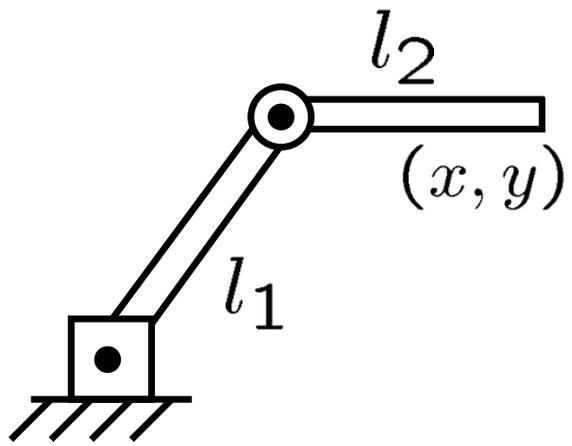
Forward kinematics

Forward kinematics

$$\mathbf{x} = (x, y)^T$$
$$\mathbf{q} = (\theta_1, \theta_2)^T$$

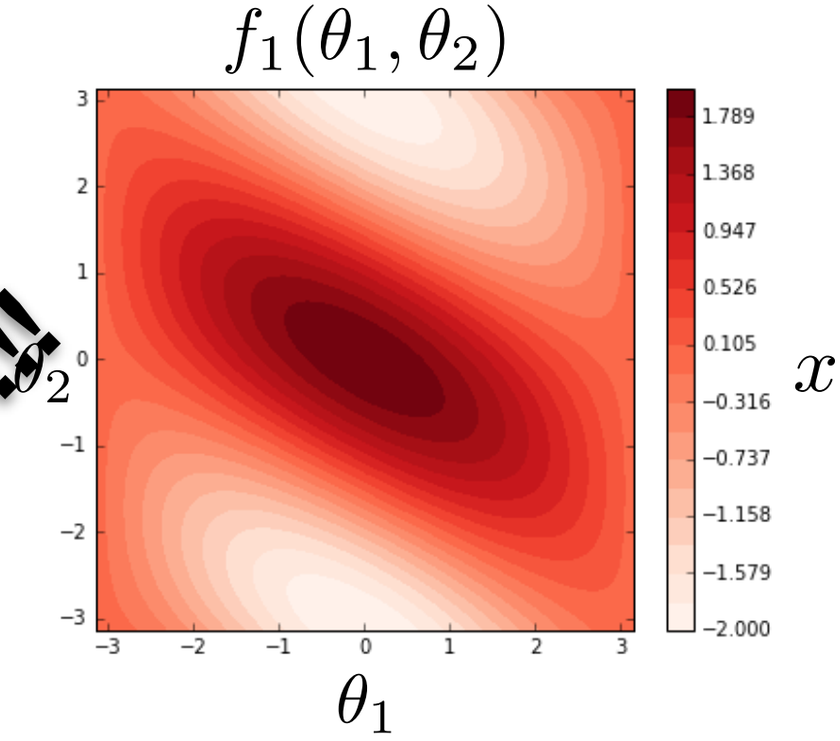


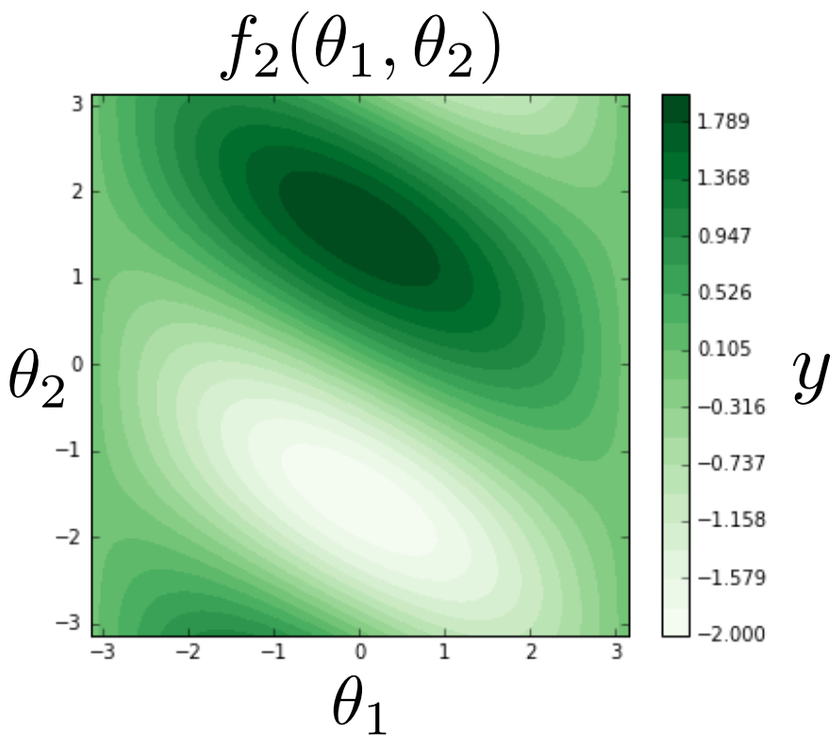
Nonlinear System !!!

Forward kinematics

$$\mathbf{x} = f(\mathbf{q})$$

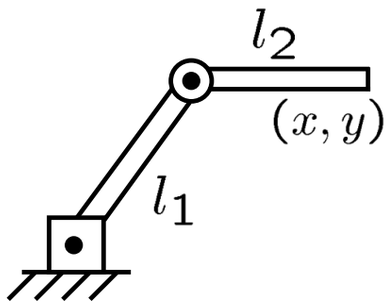
$$x = f_1(\mathbf{q}) = l_1 c_1 + l_2 c_{12}$$
$$y = f_2(\mathbf{q}) = l_1 s_1 + l_2 s_{12}$$





Inverse kinematics

Inverse kinematics



Forward kinematics

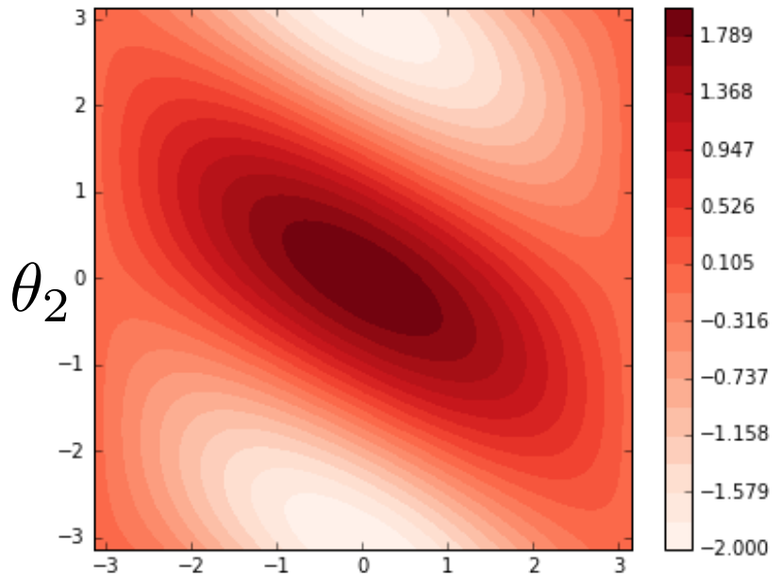
$$\mathbf{x} = f(\mathbf{q})$$

Inverse kinematics

$$\mathbf{q} = f^{-1}(\mathbf{x})$$

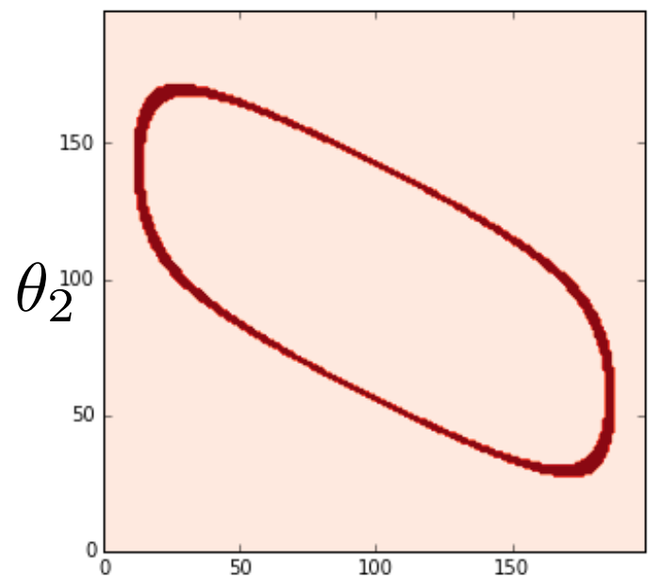
Forward kinematics

$f_1(\theta_1, \theta_2)$



Inverse kinematics

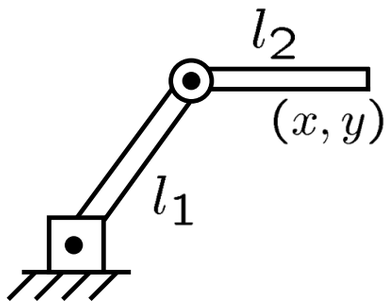
$(\theta_1, \theta_2)^T = f_1^{-1}(0.4)$



