TU Berlin Robotics WiSe 18/19
Lab Assignment #2

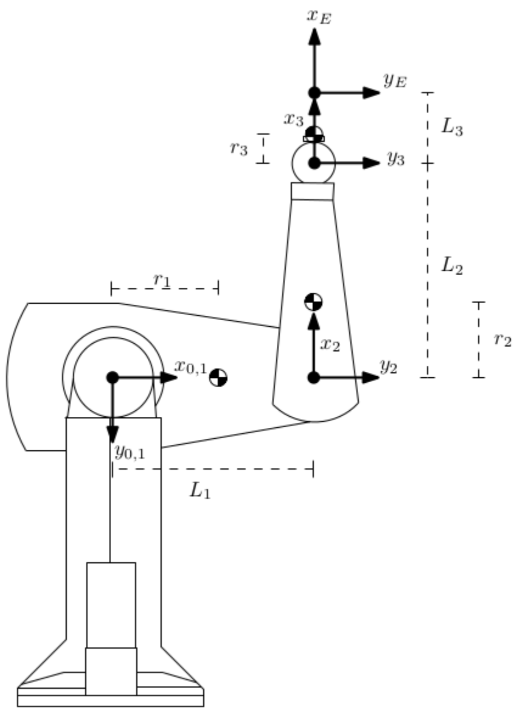


Figure 1: RRR Puma in zero configuration

Joint:	1	2	3	4	5	6
τ_{max}:	97,6 Nm	156,4 Nm	89,4 Nm	24,2 Nm	20,1 Nm	21,2 Nm

Table 1: torque limits of the PUMA robot

	A1	**A2**	**A3**	**(A4)**	**(B1)**	**B2**	**C1**	**C2**	**C3**	**C4**	**C5**
Jiaqiao Peng											
Benjamin Oesterle	x	x	x	x	x	x	x	x	x	x	
Botond Péter Sléber	x	x	x	x	x	x					
Dhananjay Mukhedkar	x	x	x		x					x	

Table 2: implementation table

A Forward Kinematics

A1 Transformation Between Frames

Implemented in: `ForwardKinematicsPuma2D::computeTX_Y()`

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	q_1
2	0	L_1	0	$q_2 - \frac{\pi}{2}$
3	0	L_2	0	q_3
4(E)	0	L_3	0	0

DH-parameters from assignment 1

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & \alpha_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

homogeneous transformation matrix

Homogenous transformations between adjacent links:

$$
{}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} s_2 & c_2 & 0 & L_1 \\ -c_2 & s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{E}T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward kinematics for the end effector:

$$
{}^{0}_{E}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T \cdot {}^{3}_{E}T = \begin{bmatrix} s_{123} & c_{123} & 0 & c_1 L_1 + s_{12} L_2 + s_{123} L3 \\ -c_{123} & s_{123} & 0 & s_1 L_1 - c_{12} L_2 - c_{123} L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

A2 End Effector Position In Operational Space

Implemented in: `ForwardKinematicsPuma2D::computeF()`

$$
F(q) = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} c_1 L_1 + s_{12} L_2 + s_{123} L_3 \\ s_1 L_1 - c_{12} L_2 - c_{123} L_3 \\ q_1 + q_2 + q_3 - \frac{\pi}{2} \end{pmatrix}
$$

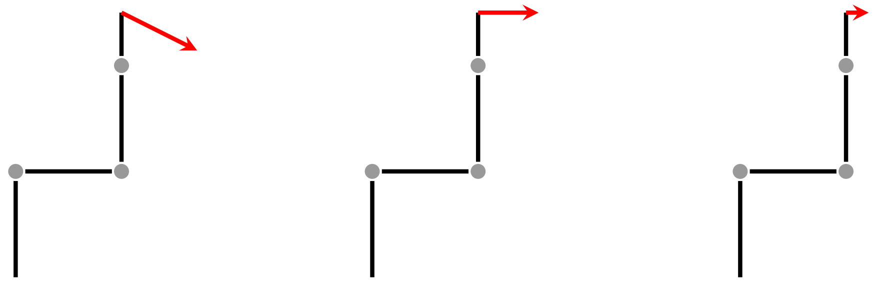
A3 Compute The End Effector Jacobian

Implemented in: `ForwardKinematicsPuma2D::computeJ()`

$$J(q) = \frac{\partial F(q)}{\partial q} = \begin{bmatrix} -s_1 L_1 + c_{12} L_2 + c_{123} L_3 & c_{12} L_2 + c_{123} L_3 & c_{123} L_3 \\ c_1 L_1 + s_{12} L_2 + s_{123} L_3 & s_{12} L_2 + s_{123} L_3 & s_{123} L_3 \\ 1 & 1 & 1 \end{bmatrix}$$

