ECE158a solution HW 1 and 2

1 HW2

1. We have seen in class that Hamming code is an error correction code that gives a simple way to add check bits and correct up to $(d_{min} - 1)/2$ errors, where d_{min} is the minimal Hamming distance between any two codewords in the code. In the following, we consider the Hamming $(7, 4)$ code, where $n = 7$ is the codeword length, and $k = 4$ is the message length.

   a) List all the codewords in the Hamming $(7, 4)$ code.

   $$\begin{gathered}0000000, 0001011, 0010110, 0011101\\0100101, 0101110, 0110011, 0111000\\1000111, 1001100, 1010001, 1011010\\1100010, 1101001, 1110100, 1111111\end{gathered}$$

   b) Pick any two codewords from the Hamming $(7, 4)$ code, compute the sum of the two codewords. Is the result a codeword in the Hamming $(7, 4)$ code.

   A:Any sum of the two codewords is a codeword in the Hamming (7, 4) code. For example, $0001011 + 0111000 = 0110011$ is a codeword.

   This property follows from the fact that Hamming code is a linear code, a code in which any linear combination (since we are in a boolean algebra coefficients are 0 and 1) of codewords is also a codeword.

   c) Suppose you receive a block of bits 1010010. Use the syndrome decoding scheme introduced in class to decode the received block. Assuming that the decoding scheme does not make a mistake in this case, please identify the original message (4 bits) and the locations of error bits in the received block (if errors occur).

   A:

   $$\begin{bmatrix}1 & 1 & 1 & 0 & 1 & 0 & 0\\1 & 0 & 1 & 1 & 0 & 1 & 0\\1 & 1 & 0 & 1 & 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}1\\0\\1\\0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix}$$

   Note that the syndrome 011 corresponds to the 4th column of the parity check matrix, which suggests that the error occurred at the 4th bit. We may then reconstruct the codeword as 1011010 and the original message bits as 1011.

   d) What is the minimal Hamming distance d_{min} of the Hamming $(7, 4)$ code? How many error bits could be corrected by the Hamming $(7, 4)$ code?

   A:The minimum Hamming distance of the Hamming (7, 4) code is $d_{min} = 3$, and it could correct up to (d_{min} - 1)/2 = 1 bit of error.

   e) Now consider a channel that transmits one bit at a time with an error probability p, and each use of the channel is independent. Suppose you send a Hamming $(7, 4)$ codeword through this channel (use the channel 7 times to send the $7-bit$ codeword), and decode the received bits with the syndrome decoding. Derive the probability that you make a mistake in decoding (i.e. the probability that the decoded message is not the original message). Also, apply $p = 0.9$ in the expression to evaluate the probability.

A: The cases where the syndrome decoding will make a mistake is when the received bits have 2 or more error bits. Therefore, it suffices to just evaluate the probability that 2 or more bits are error:

$$P_{error} = \sum_{k=2}^{7} \binom{7}{k} p^k (1-p)^{7-k} = 1 - \sum_{k=0}^{1} \binom{7}{k} p^k (1-p)^{7-k} = 1 - (1-p)^7 - 7p(1-p)^6$$

When $p = 0.9$, we have $P_{error} = 0.9999936$.

2. Suppose you are transmitting a data message 1110011 using a cyclic redundancy check (CRC) code with generator 101 lets call it message A. If you send A with checksum method with blocks of size 3 what is the checksum that you would append to the message?

A: Following the long division introduced in class, we compute the division with respect to the generator 101 since we are using boolean algebra our it would be like: Now you have 9bit

$$\begin{array}{r|r}
 & 1101001 \\
\hline
101 & 111001100 \\
 & 101\!:\!:\!:\!:\!:\!: \\
\hline
 & 100\!:\!:\!:\!:\!: \\
 & 101\!:\!:\!:\!:\!: \\
\hline
 & 010\!:\!:\!:\!: \\
 & 000\!:\!:\!:\!: \\
\hline
 & 101\!:\!:\!: \\
 & 101\!:\!:\!: \\
\hline
 & 001\!:\!: \\
 & 000\!:\!: \\
\hline
 & 010\!: \\
 & 000\!: \\
\hline
 & 100 \\
 & 101 \\
\hline
 & 01
\end{array}$$

