2021 IEEE International Conference on Robotics and Automation (ICRA 2021)
May 31 - June 4, 2021, Xi'an, China

Bias Compensated UWB Anchor Initialization using Information-Theoretic Supported Triangulation Points

Julian Blueml[1], Alessandro Fornasier[1], Stephan Weiss[1]

Abstract— **For Ultra-Wide-Band (UWB) based navigation, an accurate initialization of the anchors in a reference coordinate system is crucial for precise subsequent UWB-inertial based pose estimation. This paper presents a strategy based on information theory to initialize such UWB anchors using raw distance measurements from tag to anchor(s) and aerial vehicle poses. We include a linear distance-dependent bias term and an offset in our estimation process in order to achieve unprecedented accuracy in the 3D position estimates of the anchors (error reduction by a factor of about 3.5 compared to current approaches) without the need of prior knowledge. After an initial coarse position triangulation of the anchors using random vehicle positions, a bounding volume is created in the vicinity of the roughly estimated anchor position. In this volume, we calculate points which provide the maximal triangulation related information based on the Fisher Information Theory. Using these information theoretic optimal points, a fine triangulation is done including bias term estimation. We evaluate our approach in simulations with realistic sensor noise as well as with real world experiments. We also fly an aerial vehicle with UWB-inertial based closed loop control demonstrating that precise anchor initialization does improve navigation precision. Our initialization approach is compared to state-of-the-art as well as to an initialization without the simultaneous bias estimation.**

I. Introduction

For UAV localization, often a Global Navigation Satellite System (GNSS) is used. But in areas where there is no GNSS signal available, e.g. forest or indoor locations, some other form of localization provider needs to be available. This localization provider can for example be a set of UWB modules. UWB is a communication technique which operates in the RF (radio frequency) spectrum and as the name implies, it operates on a large band of frequency. This results in much more precise and less error prone distance measurement than other e.g. ultrasonic based systems. The position of a mobile robot can be calculated in a similar fashion as it is done in GNSS systems. The position can be computed through trilateration using at least three UWB modules which are configured to be senders (also called anchors). Similarly to the GNSS satellites for an accurate estimation of the mobile robot, the positions of these anchors have to be known as accurately as possible. Often, this is measured manually but this can be very time consuming and inaccurate, especially in wide areas and with low-quality beacons, in buildings with a large number of rooms, or in areas where the anchors are hard to reach. Even though the location measurement is accurate, biases in the signals may introduce inaccuracies in the trilateration process.

[1]The authors are with the Control of Networked Systems Group, University of Klagenfurt, Austria `j1blueml@edu.aau.at`, `{firstname.lastname}@ieee.org`

This work is supported by the EU-H2020 project BUGWRIGHT2 (GA 871260) and the BMVIT project SCAMPI (GA 878661)

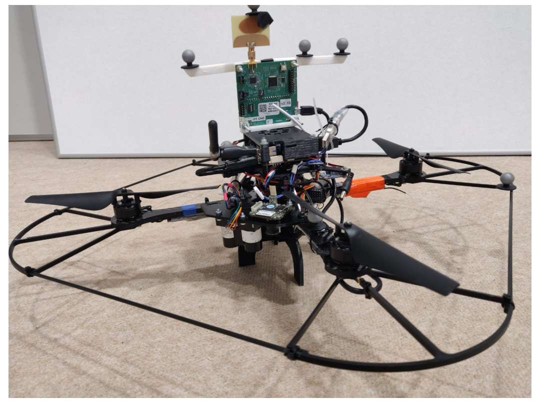

Fig. 1: UAV used for real world experiments with computation board and UWB node.

Thus, the here presented approach not only focuses on the precise anchor initialization without prior knowledge, but also on the estimation of bias terms in the raw signal. The goal is to place the anchors randomly in a room and the mobile robot, in our case a UAV, initializes the anchor positions automatically. In addition it calculates a linear bias model with a constant offset term for the distance dependent error of the UWB modules. The proposed initialization is a two stage process. First, the UAV navigates to some random points in space and records at each point a measurement to the UWB anchor which's position should be initialized. After sufficient points have been reached an initial guess of the position is performed. We leverage and extend the approach presented in [1] with a modified least squares approach to include the bias terms. The calculated position and corresponding covariance matrix is used to calculate an appropriate boundary volume which is used to construct optimal points using the Fisher Information Matrix (FIM). At these optimal points, the available information of the range-related UWB measurements is maximised to archive best trilateration results. With the information obtained by the mobile robot at these optimal points a final estimation of the position of the UWB anchor including the linear distance dependend bias and constant offset is calculated using the same modified least squares algorithm used for the trilateration from the random points.