$f_2(\theta_1, \theta_2)$

$(\theta_1, \theta_2)^T = f_2^{-1}(1.2)$

Inverse kinematics



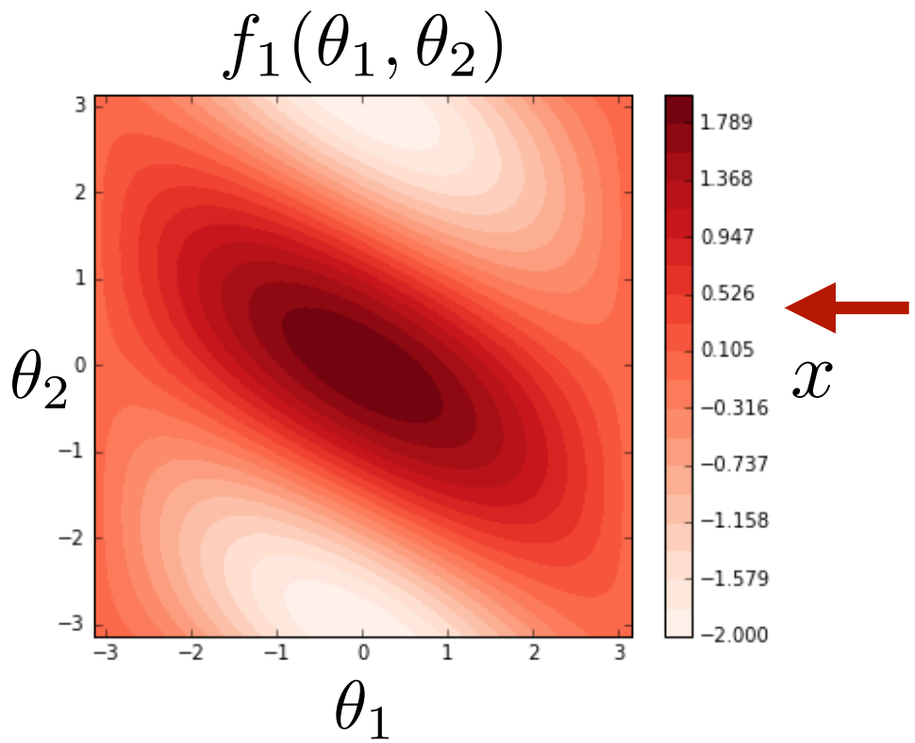
Forward kinematics

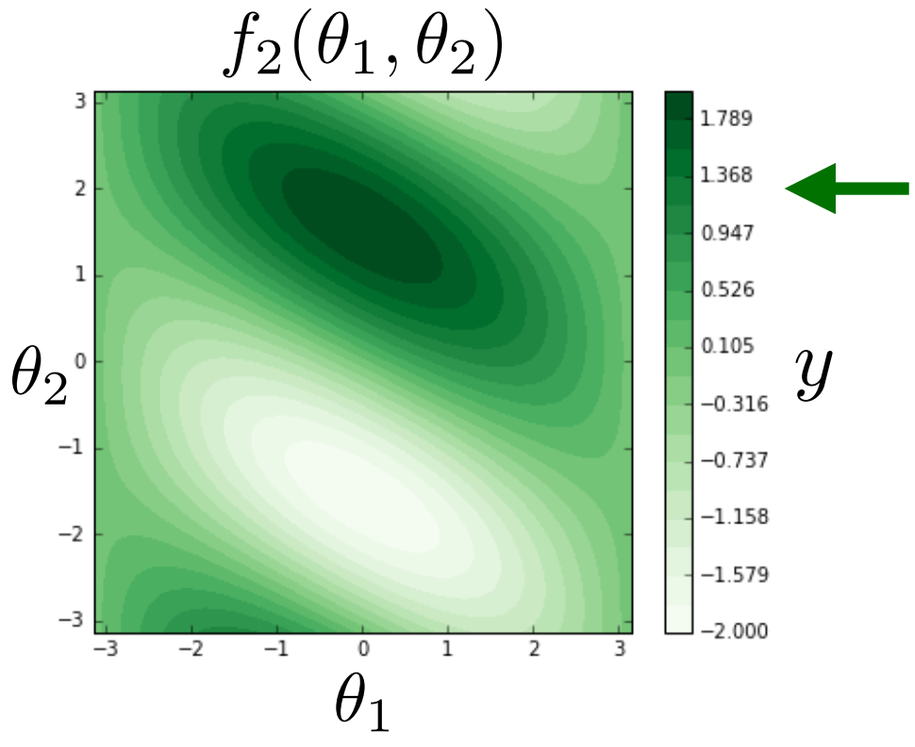
$$\mathbf{x} = f(\mathbf{q})$$

Inverse kinematics

$$\mathbf{q} = f^{-1}(\mathbf{x})$$

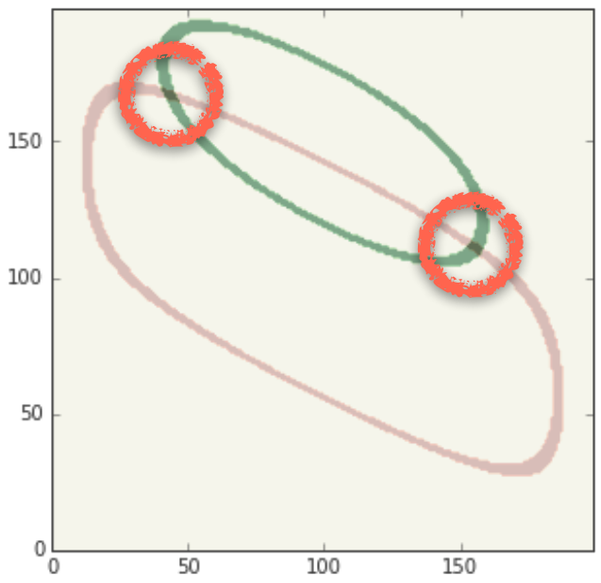
Forward kinematics





Inverse kinematics

$$(\theta_1, \theta_2)^T = f_1^{-1}(0.4)$$
$$(\theta_1, \theta_2)^T = f_2^{-1}(1.2)$$



Inverse kinematics

Forward kinematics

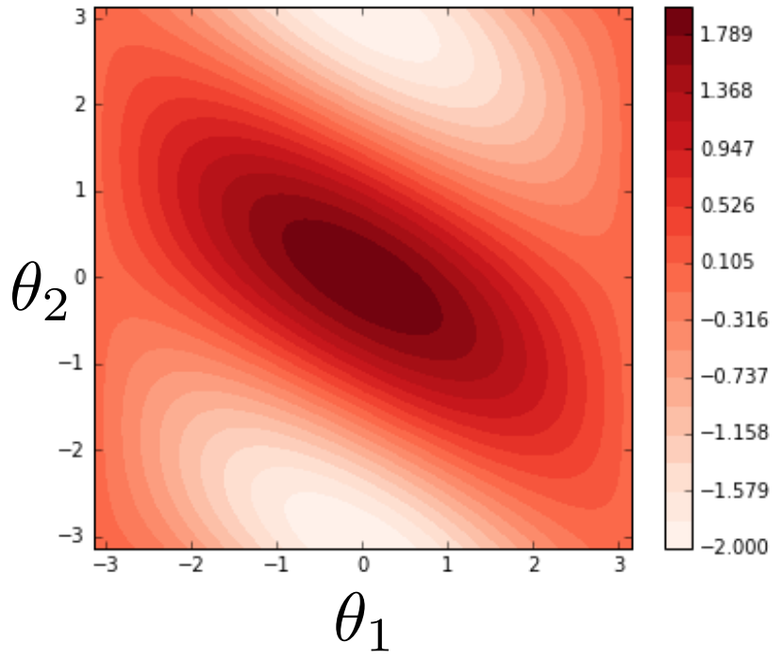
$$\mathbf{x} = f(\mathbf{q})$$

Inverse kinematics

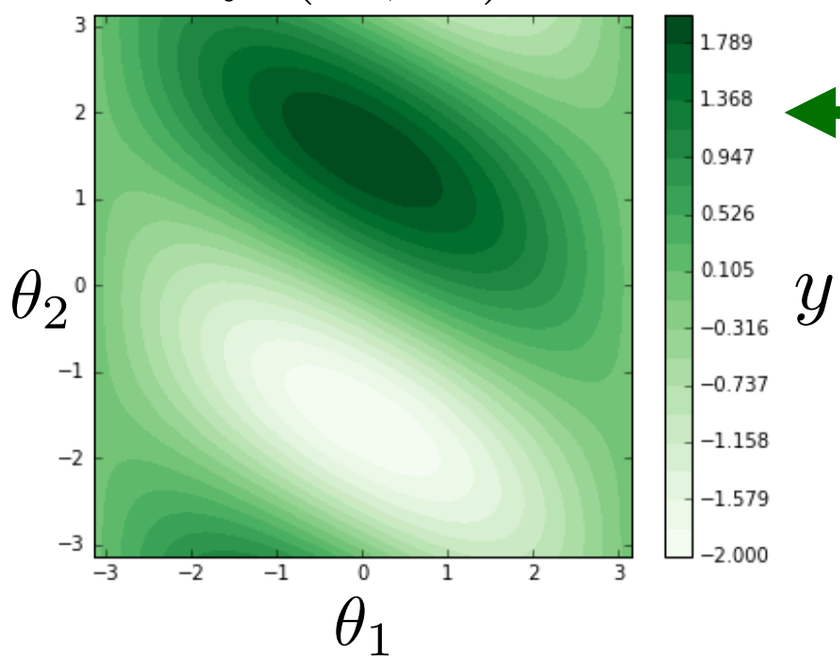
$$\mathbf{q} = f^{-1}(\mathbf{x})$$

Forward kinematics

$f_1(\theta_1, \theta_2)$

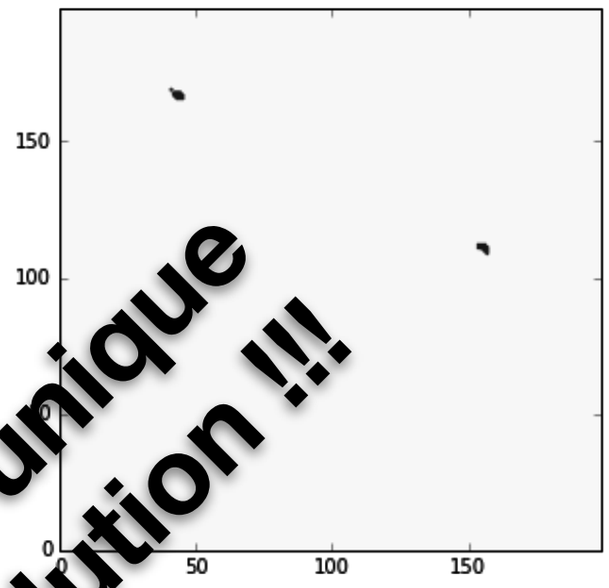


$f_2(\theta_1, \theta_2)$

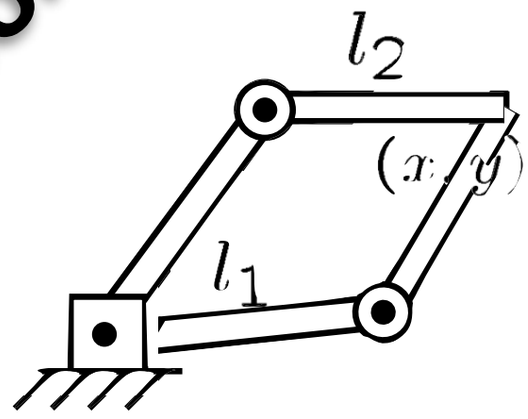


Inverse kinematics

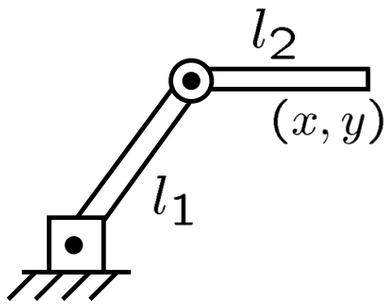
$(\theta_1, \theta_2)^T = f^{-1}((0.4, 1.2)^T)$



No unique solution !!!



