



 **TopCow**: **Top**ology-Aware Anatomical Segmentation
of the **C**ircle of **W**illis for CTA and MRA

Centerline Boundary Dice Loss for Vascular Segmentation

MICCAI 2024

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<https://github.com/PengchengShi1220/cbDice>

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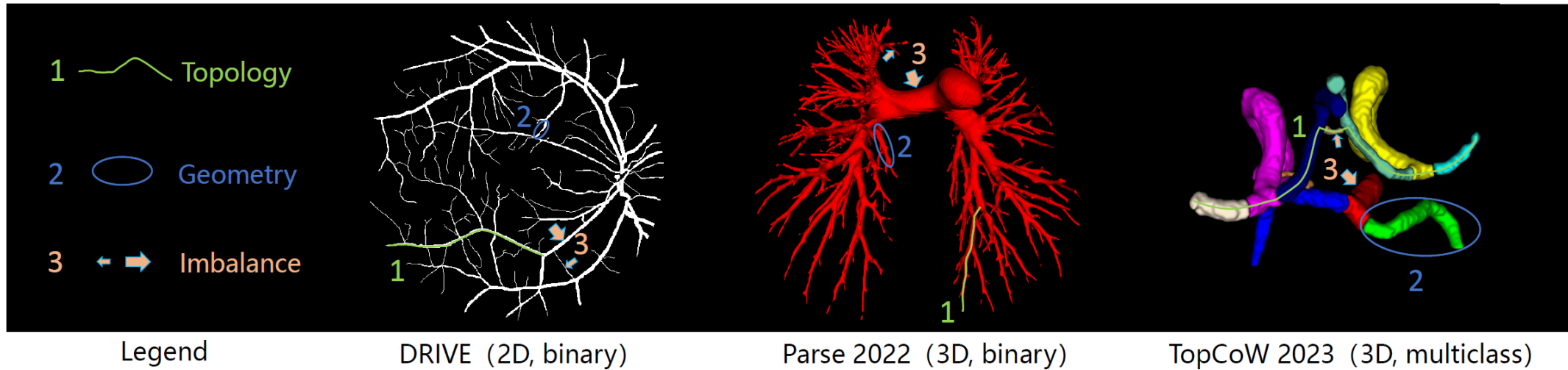
HARBIN INSTITUTE OF TECHNOLOGY, SHENZHEN



Code:



Challenges in Vascular Segmentation

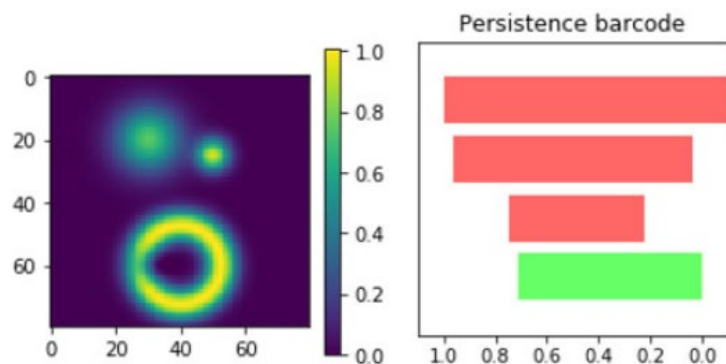


Vascular segmentation encounters three significant challenges:

- (1) Preserving the **topology** of the vascular network to enable accurate hemodynamic analysis.
- (2) Capturing the intricate **geometric morphologies** vital for diagnosing conditions like stenosis.
- (3) Ensuring consistent width adjustments within each branch to mitigate **diameter imbalance**.

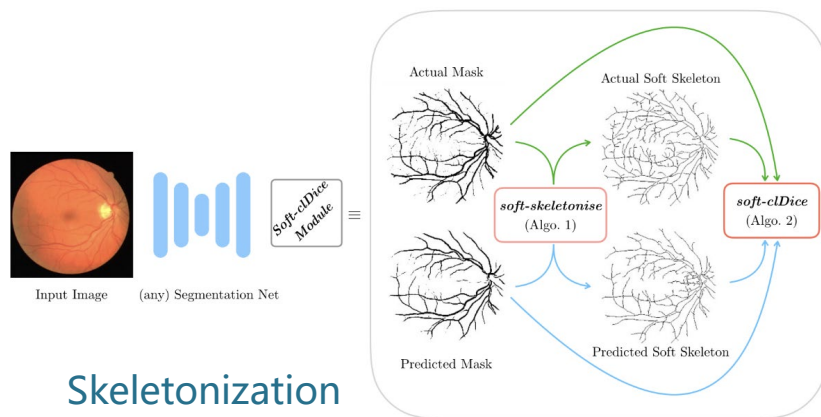
How to encompass topology, geometry, and vessel diameter consistency in vascular segmentation?