II. Related work

In GPS denied environments e.g. inside buildings, range sensors are a popular choice for localization tasks. In [2] several different indoor positioning systems and their algorithms are examined. They found out that systems using infrared, ultrasonic sound or UWB signals have the best accuracy but infrared and ultrasonic sound suffer in non line of sight situations. With these signals the error increases while with UWB signals the accuracy stays approximately the same even in non line of sight conditions. The authors of [3] propose a UWB-IMU pose estimation system. The system assumes known, fix UWB anchor positions and is reliable under multipath effects and non line of sight conditions. Ledergerber et al. [4] presented a localization system using UWB transceivers with known positions for robot localization. The system is also able to handle multiple robots simultaneously.

There is a large body of work in the area of calibrating (or initializing) positioning systems. The position of the anchors have to be known as exact as possible to reduce the localization error. Usually calibration is done manually by measuring the exact position of the UWB anchors but since this is an error prone and time-consuming procedure and also not suitable in some scenarios we want to avoid it whenever possible. Hol et al. [5] proposed a calibration method for UWB receivers for indoor positioning. First multiple UWB receivers are placed to stationary places. The same number of transmitters are placed near to the receivers. They acquire a dataset for this configuration. On this dataset a nonlinear optimization is performed. Then a transmitter is moved around the receivers and another dataset was recorded. A second nonlinear optimization was done on the second dataset with the positions obtained from the first optimization as initial values.

Another approach to perform anchor initialization is described in [6]. The goal of this paper is to provide a initialization method for dynamic anchor setups. Range only measurements are performed between mobile tag and fied anchors. The proposed approach is apparently very robust against multipath propagation because a RANSAC based outlier rejection is used before the position candidate is further refined by an Unscented Kalman Filter (UKF).

Another way to auto calibrate UWB anchors is to use range information from a receiver and estimate the position of the anchors. Therefore, the range-related information of the anchors is maximized. For maximizing information a popular tool is the FIM or its inverse which corresponds to the Cramer-Rao Lower Bound (CRLB). Cardinali et al. [7] used the Cramer-Rao Lower Bound on different UWB signals to obtain the ranging accuracy of these signals. The authors of [8] proposed an algorithm for optimal sensor placement in 2D. By maximizing the FIM the optimal sensor positions can be obtained in order to get the position of the signal transmitters.

In our work, we extend the approach of [8] to 3D and flip the problem set to determine the optimal positions of the moving module to gather most information for the triangulation of the fixed module(s).

In [6], the authors provide an initialization method for dynamic anchor setups using only the range measurements from the UWB modules. The authors apply a cascade containing an outlier removal step through RANSAC with a subsequent filtering process based on an Unscented Kalman Filter (UKF). The double use of the same information in the RANSAC and UKF step may lead to inconsistencies. In addition, the selected positions for triangulation are on a fix grid pattern and not chosen based on their information content.

With respect to the state of the art, we improve the initialization of the anchors' position in 3D and include signal bias terms to additionally improve subsequent state estimators on mobile systems using the UWB anchors as positioning system. In particular our contributions are as follows:

- the extension from 2D to 3D space and flipping of FIM/CRLB based optimal sensor placement methods [8] for range sensing modules .
- FIM/CRLB definition for the problem set with extended covariance models including distance dependency, bias terms, and correlation between measurement positions.
- the extension to initialize several UWB anchors in real-time with low computational complexity and improved models including distance dependent bias and offset terms without any prior knowledge.
- a detailed evaluation based on verified simulations and realistic real experiments including a comparison (and improvement) to a state of the art approach.
- an evaluation of the effect of the anchor initialization-precision on the navigation precision when three UWB anchors are used for on-board real-time UWB-inertial positioning control of a UAV.

III. UWB ANCHOR INITIALIZATION PROCESS

A. Coarse initial position computation

Over the entire initialization process to compute the UWB anchor positions in a 3D reference frame, we assume the mobile robot, in our case a UAV, can estimate its own pose in the 3D reference frame through other sensor modalities (e.g. vision based, with GNSS signals, laser, etc). In our real world examples, we use an Optitrack motion capture system.