Inverse kinematics



Forward kinematics

$$\mathbf{x} = f(\mathbf{q})$$

$$x = f_1(\mathbf{q}) = l_1 c_1 + l_2 c_{12}$$

$$y = f_2(\mathbf{q}) = l_1 s_1 + l_2 s_{12}$$

Inverse kinematics

$$\mathbf{q} = f^{-1}(\mathbf{x})$$

Analytical solution:

$$\theta_1 = \pm\cos^{-1}\left(\frac{\sqrt{-y^2\sin^2(x)+2\sin^2(x)+2\sin^2(x)\cos(x)}+y\cos(x)+y}{\sin^2(x)+\cos^2(x)+2\cos(x)+1}\right)$$

$$\pm\cos^{-1}\left(\frac{-\sqrt{-y^2\sin^2(x)+2\sin^2(x)+2\sin^2(x)\cos(x)}+y\cos(x)+y}{\sin^2(x)+\cos^2(x)+2\cos(x)+1}\right)$$

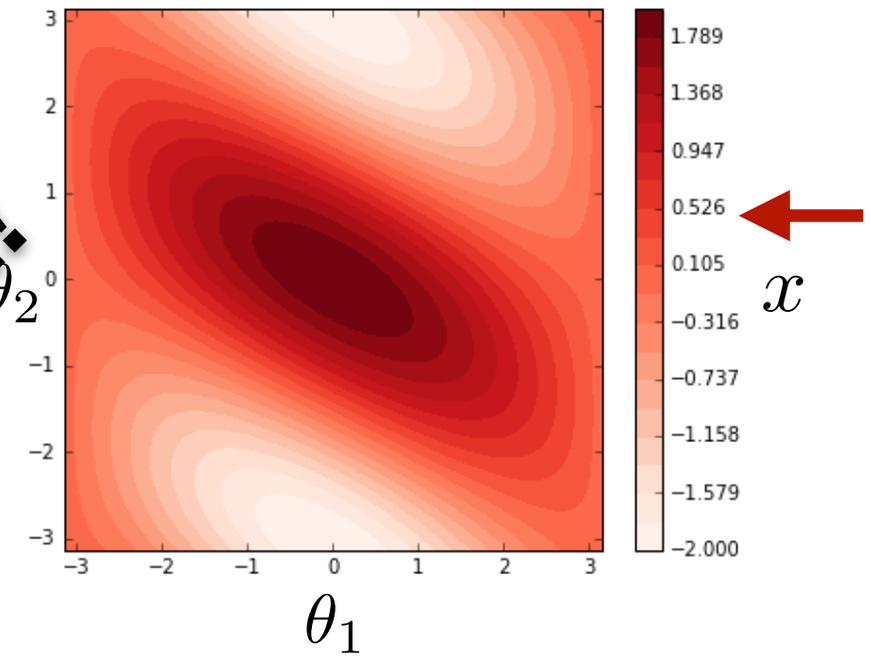
$$\theta_2 = \pm\cos^{-1}\left(\frac{y\sin(x)-2\sqrt{-y^2\cos^4\left(\frac{x}{2}\right)+2\cos(x)\cos^4\left(\frac{x}{2}\right)+2\cos^4\left(\frac{x}{2}\right)}}{\sin^2(x)+\cos^2(x)+2\cos(x)+1}\right)$$

$$\pm\cos^{-1}\left(\frac{y\sin(x)-2\sqrt{-y^2\cos^4\left(\frac{x}{2}\right)+2\cos(x)\cos^4\left(\frac{x}{2}\right)+2\cos^4\left(\frac{x}{2}\right)}}{\sin^2(x)+\cos^2(x)+2\cos(x)+1}\right)$$

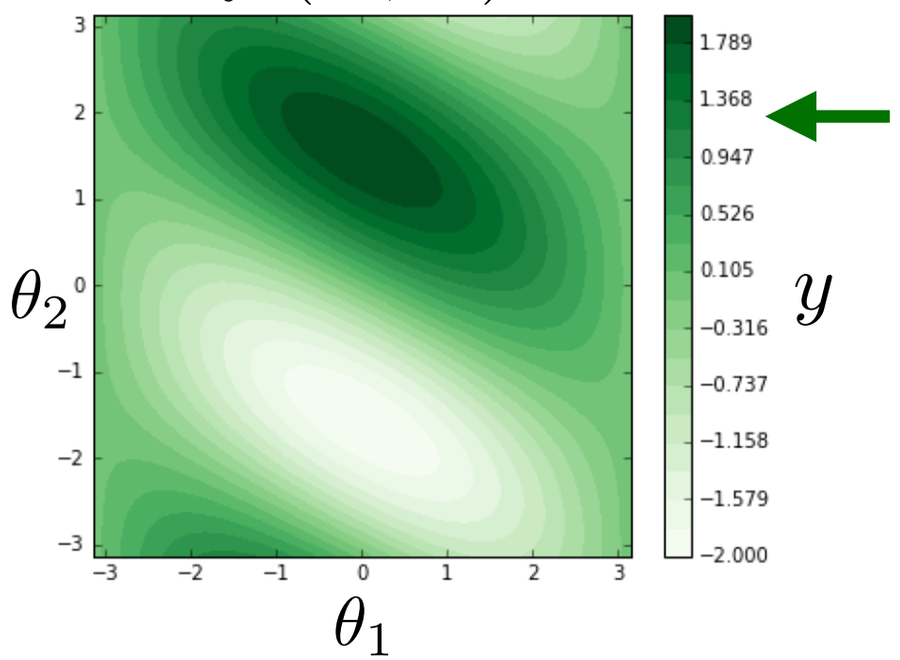
Super complicated !!!

Forward kinematics

$f_1(\theta_1, \theta_2)$



$f_2(\theta_1, \theta_2)$

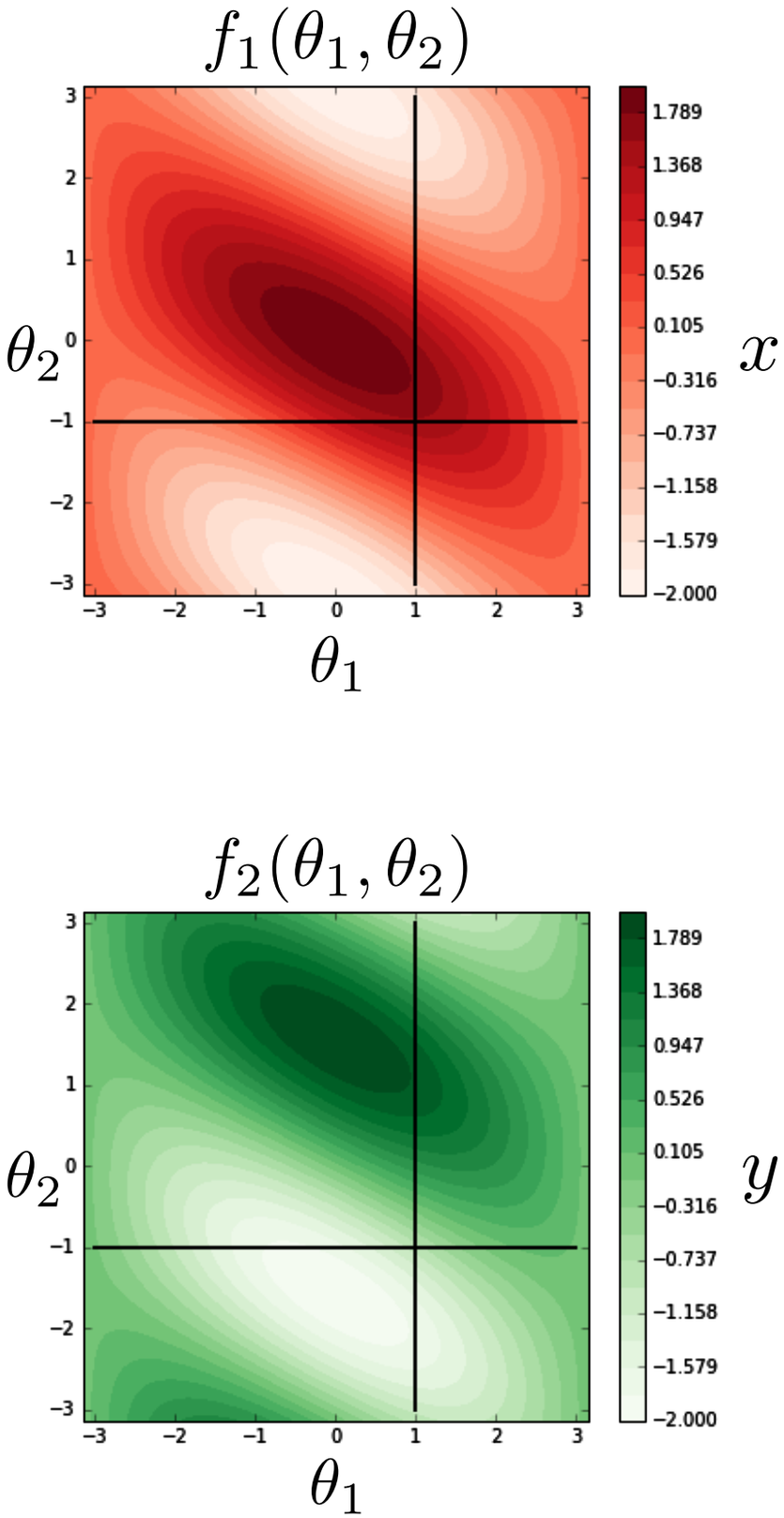


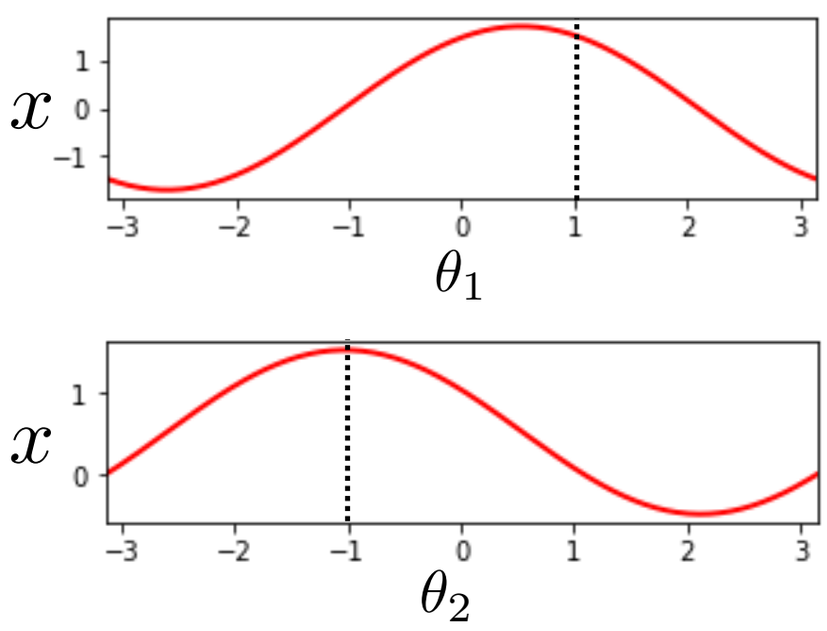
Help please!

