Math 494

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Proof.

1. Fix $P \in \mathrm{Spec}(R)$ and $X - V(E)$ open set containing P. We want to show that $\exists f \in R\, s.t.\ P \in X_f \subset X - V(E)$.

   $P \in X - V(E) \implies \exists f \in E\, s.t.\ f \notin P \implies P \in X_f$ and $f \in E, \forall P \in F(U), X$

2. We take the basis $\{X_{f_i}\}_{i\in I}$ be a cover of X. $\forall P \in \mathrm{Spec}(R), \exists X_{f_i} \ni P \implies f_i \notin P$.

   By lemma in class (every non (1) ideal is contained in a maximal ideal) we have $(\{f_i\}_{i\in I}) = R \ni 1 \implies \exists J \subset I$ finite, $g_i \in R\, s.t.\ \sum_{j\in J} g_j f_j = 1$.

   So $(\{f_j\}_{j\in I}) = R$. This forms a finite subcover.

3. Suppose $\{P\}$ is closed $\implies \{P\} = V(E), E \subset R \implies P$ is the only prime containing $E \implies P$ is maximal.

   Suppose M is a maximal ideal, then $\{M\} = V(M)$, then it is closed by assumption. ■

Proof.

1. Suppose $f, g \in \phi^{-1}(P), a, b \in R, \phi(af + bg) = \phi(a)\phi(f) + \phi(b)\phi(g) \in P$. Preimage is an ideal.

   To show that it is prime, suppose $fg \in \phi^{-1}(p) \implies \phi(fg) = \phi(f)\phi(g) \in P$. So either $\phi(f) \in P$ or $\phi(g) \in P$.

2. Say $Q \in Y_{\phi(f)} \iff \phi(f) \notin Q \iff f \notin \phi^{-1}(Q) = \phi^*(Q) \iff \phi^*(Q) \in X_f \iff Q \in \phi^*(X_f)$.

3. $f \in (\psi \circ \phi)^*(P) \iff (\psi \circ \phi)(f) \in P \iff \psi(\phi(f)) \in P \iff \phi(f) \in \phi^*(P) \iff f \in (\phi^* \circ \psi^*)(P)$. ■