To calculate the information content of a UAV position for best UWB anchor initialization based on the FIM, at least a rough estimate of the UWB anchor needs to be available. For this coarse initialization, we fly the UAV to random positions while gathering range measurements from the UWB node on the vehicle to the anchor we want to initialize. We extend the approach presented in [1] such that we can formulate a linear least squares as shown in the following even with our additional states including the distance dependent bias and constant offset. The distance from the node on the UAV to the anchor can be expressed as:

$$z^2 = (p-q)^2 = (p^2 - 2pq + q^2) \quad (1)$$

$$d_p^2 = p_x^2 + p_y^2 + p_z^2 \ , \ d_q^2 = q_x^2 + q_y^2 + q_z^2 \quad (2)$$

where $p = [p_x,\, p_y,\, p_z]^T$ describes the node position in the global frame, $q = [q_x,\, q_y,\, q_z]^T$ describe the position of the anchor, d_q the distance from the anchor to the origin of the global frame. z is the distance between node and anchor and d_p the distance from the node position to the origin of the world frame. Assuming known node (i.e. UAV) positions and no biases as done in [1], for each distance measurement between node and anchor we can then formulate a modified least squares problem as

$$\begin{bmatrix} 2p_{x(t_1)} & -2p_{y(t_1)} & -2p_{z(t_1)} & 1 \\ -2p_{x(t_2)} & -2p_{y(t_2)} & -2p_{z(t_2)} & 1 \\ \vdots & & \vdots & \\ -2p_{x(t_n)} & -2p_{y(t_n)} & -2p_{z(t_n)} & 1 \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ q_z \\ d_q^2 \end{bmatrix} = \begin{bmatrix} z_{(t_1)}^2 - d_{p(t_1)}^2 \\ z_{(t_2)}^2 - d_{p(t_2)}^2 \\ \vdots \\ z_{(t_3)}^2 - d_{p(t_3)}^2 \end{bmatrix} \tag{3}$$

which is a set of linear equations in the form of $Ax = b$ where the rows of A are a measurement at time t_i. This can be solved for the anchor position q. Although UWB sensors are said to be fairly robust against multi-path issues, they show in practice a non-negligible distance dependent bias and constant offset depending on the manufacturer. To increase the accuracy of the triangulation results, we extend the above distance model of Eq.(1) with a distance dependent bias β and a constant offset γ to better reflect the actually measured distance z_m

$$z_m = \beta z + \gamma \tag{4}$$

Following the idea in [1], we design two additional auxiliary elements β^2 and γ, and modify the previous distance term d_q^2 in Eq.(3) to include the new bias terms

$$\begin{bmatrix} -2p_{x(t_1)} & \dots & -2p_{x(t_n)} \\ -2p_{y(t_1} & \dots & -2p_{y(t_n} \\ -2p_{z(t_1)} & \dots & -2p_{z(t_n)} \\ d_{p(t_1)}^2 & \dots & d_{p(t_n)}^2 \\ 2z_{(t_1)} & \dots & 2z_{(t_n)} \\ 1 & \dots & 1 \end{bmatrix}^T \begin{bmatrix} \beta^2 q_x \\ \beta^2 q_y \\ \beta^2 q_z \\ \beta^2 \\ \gamma \\ \beta^2(d_q^2 - \gamma^2) \end{bmatrix} = \begin{bmatrix} z_{t_1}^2 \\ z_{t_2}^2 \\ \vdots \\ z_{t_n}^2 \end{bmatrix} \tag{5}$$

solving this linear set of equation in the form of $Ax = b$ allows then to solve for the anchor position q and the two bias terms β and γ. The entries of Eq.(5) are based on the randomly chosen UAV positions. In practice, this system of equations is usually not well posed yielding poor solutions. Nevertheless, the coarse direction and distance can be inferred as an initial guess to apply our information theoretic approach for optimal UAV position selection in a refinement step as detailed below.

B. FIM based optimal points calculation

The goal is to find the optimal positions where the UAV (i.e. the UWB node) has to be placed in a limited volume to best triangulate a fix UWB anchor in the global coordinate frame. Until this anchor is triangulated, we assume the UAV position is known within a bounded volume (e.g. through fusion of IMU and a visual fiducial in the volume where the fiducial is in the field of view, a traking system, an area where GNSS signals are available, etc). In order to find the optimal sensor placement, the corresponding Cramer-Rao Lower Bound (CRLB) or FIM is considered [9]. The CRLB expresses a lower bound on the variance of estimators of a deterministic parameter. By achieving this bound the unbiased estimator is said to be (fully) efficient. The FIM on the other hand captures the amount of information from the obtained measured data of an unknown parameter which gets estimated. Under the regularity conditions the variance of any unbiased estimator is at least as high as the inverse of the FIM and the following inequality holds:

$$Cov\{\hat{\theta}\} \geq FIM(\theta)^{-1} = CRLB(\theta) \tag{6}$$

where θ is the variable of the estimation problem and where

$$Cov\{\hat{\theta}\} = E\left\{(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T\right\} \tag{7}$$

