Congestion Effect for Taxi Market and its Implication for Mechanism design

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1 Introduction

As the urbanization process preceeds dramatically fast for temporary society, various social problem arises, especially in metropolitan city like Beijing etc. One of major problem is traffic jam in certain time period. Congestion could cause serious social problem. For people in congestion area, their waiting time could increase unbearably. For taxi driver, their revenue decrease in traffic jam due to low speed.

In this paper, we focus on the congestion effect for taxi market. We point out their affect in traditional society as well as modern society with help of taxi platform like Didi and Uber. Our main conclusion is that taxi platform shift the quatitive relationship between supply and demand. Moreover, we develop new mechanism for platform pricing scheme with implication from congestion effect, which we believe has its own interest.

The paper organizes as follow. In section 2, we describe model for congestion as well as supplier and consumer. In section 3, we analyse congestion effect for taxi market and positive effect of taxi platform . In section 4, we briefly discuss some heuristic for taxi platform, then we utilize result in section 3 to develop a black box pricing mechanism to minimize social cost/ In section 5, we will discuss some further direction for research.

2 Model

In this section, we'll present formal model for congestion effect. We split the whole marke into two part, one with congestion effect, the other does not. For market 1(with congestion effect), let D_1 denote the demand of taxi, p_1 denote the profit for satisfying one unit demand per unit time. The same notation D_2, p_2 is used for market 2.

Assumption 1. *Demand is invariate over time, i.e the demand level for market 1(2) is always $D_1(D_2)$*

Assumption 2. $D_1 > D_2$, $p_1 < p_2$

Assumption 2 means that demand in congestion area is greater than congestion-free market, while profit in market 1 is less, since taxi moves slower and profits are propotional to speed.

2.1 Model for Consumer and Supplier

We assume both consumer and supplier is utility maximizer. For supplier, its utility

$$U_{supply} = p$$

is just its profit

Assumption 3. $D_1 + D_2 \geqslant S \geqslant D_2$

Assumption 3 describe that supply is not enough for the whole market, which is just the case for peak period. While supply must satiesfy the clear market, otherwise more supply will flush into taxi market.

For consumer, its utility not only affects by its outcome, but also the time he spends on waiting. For simplicity, we model time effect as a linear function and

$$U_{consumer} = -p - \alpha T$$

where α represents people's value for time. Large α represents high valuation for time.

2.2 Social Welfare

When we define social welfare, we also need to consider empty rate. For the whole society, we want less empty taxi for the sake of air pollution. Thus we define

$$\text{Social Welfare} = U_{consumer} + U_{supply} - \beta R_{empty}$$

2.3 Model for Traditional Market and Taxi Platform

In the traditional taxi market, government regulate taxi fee, for consideration of social fairness and other factors. However, with the popularity of smart phone and internet, taxi platform appears. In this paper, we focus on the regulation aspect of taxi platform. With the help of taxi platform, people can increase price for taxi(increae p_i correspondingly). In order for modeling taxi platform, we define its utility function, which will only be mentioned in section 4.

$$U_{plat} = -\gamma R_{empty} - C_{cost}$$

An explanation for platform utility is that platform will charge joinin fee γ for each load, while some necessary cost need to be reduced.

3 Congestion Effect for Taxi Market

In this section, we analyse the what the impact congestion does to taxi market.

3.1 Traditional Market

In Traditional market, consumer and taxi drive do not have channel to exchange information.

Claim 1. *In traditional market, it is a dominant strategy to pay the regulation fee for taxi, which means that consumer would not increase its payment to taxi driver even it has high sensitivity to time.*

The only thing need to notice here is that increase payment would not affect supply in both market, thus it is always better to not increase outcome. The following theorem charaterize traditional market equilibrium

Theorem 1. *In traditional market, congestion free market would have supply surplus. More specifically, the supply would be*

$$S_1 = \max\{S - \frac{p_2}{p_1} D_2, 0\}$$
$$S_2 = \min\{\frac{p_2}{p_1} D_2, S\}$$

Proof. In equilibrium case, taxi driver gets equal benefit for both market, thus

$$\frac{D_2}{S_2} p_2 = p_1$$

Thus we can derive

$$S_1 = \max\{S - \frac{p_2}{p_1} D_2, 0\}$$
$$S_2 = \min\{\frac{p_2}{p_1} D_2, S\}$$

□

Before talking about taxi platform, we want to know what's the condition for maximum social welfare. We found it particularly simple to characterize.

Proposition 1. *Maximum social welfare can be achieved when*

$$p_1^{'} - \varepsilon = p_2^{'} = p^{\star}$$

Proof. Notice that congestion free market is dominant to congestion market, which indicates that its demand would always be satiesfied. Thus maximize social welfare is equavalent to reduce empty rate(which results in high consumer utility). It is easy to show that it can be achieved when price of two market is roughly equal. □

We want to mention that is not suitable to directly rise price in congestion market to p_2 since consumer's utility may not reach its climax. Thus it do not have the incentive to increase price.

3.2 Taxi Platform

With the help of taxi platform, the equilibrium of the market would shift due to information exchange. For the ease of proof, we assume that $\frac{p_2}{p_1} D_2 < S$, which is more realistic. The following are the main theorem of this article.

