TU Berlin Robotics WiSe 18/19
Lab Assignment #2

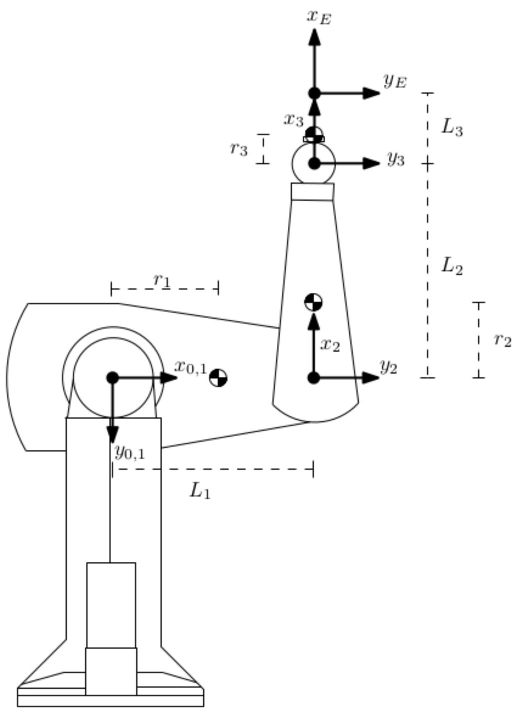

Figure 1: RRR Puma in zero configuration

Joint:	1	2	3	4	5	6
τ_{max}:	97,6 Nm	156,4 Nm	89,4 Nm	24,2 Nm	20,1 Nm	21,2 Nm

Table 1: torque limits of the PUMA robot

	A1	**A2**	**A3**	**(A4)**	**(B1)**	**B2**	**C1**	**C2**	**C3**	**C4**	**C5**
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Table 2: implementation table (we solved all tasks as a group)

A Forward Kinematics

A1 Transformation Between Frames

$$
{}^{0}_{E}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{E}T = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} s_2 & c_2 & 0 & L_1 \\ -c_2 & s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{0}_{E}T = \begin{pmatrix} c_3 & -s_3 & 0 & c_3L_3 + L_2 \\ s_3 & c_3 & 0 & s_3L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

A2 End Effector Position In Operational Space

The end-effector position and orientation is calculated:

$$
{}^{0}_{E}T(q) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{EE} = \begin{pmatrix} c_1L_1 + s_{12}L_2 + s_{123}L_3 \\ s_1L_1 - c_{12}L_2 - c_{123}L_3 \\ 0 \\ 1 \end{pmatrix}
$$

End effector orientation: $\theta = q_1 + q_2 + q_3 - \frac{\pi}{2}$

Finally,

$$
F(q) = \begin{pmatrix} c_1L_1 + s_{12}L_2 + s_{123}L_3 \\ s_1L_1 - c_{12}L_2 - c_{123}L_3 \\ q_1 + q_2 + q_3 - \frac{\pi}{2} \end{pmatrix}
$$

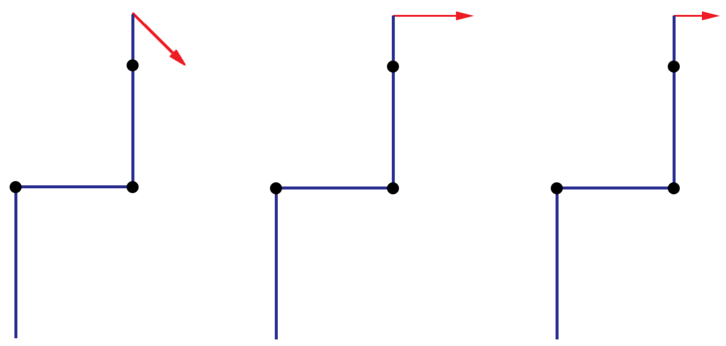
A3 Compute The End Effector Jacobian

End effector Jacobian:

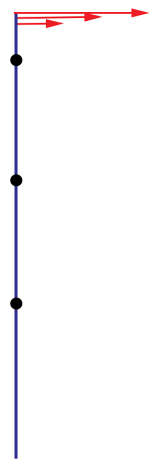
$$
\frac{\partial J(q)}{\partial q} = \begin{pmatrix} -s_1L_1 + c_{12}L_2 + c_{123}L_3 & c_{12}L_2 + c_{123}L_3 & c_{123}L_3 \\ c_1L_1 + s_{12}L_2 + s_{123}L_3 & s_{12}L_2 + s_{123}L_3 & s_{123}L_3 \\ 1 & 1 & 1 \end{pmatrix}
$$

A4 Understanding The Jacobian Matrix And Pose Singularities

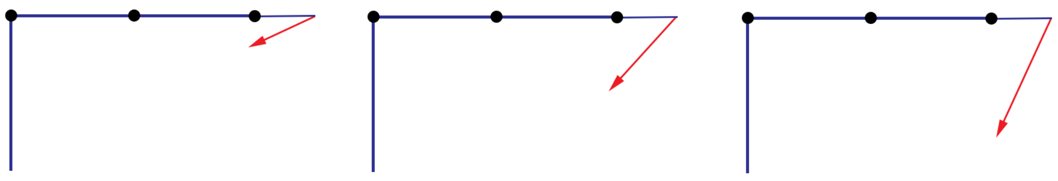
a)