$Cov\{\hat{\theta}\}$ corresponds to the covariance matrix of the estimated parameters. In the following, $FIM(\theta)$ (abbreviated as FIM) is defined as

$$FIM(\theta) = E\left\{(\nabla_\theta log\ p_\theta(z))(\nabla_\theta log\ p_\theta(z))^T\right\} \tag{8}$$

where $\nabla_\theta log\ p_\theta(z)$ denotes the gradient of the log-likelihood function with respect to the unknown parameter θ. By selecting a proper estimator the minimization of the CRLB or the maximization of the FIM leads to a decrease of the uncertainty when estimating the parameter.

1) Fisher Information Matrix for UWB anchor initialization: Let $\mathcal{I}$ denote the global reference frame and let $q = [q_x, q_y, q_z]^T$ be the position of the UWB anchor which's position needs to be refined in $\mathcal{I}$. Furthermore, let the position of the UWB node mounted on the UAV, assuming no or known offset between IMU and mounted UWB node, in $\mathcal{I}$ be $p_i = [p_{ix}, p_{iy}, p_{iz}]^T$ with $i = 1, 2, \dots, n$ the $i-th$ position of the UAV where a measurment was taken. The distance between the UWB anchor and the $i-th$ position of the UWB node on the UAV is then given by $d_i = ||q - p_i||$, where $||\cdot||$ denotes the euclidean norm. The, now noisy, measurement model from Eq. (4) is then given by

$$z_{m_i} = \beta(||q - p_i|| + \omega_i) + \gamma = \beta(d_i + \omega_i) + \gamma, i = 1 \dots n \tag{9}$$

where z_{m_i} is the $i-th$ distance measurement and ω_i as distance dependent additive noise. Usually it is assumed that the measurement noise is additive zero mean white Gaussian noise with $\omega_i \sim \mathcal{N}(0, C_i(d_i))$ and $C_i = \sigma^2(I + d_i)^2$, where I is the identity matrix (i.e. all noise sources are independent). In vector notation we have $\mathbf{z}_m = [z_{m_1}, z_{m_2}, \dots, z_{m_n}]^T$ which corresponds to the vector containing the distance measurements, the vector of the actual ranges is $\mathbf{d} = [d_1, d_2, \dots, d_n]^T$ and the corresponding measurement noise vector is $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_n]^T$. In order to obtain the Fisher Information Matrix we have to calculate

$$FIM(\theta) = E\left\{(\nabla_q log\ p_q(\mathbf{z}_m))(\nabla_q log\ p_q(\mathbf{z}_m))^T\right\} \tag{10}$$

where $p_q(z)$ is the likelihood function for the target positioning problem which is given by

$$p_q(z_m)) = \frac{1}{(2\pi)^{\frac{n}{2}}|C|^{\frac{1}{2}}} exp\left\{-\frac{1}{2}(\mathbf{z}_m - \mathbf{d})^T C^{-1}(\mathbf{z}_m - \mathbf{d})\right\} \tag{11}$$

For general Gaussian noise there is also a general expression of the Fisher Information Matrix [10]. For the estimation of the UWB module this expression is given by Eq. (12).

$$FIM(q)_{kl} = \frac{\partial \mathbf{z}_m(q)}{\partial q_k}^T C(q)^{-1} \frac{\partial \mathbf{z}_m(q)}{\partial q_l} + \frac{1}{2} tr \left\{ C^{-1}(q) \frac{\partial C(q)}{\partial q_k} C^{-1}(q) \frac{\partial C(q)}{\partial q_l} \right\} \quad (12)$$

with the indices k and l representing the three coordinate axis x, y, z respectively. Note that with our extension to use a distant dependent bias term, each covariance matrix C_i per measurement is dependent on the anchor position q. Thus, the second term in Eq. 12 needs to be considered as non-zero term.