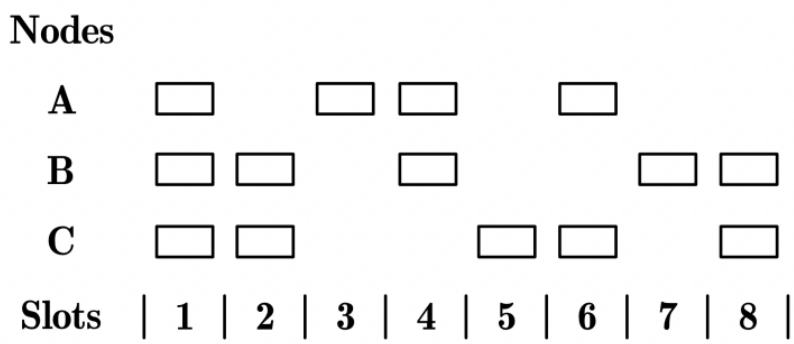
message 111001101 take the blocks of 3bits and sum them up $111 + 001 + 101 = 011$, now take a 1 complement of it and append it to our 9digit message the result will be 111001101100.

3. Suppose three nodes are communicating on a network that uses a random access protocol to mitigate collisions. Let the nodes transmit packets in slots as shown in the following diagram 1, where slot 1 begins at time t = 0. In the protocol, each node transmits a packet, and if there is a collision then the node waits for X time slots before transmitting again, where X is a random variable following the distribution $Unif\{0, 1, 2, \ldots, 2^N - 1\}$, $N = min\{m, 10\}$, and m is the number of previous collisions for that packet.

   a) How many packets has each node successfully transmitted after time slot 8 has completed?
   A: Each node has successfully transmitted one packet: A in slot 3, B in slot 7, C in slot 5.

   b) After slot 1, what was the probability that there was a collision in slot 2?
   A: In slot 1, all three nodes attempted to transmit a packet for the first time, and so each node waits either 0 or 1 slot with equal probability before transmitting again. Therefore, each node has a probability of $\frac{1}{2}$ of transmitting in slot 2, and a probability of $\frac{1}{2}$ of transmitting in slot 3. Let X_N denote the random variable which models the number of time slots node N waits before transmitting. $P(collisioninslot2) = P(X_A = 0, X_B = 0, X_C = 0) + P(X_A = 0, X_B = 0, X_C = 1) + P(X_A = 0, X_B = 1, X_C = 0) + P(X_A = 1, X_B = 0, X_C = 0) = 4(\frac{1}{2})^3 = \frac{1}{2}$

Figure 1: Q4 Diagram



c) What is the probability no node transmits in slot 9? The packet that A is trying to transmit has had 2 previous collisions (in slots 4 and 6), so in slot 6 we have that $X_A \in 0, 1, 2, 3$, since it didn't try in slots 7 and 8, it must try in either slot 9 or 10 and it is distributed uniformly, and given that A didn't transmit in slots 7 or 8, we know A will transmit in slot 9 or slot 10 with equal probability of $\frac{1}{2}$. So $P(no\,node\,transmits\,in\,slot9) = P(A\,transmits\,in\,slot10) \times P(B\,transmits\,in\,slot10) \times P(C\,transmits\,in\,either\,slot10, 11, or12) = (\frac{1}{2})(\frac{1}{2})(\frac{3}{4}) = \frac{3}{16}$

4. Consider the topology shown in Figure 2, where every device is in the same LAN. Suppose switches $1 - 4$ are learning switches. Answer the following questions:

a) Suppose host 2 is going to send a frame to host 8, and the IP address of host 8 is known to host 2. Describe the procedure required to forward the frame from host 2 to host 8. In particular, explain how the switches determine the path to forward the frame.

