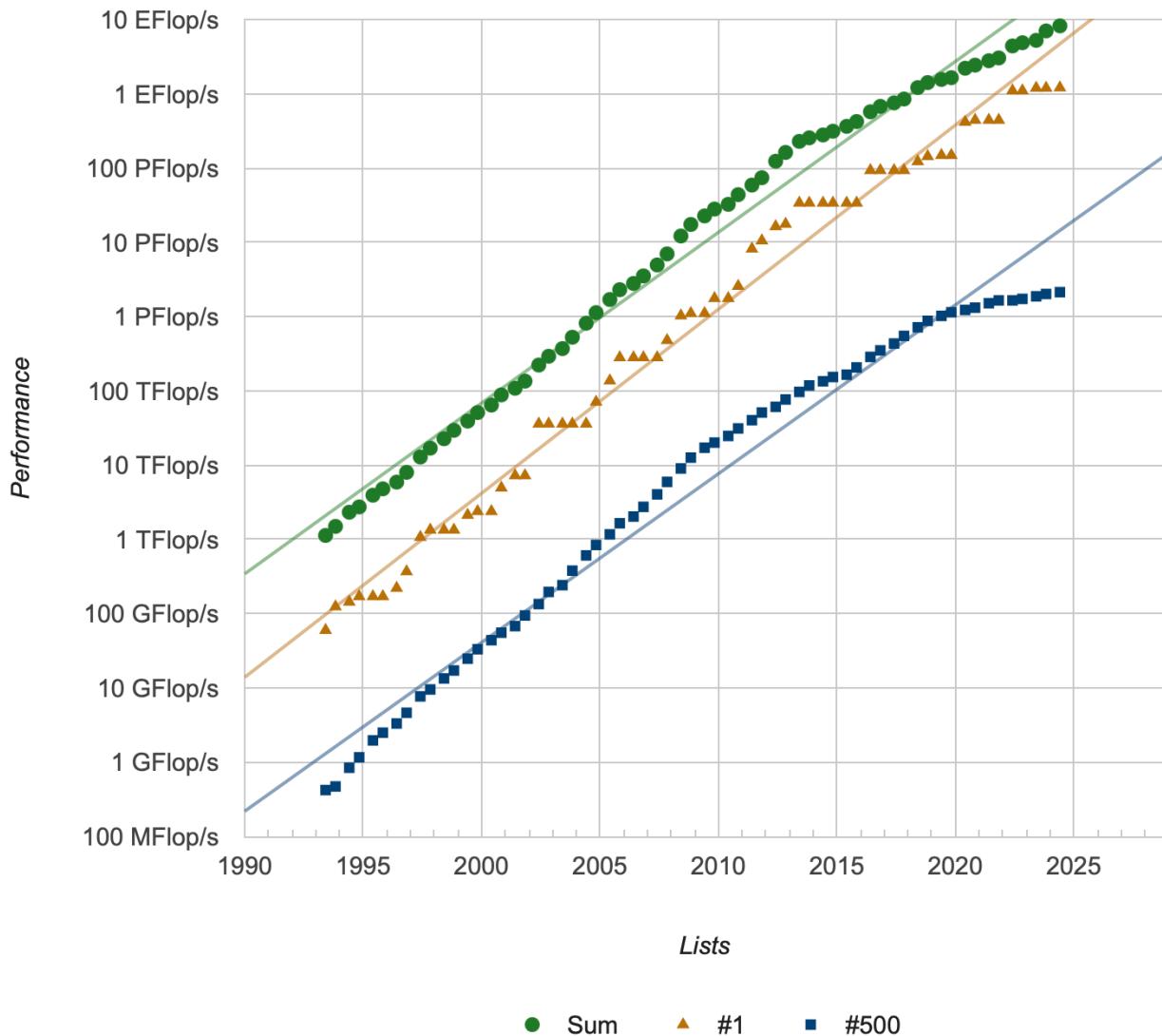
1.714 PFlop/s (RPeak)

- 1 GigaFLOPS  $\approx 10^9$ flops
- 1 TeraFLOPS  $\approx 10^{12}$  flops
- 1 PetaFLOPS  $\approx 10^{15}$  flops
- 1 ExaFLOPS  $\approx 10^{18}$  flops

# 35 years of supercomputers



<https://www.top500.org>



yottaFLOPS	$10^{24}$
zettaFLOPS	$10^{21}$
exaFLOPS	$10^{18}$
petaFLOPS	$10^{15}$
teraFLOPS	$10^{12}$
gigaFLOPS	$10^9$
megaFLOPS	$10^6$
kiloFLOPS	$10^3$
FLOPS	1

# Modeling, Scientific Computing and Sustainability

## Predictive Insights

Modeling provides essential predictive insights into complex systems, allowing to make informed decisions based on potential outcomes.

## Resource Optimisation

Through simulations, models help optimise the use of resources

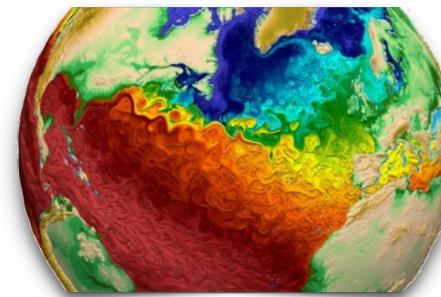
## Predictive Insights

Modeling enables the exploration of multiple scenarios, which is crucial for effective policy-making and strategic planning.