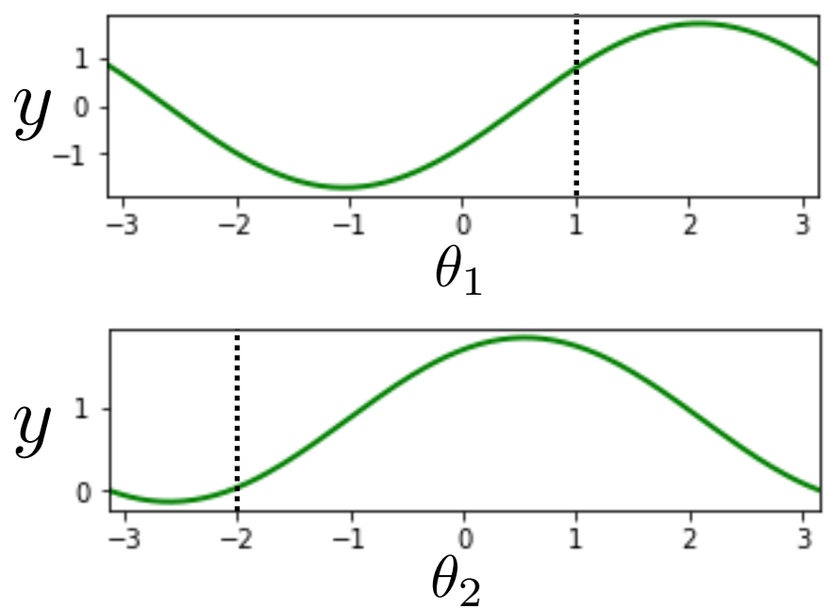
Jacobian

Jacobian

Forward kinematics

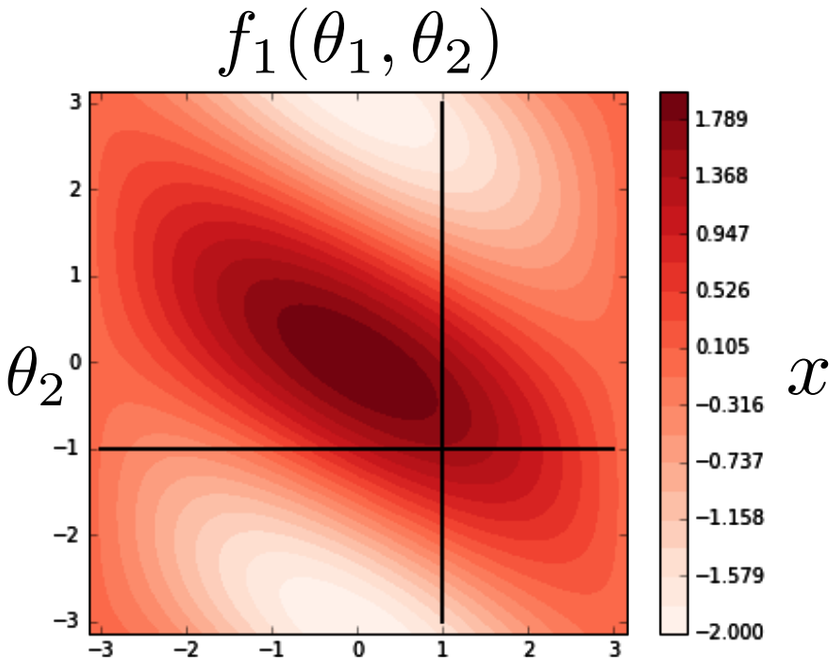


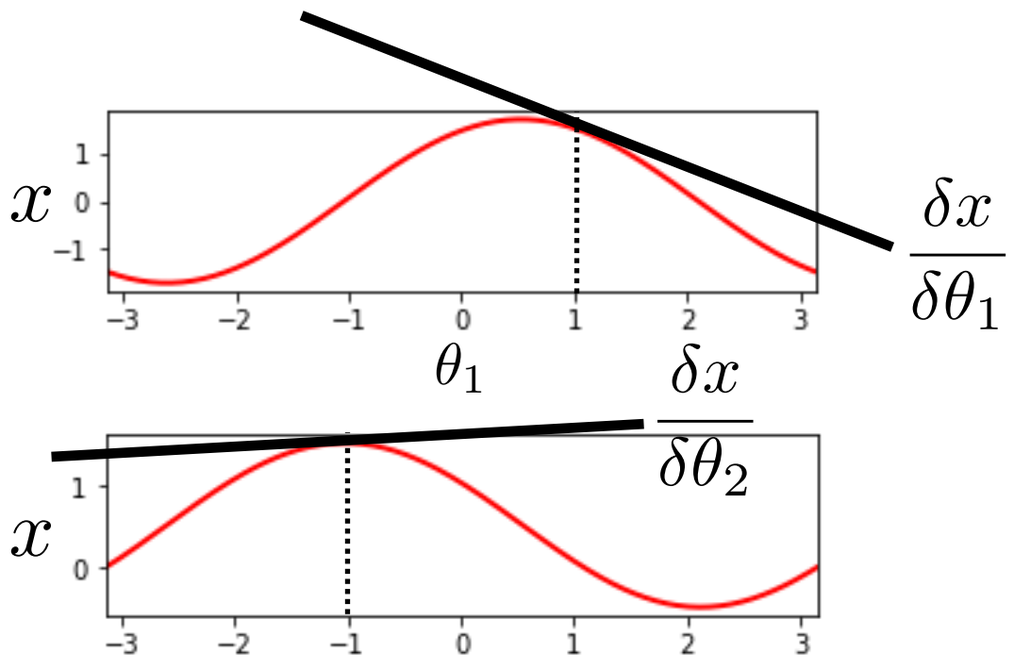


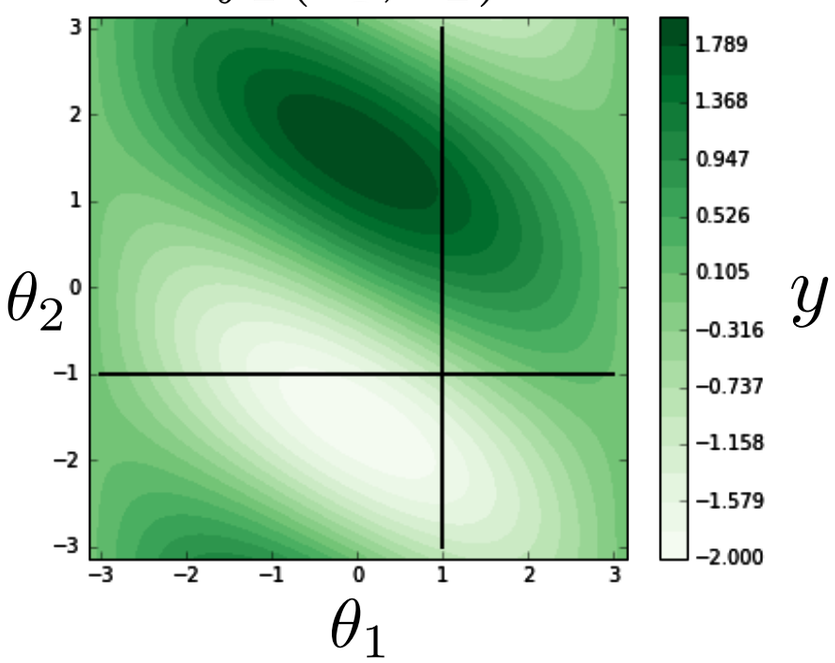


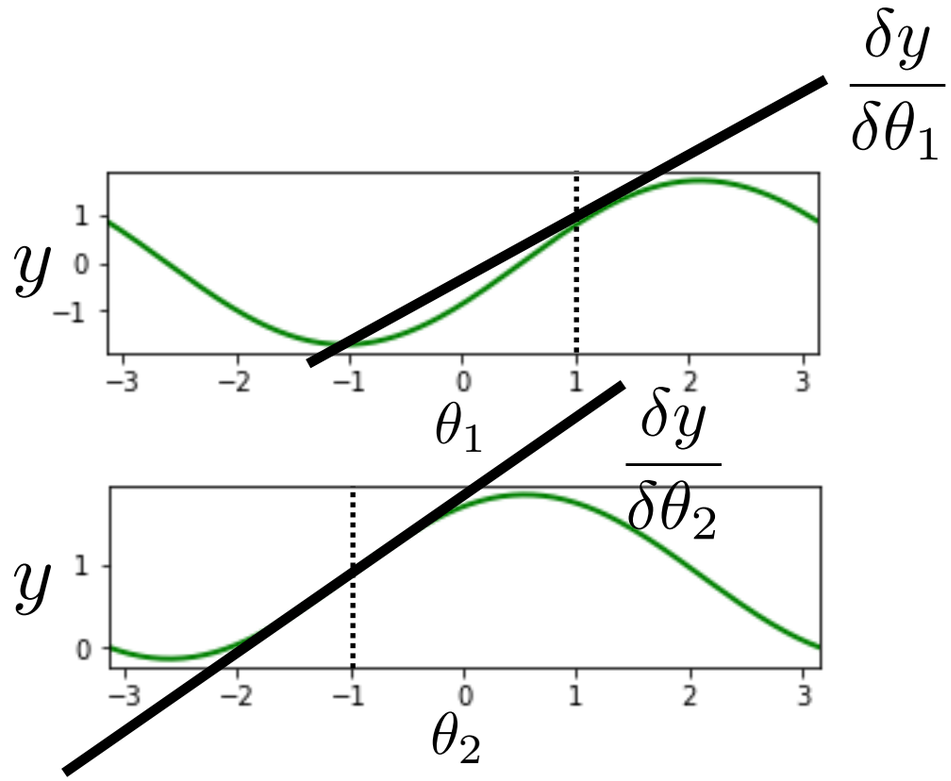
Jacobian

Forward kinematics



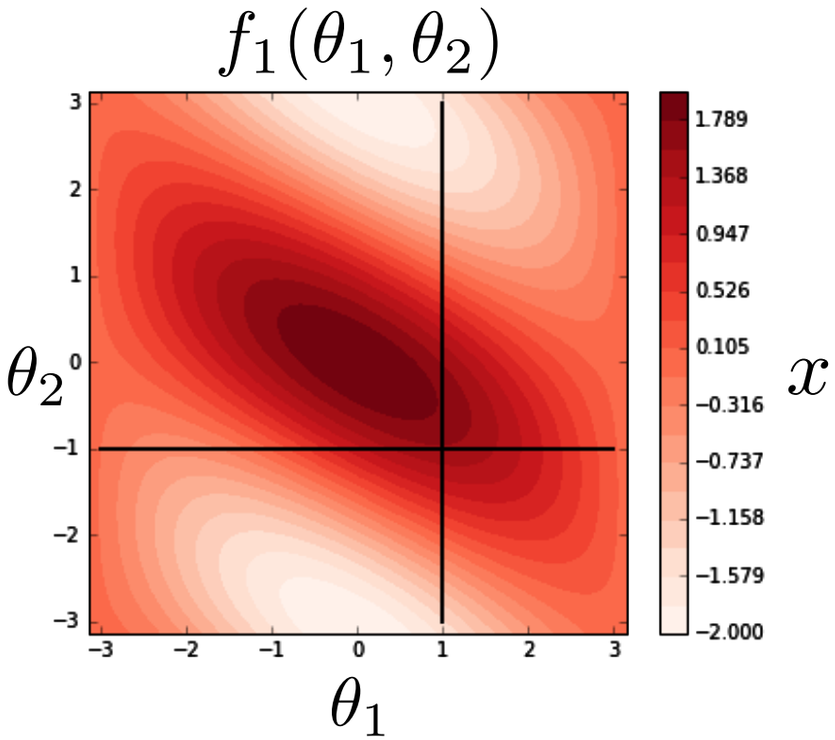


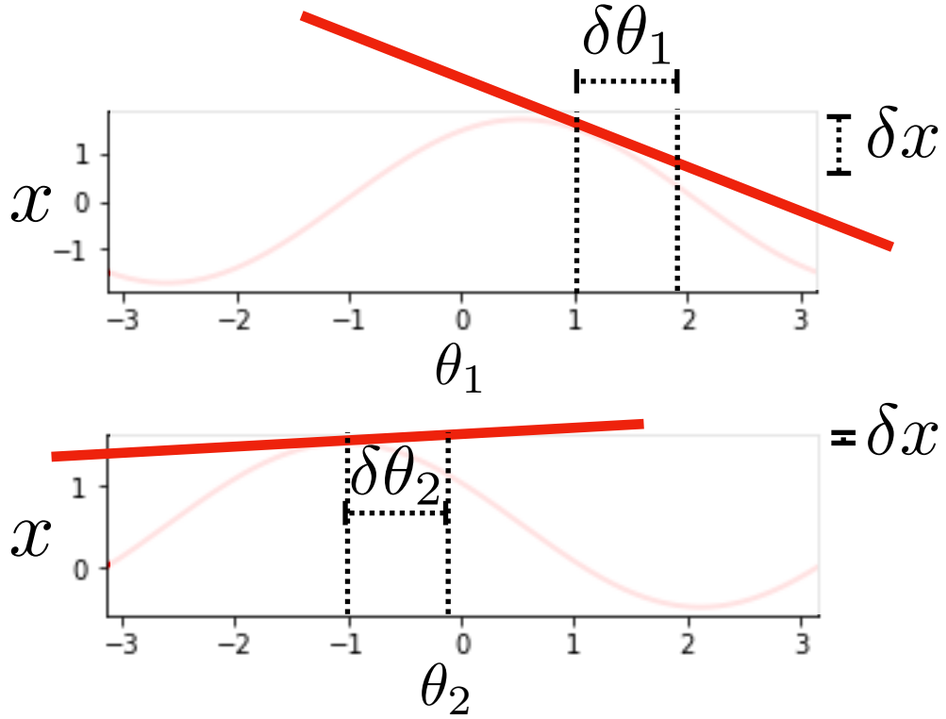




