Table A.1-1: P-values between the performance of DRVML vs. other methods on each dataset as calculated via Welch's two-sided t-test. Bold numbers denote higher performance by the proposed method. Values calculated on results shown in Table 3.

|              | KNN-EUC | LMNN  | RVML-classes | RVML-transport |
|--------------|---------|-------|--------------|----------------|
| Balance      | .0147   | .2117 | .0259        | .0424          |
| Credit       | .1340   | .4947 | .0341        | .2474          |
| Digits       | .7983   | .7751 | .1789        | .0477          |
| German       | .3481   | .2671 | .9573        | .7982          |
| Mice         | .0074   | .6304 | .0282        | .0023          |
| Segmentation | .0007   | .4423 | .1460        | .0012          |
| Wisconsin    | .0866   | .0070 | .4467        | .4752          |

## A.2 Visualization

Results of the visualization experiment are shown as Figure A.2-1.

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