# Some case studies of successful implementations



Complex ecosystems



Environmental modeling

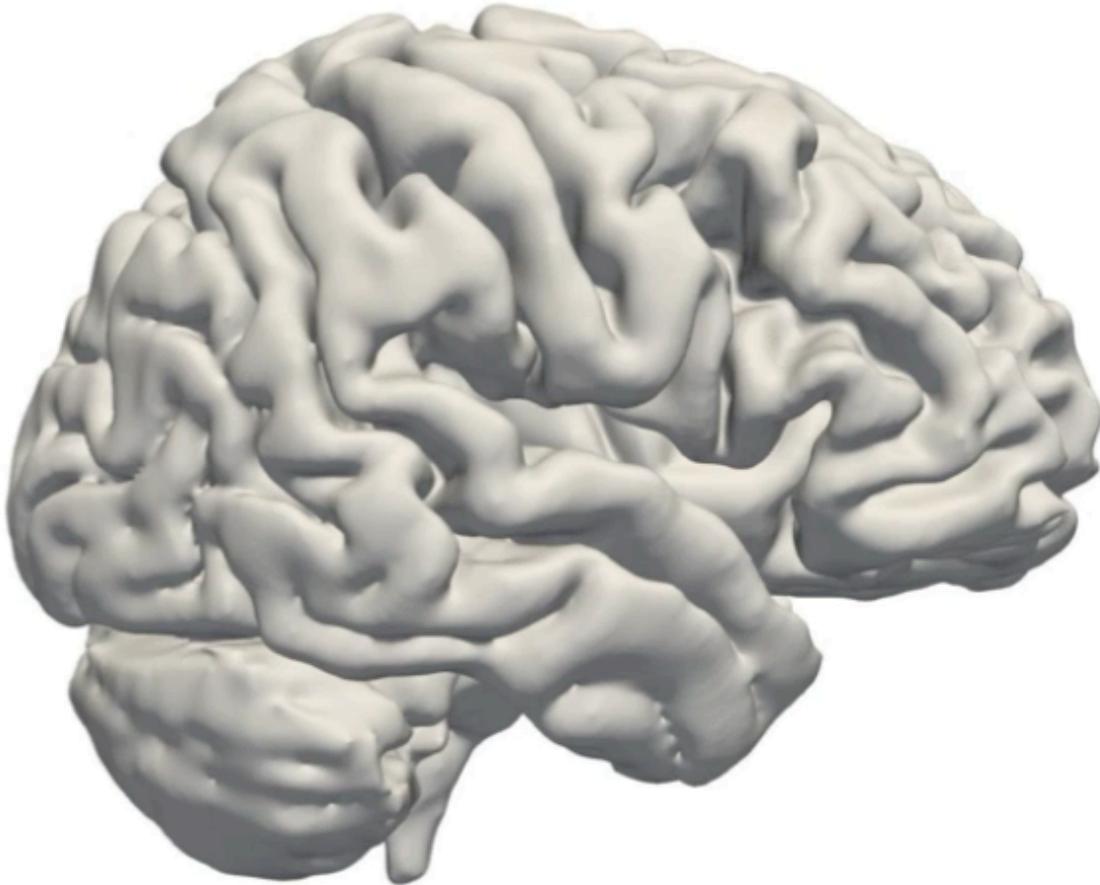


Resources exploitation and sustainable production

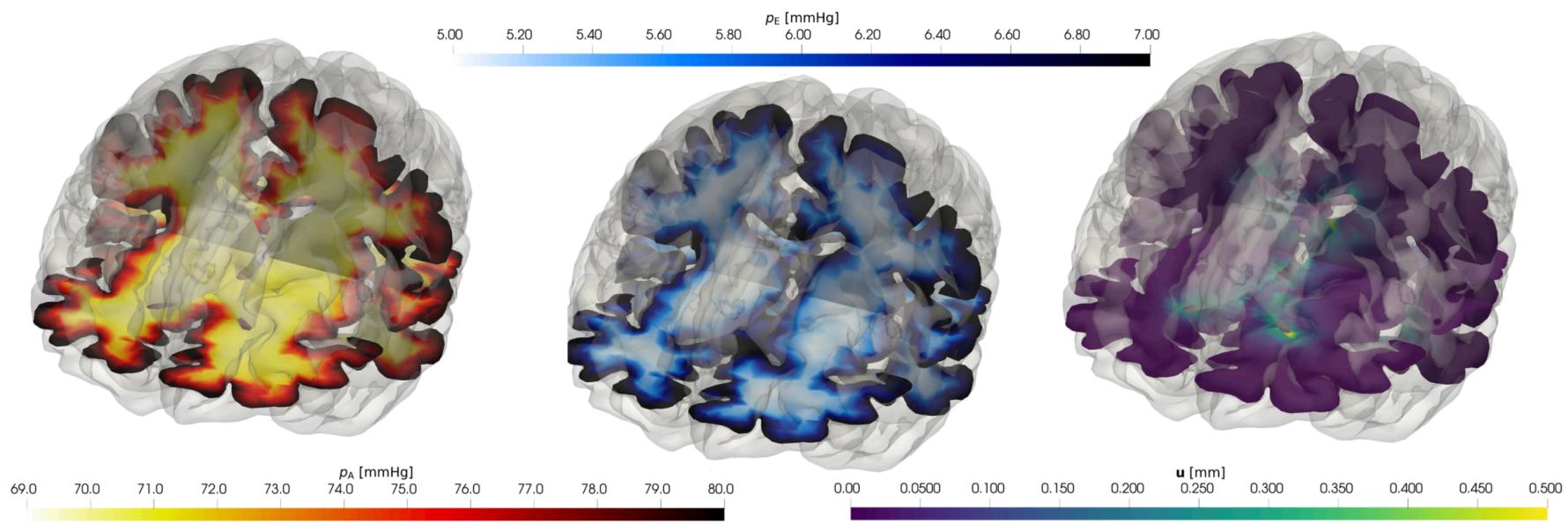


Human modeling and medicine

