Examing the effectiveness of diversity maintenance methods in decision space.

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

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I. Introduction

Many real world optimization problems requires more than one objective functions which are usually conflicting to each other. In most circumstances, the fitness improvement in one objective function may cause deterioration in other objectives. We generally denote this kind of problems as MOPs. Multi-objective Evolutionary Algorithms(MOEAs) are designed for solving MOPs. Mainstream MOEAs usually follow "posterior" schema which assumes that no user preference information are provided before and during the optimization process. Therefore, MOEAs need to obtain a set of non-dominated solutions that is sparsely cover the Pareto Front(PF). Then the decision maker can select solutions set from the obtained solutions according to user's preferences. As mentioned above, usually MOEAs only consider the diversity of solution set in objective space. Diversity in the decision space does not gain much attention. However, sometimes the diversity in decision space is also very important, especially for multi-modal multi-objective optimization problem(MMOP). MMOPs is a special kind of MOPs which contains more than one Pareto optimal solution sets(PS) corresponding to the same PF. There are many MMOPs in real-world applications such as multi-objective knapsack problems [1], game map generation problem [2] etc. An MMOP requires to cover as much PSs as possible. In many engineering problems, some solutions are hard to implement due to physical limitations. In this case, obtaining multiple PSs can help decision maker to select alternative solutions with equivalent quality in objective space. In addition, the knowledge about multiple Pareto optimal solution sets may be for better understanding of the problem structure [3].

However, maintaining the diversity in decision space is a very challenging task. In order to obtain a set of solutions that is uniformly spread along the PF, most MOEAs introduce diversity maintenance in objective space such as niching and crowding distance. Therefore equivalent solutions will be eliminated because they are too crowded in the objective space [4]. For this reason, we can infer that MOEAs do not perform well on MMOPs. In this paper, we further confirm this proposition with a series of computational experiments. To find solutions in all PS, an explicit mechanism to maintain the solution diversity in decision space is a must. This means that the algorithm should be able to maintain the diversity of the solutions in both decision and objective space. For convenience, evolutionary algorithms designed for solving MMOPs are known as Multi-modal Multi-objective Evolutionary Algorithms or MMEAs in short. Since 2005, MMEAs has begun to receive attention from community researchers. Some new algorithms are specially designed for MMOPs and some of them are modified from existing MOEAs. Although there are already some literatures summarizing the characteristics of these MMEAs, there is no comparison of the performance between them. In this paper, several popular MMEAs are tested and their performance on the test problems with different dimensional decision space are studied.

The structure of this paper is as follows. In section II, we give the formal definition of MMOPs, and carefully analyze the difficulties of solving MMOPs. In section III, we review several mainstream diversity maintenance techniques in the literatures and analyze their characteristics. Then a series of computational experiments are conducted in section IV. Finally, we conclude this paper. *

II. Multi-modal Multi-objective Optimization

A. Formal Definition

The formal definition of MMOPs are give by Tanabe [5] which can be described as follows.

An MMOP is a kind of MOP that requires to find all solutions that are equivalent to Pareto optimal solutions. Solutions $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are equivalent iff $||f(\boldsymbol{x_1}) - f(\boldsymbol{x_1})|| \leq \delta$.

Where δ is a positive number that represents the tolerance of similarity of solutions given by the decision maker. If $\delta = 0$, decision maker only accept Pareto optimal solutions and the dominated solutions should be removed in the final population. Larger δ means that decision maker can accept the suboptimal solutions that are more inferior than Pareto optimal solutions. Because it is difficult to accurately describe the relationship between the value of δ and the quality of the

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resulting solutions, in order to simplify the process of analysis, we only consider the case of $\delta = 0$.

