# Assignment 2: Random number generator and numerical integrations

*Due on Jan. 31*

1. Consider the "midsquare" generator introduced on p.5 of the notes.

   (a) Continue with the example on p.5 to compute the first 20 values with $Z_0 = 7182$.

   (b) Compute the first 30 values with $Z_0 = 1009$.

   Based on the above two examples, what observations do you make for midsquare generator?

2. Consider the LCG with $m = 16$, $a = 5$, $c = 3$ and $Z_0 = 7$. Find its first 20 values. Does this LCG have a full cycle?

3. Consider the pseudorandom number generator defined by $x_1 = 23$, $x_2 = 66$, and

$$x_n = 3x_{n-1} + 5x_{n-2} \mod 100, \quad n \geq 3.$$

   Find its first 14 values.

4. Use simulation to approximate the following integrals. Compare your estimate with the exact answer if known.

$$(a)\ \int_0^1 e^{e^x} dx, \qquad (b)\ \int_{-2}^2 e^{x+x^2} dx, \qquad (c)\ \int_{-\infty}^{\infty} e^{-x^2} dx,$$

$$(d)\ \int_0^1 \int_0^1 e^{(x+y)^2} dy dx, \qquad (e)\ \int_0^{\infty} \int_0^x e^{-(x+y)} dy dx.$$

5. Use simulation to approximate $\mathrm{Cov}(U, e^U)$, where $U$ is uniform on (0,1). Compare your approximation with the exact answer.

6. Let $U$ be uniform on (0,1). Use simulation to approximate:

   (a) $\rho\left(U, \sqrt{1-U^2}\right)$.

   (b) $\rho\left(U^2, \sqrt{1-U^2}\right)$.