Tongji University
Department of Bridge Engineering

A novel seismic fragility analysis method using BOX-COX regression and Monte Carlo Sampling

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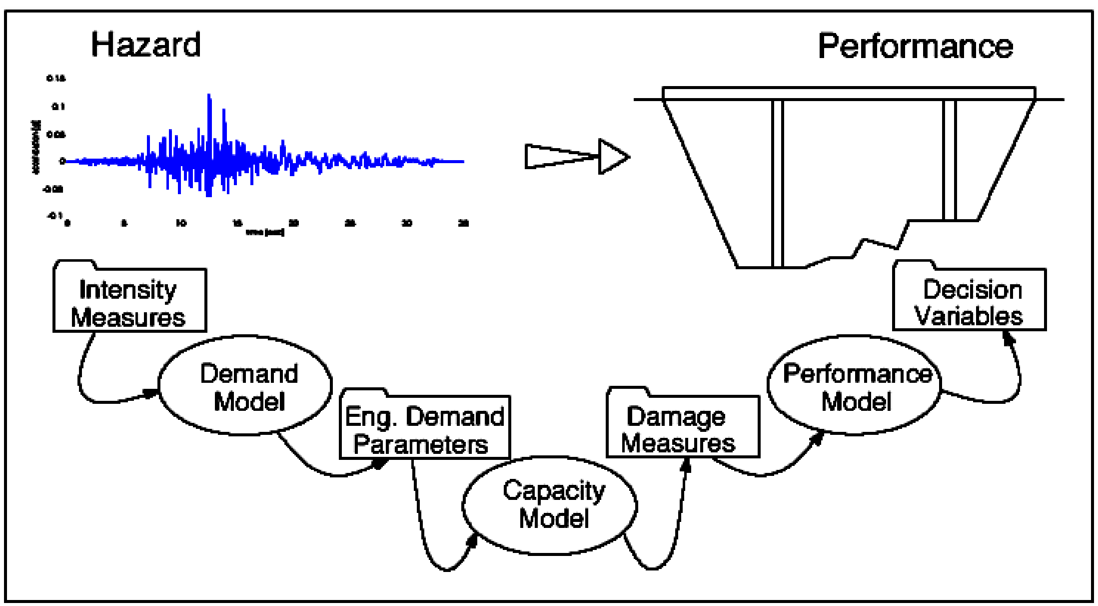
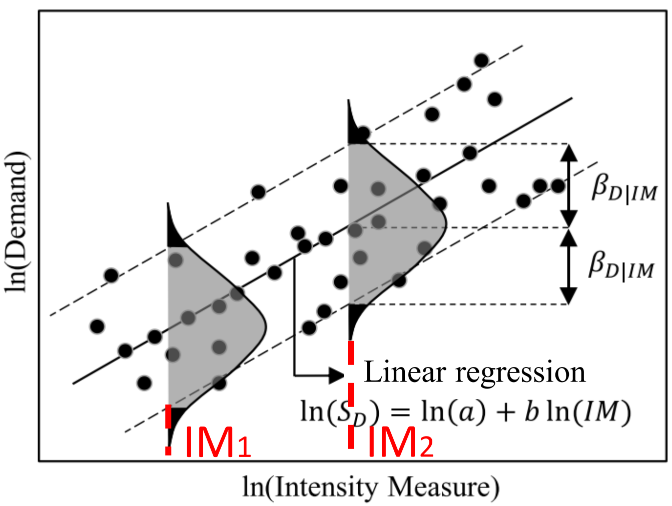
Background