A4 Understanding The Jacobian Matrix And Pose Singularities

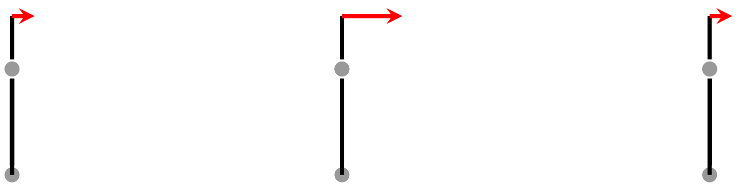
i) $q_1 = (0, 0, 0)^T$



$\dot{q}_1 = (1, 0, 0)^T$ $\qquad$ $\dot{q}_1 = (0, 1, 0)^T$ $\qquad$ $\dot{q}_1 = (0, 0, 1)^T$

The robot is **not in a singularity**, it can move in x- and y-direction.

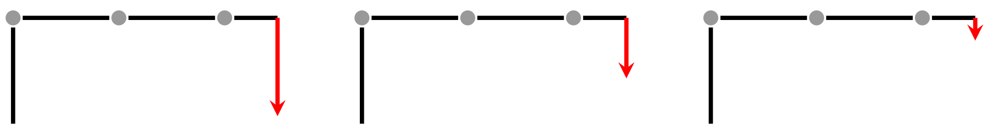
ii) $q_2 = (\frac{\pi}{2}, -\frac{\pi}{2}, 0)^T$



$\dot{q}_1 = (1, 0, 0)^T$ $\qquad$ $\dot{q}_1 = (0, 1, 0)^T$ $\qquad$ $\dot{q}_1 = (0, 0, 1)^T$

The robot is **in a singularity**, it cannot move in y-direction (all velocities are parallel).

iii) $q_3 = (0, \frac{\pi}{2}, 0.01)^T$



$\dot{q}_1 = (1, 0, 0)^T$ $\qquad$ $\dot{q}_1 = (0, 1, 0)^T$ $\qquad$ $\dot{q}_1 = (0, 0, 1)^T$

The robot is **close to a singularity**, it barely can move in x-direction anymore (all velocities point in nearly the same direction, there are still small differences though - invisible in the sketch).

B Trajectory Generation In Joint Space

B1 Generation Of Smooth Trajectories With Polynomial Splines

a)

Zero configuration: $q_a = \begin{pmatrix} 0, & 0, & 0 \end{pmatrix}^T$

Intermediate point: $q_b = \begin{pmatrix} -\frac{\pi}{4}, & \frac{\pi}{2}, & 0 \end{pmatrix}^T$

Final configuration: $q_c = \begin{pmatrix} -\frac{\pi}{2}, & \frac{\pi}{4}, & 0 \end{pmatrix}^T$

Two splines:

$u_1(t) = a_0 + a_1 t_{via} + a_2 t_{via}^2 + a_3 t_{via}^3$
$u_2(t) = b_0 + b_1(t - t_{via}) + b_2(t - t_{via})^2 + b_3(t - t_{via})^3$

Contraints:

$q(0) = q_0$

$q(t_f) = q_f$

$\dot{q}(0) = 0$

$\dot{q}(t_f) = 0$

Velocities at the intermediate point:

$\dot{q}_b(t_f) = \begin{pmatrix} -\frac{\pi}{10}, & 0, & 0 \end{pmatrix}^T [\frac{rad}{s^2}]$

Equations for computing the for the scalar values:

$a_0 = q_0$

$a_1 = \dot{q}_0$

$a_2 = \frac{3}{t_{via}^2}(u_{via} - u_0) - \frac{2}{t_{via}}\dot{u}_0 - \frac{1}{t_{via}}\dot{u}_{via}$