EE velocity, when $\dot{q}_{11} = \begin{pmatrix}1 & 0 & 0\end{pmatrix}$ $\dot{q}_{12} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}$ $\dot{q}_{13} = \begin{pmatrix}0 & 0 & 1\end{pmatrix}$ and $q_1 = \begin{pmatrix}0 & 0 & 0\end{pmatrix}$



EE velocity, when $\dot{q}_{21} = \begin{pmatrix}1 & 0 & 0\end{pmatrix}$ $\dot{q}_{22} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}$ $\dot{q}_{23} = \begin{pmatrix}0 & 0 & 1\end{pmatrix}$ and $q_2 = \begin{pmatrix}\frac{\pi}{2} & -\frac{\pi}{2} & 0\end{pmatrix}$
$|\dot{q}_{21}| > |\dot{q}_{22}| > |\dot{q}_{23}|$



EE velocity, when $\dot{q}_{31} = \begin{pmatrix}0 & 0 & 1\end{pmatrix}$ $\dot{q}_{32} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}$ $\dot{q}_{33} = \begin{pmatrix}1 & 0 & 0\end{pmatrix}$ and $q_3 = \begin{pmatrix}0 & \frac{\pi}{2} & 0.01\end{pmatrix}$

b)

In the first case the robot is not in a singularity. The robot has all degrees of freedom.

In the second case, all the velocities are paralell, so the robot is in a singularity. The robot cannot move in y direction.

In the third case, nearly all velocities are in the same direction, so the robot is close to singularity.

B Trajectory Generation In Joint Space

B1 Generation Of Smooth Trajectories With Polynomial Splines

a)

Zero configuration: $q_a = \begin{pmatrix} 0, & 0, & 0 \end{pmatrix}^T$

Intermediate point: $q_b = \begin{pmatrix} -\frac{\pi}{4}, & \frac{\pi}{2}, & 0 \end{pmatrix}^T$

Final configuration: $q_c = \begin{pmatrix} -\frac{\pi}{2}, & \frac{\pi}{4}, & 0 \end{pmatrix}^T$

Two splines:

$$u_1(t) = a_0 + a_1 t_{via} + a_2 t_{via}^2 + a_3 t_{via}^3$$
$$u_2(t) = b_0 + b_1(t - t_{via}) + b_2(t - t_{via})^2 + b_3(t - t_{via})^3$$

Contraints:

$q(0) = q_0$

$q(t_f) = q_f$

$\dot{q}(0) = 0$

$\dot{q}(t_f) = 0$

Velocities at the intermediate point:

$\dot{q}_b(t_f) = \begin{pmatrix} -\frac{\pi}{10}, & 0, & 0 \end{pmatrix}^T [\frac{rad}{s^2}]$

Equations for computing the for the scalar values:

$a_0 = q_0$

$a_1 = \dot{q}_0$

$a_2 = \frac{3}{t_{via}^2}(u_{via} - u_0) - \frac{2}{t_{via}}\dot{u}_0 - \frac{1}{t_{via}}\dot{u}_{via}$

$a_3 = -\frac{2}{t_{via}^3}(u_{via} - u_0) + \frac{1}{t_{via}^2}(\dot{u}_0 + \dot{u}_{via})$

$b_0 = q_{via}$

$b_1 = \dot{q}_{via}$

$b_2 = \frac{3}{(t_f - t_{via})^2}(u_f - u_{via}) - \frac{2}{t_f - t_{via}}\dot{u}_{via} - \frac{1}{t_f - t_{via}}\dot{u}_f$

$b_3 = -\frac{2}{(t_f - t_{via})^3}(u_f - u_{via}) + \frac{1}{(t_f - t_{via})^2}(\dot{u}_{via} + \dot{u}_f)$

Results:

$a_{10} = 0$	$a_{20} = -\frac{\pi}{4} \approx -0.79$	$b_{10} = 0$	$b_{20} = \frac{\pi}{2} \approx 1.57$
$a_{11} = 0$	$a_{21} = -\frac{\pi}{10} \approx -0.31$	$b_{11} = 0$	$b_{21} = 0$
$a_{12} = -\frac{10\pi}{125} \approx -0.25$	$a_{22} = -\frac{5\pi}{125} \approx -0.13$	$b_{12} = \frac{30\pi}{125} \approx 0.75$	$b_{22} = -\frac{15\pi}{125} \approx -0.38$
$a_{13} = \frac{2\pi}{125} \approx 0.05$	$a_{23} = \frac{2\pi}{125} \approx 0.05$	$b_{13} = -\frac{8\pi}{125} \approx -0.20$	$b_{23} = \frac{4\pi}{125} \approx 0.10$

b)

TODODODO

B2 Trajectories In Joint Space

TODO

C Operational Space Control

C1

TODO

C2

TODO

C3

TODO

C4

TODO

C5

TODO