B. Multi-Polygon Test Problem

There are several benchmark MMOPs used in existing literatures such as Two-On-One [6], Omni-test [7] , SYM-PART [8] and Multi-Polygon test problem [**?**] etc. This paper use Multi-polygon test problem for analyzing the performance of MOEAs and MMEAs because it has better flexibility than other test problems. Multi-polygon test problem is a MMOPs based on the idea of distance minimization. For example, there are four identical regular hexagons with radius r in figure 1, and the distance l between any two adjacent polygons are larger than $4r$ (Please refer to [**?**] for more details about the specification of l). With these settings, we can define a 6-objective optimization problem by:

$$\begin{cases} f_1(\boldsymbol{x}) = \min\left\{d\left(\boldsymbol{x}, A_1\right), d\left(\boldsymbol{x}, B_1\right), d\left(\boldsymbol{x}, C_1\right), d\left(\boldsymbol{x}, D_1\right)\right\} \\ \qquad\qquad\qquad\qquad \cdots \\ f_6(\boldsymbol{x}) = \min\left\{d\left(\boldsymbol{x}, A_6\right), d\left(\boldsymbol{x}, B_6\right), d\left(\boldsymbol{x}, C_6\right), d\left(\boldsymbol{x}, D_6\right)\right\} \end{cases}$$

Where $d(\boldsymbol{a}, \boldsymbol{b})$ is the Euclidean distance between vector $\boldsymbol{a}$ and $\boldsymbol{b}$. In this problem, any point inside the four hexagons is Pareto optimal solutions which means that this test problem has four equivalent PSs. One of the biggest advantage of this test problem is that the dimension of its decision space can be extended by mapping the plane to a two-dimensional subspace in any dimension space.

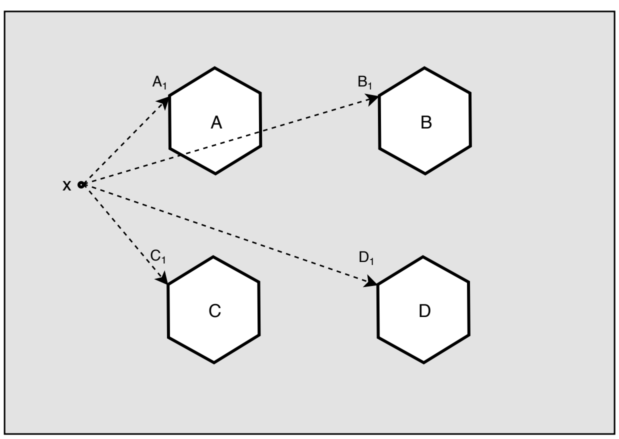


Fig. 1: An example of Multi-Polygon test problem.

III. Difficulties Analysis

For MOEAs, the difficulty of maintaining the diversity of populations in the decision space coms mainly from the following three aspects:

A. Environment Selection

For most MOEAs, the environmental selection is carried in whole population. This will cause two consequences: On one hand, as pointed out in [4], for MMOPs, two solutions that are very far apart in the decision space may be very close in the objective space. Then MOEAs tend to remove the solutions that are too crowded in the objective space, so equivalent(or slightly inferior) solutions will be eliminated.

On the other hand, in the early stage of the evolution, a relatively good solutions will quickly lead to other potential solutions which is far away from it in decision space to extinction. Figure 2 shows these two cases respectively and in both cases, the diversity of the population in decision space will decrease.

(a) Case 1

(b) Case 2

Fig. 2: The impact of environmental selection on the diversity of population in the decision space. In each case, only the red circle survive in next generation.

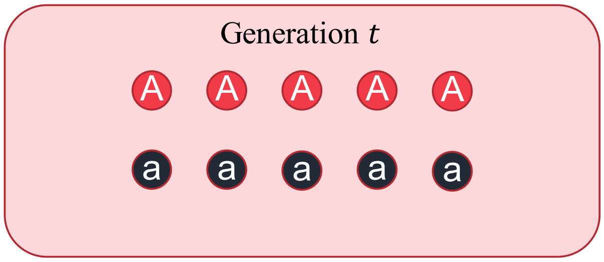
B. Genetic drift

The reduction of the diversity in decision space also related to a phenomena called Genetic Drift. Genetic drift is a phenomena in which the frequency of alleles in a population declines due to random sampling errors [9]. As shown in figure 3, in generation t, the population contains same number of alleles A and a, and we assume that only random sampling without selection, mutation and crossover from generation t to $t+1$. Then due to the random sampling error, the frequency of A becomes larger than a in generation $t+1$.

