COT 3100 Fall 2024 Homework #3
Please Consult WebCourses for the due date/time

1) (5 pts) State the quotient and remainder for the following division operations when dividing a by b:

a) a = 1234, b = 123
b) a = 6898, b = 10000
c) a = 374, b = 11
d) a = 888, b = 37
e) a = 2024, b = 32

2) (4 pts) Use the cycle method to calculate the remainder (by hand) when dividing a by b for each of the following problems:

a) $a = 3^{12767}$, b = 7
b) $a = 17^{1000000,}$ b = 32

3) (4 pts) Use the method of fast modular exponentiation (bottom up) to find the remainders when dividing a by b for each of the following problems:

a) $a = 2^{45}$, b = 23
b) $a = 3^{27}$, b = 29

4) (5 pts) Let a and b be integers such that 17 | (4a + 7b). Prove that 34 | (58a + 8b).

5) (5 pts) Determine, with proof, all ordered pairs of integers (x, y) which satisfy the equation

$$228x + 589y = 16657.$$

6) (15 pts)

   (a) Find all integer solutions to the equation 193x + 85y = 1.

   (b) Find all integer solutions to the equation 193x + 85y = 8.

   (c) Find 85^{-1} mod 193.

7) (4 pts) Let $a = 2^6 3^3 5^4 7^2$, $b = 2^3 3^7 5^1 11^6$, and $c = 2^7 3^5 5^{10} 11^5$. Determine, in prime factorized form, both gcd(a, b, c) and lcm(a, b, c).

8) (3 pts) For the numbers a, b and c listed in problem 7, determine the number of divisors each of those numbers has.

9) (5 pts) Let X = 1250! (1250 factorial). If we were to represent X in base 12, how many zeroes would that number end in?