# Computational neurosciences



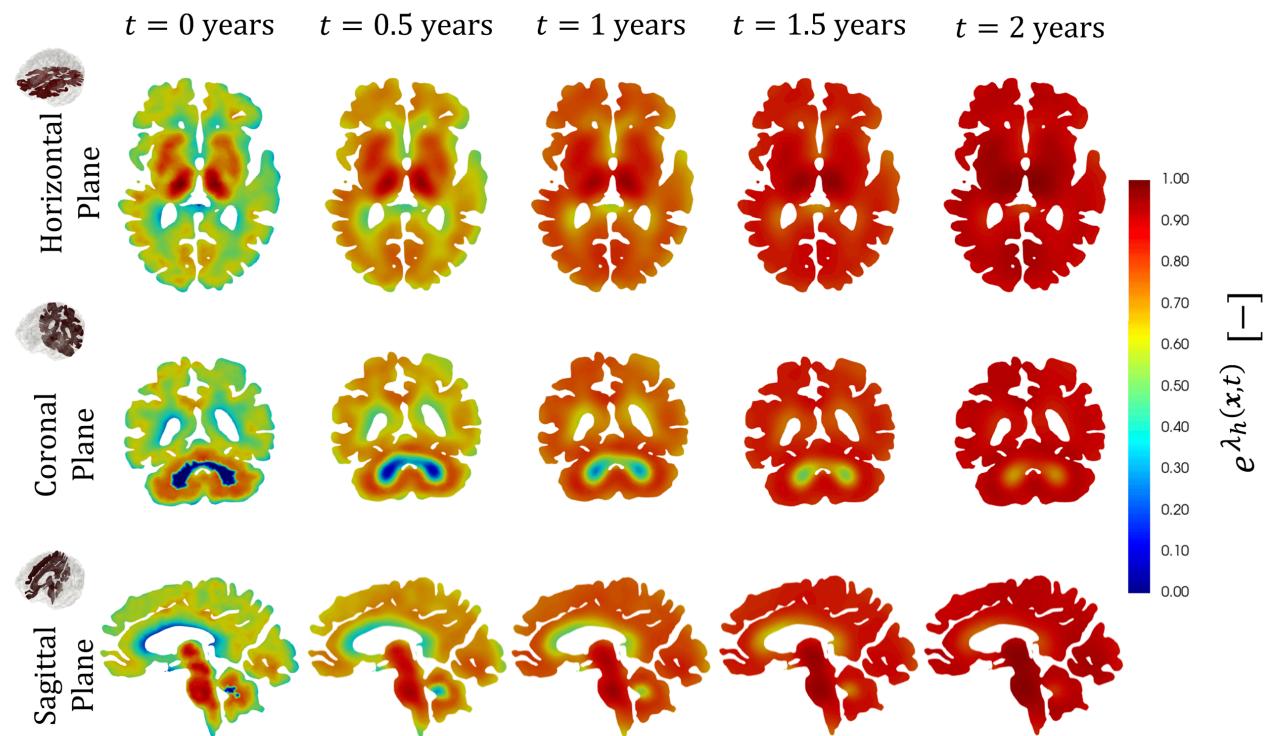
# Computational neurosciences



Arterial pressure, CSF pressure, and tissue displacement resulting from a multiple network poroelasticity simulation in physiological conditions [Corti, Antonietti, Dede', Quarteroni. M3AS, accepted (2023)]

