

a. Result comparison of linear case.

DoFs: Degree of Freedoms Nol: Number of Iterations Tpl: Time per Iteration							
Number of DoFs	Comsol MUMPS	KLU	cuSPARSE	Error:	Vectorized FEM		
					10%	1%	0.1%
3,700	230ms	15ms	50ms	Nol:	96	256	430
				Tpl:		31us	
				Total:	3ms	7.9ms	13.3ms
7,100	520ms	42ms	135ms	Nol:	184	478	781
				Tpl:		31us	
				Total:	5.7ms	14.7ms	24.1ms
18,000	900ms	120ms	380ms	Nol:	462	1185	1934
				Tpl:		70us	
				Total:	32ms	83ms	135ms
				Est:	14ms	37ms	60ms
36,000	2.0s	326ms	1.3s	Nol:	920	2397	3924
				Tpl:		170us	
				Total:	156ms	407ms	667ms
				Est:	28ms	75ms	122ms

b. Result comparison of nonlinear case.

DoFs: Degree of Freedoms Nol: Number of Iterations Tpl: Time per Iteration						
Number of DoFs	Comsol MUMPS	Error:	Vectorized FEM			Speedup @ Er=0.1%
			10%	1%	0.1%	
3,700	1.3s	Nol:	100	345	990	20
		Tpl:		65us		
		Total:	6.5ms	22ms	64ms	
7,100	3s	Nol:	173	589	1145	40
		Tpl:		65us		
		Total:	11ms	38ms	75ms	
18,000	7s	Nol:	482	1535	2631	16
		Tpl:		165us		
		Total:	80ms	253ms	434ms	
		Est:	31ms	100ms	171ms	
36,000	13s	Nol:	894	2856	4890	4
		Tpl:		340us		
		Total:	304ms	974ms	3.4s	
		Est:	58ms	186ms	318ms	

The case study was carried out on the Tesla V100 accelerating card, which has 5120 Cuda cores. The results show that when the DoFs is less than 7100, the consumed time for each iteration is fixed, determined by the sub-domain solver. For larger case with more DoFs, each Cuda core will be assigned multiple sub-domains to solve, which must be done in sequential and thus cause an increase of execution time. In this case, although the algorithm has massive parallelism, the limited hardware resource is the bottleneck.

The 'Est:' row highlighted in red is the estimated time or speedup when the hardware resource is not a problem, for example, in a high-performance computing environment that has cluster of workstations with each workstation equipped with 6 Tesla V100 accelerating card.

Also, it is noticeable that the number of iterations required increase linearly with the problem size, meaning the solution time also has a linear increase with the DoFs provided that the hardware resource is enough.