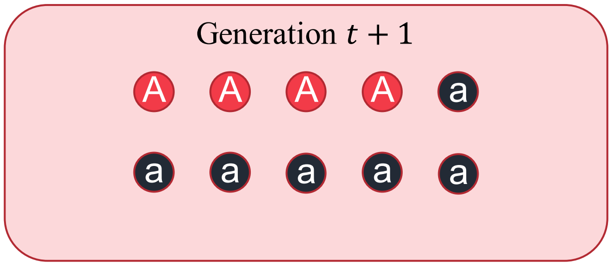
Genetic drift tend to reduce genetic variation of the population during the evolution process. Some of the alleles may even disappear due to genetic drift. Asoh [10] mathematically proved that with only random selection, as the iteration proceeds, only one of alleles in the population will survive(this process is called population convergence). And the mean convergence time is proportional to the population size. In MMOPs, equivalent solutions can be seen as alleles because they are indistinguishable during selection. Therefore it's expected that due to the existence of genetic drift, all equivalent solutions in the population will be removed after a long enough evolution time. In section IV, we use more computational experiments to study the effects of genetic drift on solving MMOPs.

C. Crossover and mutation

In evolutionary algorithms, crossover and mutation are two operation that will increase the variety of the population. These



(a) At time t, the population contains same number of alleles A and a.



(b) At time $t+1$, due to random sampling error, $frequency(A) < frequency(a)$

Fig. 3: Demonstration of genetic drift

two techniques can help the algorithms to avoid trapping in the local optima. However, since mutation and crossover have higher probability of producing individuals that are close to their parents, the distribution of the parent population will affects the distribution of the offspring population. More specifically, more offsprings will be produced where the parent population is dense. Take SBX crossover [] and polynomial mutation [] as an example.

IV. Review of State-of-the-art Techniques

In this section, we provide a short review of state-of-the-art techniques for maintain the diversity in the decision space. As pointed out by Deb [3], the key idea for solving MMOPs is to ”divide” the entire population into several subpopulation that do not affect each other as much as possible, and then let each subpopulation evolve independently. Here we list some commonly used approaches:

A. Fitness Sharing Approach

Fitness sharing was first introduced in [11] and widely adopted in MOEAs to keep the sparsity of the solutions in objective space. The key idea is to degrade the fitness of crowded solutions by sharing their fitness to neighborhood solutions. In order to solve MMOPs, fitness sharing in both decision and objective space need to be considered simultaneously. DNEA [12] used two fitness sharing function both in decision space and objective space. Fitness sharing approach is hard to apply because the sharing radius is problem dependent. In addition, combining fitness sharing in two spacing is also a challenging task.

B. Crowding Distance Approach

Crowding distance often used as a second selection criteria in Pareto-dominance based algorithms such as NSGA-II [13]. Several MMEAs such as Omni-optimizer [14] and MO_Ring_PSO_SCD [15] design special types of crowding distance that take both the decision and objective space into account.

C. Restrict Environmental Selection Approach

As discussed in section III-A, carrying the environmental selection in whole population is harmful to the population diversity in decision space. Therefore one remedy is to restrict the environmental selection in part of solutions. More specifically, a solution should only compete with its neighborhood solutions in decision space. The neighborhood relationship may either distance based or index based. For example, in MOEA/D-AD [16], a solution x is only compete with the closest L solutions in decision space by scalarizing function.

V. Experimental Studies

VI. Conclusions

Acknowledgment

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