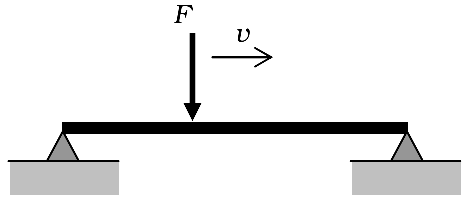
DYNAMICS OF STRUCTURES

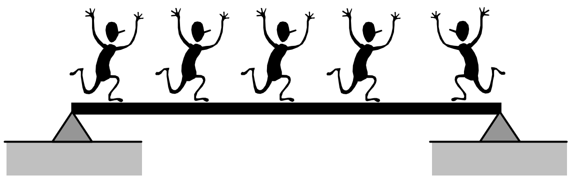
Example Sheet No. 2

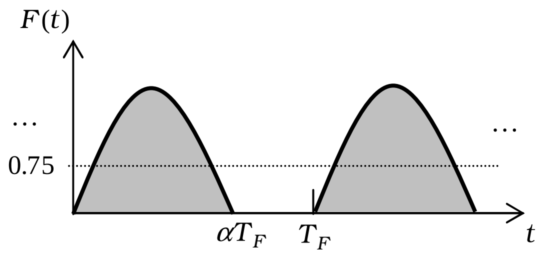
(For steel take E=200 GN/m^2 and for concrete E=14 GN/m^2)

1- A point load $F = 1\,\mathrm{kN}$ moves along with constant speed $\upsilon = 10$ m/s on a simply supported beam of length $l = 10\pi$ m as shown in the figure. The beam is made of concrete, has a rectangular section of height 1 m and an average density of 2,800 kg/m^3. Determine the deflection of the beam as a function of time, the dynamic magnification factor and the maximum bending moment at centre section.



2- A concrete ribbed slab floor spans 9 m and has an average mass of 500 kg/m^2. The floor is simply supported on either side and has a natural frequency of vibration of 6.3 Hz. The floor is to be used for aerobics and other similar rhythmic activities at frequencies ranging from 1.5 Hz to 2.5 Hz and with contact ratios α between 0.5 and 1. During these activities the average imposed load will remain below 0.75 kN/m^2 (before dynamic magnification) and the damping ratio can be taken to be 3%.

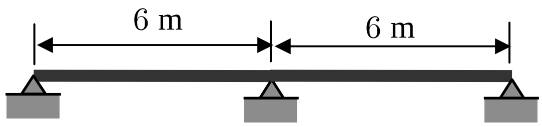




(a) Determine the maximum possible resonant displacement and the resulting peak acceleration and bending moment per unit width.

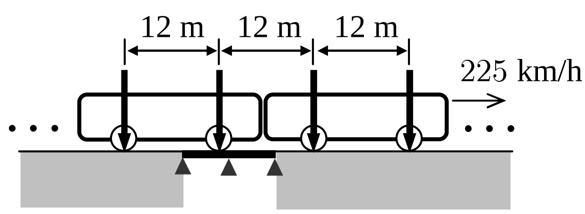
(b) If the floor has been designed for a service load of 5kN/m^2, determine its suitability for the proposed use.

3- A 12 m long rail bridge has the continuous beam configuration shown in the figure, an average flexural stiffness $EI = 5\,\mathrm{GNm}^2$, a mass per unit length of 10,0000 kg/m and a damping ratio of 2%.



(a) Using Rayleigh's method with an appropriate sinusoidal function, obtain its fundamental frequency of vibration.

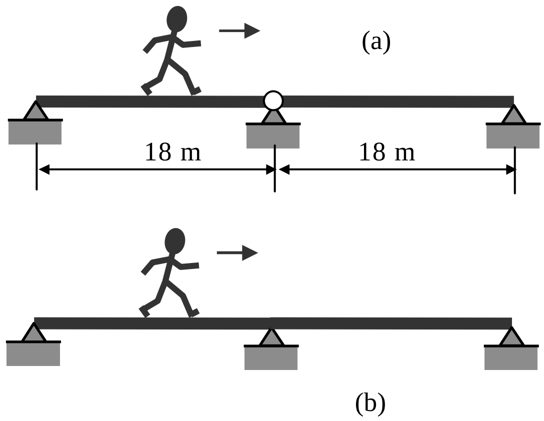
(b) Determine the equivalent modal force that results from the motion of a train consisting of an 'infinite' number of $100\,\mathrm{kN}$ point loads separated by equal distances of 12 m and



travelling at a constant speed of 225 km/h.

(c) Obtain the dynamic magnification factor for the above train of loads.

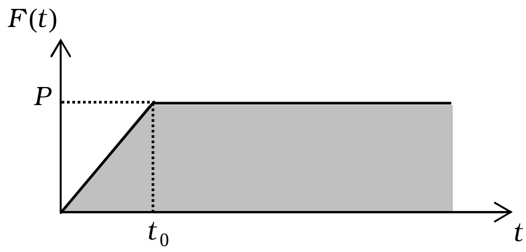
4- A footbridge spans 36 m in two equal simply supported sections as shown. The total mass of each section is 12 600 kg, the equivalent uniform flexural stiffness EI is $2\ 10^8$ N m^2 and the logarithmic decrement damping is 0.05.



(a) Determine the peak displacement and peak acceleration produced by a 700 N pedestrian walking in step with the natural frequency of the footbridge and producing a pulsating force of 180 N.

(b) Consider the case where the two sections of the footbridge are connected together to produce a continuous beam.

5- A load is applied on a structure in such a way that its magnitude increases linearly with time until a maximum value P is reached at time t_0 (see figure). Determine the dynamic magnification factor as a function of t_0 and the natural period of vibration of the structure T.



6- The infinitely long UDL w shown in the figure moves along at constant speed v and enters a simply supported bridge of length ℓ, total mass $2m$ and natural period of vibration T. Determine the resulting vibration and the dynamic magnification factor.