Topology:



Persistent Homology

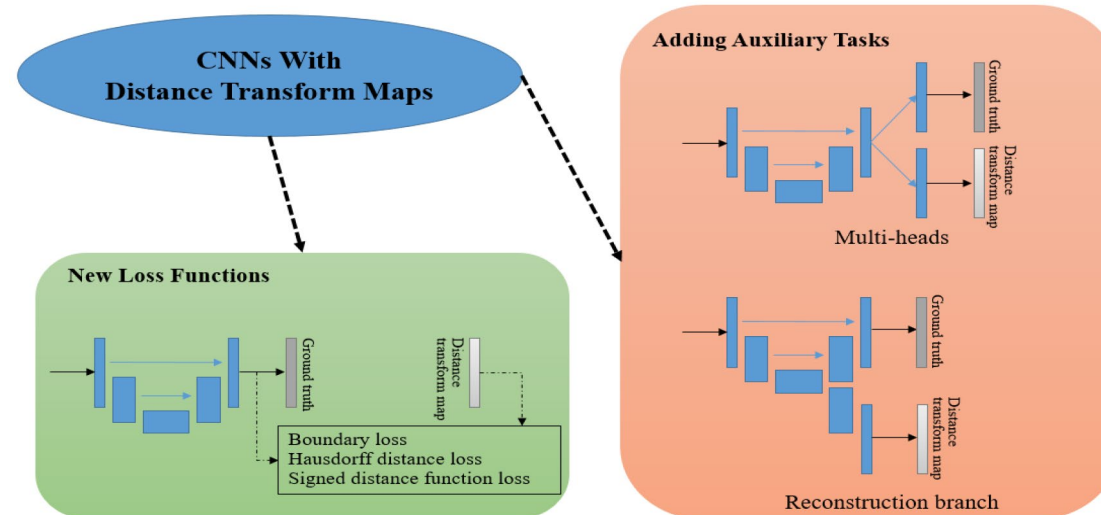
2020, TPAMI, A Topological Loss Function for Deep-Learning Based Image Segmentation Using Persistent Homology



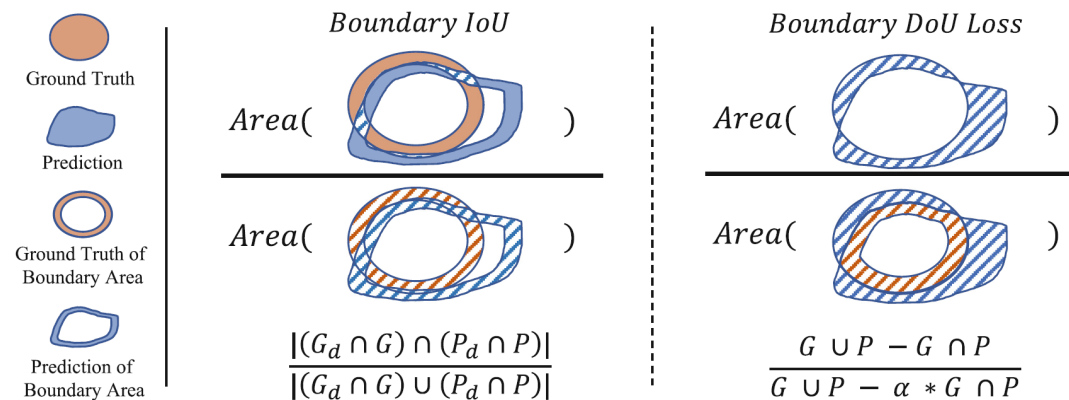
Skeletonization

2021, CVPR, cDice - a Novel Topology-Preserving Loss Function for Tubular Structure Segmentation

Geometry:



2020, MIDL, How Distance Transform Maps Boost Segmentation CNNs: An Empirical Study