2) Optimality criteria: There are several optimality criteria for the Fisher Information Matrix to maximize the gathered information. Some of them are described in [11]:

- D-optimum design: the determinant of the FIM gets maximized: $\left(arg \max_{\theta \in \Re^n} |FIM(\theta)| \right)$.
- A-optimum design: the trace of the inverse of the FIM gets minimized: $\left(arg \min_{\theta \in \Re^n} tr \left(FIM(\theta)^{-1} \right) \right)$.
- E-optimum design: the smallest eigenvalue of the FIM gets maximized: $\left(arg\, max \min_{\theta \in \Re^n} eigv \left(FIM(\theta) \right) \right)$.

For this paper the D-optimum design is chosen. It minimizes the volume of the multi-dimensional uncertainty ellipsoid for the parameters to be estimated for a given model. The A-optimum design minimizes the trace of the CRLB which results in minimizing the average variance of the estimates. The E-optimum design maximizes the smallest eigenvalue of the Fisher Information Matrix which means that the length of the largest axis of the uncertainty ellipsoid gets minimized. The main advantage of D-optimum design is that it is scale invariant in the parameters and it is also invariant to linear transformations. A-optimum design and E-optimum design are not invariant to these transformations. The disadvantage of D-optimum design is that if no global optimum is found the obtained D-optimum design can be erroneous. This is due to the fact that the uncertainty ellipsoid can get minimized in one dimension while in the other dimension we do not have information at all. In other words, the uncertainty ellipsoid is very small in on direction while it is very large in the other direction. Due to the computational constraints we have on the UAV and the benefit of the D-optimum of not requiring to compute a matrix inverse, it is, however, still our favorite choice; the E-optimum design needs to compute the eigenvalues of the FIM and the A-optimum design needs to inverse the FIM.

Under certain assumptions, the maximization of the FIM determinant could be solved analytically. As an example [12] assumes that the measurement points are only on a circle and the source is in the middle of the circle. This gives an optimal sensor placement when the sensors are placed in $2\pi\ i/n;\ i = 1, 2, \ldots, n$ angles around the source on the circle. With this approach the number of sensors placed around the source can be arbitrary. In [13] this approach gets extended to 3D. Again assumptions are made in order to get an analytical solution. The sensors are now placed on a sphere and the source is placed in the middle of it. This sphere gets intersected with a hyperboloid. The sensors are then placed on the intersection area. Since we do not want to make any assumption on the position of the range module and the measuring point e.g. we want to place the measurement point freely in a certain area and the source can be placed anywhere in a certain location, we calculate the maximum of the FIM determinant numerically using the previous coarse initialization of the anchor as a rough estimate of q. For simulation purposes the Global Optimization Toolbox of MATLAB is used.

As a toy example to demonstrate the functioning of our approach, in Fig. 2, we assume that the UAV is only allowed to move in a volume of $1 \times 1 \times 1m$ and we would like to achieve best UWB anchor-position initialization by only flying the UAV to five positions. Furthermore, we assume distance dependent covariance matrix. The true location of the UWB anchor to be estimated is set to $[1.5, 1, 0]^T m$. As it is intuitive, the optimal positions for the UAV to fly to within the allowed volume are at the corners of the cube closest to the anchor.

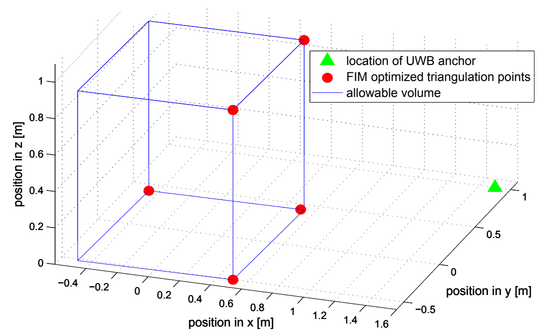

Fig. 2: Optimal measurement points for 5 measurements and 1 UWB anchor at $[1.5, 1, 0]^T$ for a distance dependent covariance matrix.

This toy example also highlights the low sensitivity of the selection of the optimal points in the volume with respect to the UWB anchor position: Already a rough initial direction and distance of the anchor with respect to the volume is suffcient to converge to the depicted result in Fig. 2. Or in other words, to make the optimal points be placed at different locations than depicted in Fig. 2, the true anchor position needs to drastically change. Furthermore, on distance independent covariance matrices, [14] proposed to transform the FIM to spherical coordinates to emphasize that the FIM depends on the *angle* between the range vectors. Adding the distance dependent element essentially adds the requirement "closer is better" – again without the need of very precise initial position information of the anchor. This low sensitivity of the optimal point placement with respect to the anchor point is in favor our our coarse initialization still being sufficiently accurate to generate informative points in a volume for the subsequent anchor-position refinement.