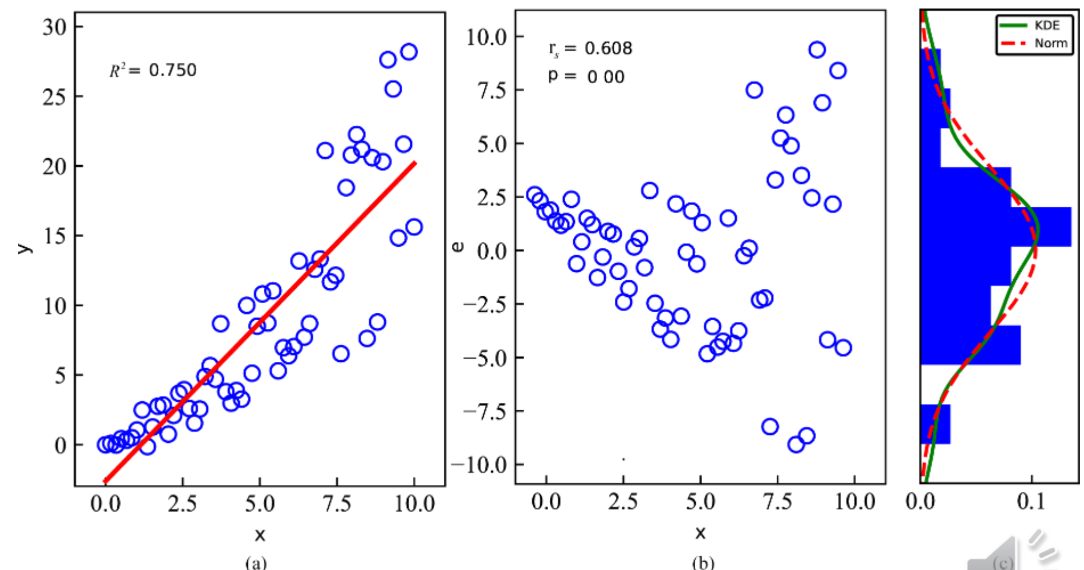
- The analytical fragility curve is one of the most popular tools to estimate potential seismic risk of bridges.
- During seismic fragility analysis, probabilistic seismic demand model (PSDM) needs to be developed. So that the relationship between ground motion intensity measure (IM) and engineering demand parameter (EDP) can be established.
- The cloud method has become one of fundamental approaches to build up PSDM owing to computational efficiency.



Background

- As a result of using linear regression, the cloud method relies on three assumptions: normality, linearity and homoscedasticity.
- But the assumptions are often inconsistent with the actual outcomes of the PSDM.
- It's essential to put forward a fragility analysis method which can be free from the aforementioned assumptions.





Introduction of BOX-COX regression

The least squares linear (LSL) regression model

$$y = \alpha_0 + \alpha_1 x + e$$

In 1964, Box and Cox proposed a method to handle nonlinearity, non-normality and heteroskedasticity by data transformation. It is written as

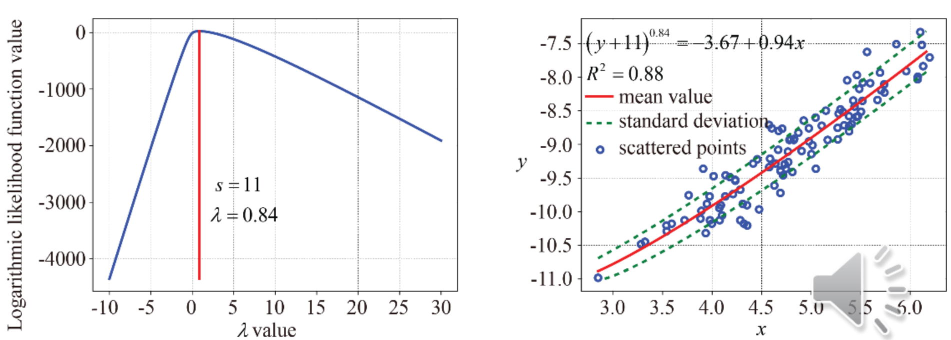
$$y^{(\lambda)} = \begin{cases} \dfrac{(y+s)^{\lambda} - 1}{\lambda}, & \lambda \neq 0 \\ \ln(y+s), & \lambda = 0 \end{cases}$$