2021, CVPR, Boundary IoU

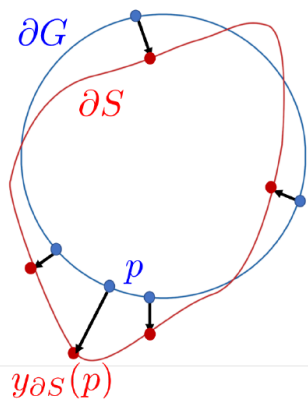
2023, MICCAI, Boundary DoU

Imbalance:

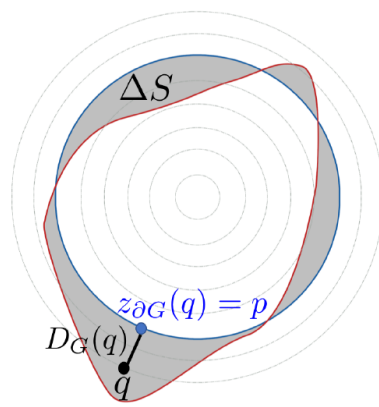
$$\text{GDL} = 1 - 2 \frac{\sum_{l=1}^2 w_l \sum_n r_{ln} p_{ln}}{\sum_{l=1}^2 w_l \sum_n r_{ln} + p_{ln}}$$

$$w_l = 1/(\sum_{n=1}^N r_{ln})^2$$

2017, DLMIA, Generalised Dice overlap as a deep learning loss function for highly unbalanced segmentations

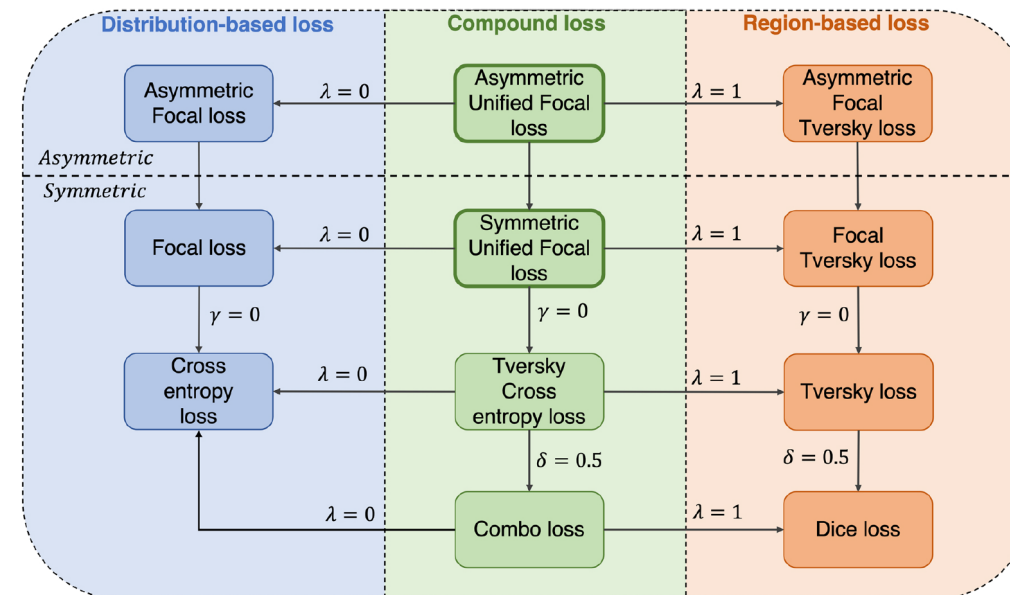


(a) Differential



(b) Integral

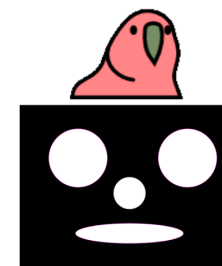
2019, MIDL, Boundary loss for highly unbalanced segmentation



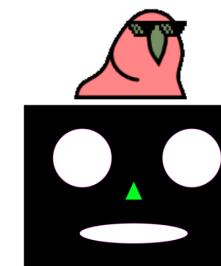
2022, CMIG, Unified Focal loss: Generalising Dice and cross entropy-based losses to handle class imbalanced medical image segmentation

blob loss: instance imbalance awareness

how blob loss changes loss functions' perception of sleep paralysis demons



w/o blob loss



with blob loss


2023, IPMI, blob loss: Instance Imbalance Aware Loss Functions for Semantic Segmentation


Limitations of centerline Dice (clDice)


$$T_{\text{prec}}(S_P, V_L) = \frac{|S_P \cap V_L|}{|S_P|}; \quad T_{\text{sens}}(S_L, V_P) = \frac{|S_L \cap V_P|}{|S_L|}$$

$$\text{clDice}(V_P, V_L) = 2 \times \frac{T_{\text{prec}}(S_P, V_L) \times T_{\text{sens}}(S_L, V_P)}{T_{\text{prec}}(S_P, V_L) + T_{\text{sens}}(S_L, V_P)}$$

V_P : The predicted segmentation mask. 