$a_3 = -\frac{2}{t_{via}^3}(u_{via} - u_0) + \frac{1}{t_{via}^2}(\dot{u}_0 + \dot{u}_{via})$

$b_0 = q_{via}$

$b_1 = \dot{q}_{via}$

$b_2 = \frac{3}{(t_f - t_{via})^2}(u_f - u_{via}) - \frac{2}{t_f - t_{via}}\dot{u}_{via} - \frac{1}{t_f - t_{via}}\dot{u}_f$

$b_3 = -\frac{2}{(t_f - t_{via})^3}(u_f - u_{via}) + \frac{1}{(t_f - t_{via})^2}(\dot{u}_{via} + \dot{u}_f)$

Results:

$a_{10} = 0$	$a_{20} = -\frac{\pi}{4} \approx -0.79$	$b_{10} = 0$	$b_{20} = \frac{\pi}{2} \approx 1.57$
$a_{11} = 0$	$a_{21} = -\frac{\pi}{10} \approx -0.31$	$b_{11} = 0$	$b_{21} = 0$
$a_{12} = -\frac{10\pi}{125} \approx -0.25$	$a_{22} = -\frac{5\pi}{125} \approx -0.13$	$b_{12} = \frac{30\pi}{125} \approx 0.75$	$b_{22} = -\frac{15\pi}{125} \approx -0.38$
$a_{13} = \frac{2\pi}{125} \approx 0.05$	$a_{23} = \frac{2\pi}{125} \approx 0.05$	$b_{13} = -\frac{8\pi}{125} \approx -0.20$	$b_{23} = \frac{4\pi}{125} \approx 0.10$

b)