Applying the BOX-COX transformation to the response variable of linear regression model leads to the Box-Cox regression model

$$y^{(\lambda)} = \alpha_0' + \alpha_1' x + e'$$

The optimal λ is determined by maximum likelihood estimation：

$$L(\lambda) = (\lambda - 1)\sum_{i=1}^{n} \ln(y_i) - \frac{n}{2}\ln\left\{\sum_{i=1}^{n} \frac{[y_i^{(\lambda)} - (\alpha_0' + \alpha_1' x_i)]^2}{n-2}\right\}$$



(a) (b)

Novel seismic fragility analysis method

Cloud Method

PSDM:

Least squares linear regression

$$\ln(D) = a + b\ln(IM) + e, \quad e \sim N(0, \beta_D^2)$$

Conditional probability:

$$P[DS_j \mid \mathrm{IM}] = \Phi\left[\frac{\ln(S_D) - \ln(S_C)}{\sqrt{\beta_D^2 + \beta_C^2}}\right]$$

$$\beta_D = \sqrt{\frac{\sum_{i=1}^{n}[\ln(D_i) - a - b\ln(IM_i)]^2}{n-2}}$$

Novel Method

Box-Cox regression

$$\ln(D)^{(\lambda)} = a' + b'\ln(IM) + e', \quad e' \sim N(0, \beta_D'^2)$$

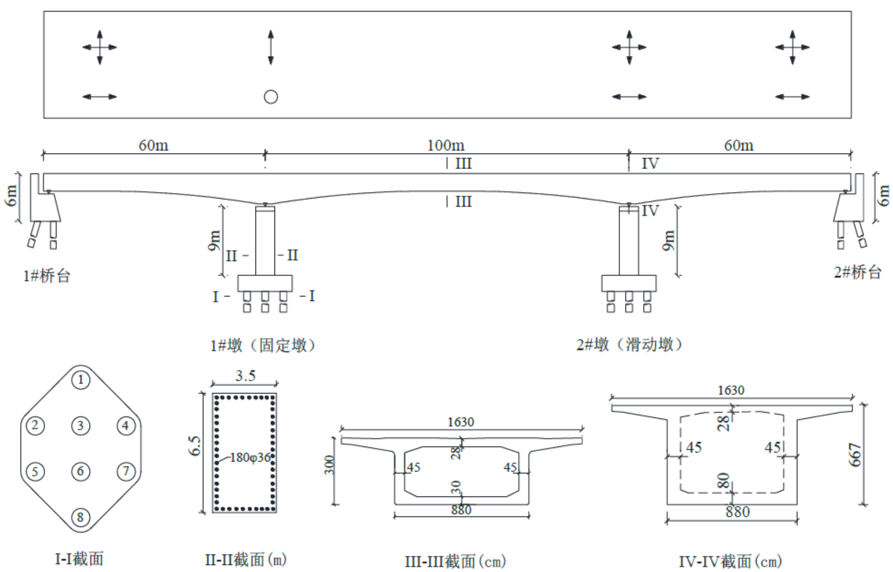
$$P[DS_j \mid IM] = \frac{\sum_{k=1}^{10^5} I[\ln(D_k) \geq \ln(C_k)]}{10^5}$$

$$I[\ln(D) \geq \ln(C)] = \begin{cases} 0, & \ln(D) < \ln(C) \\ 1, & \ln(D) \geq \ln(C) \end{cases}$$