V_L : The ground truth mask. 

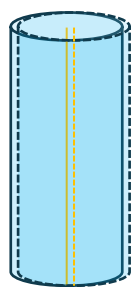
S_P : The skeleton extracted from V_P . 

S_L : The skeleton extracted from V_L . 

Each of these elements is a **binary** image.

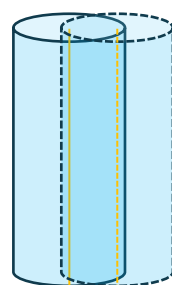
Neglect the boundary FN and FP:

Fully overlap



clDice=1

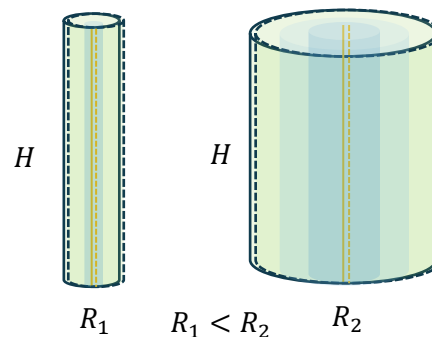
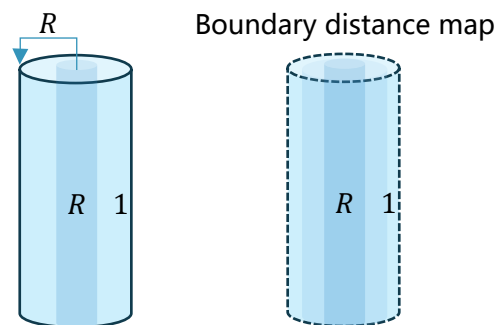
Boundary overlap



clDice=1

The clDice is insensitive to whether the skeleton line is close to the **boundary** or **centered** within the vessel.

Add boundary awareness:



$$V_1 = \pi R_1^2 \cdot H \quad V_2 = \pi R_2^2 \cdot H$$

$$\frac{V_2}{V_1} = \frac{R_2^2}{R_1^2}$$

Mitigate imbalance:

clDice loss needs to **be combined with Dice loss**, and as a result, it cannot avoid the bias towards larger targets.