Since host 2 only knows the IP address of host 8, but not the MAC address, host 2 would send out an ARP request asking for host 8's MAC address. The followings describe the process:

i. The ARP request is first sent to switch 2.
ii. Switch 2 records the information of host 2 (host 2 MAC, port) in its switch table, and then broadcasts the ARP request to hosts 1 and 3, and switch 1.
iii. Hosts 1 and 3 have no information on host 8, hence ignore the ARP request.
iv. Switch 1 records the information of host 2 (host 2 MAC, port) in its switch tableand then broadcasts the ARP request to switches 3 and 4.
v. Switches 3 and 4 both record the information of host 2 (host 2 MAC, port), and then broadcast the ARP request (switch 3 sends to hosts 4,5,6, and switch 4 sends to hosts 7,8,9).
vi. Hosts 4,5,6,7,and 9 have no information on host 8, hence ignore the ARP request. On the other hand, host 8 first records the information of host 2 into its own ARP table (host 2 IP, host 2 MAC, port), and then sends an ARP reply destined to host 2.
vii. The ARP reply is sent to switch 4, since host 8 knows the packet from host 2 comes from this interface.
viii. Switch 4 received the ARP reply and record the information of host 8 (host 8 MAC, port). Switch 4 then sends the ARP reply to switch 1, since switch 4 has an entry matching the destination (host 2) MAC address, which corresponds to the port connected to switch 1.
ix. Similar to the previous step, the ARP reply passes through switches 1, 2 and then goes to host 2, with each switch recording the information of host 8 and then passing through the interface that leads to host 2. Finally host 2 records the information of host 8 in its own ARP table.
x. Now switches 1, 2, and 4 have switch table entries that could pass packets from host 2 to host 8.

b) Suppose host 2 has sent the frame to host 8, and host 8 then sent another frame to host 2. Write down the switch table (or MAC address table) for each switch.

The switch tables are shown as below. In the following tables, each port is denoted by the end hosts connected by that interface. For example, at switch 1, the port that connects to switch 2 is denoted as (switch 1 - switch 2).

MAC address	Port
Host 2 MAC	Switch 1 - Switch 2
Host 8 MAC	Switch 1 - Switch 4

Table 1: switch 1

MAC address	Port
Host 2 MAC	Switch 2 - Host 2
Host 8 MAC	Switch 2 - Switch 1

Table 2: switch 2

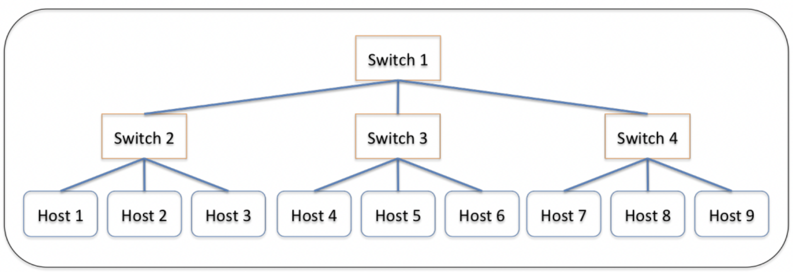
MAC address	Port
Host 2 MAC	Switch 3 - Switch 1

Table 3: switch 3

MAC address	Port
Host 2 MAC	Switch 4 - Switch 1
Host 8 MAC	Switch 4 - Host 8

Table 4: switch 4

Figure 2: Tree topology



2 HW1 Q4

a) $C \times d_{prop} == (100 \times 10^6) \times (\frac{3\times10^7}{2\times10^8}) = 1.5 \times 10^7 bits$

b) The propagation delay d_{prop} is the duration for which a bit stays on the link, the capacity C is (ideally) the maximum number of bits entering the link in an unit of time, so the product $C \times d_{prop}$ has the physical interpretation that it is the maximum number of bits on the fly.

c) The transmission delay is $d_{tran} = \frac{3\times8\times10^9}{1\times10^8} = 240s$ considering 1 byte is 8 bit. The total delay is $d_{prop} + d_{tran} = 240.15$