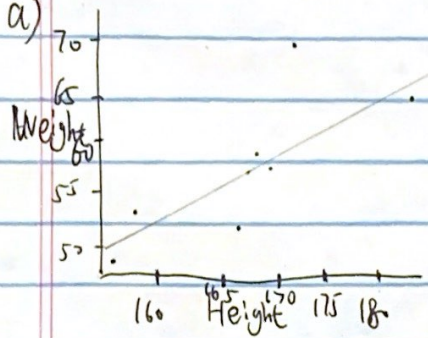
Q2

a)




b) Yes, it appears a linear trend

c) $\bar{x} = \frac{1}{8}(169.6 + 166.8 + 181.1 + 158.4 + 165.6 + 166.7 + 156.5 + 168.1) = 166.6$

$\bar{y} = \frac{1}{8}(71.2 + 58.2 + 64.5 + 53 + 52.4 + 56.8 + 49.2 + 57.9) = 57.9$

$SS_{xx} = \sum_{i=1}^{8} (x_i - \bar{x})^2 = (169.6 - \bar{x})^2 + (166.8 - \bar{x})^2 + (181.1 - \bar{x})^2 + (158.4 - \bar{x})^2 + \dots + (168.1 - \bar{x})^2$

$= 391.8$

$SS_{xy} = \sum_{i=1}^{8} (x_i - \bar{x})(y_i - \bar{y}) = (169.6 - \bar{x})(71.2 - \bar{y}) + (166.8 - \bar{x})(58.2 - \bar{y}) + \dots$

$+ (168.1 - \bar{x})(57.9 - \bar{y}) = 269.1$

(d) $\hat{\beta}_1 = SS_{xy} / SS_{xx} = 269.1 / 391.8 \approx 0.68683$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 57.9 - 269.1/391.8 \cdot 166.6 = -56.526$

(e)

(f) for 1cm increase in height, we would estimate an average increase of 0.68683 kg.

(g) $\hat{\beta}_0$ doesn't have a meaningful interpretation. Since height can't be zero.

Q3

a) $e_i = y_i - \hat{y}_i = y_i - (\hat{\beta}_1 x_i)$

$RSS = \sum e_i^2 = \sum (y_i - \hat{\beta}_1 x_i)^2$

b) $\frac{\partial}{\partial \beta_1} RSS = 0$

$\frac{\partial}{\partial \beta_1} [\sum (y_i^2 - 2 y_i \hat{\beta}_1 x_i + \hat{\beta}_1^2 x_i^2)]$

$= \frac{\partial}{\partial \beta_1} [\sum y_i^2 - 2 \sum y_i \hat{\beta}_1 x_i + \sum \hat{\beta}_1^2 x_i^2]$

$\therefore \frac{\partial}{\partial \beta_1} RSS = 0$

$\frac{\partial}{\partial \beta_1} [\sum (y_i - \hat{\beta}_1 x_i)^2] = 0$

$2 \sum (y_i - \beta_1 x_i)(-x_i) = 0$

$\sum x_i y_i - \sum \beta_1 x_i^2 = 0$

$\beta_1 \sum x_i^2 = \sum x_i y_i$

$\frac{1}{n}(\beta_1 \sum x_i^2) = \frac{1}{n} \sum x_i y_i$

$\beta_1 (\sum x_i^2 / n) = \bar{x} \bar{y}$

$\beta_1 = \frac{\sum x_i y_i}{\sum x_i^2} = \sum \frac{y_i}{x_i}$

c) $E(\hat{\beta}_1) = E(\sum \frac{y_i}{x_i}) = \sum E(y_i/x_i) = \sum E(y_i)/E(x_i) = \sum \beta_1 x_i / x_i = \beta_1$

Unbiased

Question 4

a) True

b) False, Corrections: If height is fixed, for a 1kg increase in weight, we would estimate an average increase of 0.37 $kg/m^2$ in BMI

c) False; An estimator is unbiased if its expected value is equal to the true value of the parameter.

Q5

a) $E(Y|x) = 22991.85 + 40.61 \cdot X_{OPBPC} + 1867.46 \cdot X_{OPRC} + 2353.96 X_{OPSLAKE}$

b) $43 \times 4$

c) OPRC and OPSLAKE

d) $\hat{y} = 22991.85 + 40.61 \times 26.47 + 1867.46 \times 15.33 + 2353.96 \times 26.46 = 114980.740$

Residual $= |118144 - 114980.740| = 3163.2599$ (acre-feet)