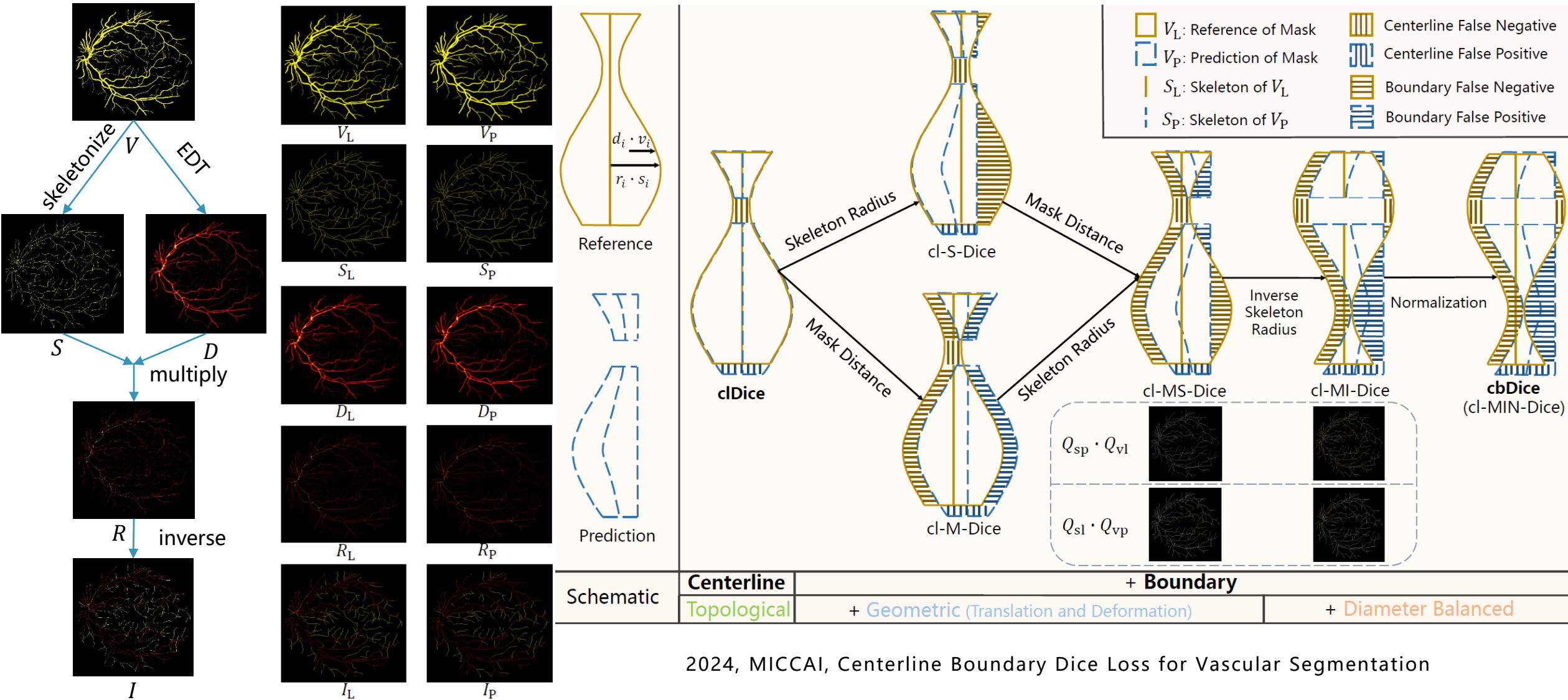
$$\mathcal{L}_c = (1 - \alpha)(1 - \text{softDice}) + \alpha(1 - \text{softclDice})$$

$$\alpha \in [0, 0.5]$$

The segmentation towards vessels with **larger diameters** due to the **volume-based** nature of the Dice metric.

The volume's sensitivity to r^2 suggests that a weighting scheme **inversely proportional to r^2** can mitigate the bias.

Centerline boundary Dice loss



Centerline boundary Dice loss

$$T_{\text{prec}}(S_P, S_L, V_L) = \frac{|Q_{\text{sp}} \cap Q_{\text{vl}}|}{|Q_{\text{sp}} \cap Q_{\text{spvp}} \cap (U - S_L)| + |Q_{\text{sp}} \cap Q_{\text{sylv}}|}$$

$$T_{\text{sens}}(S_L, S_P, V_P) = \frac{|Q_{\text{sl}} \cap Q_{\text{vp}}|}{|Q_{\text{sl}} \cap Q_{\text{sylv}} \cap (U - S_P)| + |Q_{\text{sl}} \cap Q_{\text{spvp}}|}$$

$$\text{cl-X-Dice}(V_P, V_L) = \frac{2 \times T_{\text{prec}}(S_P, S_L, V_L) \times T_{\text{sens}}(S_L, S_P, V_P)}{T_{\text{prec}}(S_P, S_L, V_L) + T_{\text{sens}}(S_L, S_P, V_P)}$$

Metric	cl-D		cl-S-D		cl-M-D		cl-MS-D		cl-MI-D		cl-MSN-D		cl-MIN-D	
Dim	2D	3D	2D	3D	2D	3D	2D	3D	2D	3D	2D	3D	2D	3D
Q_{sl}	S_L	S_L	R_L	R_L^2	S_L	S_L	R_L	R_L^2	I_L	I_L^2	$R_{L,N}$	$R_{L,N}^2$	$I_{L,N}$	$I_{L,N}^2$
Q_{sp}	S_P	S_P	R_P	R_P^2	S_P	S_P	R_P	R_P^2	I_P	I_P^2	$R_{P,N}$	$R_{P,N}^2$	$I_{P,N}$	$I_{P,N}^2$
Q_{vl}			V_L				D_L						$D_{L,N}$	
Q_{vp}			V_P				D_P						$D_{P,N}$	
Q_{sylv}			S_L				R_L						$R_{L,N}$	
Q_{spvp}			S_P				R_P						$R_{P,N}$	

Examples:

Reference	Prediction 1	Prediction 2	Metric(V_{P1}, V_L)	Metric(V_{P2}, V_L)	Difference
			$D=0.892$ $\text{cl-D}=1.000$ $\text{B-DoU}=0.803$ $\text{cl-M-D}=0.828$ $\text{cb-D}=0.837$	$D=0.785$ $\text{cl-D}=0.889$ $\text{B-DoU}=0.642$ $\text{cl-M-D}=0.603$ $\text{cb-D}=0.632$	0.107 0.111 0.161 0.225 0.205
	(a) Translation				
			$D=0.857$ $\text{cl-D}=1.000$ $\text{cb-D}=0.974$	$D=0.889$ $\text{cl-D}=1.000$ $\text{cb-D}=0.958$	-0.032 0 0.016
	(b) Deformation		$\text{cl-D}+D=1.857$ $2\text{cb-D}+D=2.805$	$\text{cl-D}+D=1.889$ $2\text{cb-D}+D=2.805$	-0.032 0
			$D=0.795$ $\text{cl-D}=0.667$ $\text{cb-D}=0.587$	$D=0.507$ $\text{cl-D}=0.667$ $\text{cb-D}=0.730$	0.288 0 -0.143
	(c) Diameter imbalance		$\text{cl-D}+D=1.462$ $2\text{cb-D}+D=1.969$	$\text{cl-D}+D=1.174$ $2\text{cb-D}+D=1.967$	0.288 0.002

Theorem proof of cl-X-Dice

$$\begin{aligned} T_{\text{prec}}(S_P, S_L, V_L) &= \frac{|Q_{\text{sp}} \cap Q_{\text{vl}}|}{|Q_{\text{sp}} \cap Q_{\text{spvp}} \cap (U - S_L)| + |Q_{\text{sp}} \cap Q_{\text{slvl}}|} \\ T_{\text{sens}}(S_L, S_P, V_P) &= \frac{|Q_{\text{sl}} \cap Q_{\text{vp}}|}{|Q_{\text{sl}} \cap Q_{\text{slvl}} \cap (U - S_P)| + |Q_{\text{sl}} \cap Q_{\text{spvp}}|} \\ \text{cl-X-Dice}(V_P, V_L) &= \frac{2 \times T_{\text{prec}}(S_P, S_L, V_L) \times T_{\text{sens}}(S_L, S_P, V_P)}{T_{\text{prec}}(S_P, S_L, V_L) + T_{\text{sens}}(S_L, S_P, V_P)} \end{aligned}$$

Theorem 1. For vertical translations along skeleton lines without deformation, cl-M-Dice coefficient is sensitive to translations of mask V within radius R , whereas clDice remains invariant.

Proof. In 2D, cl-M-Dice is defined thus (extendable analogously to 3D):

$$T_{\text{prec}}(S_P, S_L, V_L) = \frac{|S_P \cap D_L|}{|R_P \cap (U - S_L)| + |S_P \cap R_L|} \quad (1)$$

$$T_{\text{sens}}(S_L, S_P, V_P) = \frac{|S_L \cap D_P|}{|R_L \cap (U - S_P)| + |S_L \cap R_P|} \quad (2)$$

Under vertical translations maintaining constant radius, $|S_P \cap R_L|$ equals $|S_L \cap R_P|$. This reduces cl-M-Dice's denominator to $|R_P|$ (and analogously for R_L), making its sensitivity dependent solely on the numerator. Hence, cl-M-Dice reacts to spatial displacements of V within R . Conversely, clDice, assessing overlap between S and V , is not influenced by these variations.

Theorem 2. cl-S-Dice, unlike clDice, is sensitive to radius variations at the skeleton under deformation without perpendicular translation. In cases of complete overlap, cl-S-Dice equals clDice with a value of 1.

Proof. In 2D, cl-S-Dice is defined thus (extendable analogously to 3D):

$$T_{\text{prec}}(S_P, S_L, V_L) = \frac{|R_P \cap V_L|}{|R_P|}, \quad T_{\text{sens}}(S_L, S_P, V_P) = \frac{|R_L \cap V_P|}{|R_L|} \quad (3)$$

For clDice $\neq 1$ (partial overlap), changes in radius (R_P, R_L) affect both $|R_P \cap V_L|$ and $|R_L \cap V_P|$. Specifically, with $S = \{s_i, b_j^s \mid i \in [1, n], j \in [1, m]\}$ and $R = \{r_i \cdot s_i, b_j^s \mid i \in [1, n], j \in [1, m]\}$, variances in r_i at any s_i modify cl-S-Dice. When clDice = 1 (complete overlap), $|R_P \cap V_L| = |R_P|$ and $|R_L \cap V_P| = |R_L|$, aligning cl-S-Dice with clDice, highlighting cl-S-Dice's sensitivity to radius changes in other scenarios.

