ECE-269

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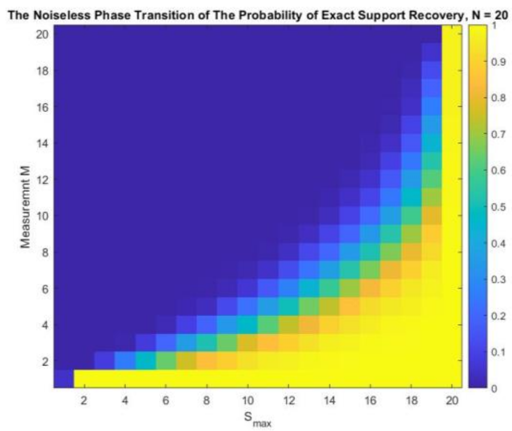
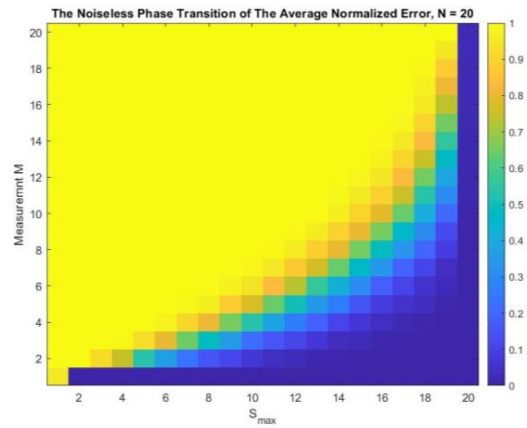
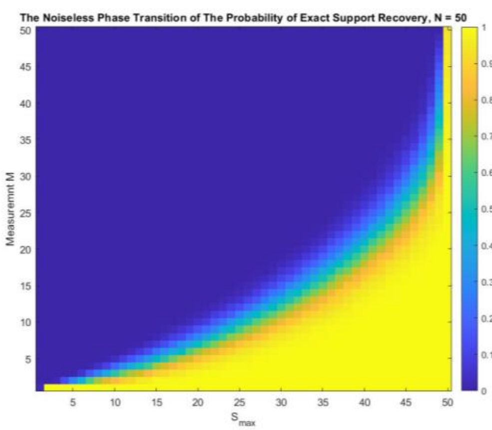
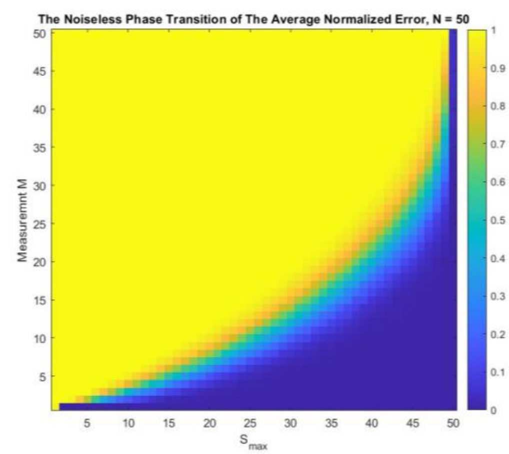
HW3 Report

