

## Swimming

$$\begin{aligned}
 1. \quad ds &= \sum_{i=1}^n (v_i + v \cdot \sin a_i) \cdot t_i = \sum_{i=1}^n (v_i + v \cdot \sin a_i) \cdot \frac{s_i}{v \cdot \cos a_i} \\
 &= \sum_{i=1}^n \left( s_i \cdot \tan a_i + \frac{v_i s_i}{v \cdot \cos a_i} \right) \\
 2. \quad \sum_{i=1}^n t_i &= \sum_{i=1}^n \frac{s_i}{v \cdot \cos a_i} = T \\
 3. \quad a_i &\in [0, \frac{\pi}{2})
 \end{aligned}$$

目标:  $\{a_i\} = \arg \max_a ds(a_i), i = 1, \dots, n$

$$L = ds + \lambda \left( \sum_{i=1}^n \frac{s_i}{v \cdot \cos a_i} - T \right)$$

极值点:

$$\circ \quad \nabla_a L = 0$$

对于  $a_i$

$$s_i \cdot \sec^2 a_i \cdot \left( 1 + \frac{v_i}{v} \sin a_i \right) + \frac{\lambda s_i}{v} \sec^2 a_i = 0$$

$$1 + \frac{v_i}{v} \sin a_i - \frac{\lambda}{v} = 0$$

$$a_i = \arcsin \left( \frac{\lambda - v}{v_i} \right)$$

$$\circ \quad \text{代入(2)式计算} \lambda, \text{从而得到} a_i$$