Implementation details

Differentiable Binarization:

To obtain a differentiable binarized predicted probability map of the foreground, follow these steps:

```
y_pred_fore = y_pred[:, 1:]  
y_pred_fore = torch.max(y_pred_fore, dim=1, keepdim=True)[0] # C foreground channels -> 1 channel  
y_pred_binary = torch.cat([y_pred[:, :1], y_pred_fore], dim=1)  
y_prob_binary = torch.softmax(y_pred_binary, 1)  
y_pred_prob = y_prob_binary[:, 1] # predicted probability map of foreground
```

Skeletonization:

Topology-preserving skeletonization: "A Skeletonization Algorithm for Gradient-based Optimization" (ICCV, 2023). This method ensures high topological accuracy but operates at a slower speed.

Morphological skeletonization: "cDice - a Novel Topology-Preserving Loss Function for Tubular Structure Segmentation" (CVPR, 2021). This method runs faster but offers lower topological accuracy.

Distance map:

We implement the Euclidean distance transform (EDT) using a **GPU-accelerated** approach with the **cupy** and **cuCIM** library.

```
pip install monai  
  
# For CUDA 12.x  
pip install cucim-cu12  
pip install cupy==12.3
```

Loss function:

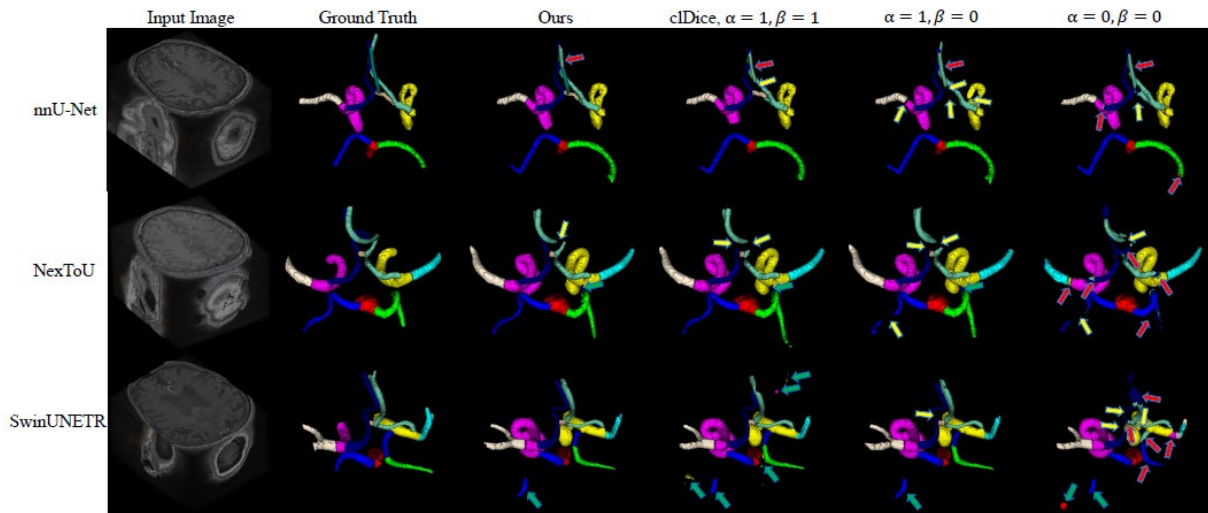
$$\mathcal{L} = 0.5 \times \text{CE} + \frac{\alpha}{2 \times (\alpha + \beta)} \cdot \text{Dice} + \frac{\beta}{2 \times (\alpha + \beta)} \cdot X$$

where X denotes cl-X-Dice or B-DoU. Parameters α and β are non-negative numbers; with both set to 0, it reverts to CE loss.