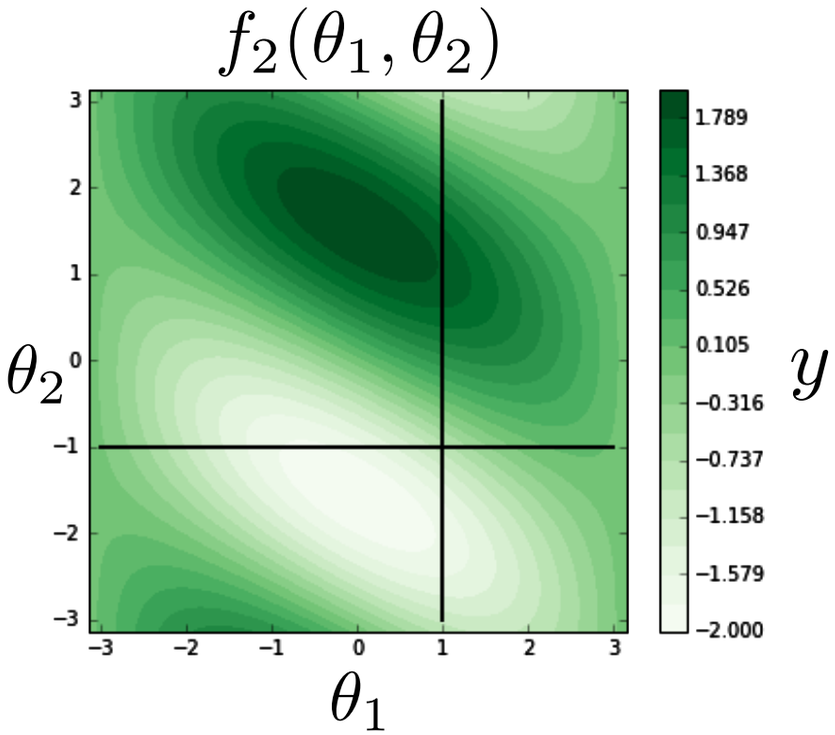
Jacobian

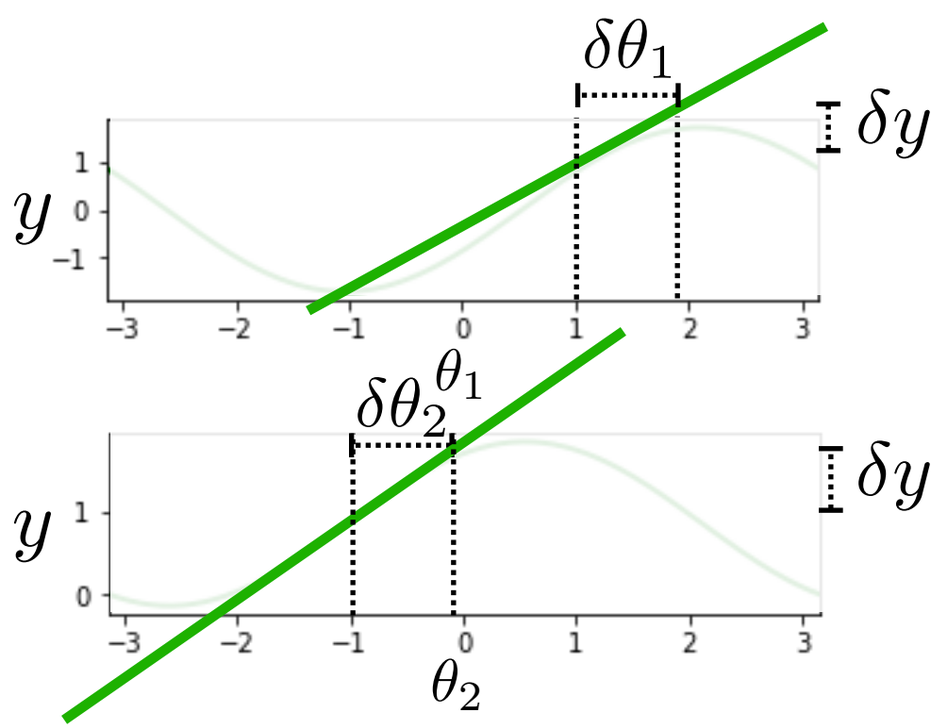
Forward kinematics





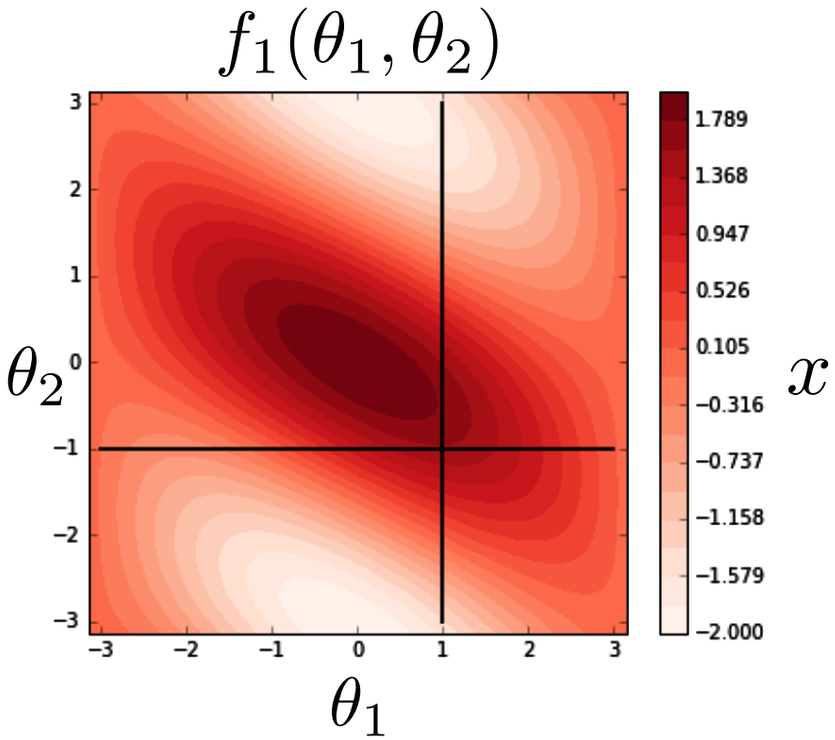
$$J = \begin{bmatrix} \frac{\delta x}{\delta \theta_1} & \frac{\delta x}{\delta \theta_2} \\ \frac{\delta y}{\delta \theta_1} & \frac{\delta y}{\delta \theta_2} \end{bmatrix}$$

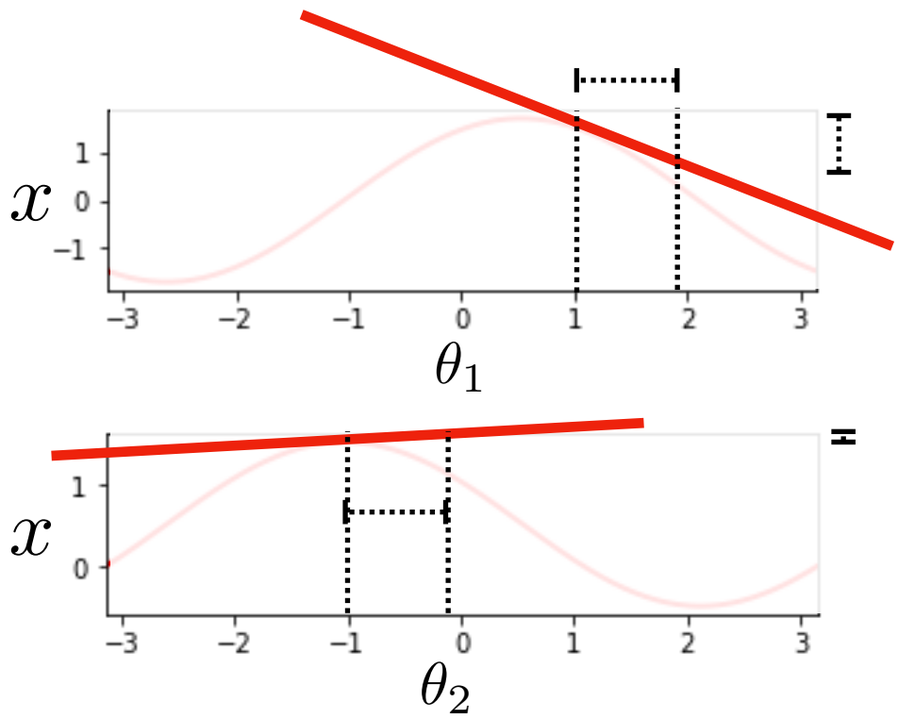




Jacobian

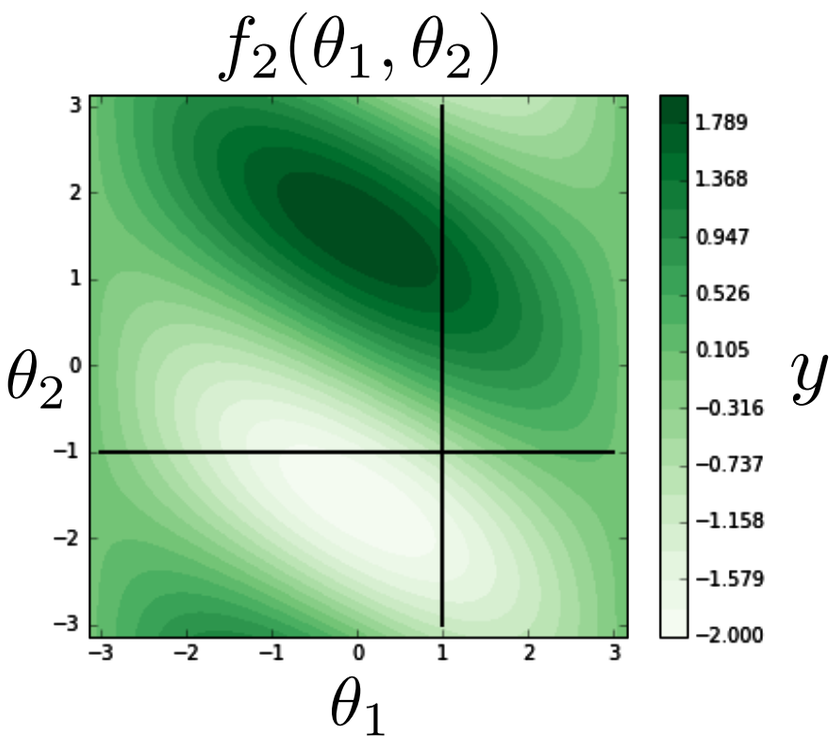
Forward kinematics

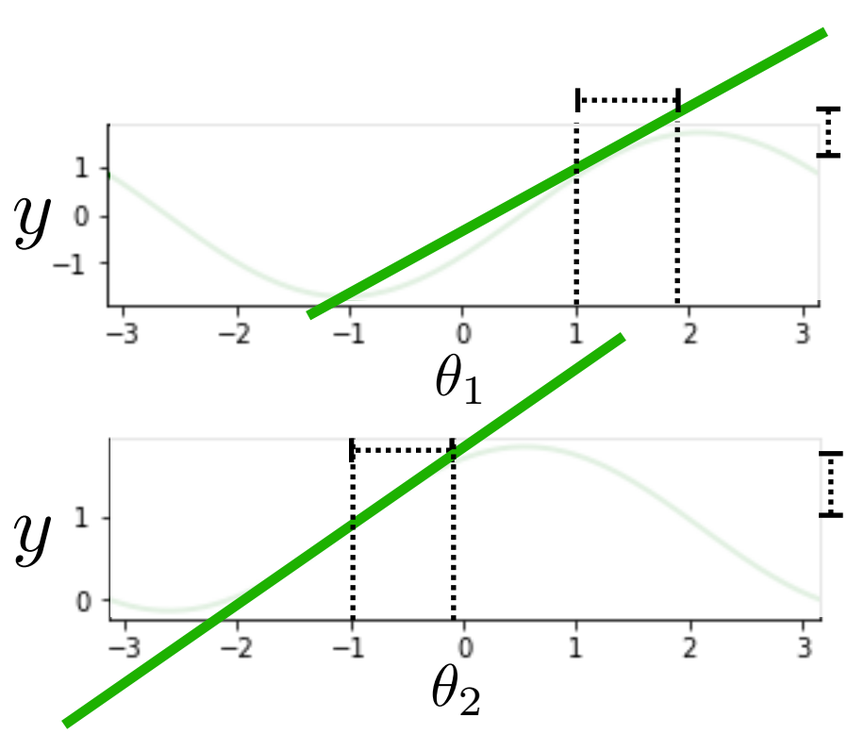




$$\begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} \frac{\delta x}{\delta \theta_1} & \frac{\delta x}{\delta \theta_2} \\ \frac{\delta y}{\delta \theta_1} & \frac{\delta y}{\delta \theta_2} \end{bmatrix} \begin{bmatrix} \delta \theta_1 \\ \delta \theta_2 \end{bmatrix}$$

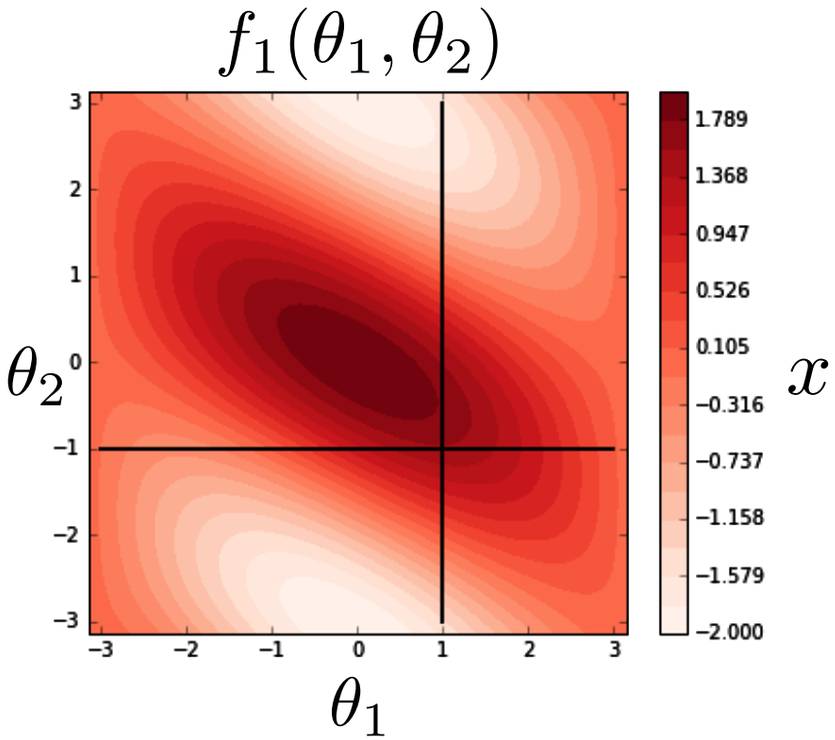
$\delta \mathrm{x}$ J δq

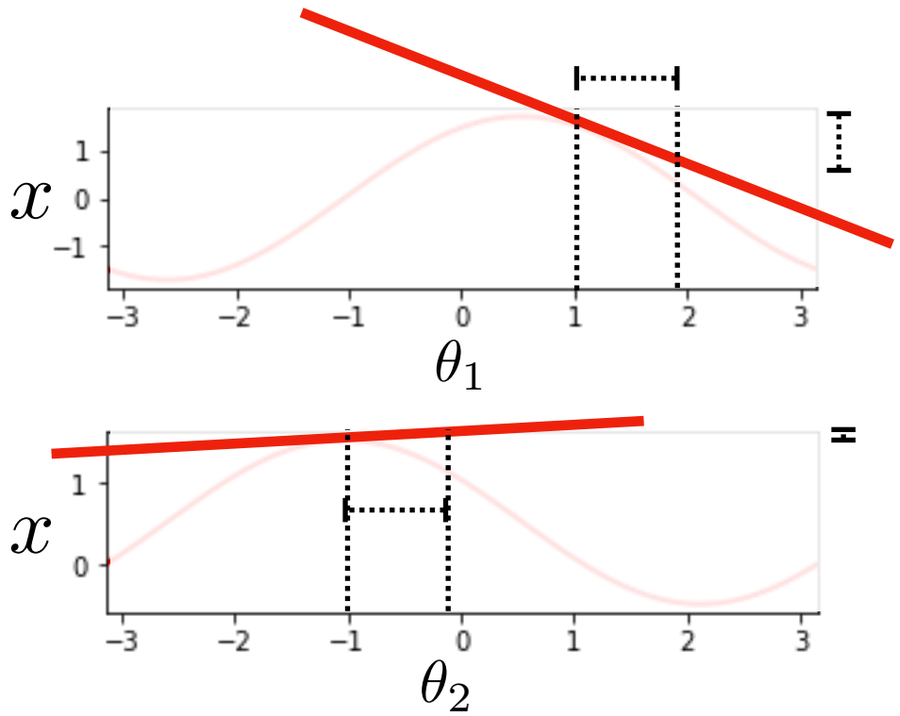




Jacobian

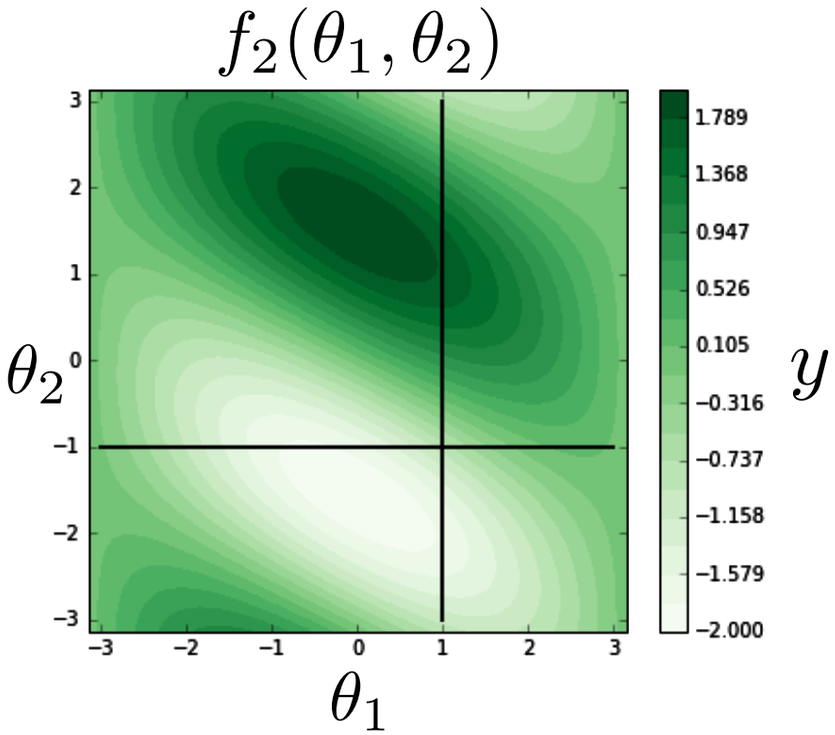
Forward kinematics

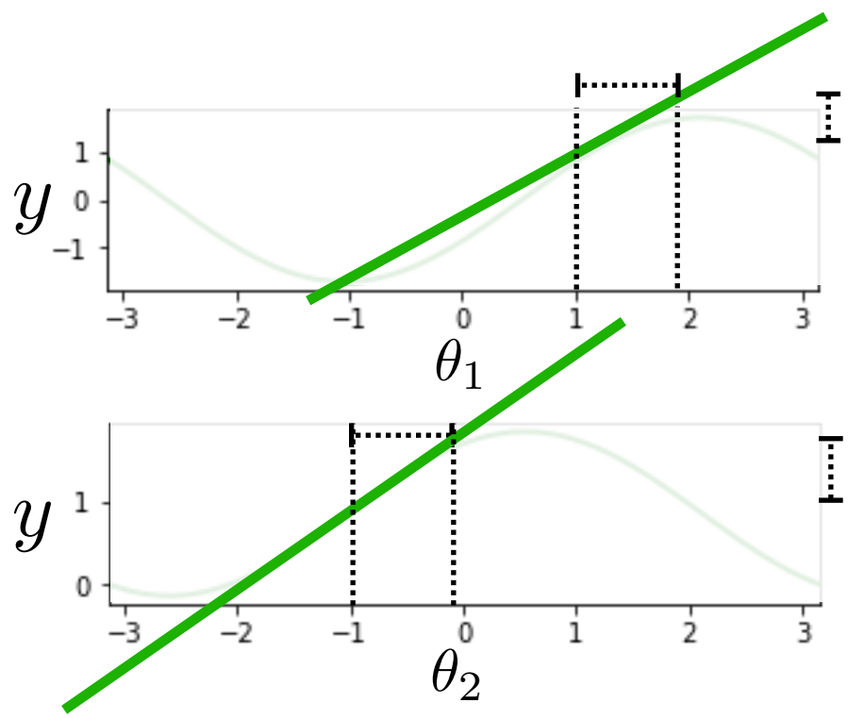




$$\begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} \frac{\delta x}{\delta \theta_1} & \frac{\delta x}{\delta \theta_2} \\ \frac{\delta y}{\delta \theta_1} & \frac{\delta y}{\delta \theta_2} \end{bmatrix} \begin{bmatrix} \delta \theta_1 \\ \delta \theta_2 \end{bmatrix}$$

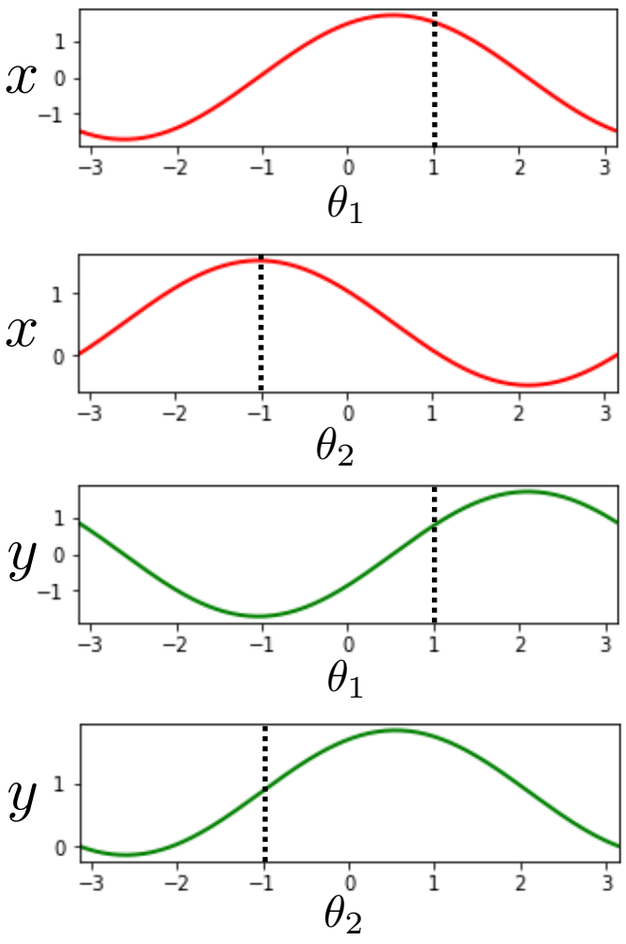
$\delta \mathrm{x}$ J δq





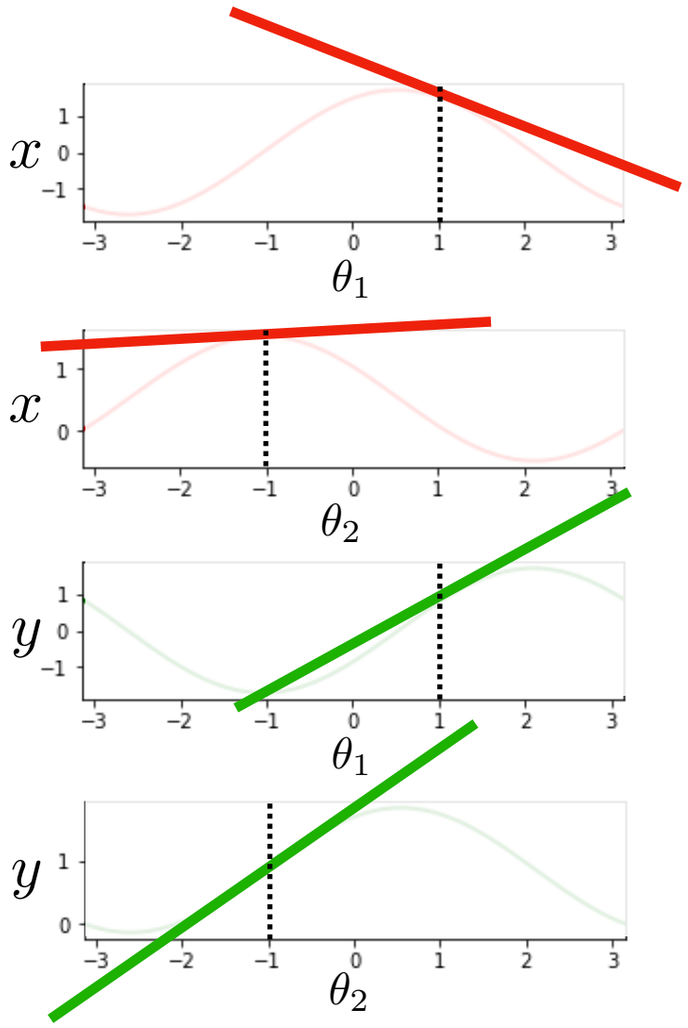
Easy to invert !!!

Forward kinematics vs. Jacobian



Linearize

Both are forward kinematic models !!!



Forward kinematics

$$\mathrm{x} = f(\mathrm{q})$$

Input & output: Absolute values !!!

Jacobian

$$\delta\mathrm{x} = J\ \delta\mathrm{q}$$

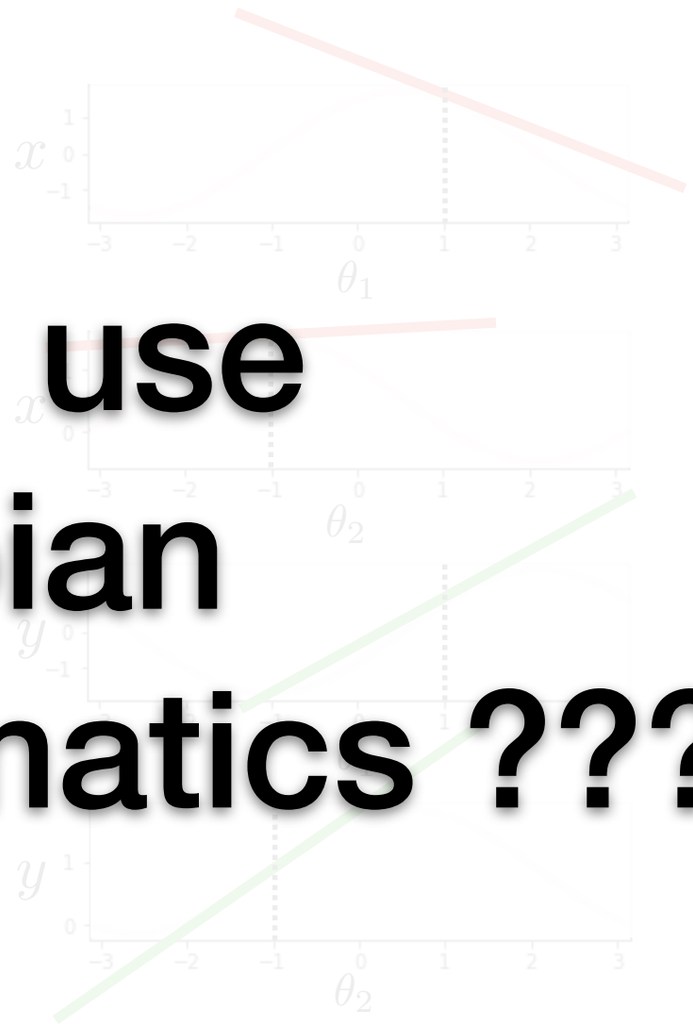
Input & output: Delta values !!!

Pro: We can compute x for given q directly
Con: Inverting f is super complicated

Con: Many steps required to reach x,q
Pro: Inverting J is super easy

Forward kinematics vs. Jacobian

Linearize



But how to use the Jacobian for inverse Kinematics ???

Forward kinematics

$$\mathrm{x} = f(\mathrm{q})$$

Input & output:
Absolute values !!!

Pro: We can compute x for given q directly
Con: Inverting f is super complicated

Jacobian

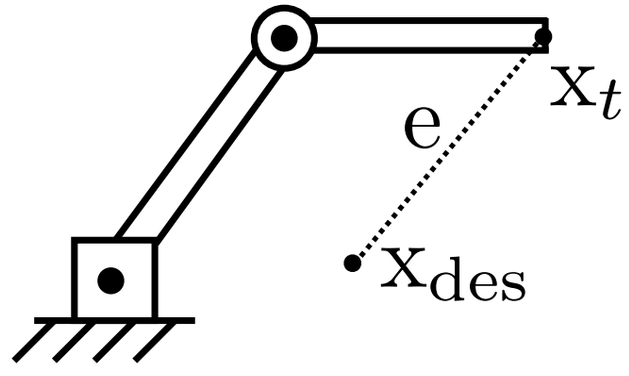
$$\delta\mathrm{x} = J\ \delta\mathrm{q}$$

Input & output:
Delta values !!!

Con: Many steps required to reach x,q
Pro: Inverting J is super easy

Inverted Jacobian for inverse kinematics

Algorithm for
P-Controller in operational space:



Repeat:

Compute $\mathrm{x}_t = f(\mathrm{q}_t)$ **(using forward model)**

Compute $\mathrm{e} = (\mathrm{x}_{\mathrm{des}} - \mathrm{x}_t)$ **(i.e. the error)**

Set $\delta \mathrm{x} = -k_p \mathrm{e}$ **(small step to reduce the error)**

Compute J **(using** q_t**)**

Compute J^{-1} **(much easier than inverting f)**

Compute $\delta \mathrm{q} = J^{-1}\ \delta_x$ **(this is the inverse kinematics part)**

Compute $\mathrm{q}_{t+1} = \mathrm{q}_t + \delta \mathrm{q}$ **(i.e. move robot)**

Let’s make it even more simple

Use Jacobian with forces

Work = Force * Distance

Work = Torque * Angle

$$\mathbf{F}^T \, \delta\mathbf{x} = \tau^T \delta\mathbf{q}$$

using $\delta\mathbf{x} = J(\mathbf{q}) \, \delta\mathbf{q}$

$$\mathbf{F}^T \, J\delta\mathbf{q} = \tau^T \delta\mathbf{q}$$

divide by $\delta\mathbf{q}$

$$\mathbf{F}^T \, J = \tau^T$$

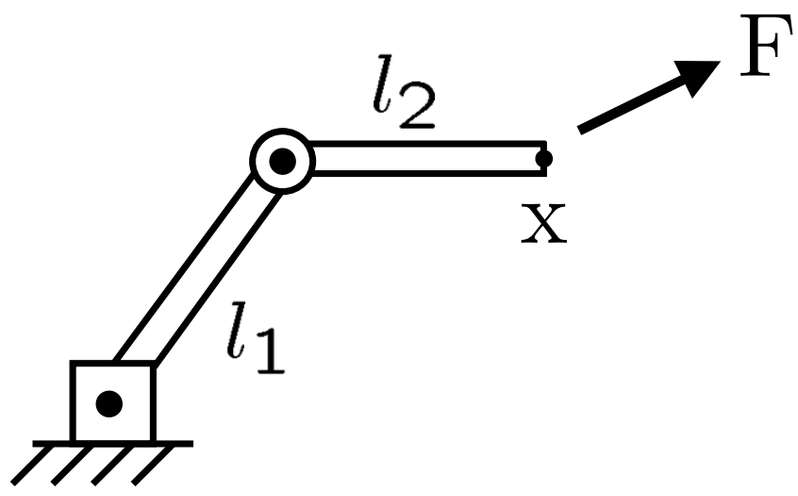
using $(A \; B)^T = B^T \; A^T$

$$\tau = J^T \mathbf{F}$$

Super simple !!!

Use Jacobian with forces

$$\tau = J^T \mathbf{F}$$



Build controller using forces!

Example:
P-Controller in operational space
with gravity compensation

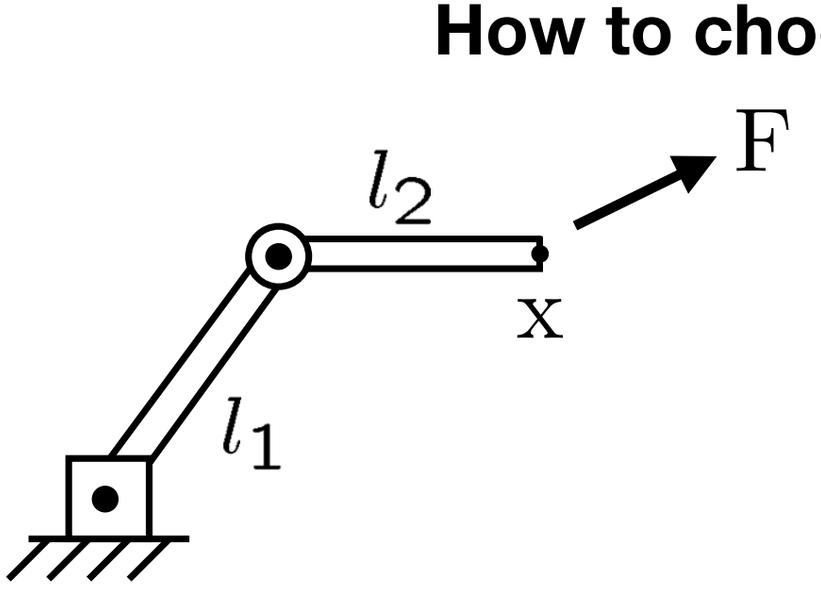
$$\tau = \mathbf{k}\, J^T \mathbf{F} + G(\mathbf{q})$$

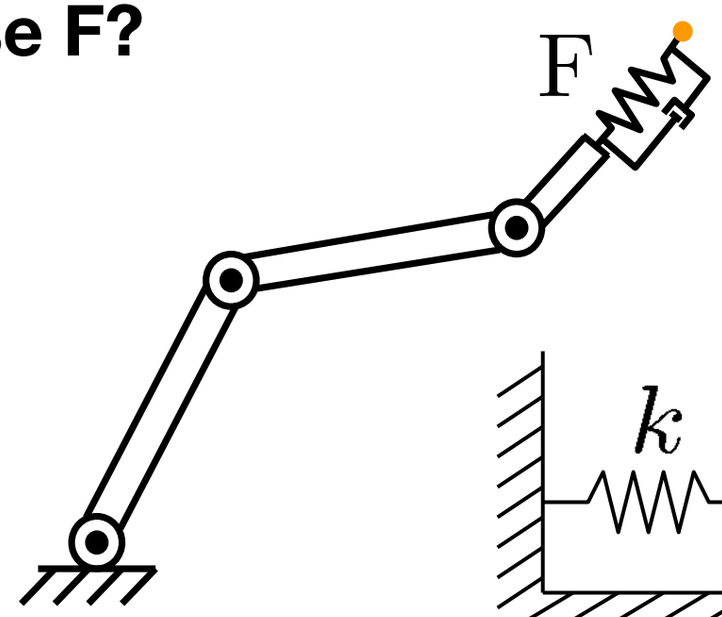
Super simple !!!

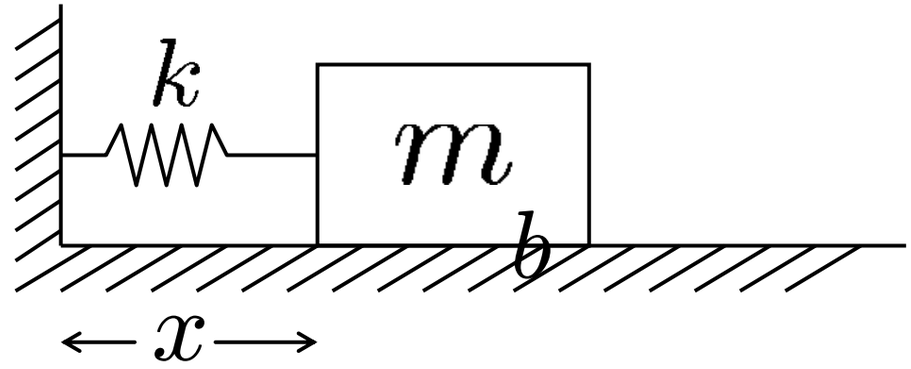
We will now use this!

Operational space control

How to choose F?



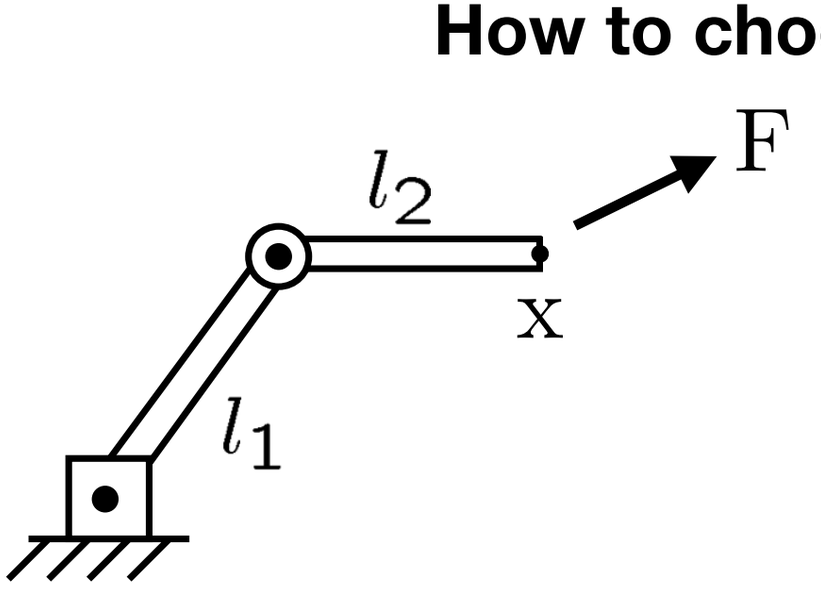


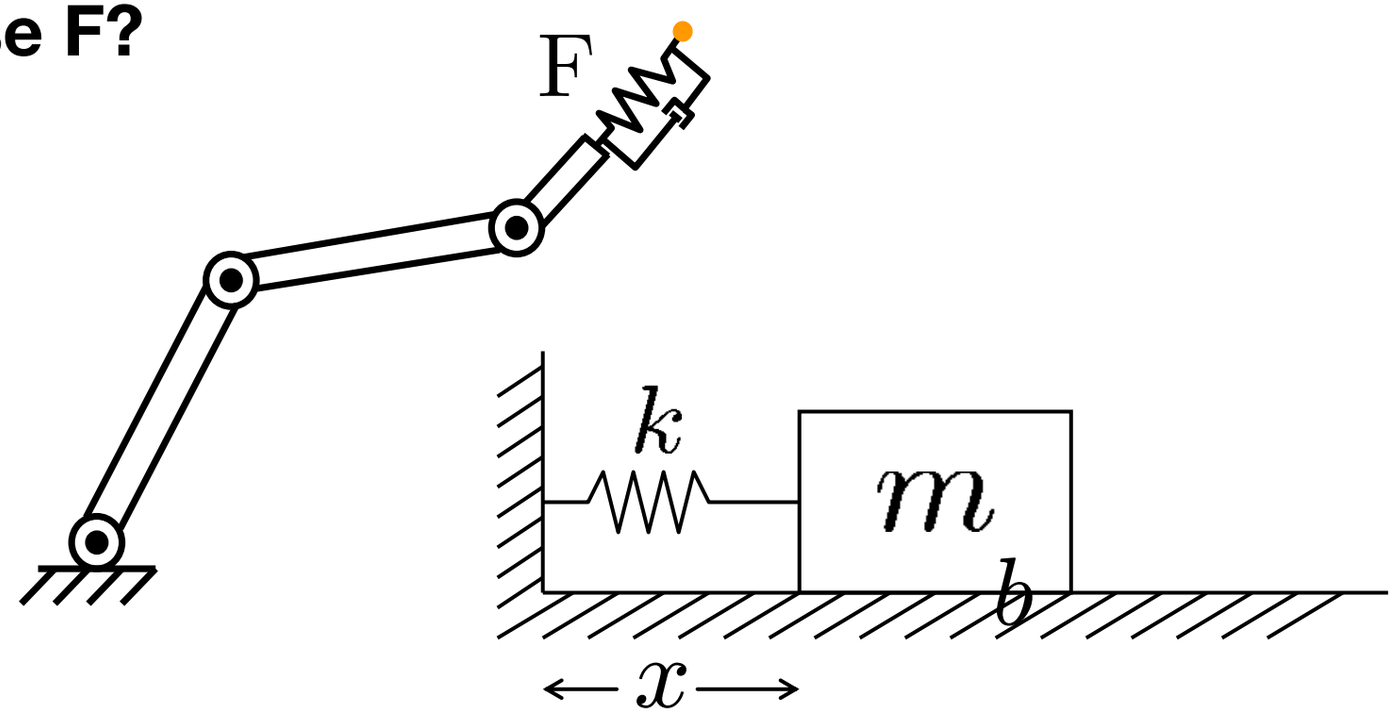


$$f = -k_p x - k_v \dot{x}$$

Operational space control

How to choose F?





$$f = -k_p x - k_v \dot{x}$$

$$f = m\ddot{x} + b\dot{x} + kx$$

Read the book!