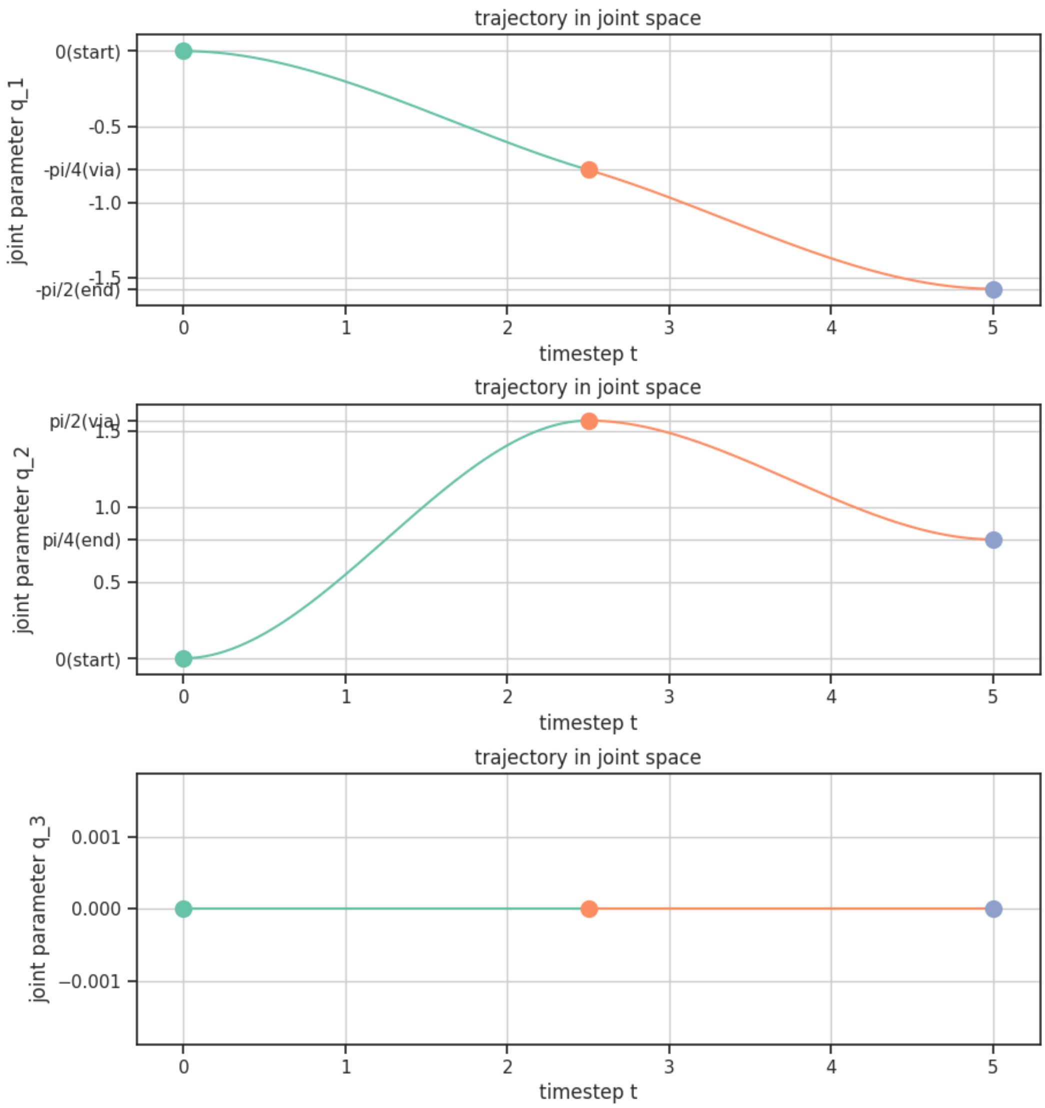
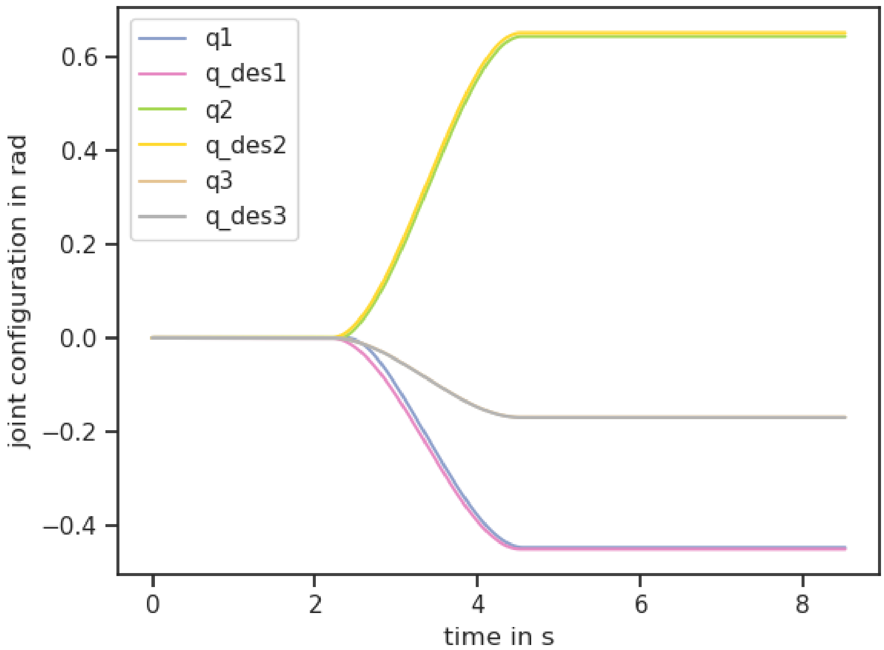
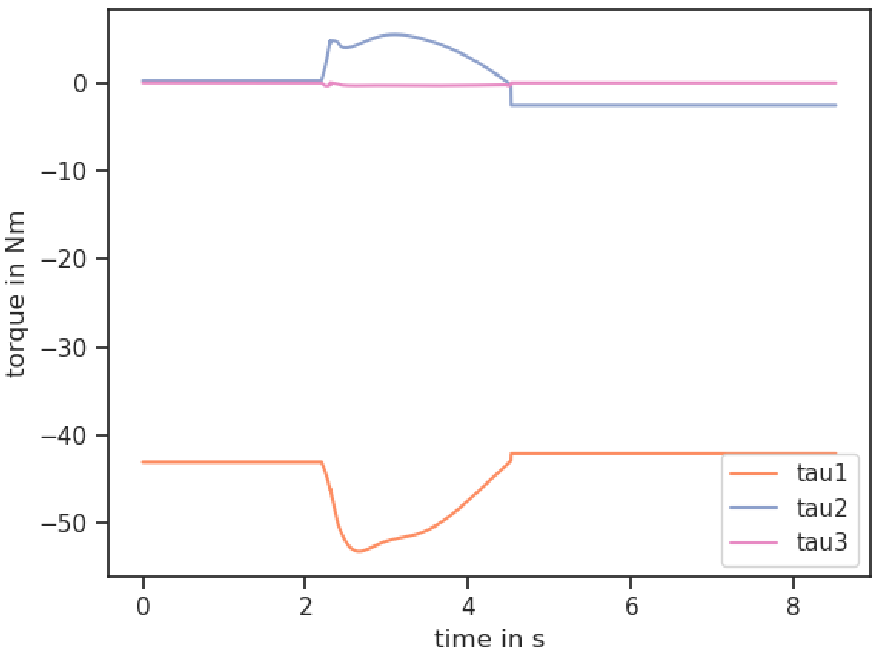


Figure 2: joint configuration q over time

B2 Trajectories In Joint Space

B3





C Operational Space Control

C3

`kp[0]`	`kp[1]`	`kp[2]`	`kv[0]`	`kv[1]`	`kv[2]`
2500	1500	400	20	100	40

Table 3: our positional and velocity gains after tuning

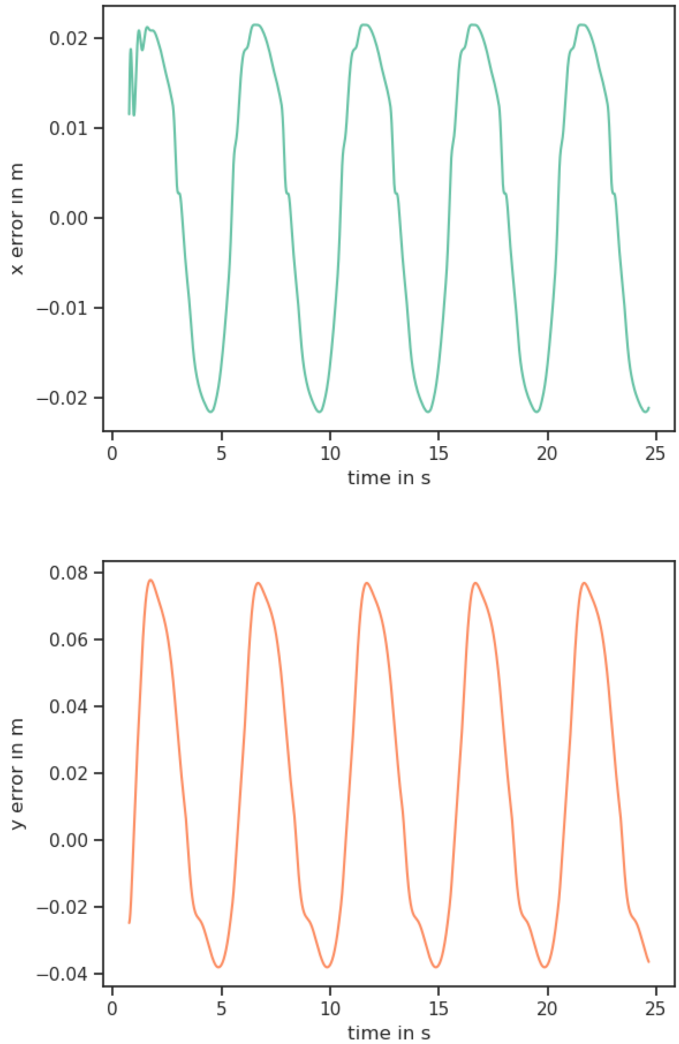