Code:



Qualitative and quantitative results



Comparative visualization of results on the TopCoW 2023 dataset. Yellow arrows mark areas of segmentation **false negatives (FN)**, green arrows point to **false positives (FP)**, and red arrows identify areas of **misclassification**.

$$\mathcal{L} = 0.5 \times \text{CE} + \frac{\alpha}{2 \times (\alpha + \beta)} \cdot \text{Dice} + \frac{\beta}{2 \times (\alpha + \beta)} \cdot X$$

nnU-Net framework, No Deep Supervision, No Mirroring for TopCoW2023

Network	Loss		Parse 2022 (50 epochs)				TopCoW 2023 (100 epochs, classes average)						
	X	α	β	Dice	clDice	β^{err}	NSD	Dice(L)	Dice(S)	clDice	β^{err}	NSD(L)	NSD(S)
nnU-Net	—	0	0	82.55	68.91	331.5	78.51	81.51	7.012	90.10	—	89.78	14.08
	—	1	0	85.05	80.11	277.7	86.04	84.03	0	88.22	—	91.90	0
	clDice	1	1	85.30	80.23	263.4	86.20	84.21	38.46	90.72	—	92.45	47.85
	cbDice	1	1	85.46	80.76	266.1	86.57	84.11	41.55	91.34	0.96	92.21	50.42
	cbDice	1	2	84.91	80.02	275.8	85.98	84.01	43.38	91.95	0.98	91.99	54.11
	cbDice	1	3	84.97	78.98	285.6	85.26	84.18	42.68	90.63	0.95	92.67	51.76
SwinUNETR	—	0	0	78.91	58.53	508.7	70.80	61.22	0	88.64	—	70.64	0
	—	1	0	81.94	69.87	496.9	78.33	83.19	37.32	90.18	1.40	89.55	46.36
	clDice	1	1	82.06	70.12	499.4	78.80	83.16	34.72	90.03	1.31	90.73	43.65
	cbDice	1	1	82.19	71.04	476.5	79.36	82.29	37.34	90.21	1.43	89.51	46.49
	cbDice	1	2	81.88	69.96	469.0	78.89	82.86	38.38	90.56	1.29	90.70	48.37
	cbDice	1	3	81.59	68.62	477.5	77.54	83.09	38.85	90.24	1.27	90.67	47.52
NexToU	—	0	0	81.33	64.80	387.7	75.35	56.33	0	89.93	—	67.79	0
	—	1	0	85.08	79.39	288.5	85.49	84.52	42.37	90.44	0.99	92.30	51.82
	clDice	1	1	84.88	79.27	274.3	85.23	84.30	43.90	90.19	0.66	92.02	53.31
	cbDice	1	1	85.19	80.07	248.95	85.88	84.32	44.51	90.72	0.68	92.18	55.62
	cbDice	1	2	85.22	79.72	216.2	85.56	84.37	47.39	90.45	0.65	92.37	57.71
	cbDice	1	3	85.05	79.18	289.6	85.31	84.21	48.43	90.89	0.66	92.30	58.91

Time Comparisons:


Configuration	Trainer	Skeletonization Type	Epoch Time (s)
Default (CE_DC)	nnUNetTrainerNoMirroring_3d_fullres	N/A	70.3
CE_DC_CLDC	nnUNetTrainer_CE_DC_CLDC_NoMirroring_3d_fullres	Morphological (iter_=10)	86.9
CE_DC_CBDC	nnUNetTrainer_CE_DC_CBDC_NoMirroring_3d_fullres	Morphological (iter_=10)	92.6
CE_DC_CLDC	nnUNetTrainer_CE_DC_CLDC_NoMirroring_3d_fullres	Topology-Preserving	318.2
CE_DC_CBDC	nnUNetTrainer_CE_DC_CBDC_NoMirroring_3d_fullres	Topology-Preserving	324.0

2024, MICCAI, Centerline Boundary Dice Loss for Vascular Segmentation

TopCoW2024 (CTA and MRA, Deep Supervision, No Mirroring, 100epochs):

Network	Loss			Dice	Dice(Large)	Dice(Small)
	X	α	β			
nnU-Net	--	1	0	<u>72.88</u>	85.82	43.77
	clDice	1	1	72.62	83.89	<u>47.26</u>
	cbDice	1	1	73.57	<u>84.95</u>	47.98



 **TopCow**: **Top**ology-Aware Anatomical Segmentation
of the **C**ircle of **W**illis for CTA and MRA

Thanks for your attention!

<https://github.com/PengchengShi1220/cbDice>
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Code:

