Solution to Q4, Question sheet 3

From Q3, we know

$$\mathbf{K}=\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}=\begin{bmatrix} 5 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}\times 10^7 \text{ N/m}$$

$$\mathbf{M}=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\times 10^5 \text{ kg}$$

a) Consider only the 1st mode increasing linearly with the height [Refer to Q3 a]

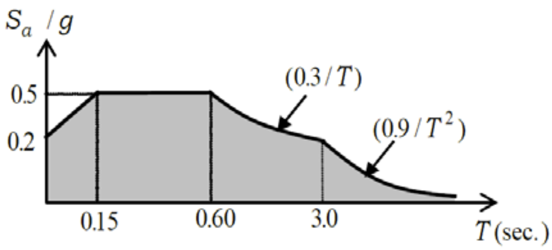
The approximation to 1st mode:

Modal shape: $\mathbf{v}_1=[1\ 2\ 3]^T$

Natural frequency: $\omega_1 = 5.94$ 1/s

Period: $T_1=2\pi/\omega_1=1.058$**s**

Modal mass: $\overline{m}_1 = \mathbf{v}_1^T\mathbf{M}\mathbf{v}_1 = 1.7\times 10^5$ kg



(1) $S_a(T_1)$ and $S_u(T_1)$

From the Eurocode 8 diagram given for the question:

$S_a(T_1)=0.3/T_1 * g=2.836$ ($g=10$ ms^{-2})

$S_u(T_1)= S_a(T_1)/\ \omega_1^2 =0.08$

(2) Load participation factor: l_1

$$l_1 = \mathbf{v}_1^T\mathbf{Md}/\overline{m}_1$$

where: Earthquake input direction vector: $\mathbf{d}=[1\ \ 1\ \ 1]^T$

$$\mathbf{Md}=[2\ \ 1.5\ \ 1]^T\times 10^5$$

So: $l_1 = 8/17$

(3) Maximum displacement $U_{1,max}$

$$\mathbf{U}_{1,\max} = l_1 S_a(\mathrm{T}_1)\mathbf{v}_1 = \{0.038\ 0.076\ 0.114\}^{\mathrm{T}}\ \ (\mathrm{m})$$

(4) Base shear force, Floor forces, Overturning moment (Following Eurocode 8)

The total mass of the structure, $m=(2+1.5+1)\times10^5$ kg

$V_b=m\ S_a(T_1)=12.8\times10^5$ N

The heights of the three floors: $\{Z_1, Z_2, Z_3\}=\{3,6,9\}$m

The floor forces:

$$\mathrm{F_i}=\mathrm{V_b}\frac{\mathrm{m_i z_i}}{\sum_{i=1}^{3}\mathrm{m_i z_i}} \quad \text{where} \quad \sum_{i=1}^{3}\mathrm{m_i z_i} = 24\times 10^5$$

So: $\{F_1, F_2, F_3\}=\{3.2, 4.8, 4.8\}\times10^5$ N

Overturning moment $M_b= F_1\times Z_1+ F_2\times Z_2 + F_3\times Z_3 = 81.6\times10^5$ Nm

b) Consider all the three modes

Use the following Matlab code to find out the natural frequencies and modal shapes:

```
K=[ 5 -2 0; -2 3 -1; 0 -1 1]*1e7;  %Stiffness matrix
M=[2 0 0; 0 1.5 0; 0 0 1]*1e5;     %Mass matrix
[EV,w2]=eig(K,M);                  %Function to solve eigenvalue problem
W=sqrt(w2)                         %Natural frequencies (diagonals)
EV(:,1)=EV(:,1)/EV(1,1)            %Scale the modal shapes
EV(:,2)=EV(:,2)/EV(1,2)
EV(:,3)=EV(:,3)/EV(1,3)
```

Note: W – Natural frequencies; EV – Modal shapes

Mode No.	**1**	**2**	**3**
Natural Freq. ω (1/s)	**5.928**	**12.68**	**18.82**
Period T (s)	**1.06**	**0.5**	**0.333**
Modal Shape V^T	**{1, 2.149, 3.313}**	**{1, 0.893, -1.473}**	**{1, -1.042, 0.410}**
Modal Mass $\bar{m}$ (kg)	**19.9×10^5**	**5.366×10^5**	**3.796×10^5**
$S_a(T)$	**0.3/T*g=2.83**	**0.5*g=5**	**0.5*g=5**
$S_u(T)= S_a(T)/\omega^2$	**0.081**	**0.0311**	**0.0141**
Load participation factor: $l_i = \mathbf{v}_i^T\mathbf{M}\mathbf{d}/\bar{m}_i$	**0.4289**	**0.3480**	**0.2231**
Max displacement Vector (m) $U_{i,max}= l_i S_u(T_i)\mathbf{v}_i$	**{0.0345,0.0742,0.1144}**	**{0.0108,0.0097, -0.0159}**	**{0.0031, -0.0033,0.0013}**
Total max displacement vector U_{max} (using SRSS)	$\mathbf{U}_{max} = \sqrt{\mathbf{U}_{1,max}^2 + \mathbf{U}_{2,max}^2 + \mathbf{U}_{3,max}^2} = \{0.036,\ 0.079,\ 0.116\}$ **(m)**		
Max floor force Vector $F_{i,max}= l_i S_a(T_i)\mathbf{M}\mathbf{v}_i$	**$\{2.43,3.91,4.02\}\times10^5$ N**	**$\{3.48,2.33,-2.58\}\times10^5$ N**	**$\{2.23,-1.74,0.46\}\times10^5$ N**
Total max force F_{max} (using SRSS)	$\mathbf{F}_{max} = \sqrt{\mathbf{F}_{1,max}^2 + \mathbf{F}_{2,max}^2 + \mathbf{F}_{3,max}^2} = \{4.79,\ 4.88,\ 4.79\}\times10^5$ **N**		
Base shear force V_b	$V_b = \mathbf{F}_{max,1} + \mathbf{F}_{max,2} + \mathbf{F}_{max,3} = 14.46\times10^5$ **N**		
Overturning Moment M_b	**$M_b= F_{1,max}\times Z_1+ F_{2,max}\times Z_2 + F_{3,max}\times Z_3=86.76\times10^5$ Nm**		