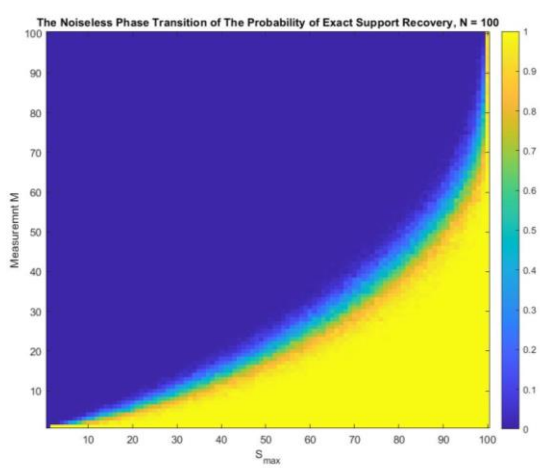
Part 3. The Noiseless Case

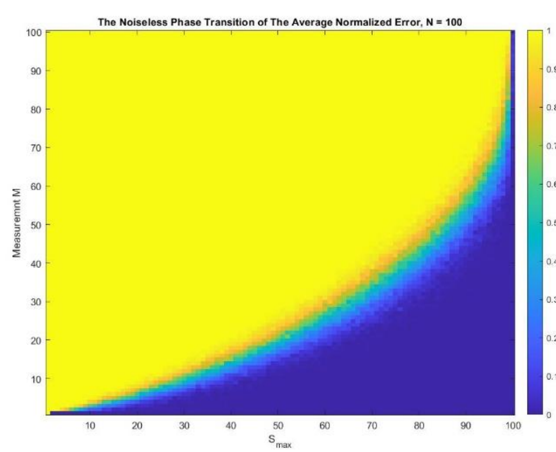










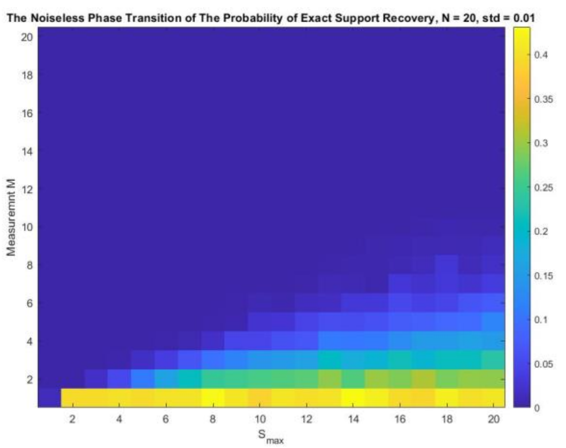


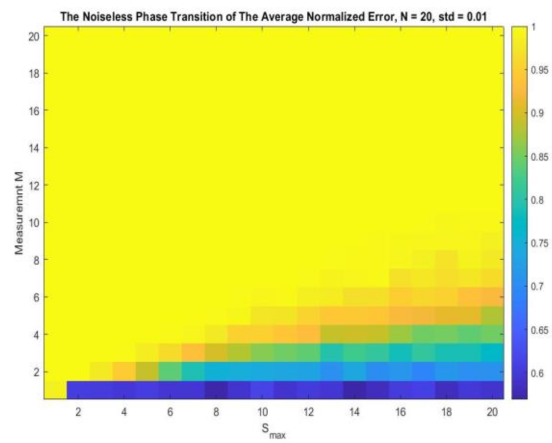
Part 3.

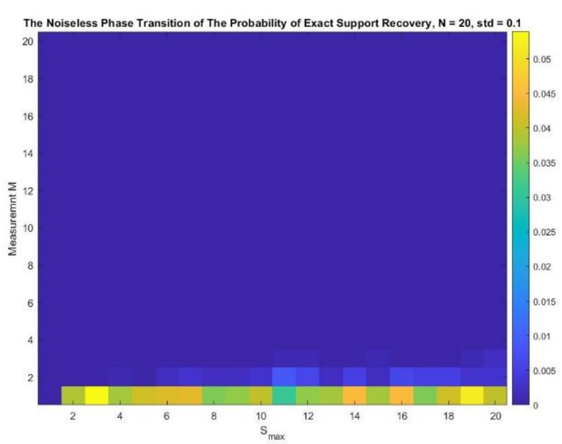
From the phase transition plots of both the probability of exact support recovery and the average normalized error with $N = 20, 50, 100$, a sharp transition region can be observed on each plot where $S_{MAX} = M$ or $S_{max} > M$. We can see that given a specific M, the probability of a successful recovery is higher when $S_{max} \geq M$. In other words, to obtain a more ideal recovery, letting $S_{max} \gg M$ could be an unwise but effective way.