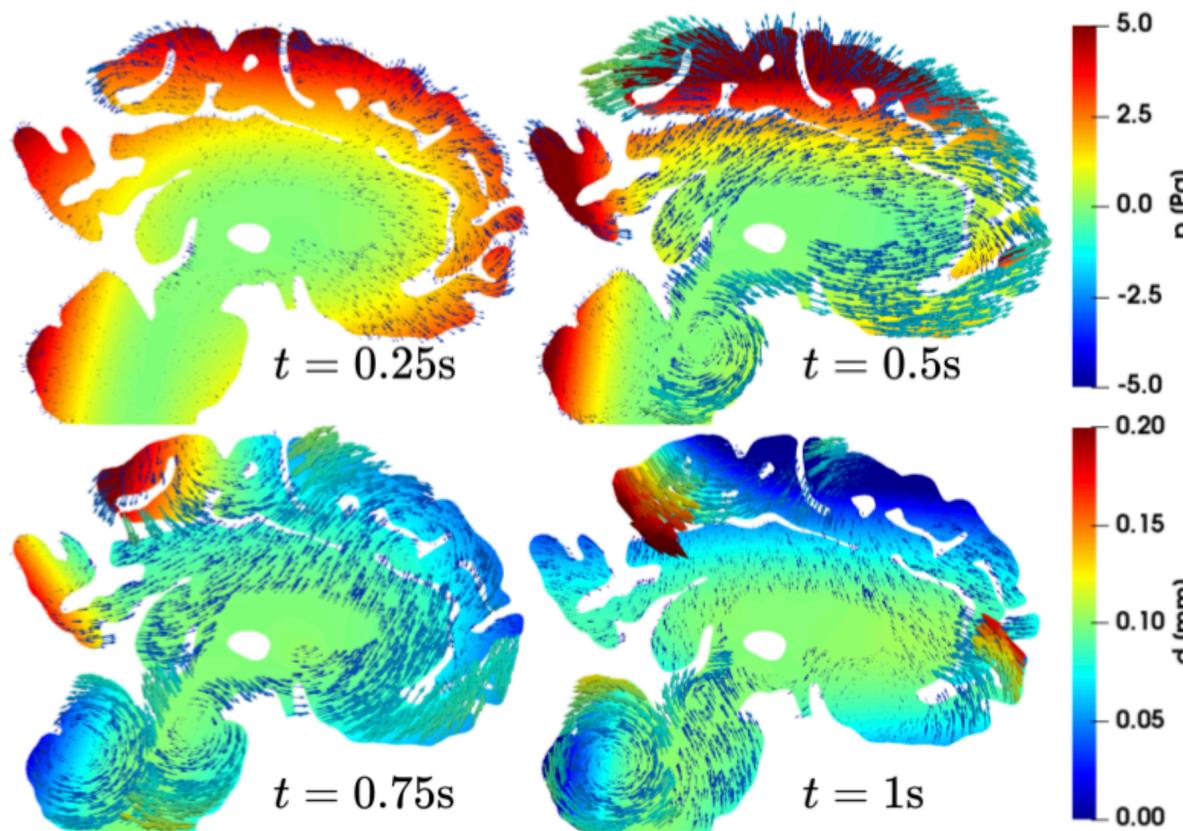
<https://brainum.mox.polimi.it>

# Computational neurosciences



<https://brainum.mox.polimi.it>

# Computational neurosciences

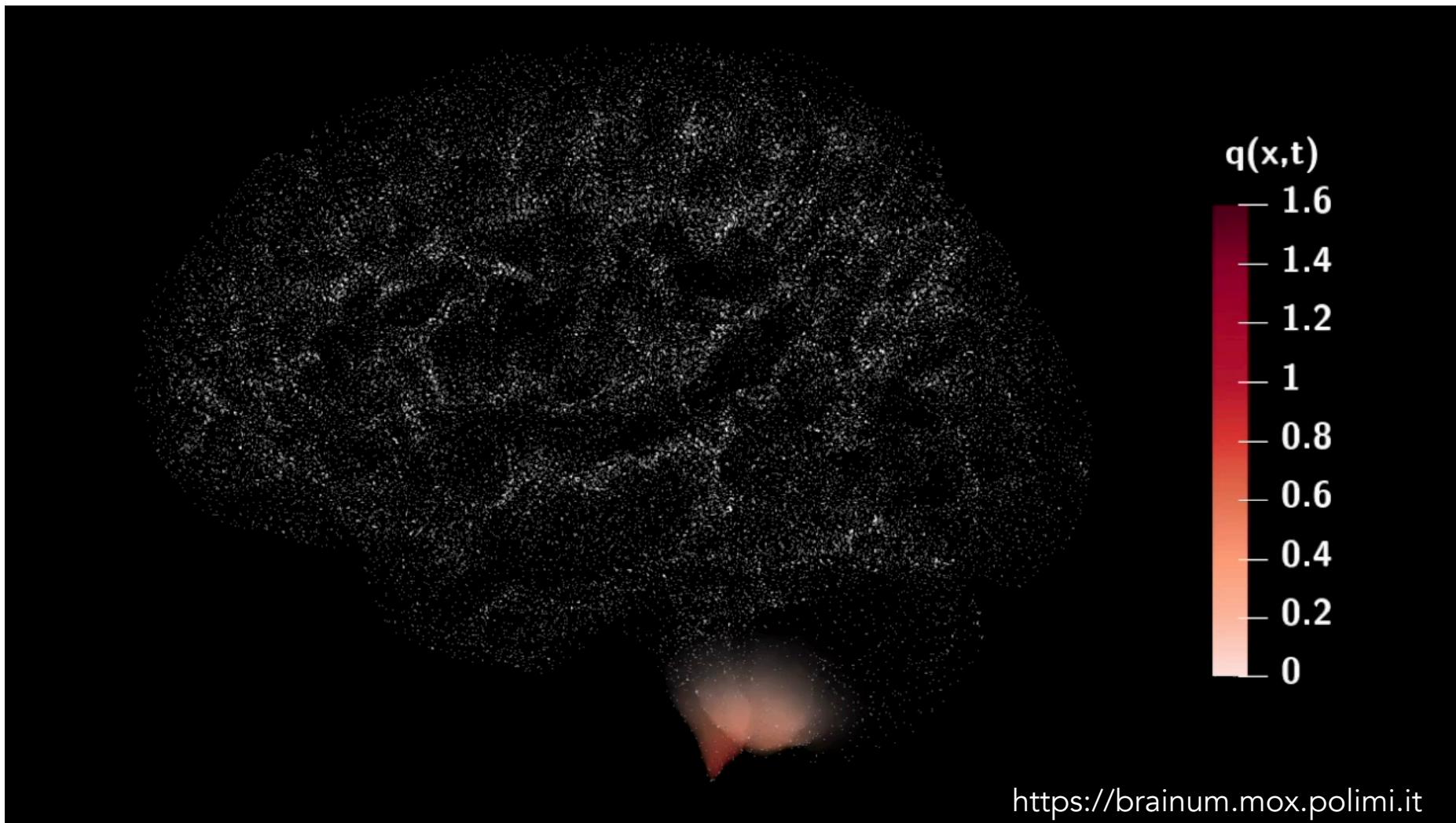


Intracranial pressure and parenchymal displacement obtained simulating the coupling between the CSF in the brain ventricles and the surrounding poroelastic tissue  
[Fumagalli, Corti, Parolini, Antonietti. arXiv:2310.07651 (2023)]

<https://brainum.mox.polimi.it>

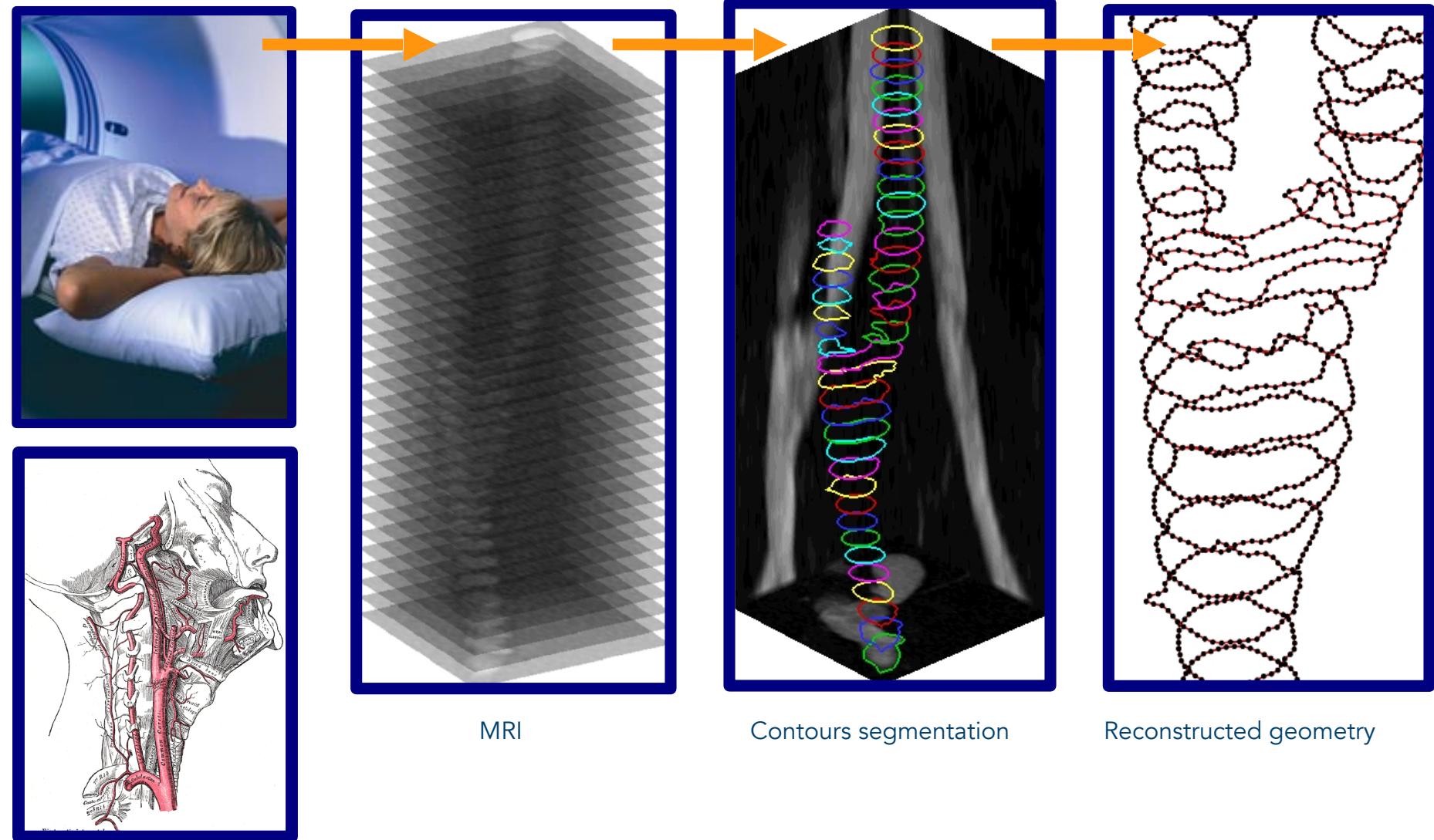
# Computational neurosciences

Patterns of  $\alpha$ -synuclein concentration at different stages of Parkinson's disease with volume rendering



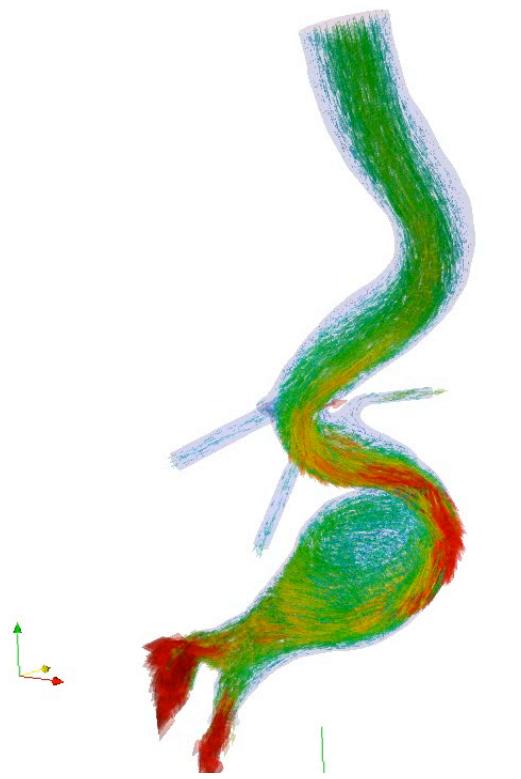
<https://brainum.mox.polimi.it>

# Cardiovascular mathematics

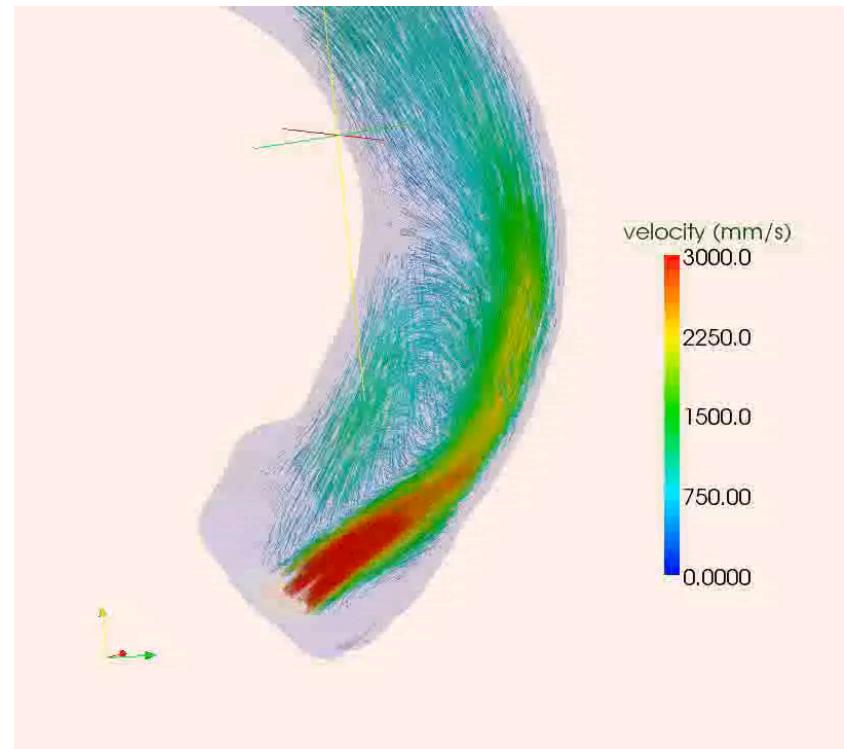


<http://mox.polimi.it>

# Blood flow simulations



Velocity in an abdominal aneurysm

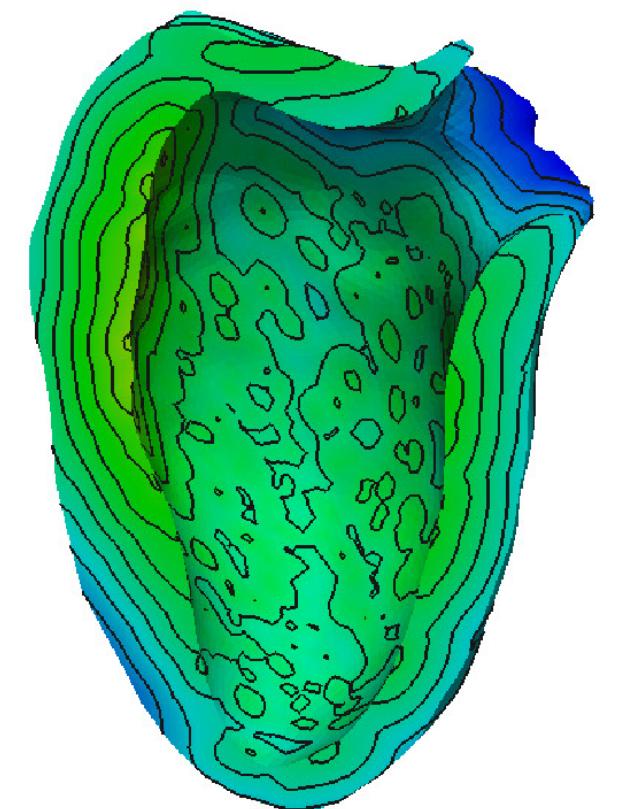
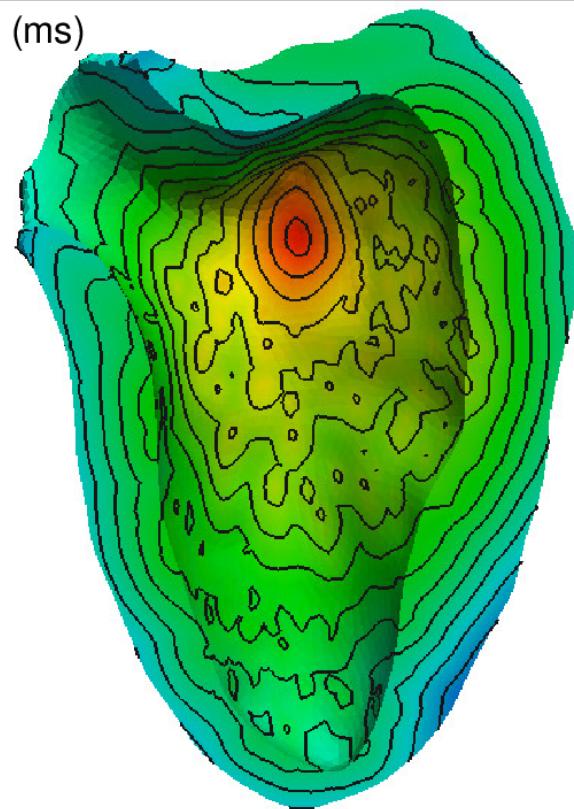
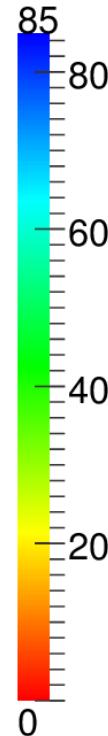


Velocity streamlines in a human carotid  
(bicuspid valve)

<http://mox.polimi.it>

# Electrical activity of the heart

Activation times (ms)



Map of activation time

# Extreme natural events

Italy has a high exposure to natural risks: earthquakes, floods, landslides, volcanic eruptions, fires. Our territory is fragile.

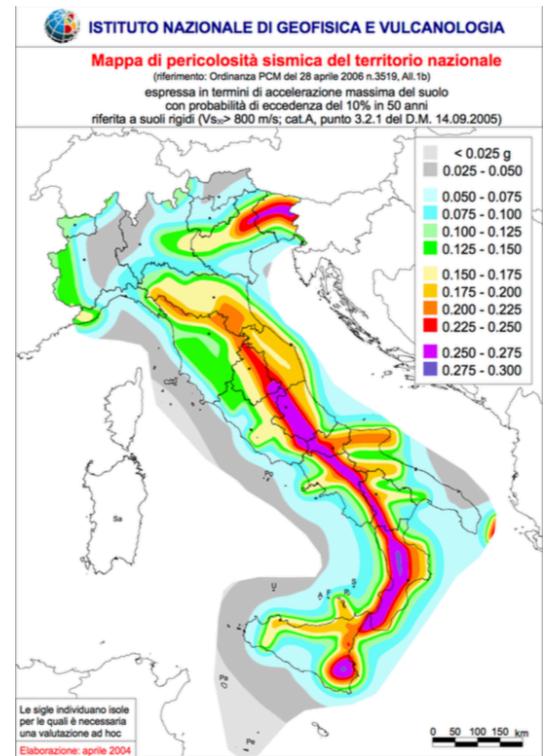


Hotel Duca degli Abruzzi  
(L'Aquila)