C. Distance dependent and position correlated covariance

The above toy example included the constraint that no two positions are allowed to be selected at the same locations. Consider again the above mentioned distance dependent covariance matrix $C_i = \sigma^2(I + d_i)^2$. One can see that it depends explicitly on the distance between the anchor and the measurement points (i.e. on $d_i = ||q - p_i||$). When the determinant of the FIM gets maximized, all positions of the measurement points tend to collapse over the range module since the distance dependent measurement error gets reduced as much as possible. This means that we have to define constraints for the optimization algorithm. In reality, and given the requirement of a base-line for later trilateration of the anchor position through use of the UAV positions, the measurement points are more correlated the closer they are to each other. This has to be considered in the covariance matrix for the FIM. For the correlated covariance matrix the squared exponential covariance is used. It is defined as follows per element:

$$C_{c_{i,j}} = \sigma^2 \, exp\left(-\frac{(p_i - p_j)^2}{2l^2}\right), \tag{13}$$

where l is the length-scale. The length-scale indicates the smoothness of the function. Large length-scale values characterize slow changing functions while small values characterize functions which can change quickly.
By combining the distance depended covariance matrix and the correlated covariance matrix one obtains

$$C = \sigma^2 \left(I + \delta\left(\mathbf{d}\right)\right)^2 + C_c \tag{14}$$

D. Refined anchor positioning and bias calculation

Using this definition of the covariance matrix in the proposed D-optimum FIM optimizer, we take the UAV positions correlation into account and can ensure well spaced measurement points in the defined volume. Once the optimal positions are defined in the volume we re-solve Eq. 5 for the refinement of the anchor position and at the same time for the bias terms. The anchor position and bias terms are later used in the closed loop tightly coupled UWB-inertial based control of the UAV.

IV. Results

A. Simulation results

Using the process described in Section III we simulated UWB range measurements to different locations using our distant dependent bias model from Eq. (9) with $\beta = 0.0049$ and $\gamma = 0.0951$. These values result from static tests with real hardware. For the standard deviation of the added noise, we did a sweep from σ starting at $0.02m$ to $0.2m$ in $0.02m$ steps. Each σ step consists of 200 individual simulation runs in order to obtain statistically relevant results. Fig. 3 shows the results. We noticed that our bias compensation significantly improved the results: for e.g. $\sigma = 0.1m$ the mean initialization error without bias consideration was $0.29m$ whereas it dropped to $0.13m$ using our model including the bias terms. Similarly, the error dropped from $0.58m$ to $0.23m$ for $\sigma = 0.2m$ with increasing improvements at higher noise values.

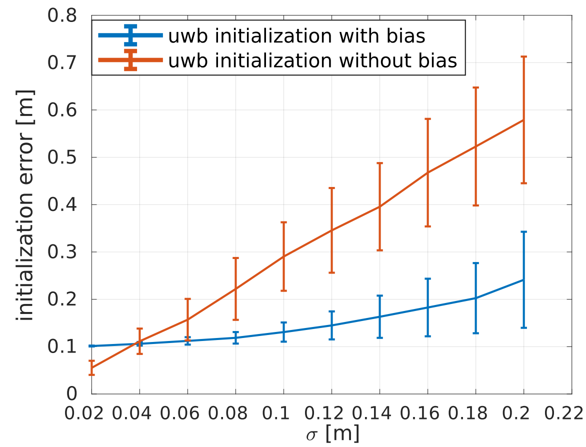

Fig. 3: Error statistics for our proposed bias compensated UWB anchor position initialization in Eq. (5) versus the one proposed in [1] without bias compensation in Eq. (3).

In Fig. 4, we show the complete initialization procedure showing the true position of the anchor (red triangle), the randomly selected initial triangulation points (green "x") with the coarse initial anchor estimation resulting from using these positions in Eq. (5) (green triangle), the subsequently selected volume within which information theoretic optimal triangulation positions are chosen (blue "x"), and the refined anchor position estimation based on these optimal position using again Eq. (5) (blue triangle). As a comparison and demonstration of the effect of taking our suggested bias compensation into account, the figure also shows the triangulated anchor position using the optimal points but Eq. (3) without modelling the bias (black triangle).