Theorem 2. *With taxi platform, there exists unique Nash equilibrium, more specifically,*

$$\begin{aligned}
&\textit{if} \quad \frac{\sqrt{\alpha D_1 D_2 p_2} + p_2 D_2}{S} > p_2 \\
&\quad p_1^{'} = p_2^{'} = \frac{\alpha p_1 p_2}{S - D_2} \\
&\textit{else} \\
&\quad p_1^{'} = \frac{\sqrt{\alpha D_1 D_2 p_2} + p_2 D_2}{S}, \; p_2^{'} = p_2
\end{aligned}$$

Proof. Consumer's utility is

$$u(p) = -p - \alpha E[T] \tag{1}$$

where

$$E[T] = \frac{D_2}{S_1} = \frac{D_2}{S - p_2/p_1 D_2} \tag{2}$$

substitutes (2) to (1) and take derivative, we obtain that maximum revenue achieved when

$$\frac{du}{dp} = 0$$

we can get that

$$p_1^{\star} = \frac{\sqrt{\alpha D_1 D_2 p_2} + p_2 D_2}{S}$$

If $p_1^{\star} < p_2$, this is just the equilibrium. For the other case, market 2 is dominant o market 1, then we know that

$$p_1^{'} = p_2^{'} = \frac{\alpha p_1 p_2}{S - D_2}$$

□

Notice that this theorem implies that when people are more care about time, maximum social welfare could be achieved. On the otherside, with low time sensitivity, people in congestion market would not increase their price much. However, the occurance of platform would necessary increase social welfare.

4 Mechanism Design for Taxi Platform

In this section, we would first talk about how taxi platform can further increase social welfare by recompensating for congestion market and give a sufficent condition for taxi platform to make more money through compensation. Next, we analyse the effect of different pricing and design corresponding mechanism to avoid further empty load.

4.1 Subsidy for Consumer could Improve Social Welfare

In the following paragraph, we would focus on the second case, i.e. stable equilibrium does not reach maximum social welfare. Our observation is that subsidy should be designed to increase consumer's marginal utility. More specifically, the subsidy policy should not compesate for each ride unconditionally, for which could only save passenger's cost, but could not increase equilibrium price. We have the following proposition.

Proposition 2. *For price $p < p_2$, taxi platform should subsides s unit money for per unit money spend, where*

$$s = \max\{0, \frac{(Sp_2 - p_2 D_2)^2}{D_1 D_2 p_2 \alpha} - 1\}$$

Proof. The proposition follows from simple calculation. With subsidy, we can imagine that $\alpha \leftarrow (1+s)\alpha$ equivalently, make marginal profit for consumer be zero, we have

$$\sqrt{(1+s)\alpha D_1 D_2 p_2} = p_2 D - p_2 S$$

from which we can derive our proposition. □

Notice that in the above price, platform compesates for consumer's cost, while reduce the empty rate. There is a trade off between subsidy and empty rate for taxi platform to make more money. Formally, we have the following sufficient condition.

Theorem 3. *Platform can increase its revenue as well as social welfare by compensating for marginal consuming cost if*

$$\frac{dS_1}{p_1^{'}} > \frac{S_1}{\gamma}$$

Proof. The sufficient condition simply says that the cost of compensate($dp_1^{'} S_1$) is less than decrease of empty rate($R_{empty}\gamma$), if taxi platform fully compensates. □

This condition is really a rough one and can be improved substantially.(Fully charaterize may need to calculate the extrem point of an integral and I'm sorry that I do not have time to finish it)

4.2 Implication for Pricing Scheme

A lesson we can learn from the above analyse is that in order to maximize social welfare, we do not want price vary much. In another word, we want every price is the same. However, in realistic world, people's valuation for time α is different. For the same analyse as theorem 2, different price could result in empty load(For example, taxi drive may wait for high price passenger and leave low price passenger waiting) and lead to waste of social resources. In order to eliminate this effect, we propose the following mechanism.

Theorem 4. *The following mechanism would eliminate price difference while preserve consumer's utility.*

- *Use mean α to calculate market equilibrium and price p*
- *Upon receives a p_i from passenger i, calculates α_i from theorem 2, as well as its expected waiting time T_i.*
- *Broadcast passenger i's demand for $F^{-1}(T)$ taxi driver and pay p for taxi driver.*

where $F : N \to T$ is a function map the number of noticed taxi to the time of passenger's waiting time. This function take any form and it is not our focus. We can learn this function from available data

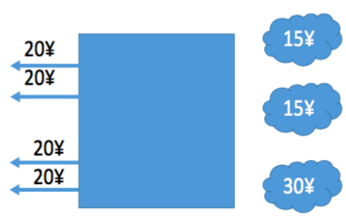

Figure 1: The Mechanism

Proof. The correctness of this mechanism is directly. Moreover, truthfully report price would be the best strategy given other plays equilibrium. □

This algorithm is more like a black box, which eliminate the different of price and transform consumer's demanding for time to broadcast number, which can be learned by various method

5 Discussion

In this section, we will talk about some further direction to improve the research.

- Notice we define consumer's utility to be linear with their waiting time T. If we extend this function to be nonlinear, like a concave function, we believe the analyses would be completely different. Consumer would dynamically change their price as they wait longer. Furthermore, we would not only care about consumer's expected waiting time due to dynamic pricing scheme. The incurrance of randomness and dynamic makes this problem much more complex.
- What if we have multi-level congestion. Does the equilibrium always exists, can we easily characterize it?
- Finally, we do not consider the effect of more taxi flush into congestion area, which may induce more traffic jam. How to model this effect, and when the social welfare can achievd its climax?

6 Conclusion

In this paper, we model the congestion effect for taxi market. In traditional society, this could cause serious empty load, while with the help of taxi platform, the equilibrium status could improve social welfare. With recompensation, taxi platform is possible to improve equilibrium price as well as making more profit. Finally, we apply congestion effect and develop a novel mechanism to eliminate impacts of different price.

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