Figure 3: position error $e = x - x_d$ over time

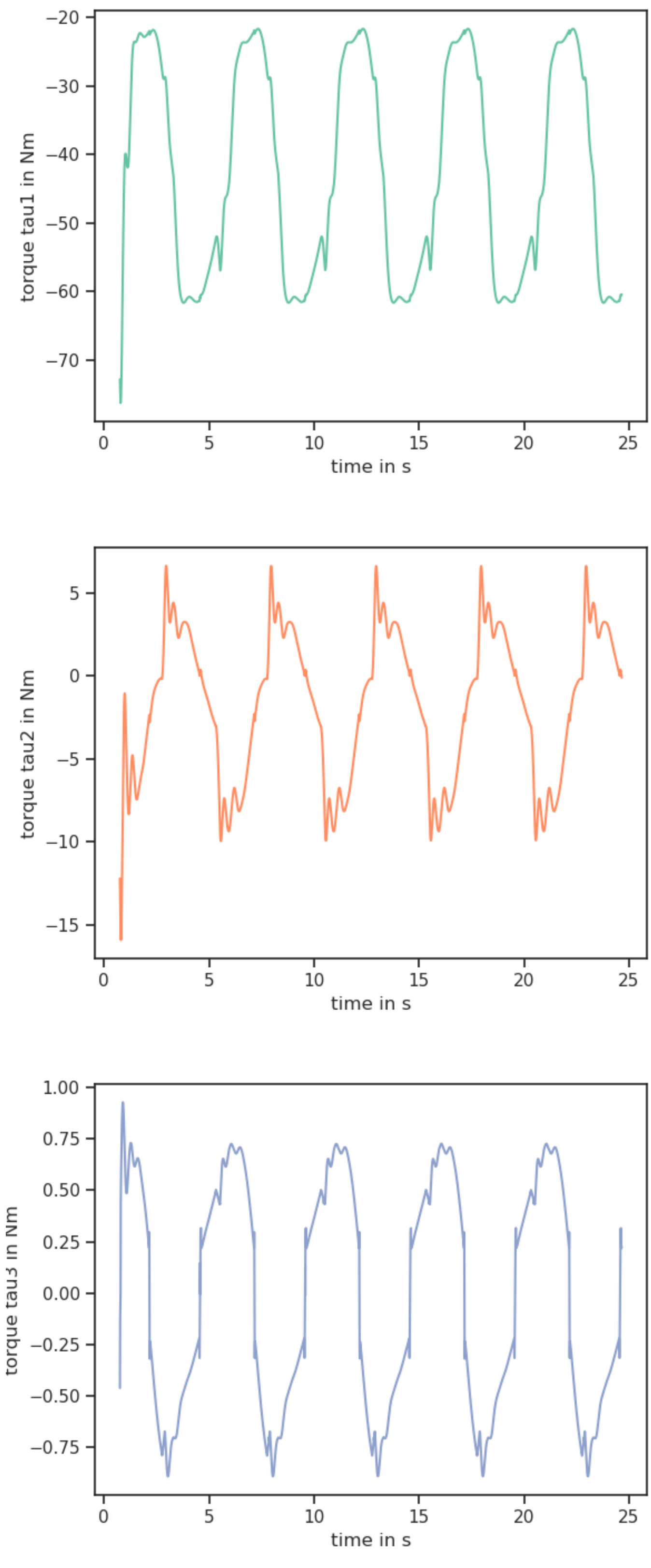


Figure 4: torque τ over time

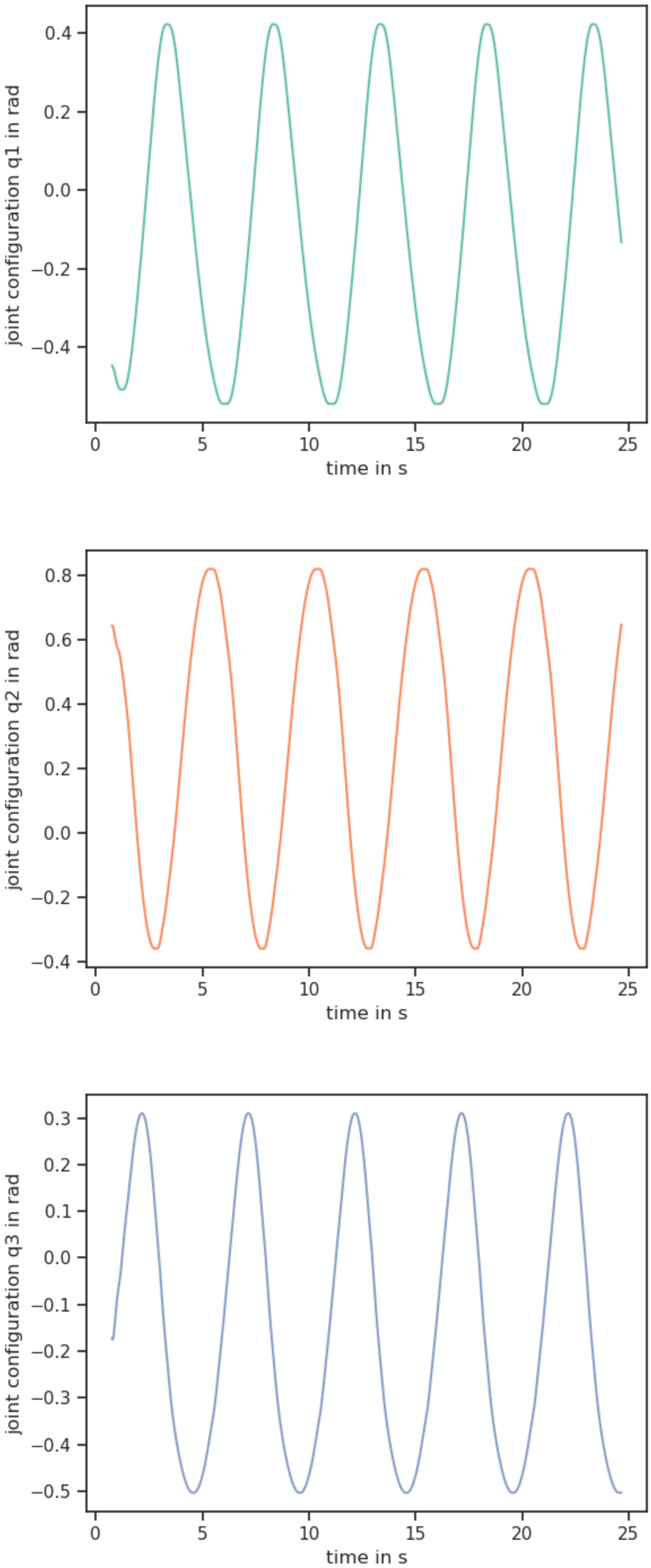


Figure 5: angle θ over time

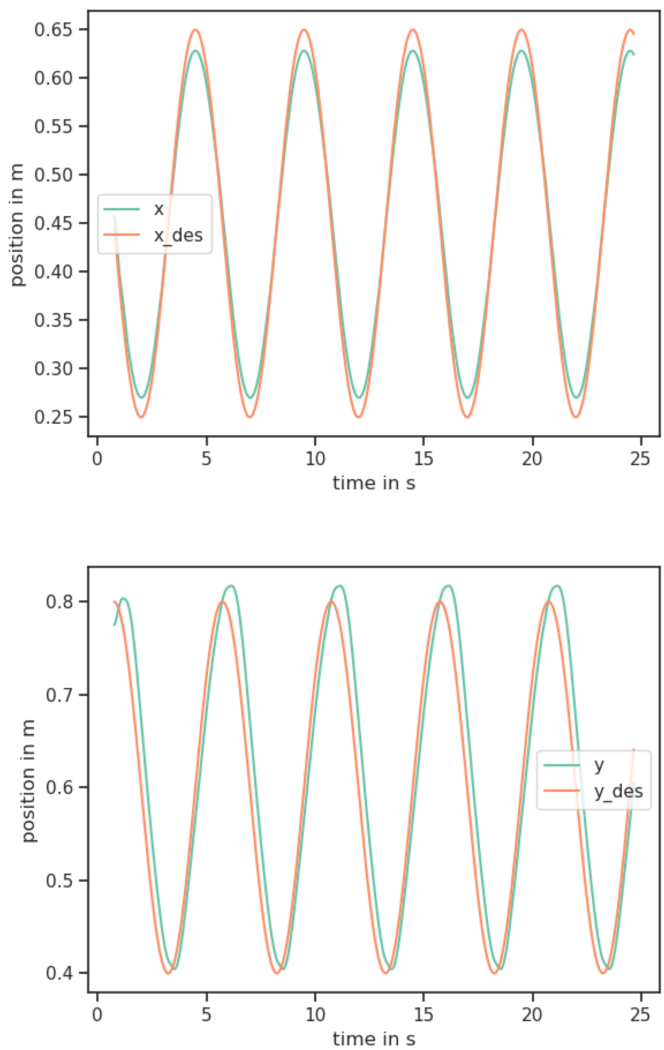


Figure 6: position x and desired position x_d over time