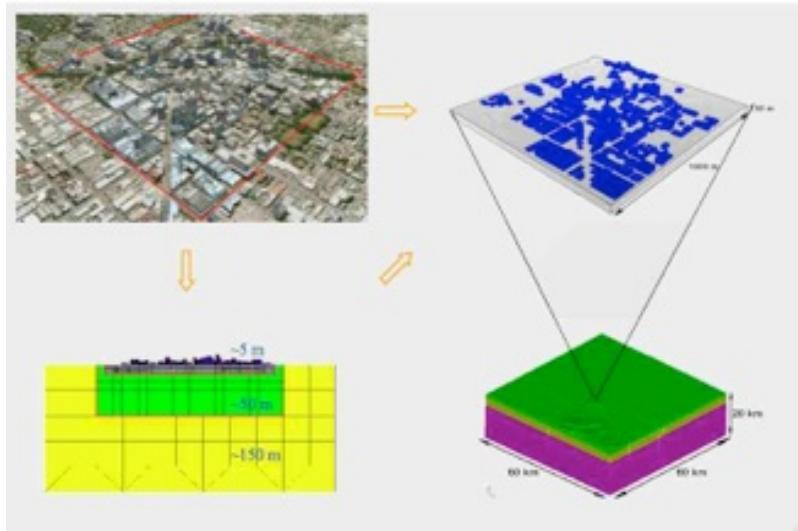
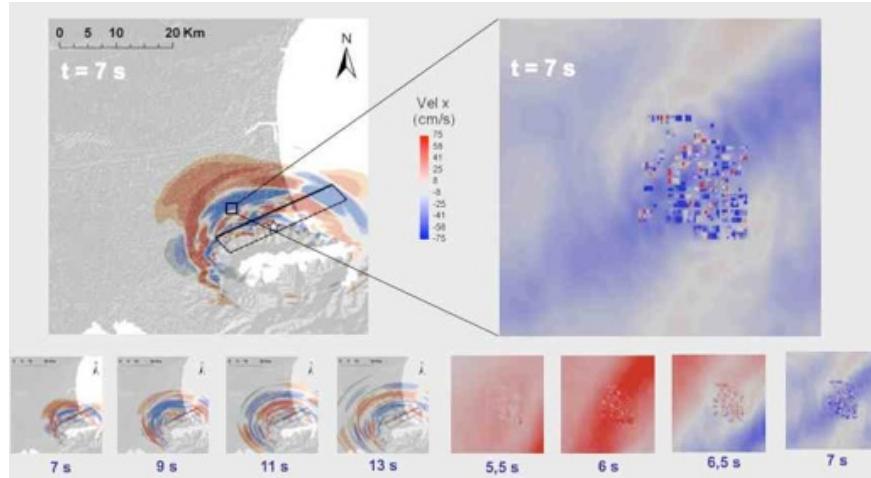
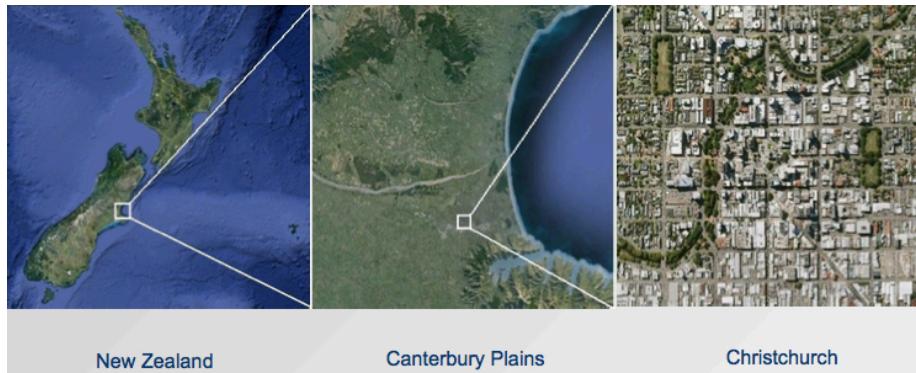
Courtesy of Prof. Paolucci

Edificio massiccio su  
pilastri sottili,  
altezza irregolare  
dei pilastri, presenza  
di corpi strutturali  
di altezza diversa.



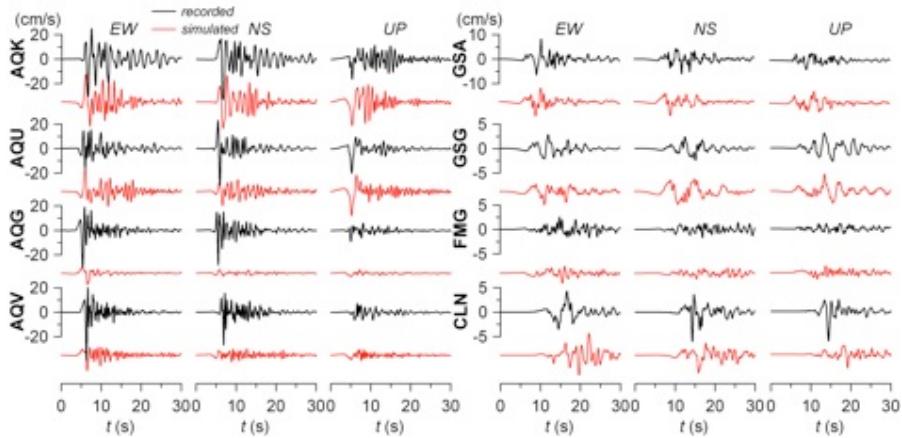
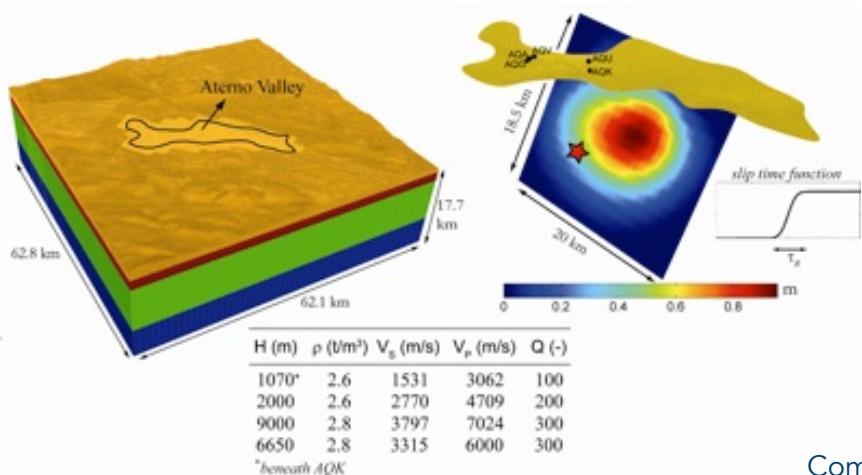
The probabilistic seismic  
hazard map for Italy.