B. Real world results

We further performed a series of real experiments to demonstrate the use of our approach with real hardware and even for subsequent UWB-inertial closed loop control of a UAV. For all real experiments, we use an Asctec Hummingbird quadrotor (Fig. 1) equipped with a flight computer (Odroid XU4) and a UWB module (DecaWave TREK1000). Furthermore, three UWB modules (DecaWave TREK1000) are placed arbitrarily in the environment. The UWB distance measurements have a standard deviation of $0.09m$. We use an Optitrack motion capture system to obtain the UAV position for all our process steps. We compare our real world results to the ones reported in [1] where the authors move a UAV on random trajectories to add range measurements whenever they improve the condition number of the matrix in Eq. 3 consisting of previous measurements. New measurements are added up to a maximum number of measurements or until a certain quality of the matrix' condition number is reached.

In a first experiment, we performed 120 initializations as reported in [1]. The mean initialization distance error using our bias compensated method in Eq. 5 is $0.0984m \pm 0.0401m$. Not using the bias compensation but with our

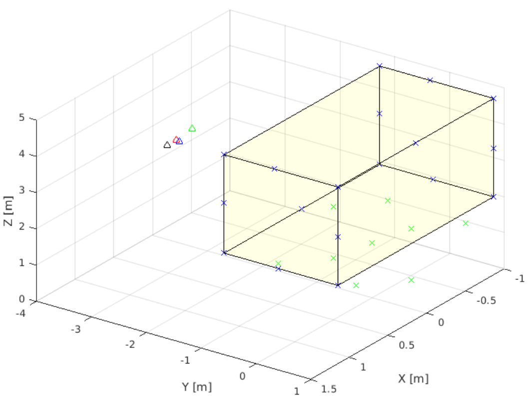

Fig. 4: Our proposed initialization procedure first using random triangulation points (green x) for coarse anchor initialization (green triangle) and subsequently for the FIM optimization to find optimal triangulation points (blue x) within a volume for position refinement (blue triangle). Also, the consideration of bias terms has an important positive performance impact (blue versus black triangle). Ground truth is the red triangle.

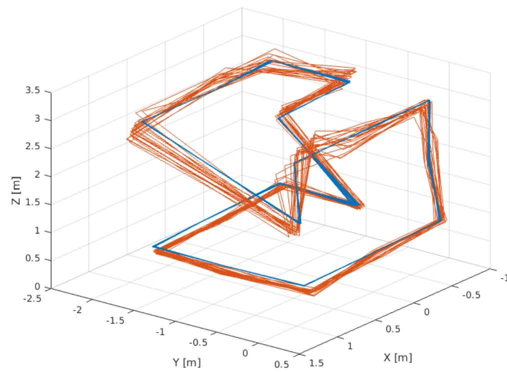

Fig. 6: Flying 20 times through 18 waypoints using a tightly coupled UWB-inertial EKF based on the anchor initializations of our proposed method. Ground trugh (blue) is obtained from an Optitrack system.

suggested method on FIM based triangulation position optimization we achieve a mean initialization distance error of $0.1417m \pm 0.0344m$. In contrast, the random approach based on the matrix condition number without considering biases in [1] reports an error of $0.3444m \pm 0.1326m$ (over 40 runs). Our approach shows an improvement by a factor of nearly 3.5. Fig. 5 shows the initialization results of our approach with bias consideration.

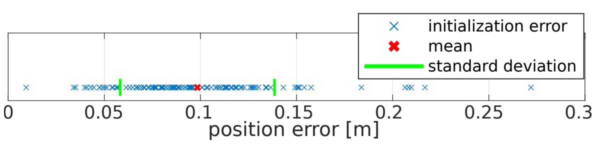

Fig. 5: Error statistics over 120 runs of UWB initialization

Additionally two more experiments where performed, a hovering test and an trajectory tracking test using a tightly coupled UWB-inertial EKF based on the anchor position initialized by our proposed method. Ground truth is optained by our Optitrack system. The mean tracking error for the trajectory following was $0.19m$ with a standard deviation of $0.0997m$ while flying 20 times a mission with 18 waypoints (Fig. 6). In the hovering test, the UAV was sent to the height of 1m and was hovering there for 60 seconds. We used five different pose estimators on the UAV for closed loop control: i) Optitrack as a reference (ref), ii) UWB measurements with correctly initialized anchor positions but without a bias model (u-gt), iii) UWB measurements with estimated anchor positions using our FIM optimization but without a bias model (u-est), iv) UWB measurements with estimated anchor positions using our FIM optimization and proposed bias model (u-bias), v) UWB measurements with estimated anchor positions using the approach in [1]. Tab. I shows the RMSE for all setups. For the method proposed in [1] and with our best tuning knowledge applied, we still got to an RMSE of $0.713m$. Unfortunately, the authors in [1] did not report the performance purely navigating based on UWB-inertial estimation in their work. Interestingly, all other UWB based setups show similar performance despite the improved UWB positioning and bias compensation through our method. With an RMSE of over 1cm even with Optitrack measurement, we assume that this is due to the low controller performance of the UAV shadowing estimation accuracy.

