UNIVERSITY OF CALIFORNIA, SAN DIEGO

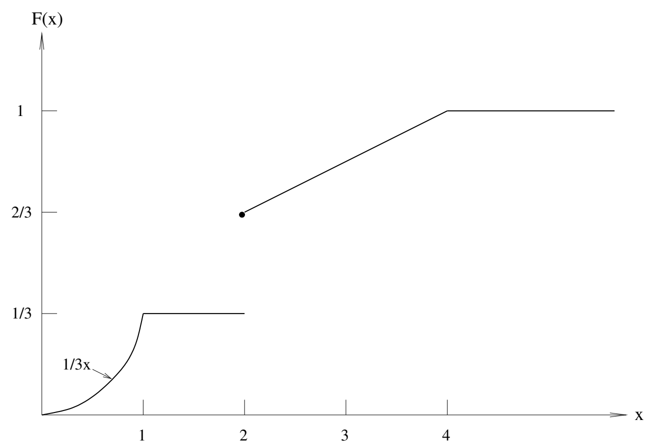
Electrical & Computer Engineering Department

ECE 250 - Winter Quarter 2022

Random Processes

Problem Set #2 Due Friday, January 21, 2022 at 11:59pm
Submit solutions to Problems 3, 4, 7, 9 only

1. *Probabilities from a cdf.* Let X be a random variable with the cdf shown below.



Find the probabilities of the following events.

(a) $\{X = 2\}$.

(b) $\{X < 2\}$.

(c) $\{X = 2\} \cup \{0.5 \le X \le 1.5\}$.

(d) $\{X = 2\} \cup \{0.5 \le X \le 3\}$.

2. *Gaussian probabilities.* Let $X \sim N(1000, 400)$. Express the following in terms of the Q function.

(a) $\mathsf{P}\{0 < X < 1020\}$.

(b) $\mathsf{P}\{X < 1020 | X > 960\}$.

3. *Laplacian.* Let $X \sim f(x) = \frac{1}{2}e^{-|x|}$.

(a) Sketch the cdf of X.

(b) Find $\mathsf{P}\{|X| \le 2 \text{ or } X \ge 0\}$.

(c) Find $\mathsf{P}\{|X| + |X-3| \le 3\}$.

(d) Find $\mathsf{P}\{X \geq 0 \,|\, X \leq 1\}$.

4. *Distance to the nearest star.* Let the random variable N be the number of stars in a region of space of volume V. Assume that N is a Poisson r.v. with pmf

$$p_N(n) = \frac{e^{-\rho V}(\rho V)^n}{n!}, \qquad \text{for } n = 0, 1, 2, \ldots,$$

where ρ is the "density" of stars in space. We choose an arbitrary point in space and define the random variable X to be the distance from the chosen point to the nearest star. Find the pdf of X (in terms of ρ).

5. *Uniform arrival.* The arrival time of a professor to his office is uniformly distributed in the interval between 8 and 9 am. Find the probability that the professor will arrive during the next minute given that he has not arrived by 8:30. Repeat for 8:50.

6. *Lognormal distribution.* Let $X \sim \mathrm{N}(0, \sigma^2)$. Find the pdf of $Y = e^X$ (known as the *lognormal* pdf).

7. *Random phase signal.* Let $Y(t) = \sin(\omega t + \Theta)$ be a sinusoidal signal with random phase $\Theta \sim U[-\pi, \pi]$. Find the pdf of the random variable $Y(t)$ (assume here that both t and the radial frequency ω are constant). Comment on the dependence of the pdf of $Y(t)$ on time t.

8. *Quantizer.* Let $X \sim \exp(\lambda)$, i.e., an exponential random variable with parameter λ and $Y = \lfloor X \rfloor$, i.e., $Y = k$ for $k \leq X < k + 1$, $k = 0, 1, 2, \ldots$. Find the pmf of Y. Define the quantization error $Z = X - Y$. Find the pdf of Z.

9. *Gambling.* Alice enters a casino with one unit of capital. She looks at her watch to generate a uniform random variable $U \sim \mathrm{unif}\,[0, 1]$, then bets the amount U on a fair coin flip. Her wealth is thus given by the r.v.

$$X = \begin{cases} 1 + U, & \text{with probability } 1/2, \\ 1 - U, & \text{with probability } 1/2. \end{cases}$$

Find the cdf of X.