# ChristChurch (NZ) earthquake, 22.02.2011, Mw 6.2

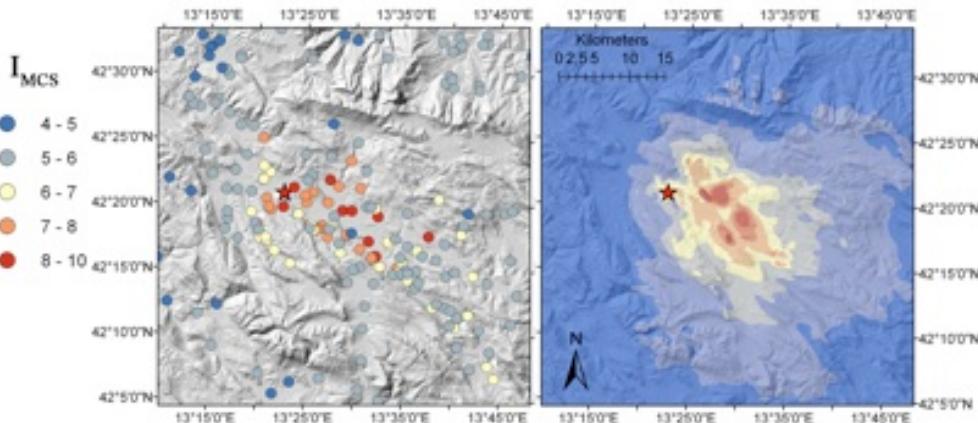


<http://speed.mox.polimi.it/>

# L'Aquila (IT), 06.04.2009 Mw 6.3

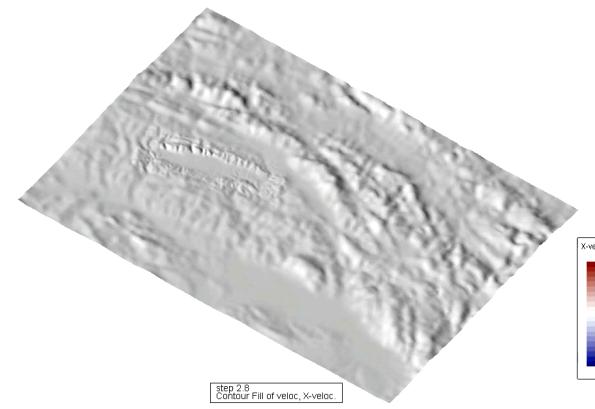


Comparison of the synthetics and strong motion recordings at 8 selected sites in terms of three component velocity time histories (0.1-2.5 Hz).



PGV<sub>GM</sub>  
(cm/s)

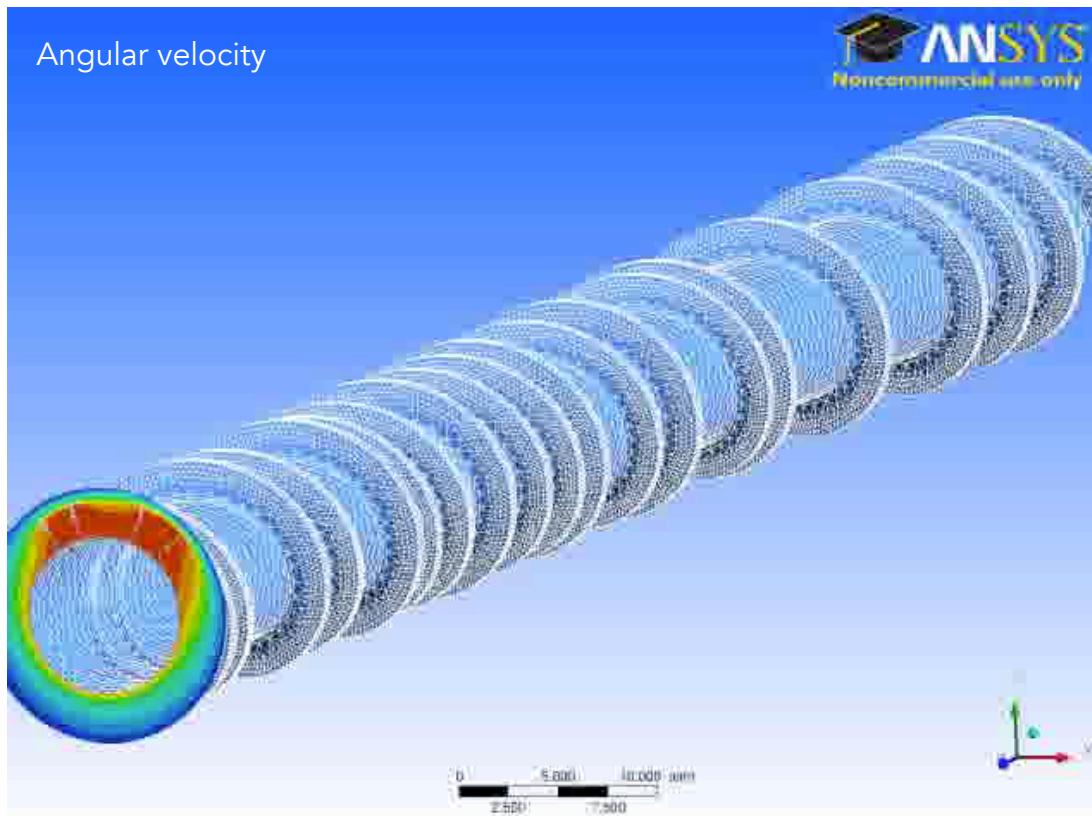
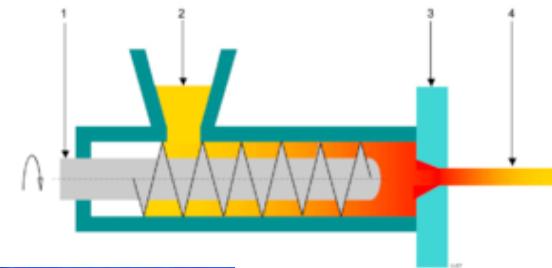
< 5
5 - 10
10 - 15
15 - 20
20 - 25
25 - 30
> 30



<http://speed.mox.polimi.it/>

# Advanced manufacturing processes

Extrusion is a process used to create objects of a fixed cross-sectional profile. A material is pushed through a die of the desired cross-section.



# Sport competitions