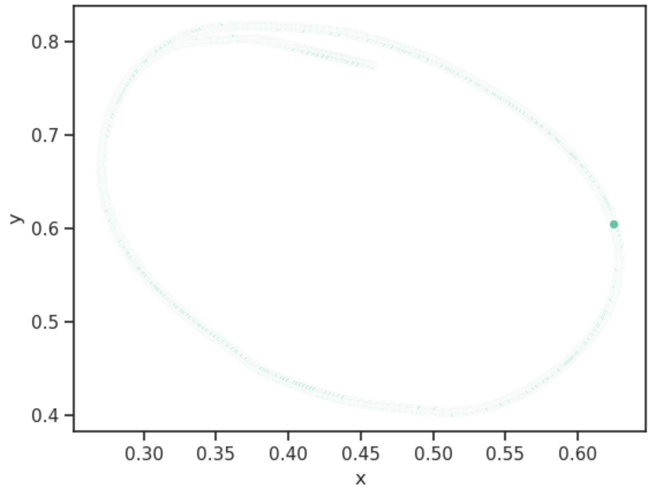


Figure 7: position x in the 2D plane

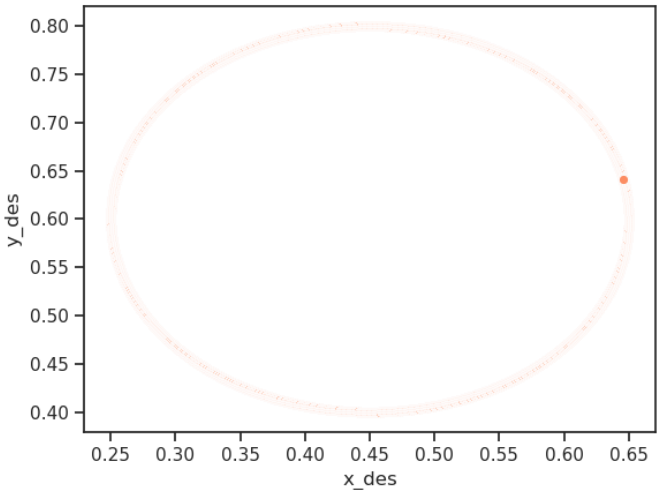


Figure 8: desired position x_d in the 2D plane

C4

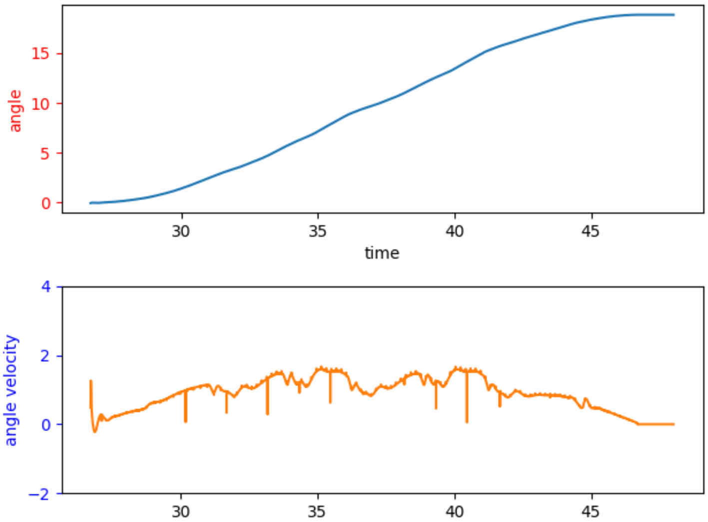


Figure 9: angle β and angular velocity $\dot{\beta}$ over time