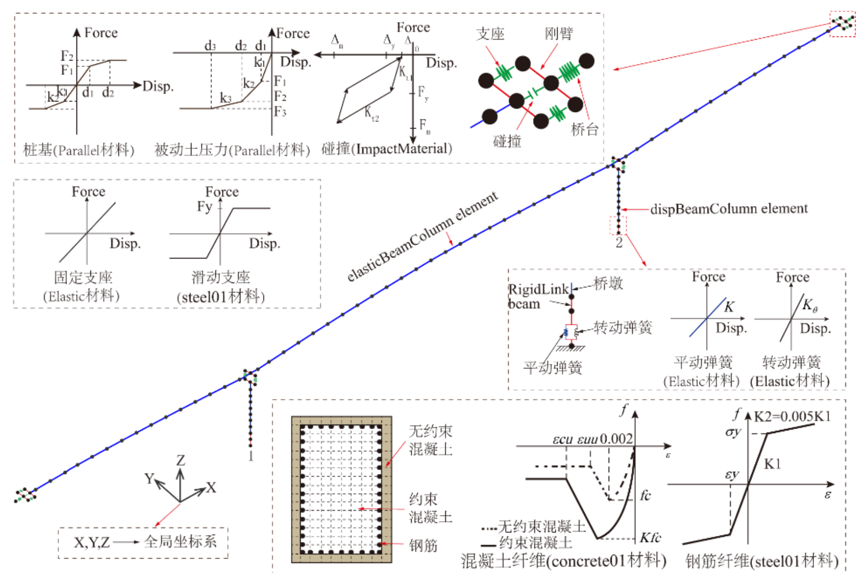
$$\ln(D) = \begin{cases} \left[\lambda \ln(D)^{(\lambda)} + 1\right]^{\frac{1}{\lambda}} - s, & \lambda \neq 0 \\ \exp\left[\ln(D)^{(\lambda)}\right] - \mathrm{s}, & \lambda = 0 \end{cases}$$

Case study

- A typical concrete continuous girder bridge with three spans of (60 m + 100 m+ 60 m) was selected as a case project.
- A detailed three-dimensional finite element model of the bridge was generated in OpenSees platform.



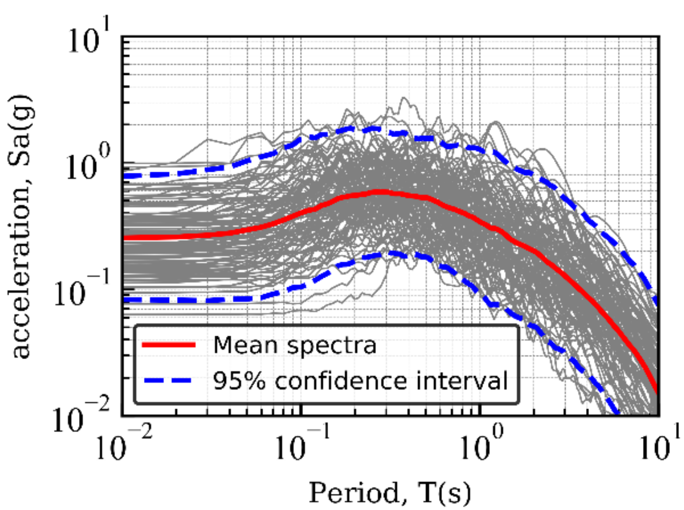
Case study bridge



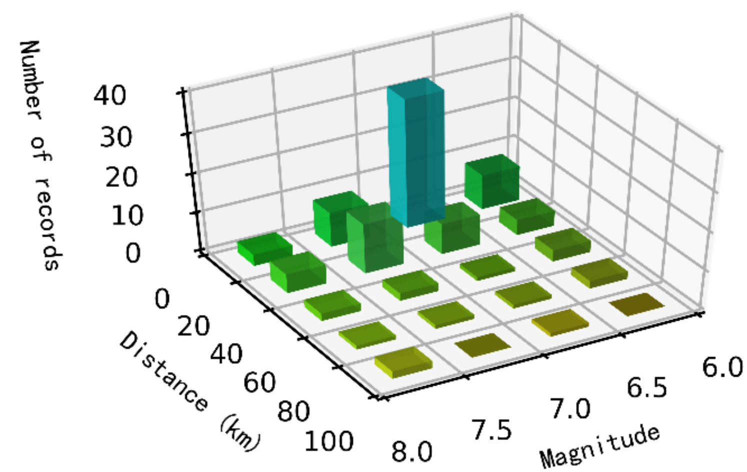
Numerical model of the prototype bridge

Case study

- The accuracy and reliability of fragility curves greatly depend on the ground motions used for nonlinear time history analyses.
- The algorithm proposed by Baker and Lee (2018) can select a set of ground motions from a database while meeting the probability distribution of spectral acceleration values generated by the ground motion prediction model (GMPM).