	ref	u-gt	u-est	u-bias	[1]
RMSE [m]	0.012	0.025	0.029	0.028	0.713

TABLE I: Results of the hovering experiment

V. Conclusion

In this paper, we addressed the problem of accurate UWB anchor initialization without prior knowledge using the FIM for information-optimized triangulation-position selection and using a distance dependent bias model for the UWB measurements to improve the final triangulation accuracy. Our approach is based on two steps where we first use randomized triangulation points for a coarse anchor initialization and bias estimation. These values serve then for a FIM based optimization to generate optimal triangulation points used in a refinement step for anchor position and measurement biases. The result has a 3.5 times lower position error compared to state of the art and reaches an anchor initialization accuracy of $9.8cm$. The proposed approach can be applied sequentially or as a lump-sum optimization to multiple anchors to use their initialized positions for subsequent UAV flight based on on-board, real-time UWB-inertial state estimation. We showed real flight following a trajectory with an RMSE of 19cm and a hover performance of under 3cm RMSE greatly superseding previous approaches.

REFERENCES

[1] K. Hausman, S. Weiss, R. Brockers, L. Matthies, and G. S. Sukhatme. Self-calibrating multi-sensor fusion with probabilistic measurement validation for seamless sensor switching on a uav. In *2016 IEEE International Conference on Robotics and Automation (ICRA)*, pages 4289–4296, May 2016.

[2] Klaithem Al Nuaimi and Hesham Kamel. A survey of indoor positioning systems and algorithms. In *Innovations in information technology (IIT), 2011 international conference on*, pages 185–190. IEEE, 2011.

[3] J. D. Hol, F. Dijkstra, H. Luinge, and T. B. Schon. Tightly coupled uwb/imu pose estimation. In *2009 IEEE International Conference on Ultra-Wideband*, pages 688–692, Sept 2009.

[4] A. Ledergerber, M. Hamer, and R. D'Andrea. A robot self-localization system using one-way ultra-wideband communication. In *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 3131–3137, Sept 2015.

[5] Jeroen D Hol, Thomas B Schön, and Fredrik Gustafsson. Ultra-wideband calibration for indoor positioning. In *Ultra-Wideband (ICUWB), 2010 IEEE International Conference on*, volume 2, pages 1–4. IEEE, 2010.

[6] Byungjae Park and Sejin Lee. Robust range-only beacon mapping in multipath environments. *ETRI Journal*, 42(1):108–117, 2020.

[7] Roberta Cardinali, Luca De Nardis, Maria Gabriella Di Benedetto, and Pierfrancesco Lombarde. Uwb ranging accuracy in high- and low-data-rate applications. *IEEE Transactions on Microwave Theory and Techniques*, 54(4):1865–1875, 4 2006.

[8] David Moreno-Salinas, Antonio M Pascoal, and Joaquin Aranda. Optimal sensor placement for multiple target positioning with range-only measurements in two-dimensional scenarios. *Sensors*, 13(8):10674–10710, 2013.

[9] Harry L Van Trees and Kristine L Bell. *Detection estimation and modulation theory, pt. I.* Wiley, 2013.

[10] Steven M. Kay. *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1993.

[11] Dariusz Ucinski. *Optimal measurement methods for distributed parameter system identification*. CRC Press, 2004.

[12] David Moreno-Salinas, Antonio M Pascoal, and Joaquin Aranda. Optimal sensor placement for multiple target positioning with range-only measurements in two-dimensional scenarios. *Sensors*, 13(8):10674–10710, 2013.

[13] David Moreno-Salinas, Antonio Pascoal, and Joaquin Aranda. Optimal sensor placement for acoustic underwater target positioning with range-only measurements. *IEEE Journal of Oceanic Engineering*, 41(3):620–643, 2016.

[14] Bruno Ferreira, Aníbal Matos, and Nuno Cruz. Optimal positioning of autonomous marine vehicles for underwater acoustic source localization using toa measurements. In *Underwater Technology Symposium (UT), 2013 IEEE International*, pages 1–7. IEEE, 2013.