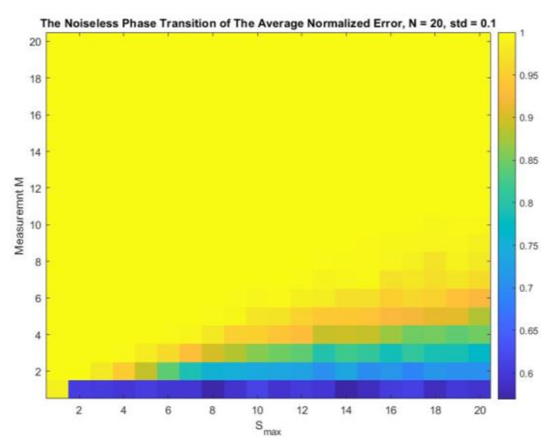
Part 4. The Noisy Case

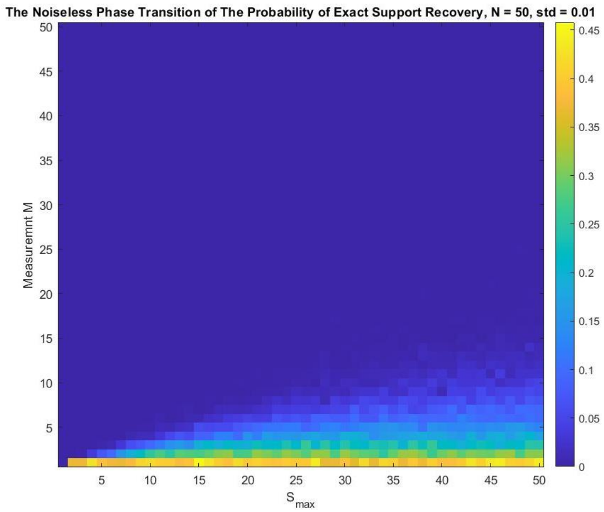
(a) The sparsity is known.

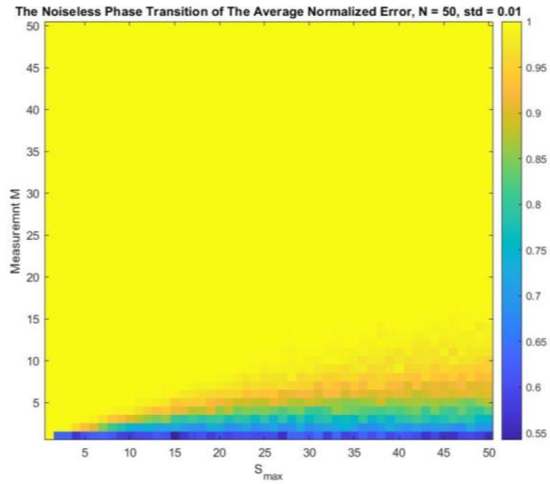


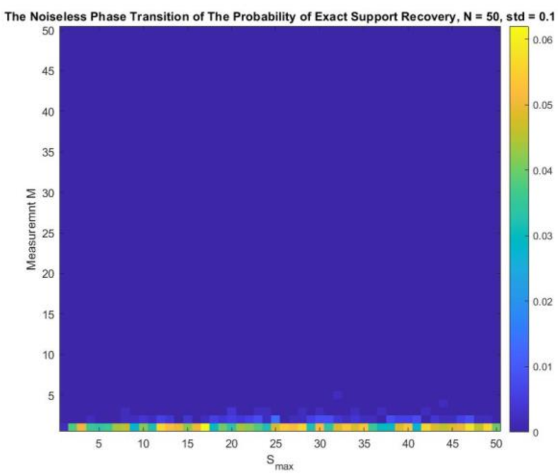


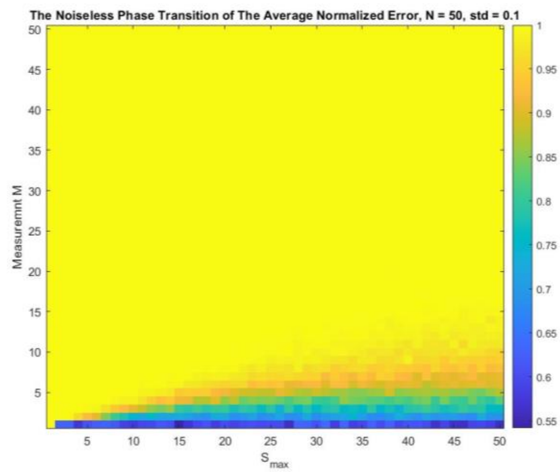


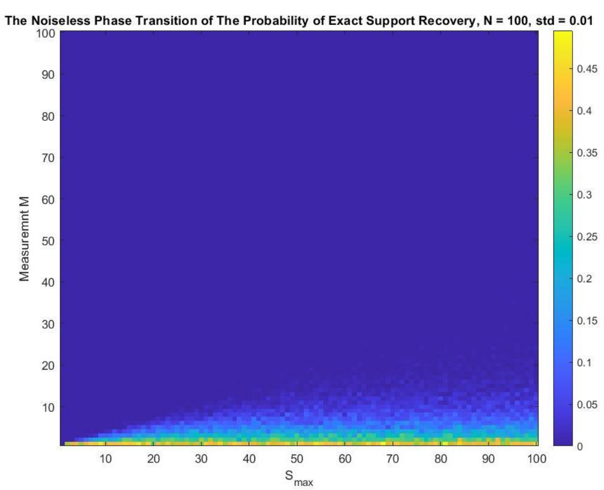


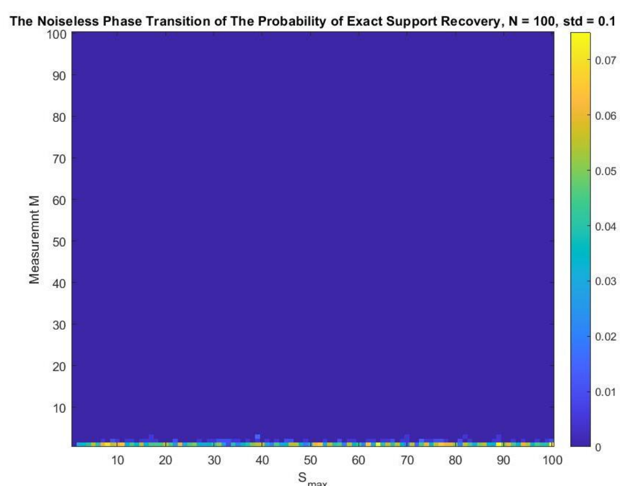

The Noiseless Phase Transition of The Probability of Exact Support Recovery, N = 50, std = 0.01
Measuremnt M
S_{max}

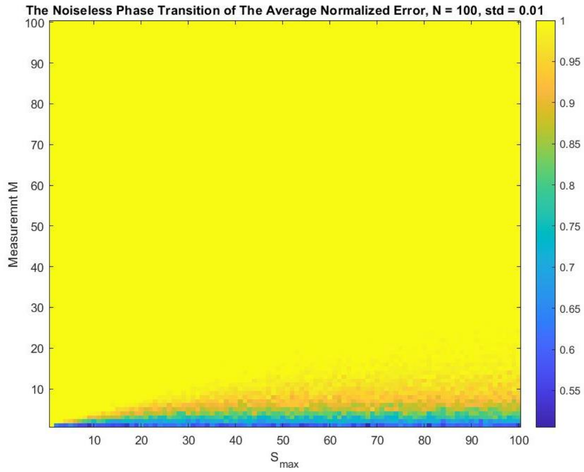

The Noiseless Phase Transition of The Average Normalized Error, N = 50, std = 0.01
Measuremnt M
S_{max}

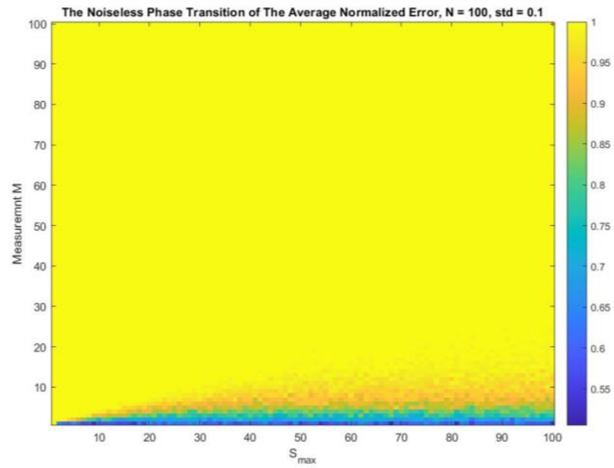

The Noiseless Phase Transition of The Probability of Exact Support Recovery, N = 50, std = 0.1
Measuremnt M
S_{max}


The Noiseless Phase Transition of The Average Normalized Error, N = 50, std = 0.1
Measuremnt M
S_{max}


The Noiseless Phase Transition of The Probability of Exact Support Recovery, N = 100, std = 0.01
Measuremnt M
S_{max}

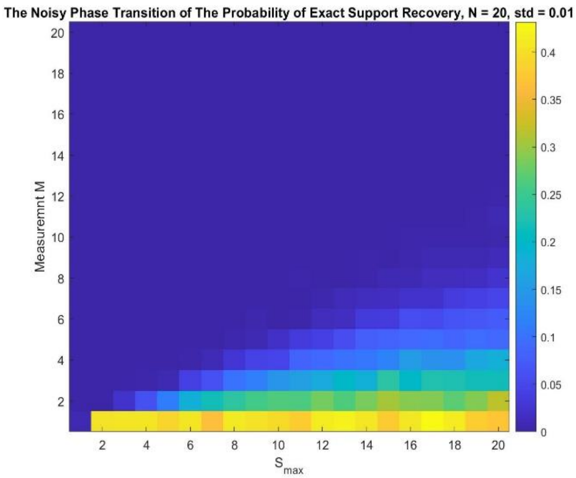

The Noiseless Phase Transition of The Probability of Exact Support Recovery, N = 100, std = 0.1
Measuremnt M
S_{max}

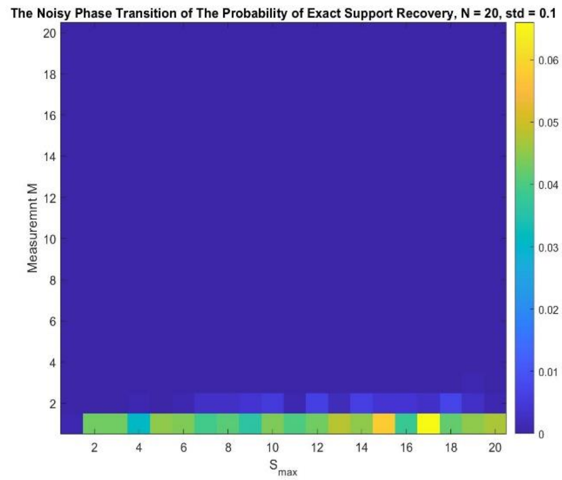


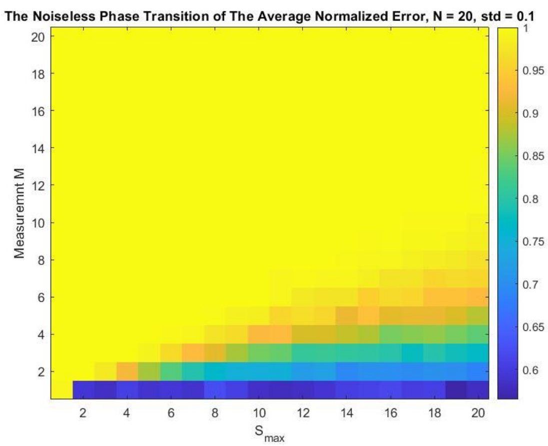


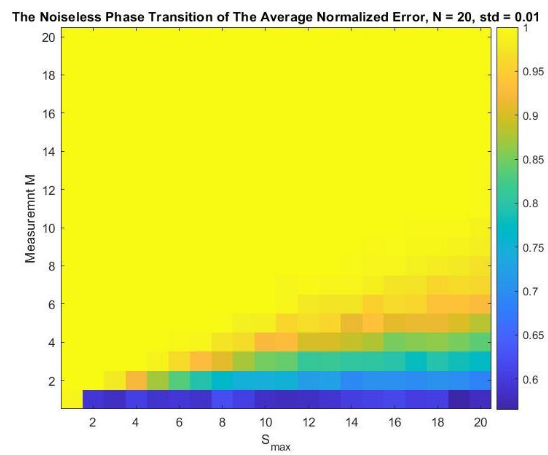
From the plots shown above, we can see that the negative impact of the (additive) noise on the recovery of sparse vector (x). After adding noise into the measurement, even we let $S_{max} \gg M$, the probability of exact support recovery is still low, which is less than 0.5 (in the case of $N = 20, std = 0.01$). Moreover, a strong noise with large standard deviation may unfortunately prevent the support recovery implementing OMP, which can be observed by comparing the cases $std = 0.1$ and $std = 0.01$.

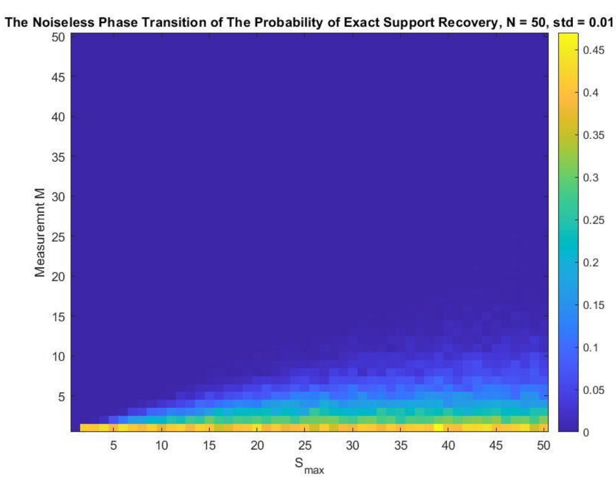
(b) When the $||n||_2$ is known

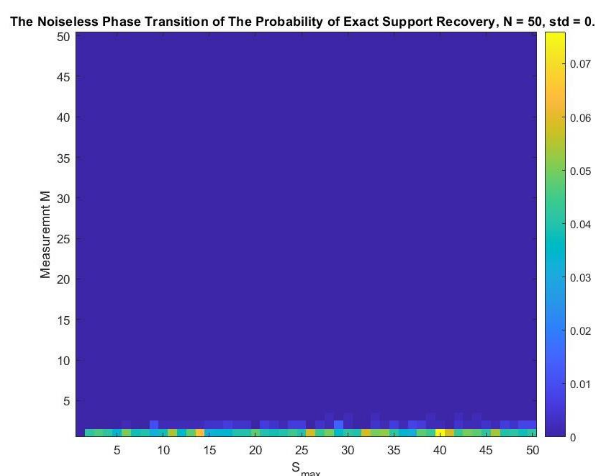


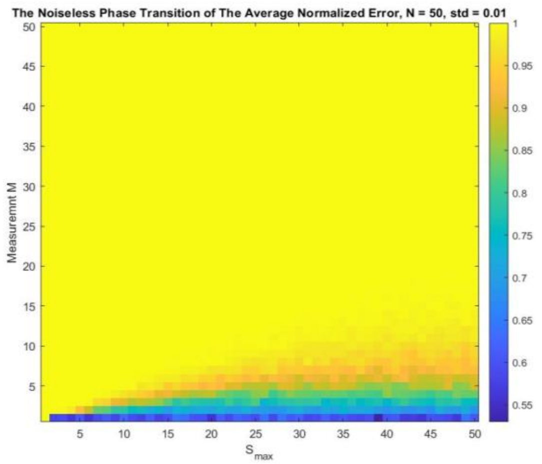


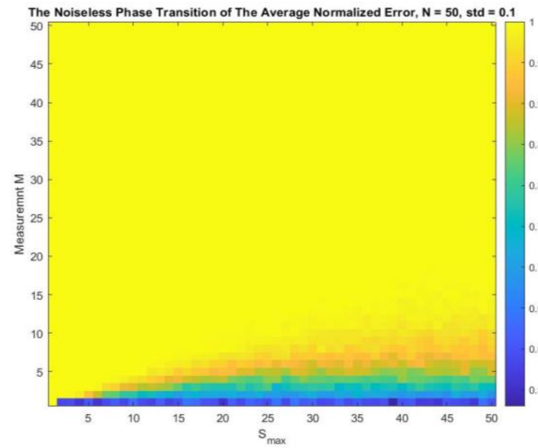


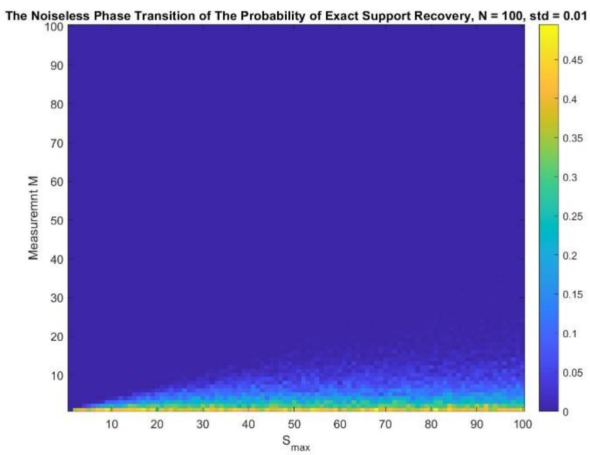


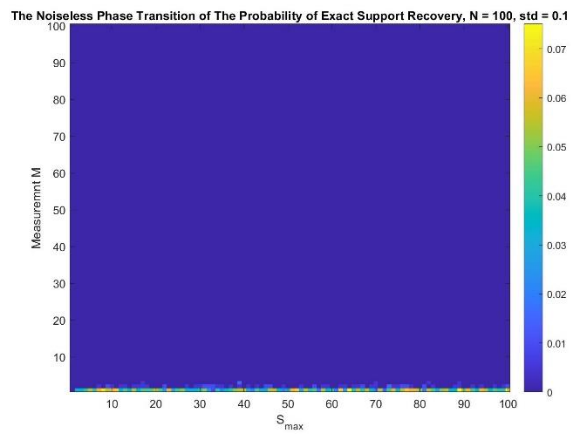


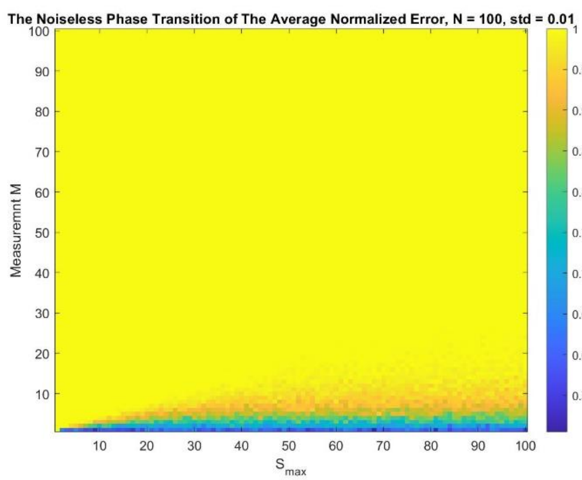


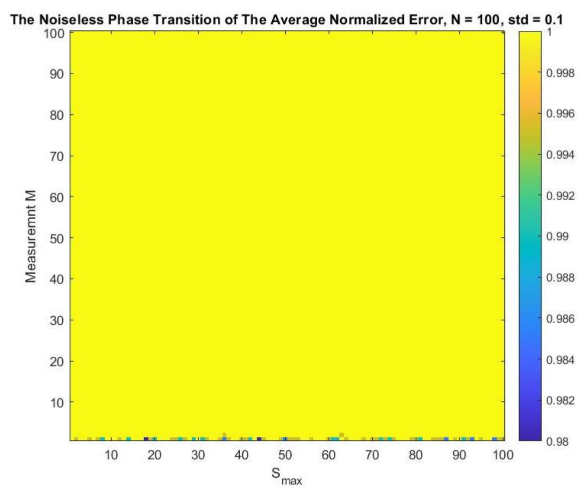








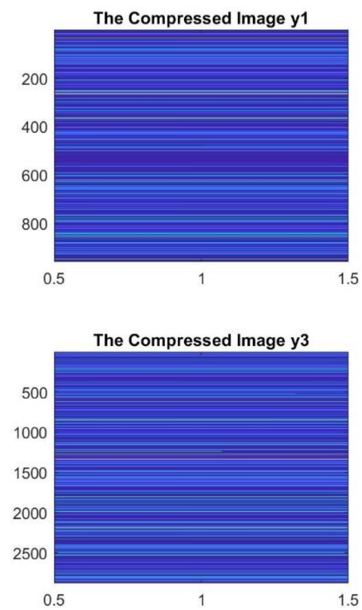




Form the plots shown above, we can see the tremendous impact of the noise introduced to the measurement on the recovery. Compare to the two different strategies of stopping OMP implemented in part (a) and part (b), both termination strategies have very close performance when the standard deviation of noise is low (like 0.01). However, when the std of noise is high (such as 1), the strategy used in part (b) has a slightly better performance than the one does in part (a). In my view, the reason of it is that the strategy in part (b) provided an exact threshold instead of simply looping the algorithm by a given times.

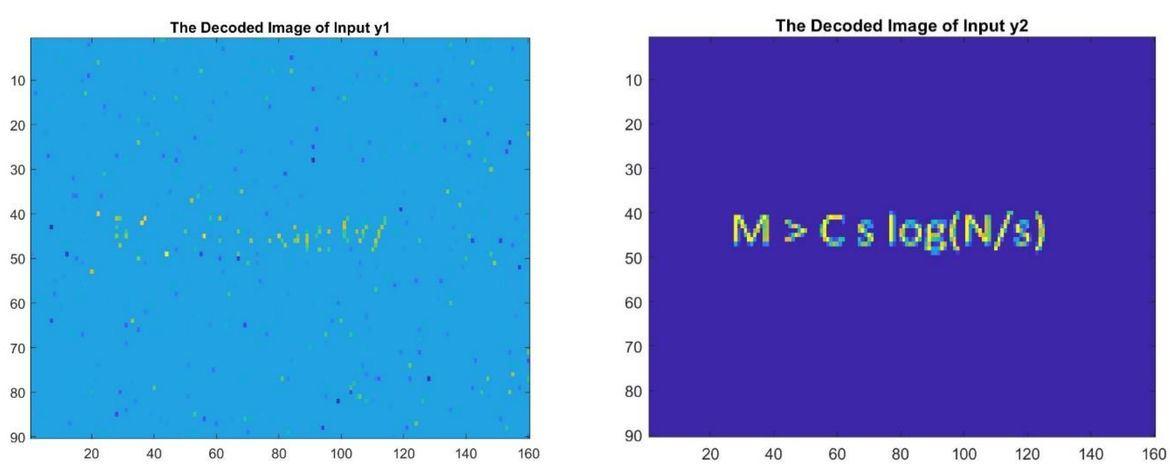
Part 5. Decode a Compressed Message

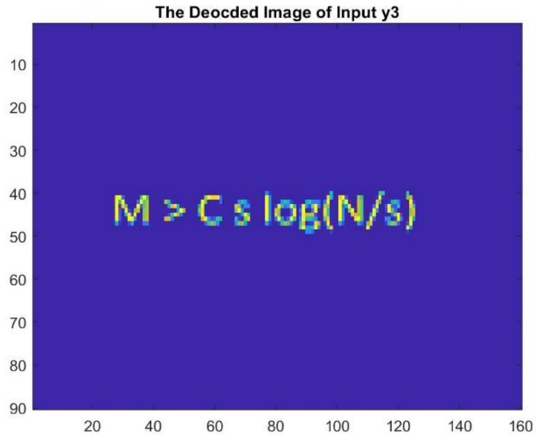
Part (a)



From the plot shown above, the message cannot be shown by simply display the compressed images.

Part (b)





From the images shown above, the compressed image Y1 can not recover the message by implementing OMP, while both Y2 and Y3 decode clear output message. Since the size of Y3 is larger than Y2 which is more sufficient for recovery, the quality of decaded message of Y3 is slightly better than Y2.