Acceleration response spectra (5% damping)

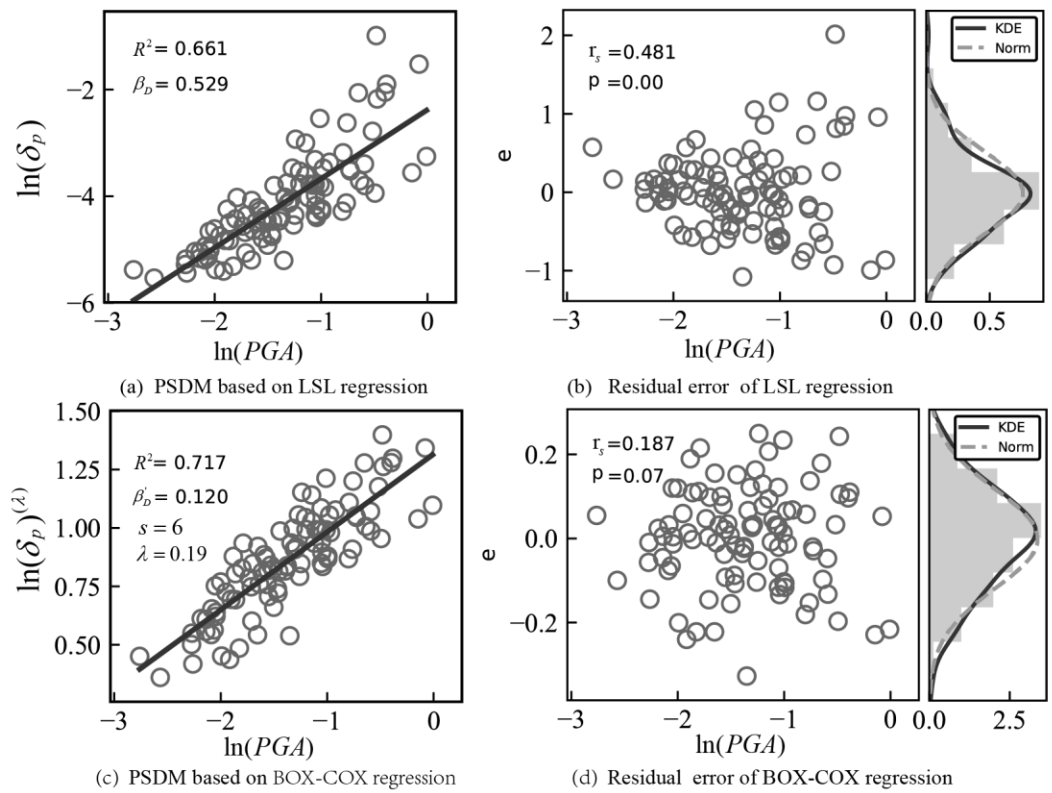


Statistics of magnitude and the closest distance

Case study

- Passive abutment displacement, active abutment displacement, bearing displacement and drift ratio of piers are selected as EDPs. Moreover, the peak ground acceleration (PGA) is considered as IM to decrease the dispersion of PSDM.
- To evaluate the normality of the residual error, the histogram of frequency distribution is plotted, kernel density estimation (KDE) curve and fitting normal distribution curve are obtained.
- Determination coefficient, which is square of linear correlation coefficient is adopted to evaluate the linearity of regression model.
- The rank correlation coefficient method is used to identify heteroscedasticity

$$r_s = 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} d_i^2$$



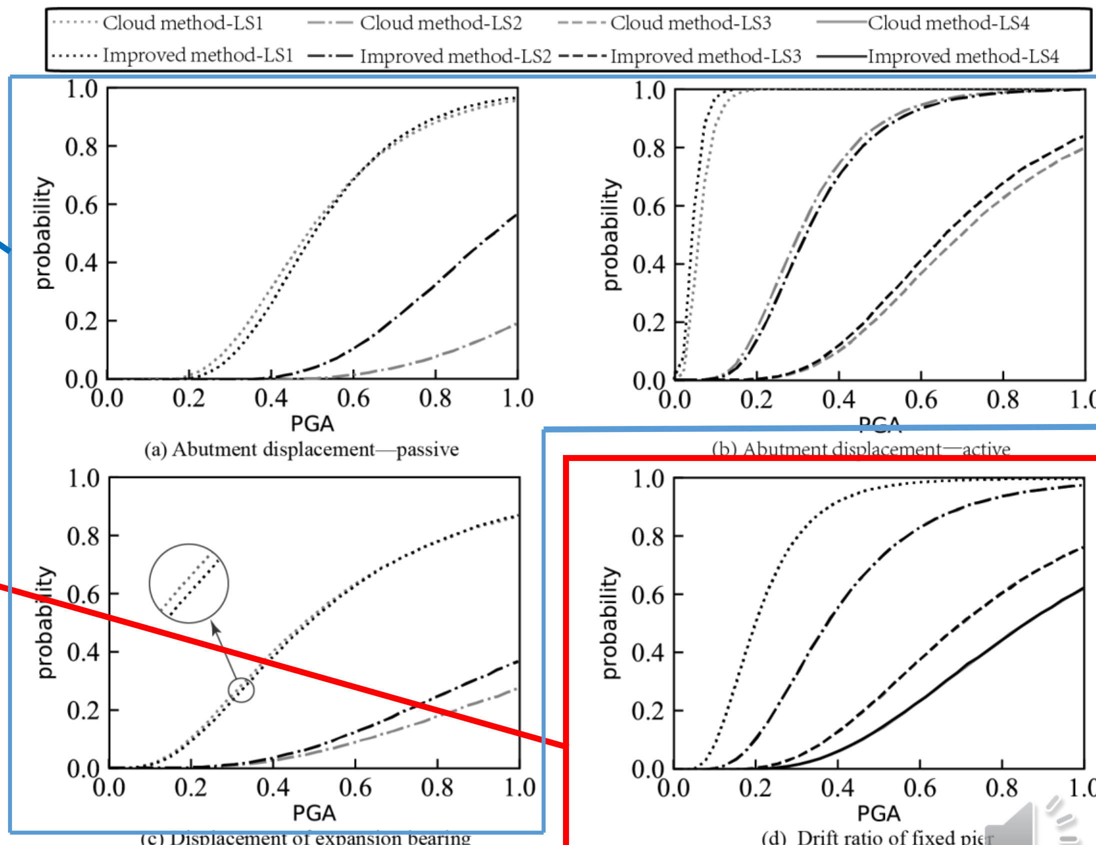
Comparison of LSL regression and BOX-COX regression

Case study

EDP	LSL regression			BOX-COX regression				
	R^2	r_s	p	a	λ	R^2	r_s	p
δ_p (m)	0.661	0.481	0.00	6	0.19	0.717	0.187	0.07
δ_a (m)	0.639	0.254	0.01	7	-0.29	0.651	0.153	0.13
δ_b (m)	0.543	0.184	0.07	4	0.58	0.544	0.044	0.67
D_r (%)	0.574	0.017	0.86	8	0.99	0.574	0.018	0.86

- **The structural fragility curves calculated using the cloud method may lead to errors when the PSDM shows heteroscedasticity, and the errors increase with the increasing damage degree.**

- When the cloud method satisfies all three assumptions, identical fragility curves are obtained using both methods.



Comparison of component fragility curves

Thanks！

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