Домашнее задание "Приложения квадратичных форм" ИУ6-22Б Лыгяев Иван

Вариант 19.

а) $5x^2+8y^2-4xy+16\sqrt{5}x+8\sqrt{5}y-44=0$

$F(x,y)=5x^2+8y^2-4xy$

$A=\begin{pmatrix}5 & -2\\ -2 & 8\end{pmatrix}$

1) $|A-\lambda E|=0$

$\begin{vmatrix}5-\lambda & -2\\ -2 & 8-\lambda\end{vmatrix}=\lambda^2+40-13\lambda-4=\lambda^2-13\lambda+36$

$\left[\begin{array}{l}\lambda_1=9\\ \lambda_2=4\end{array}\right.$

2) $\lambda_1=9$

$\begin{pmatrix}-4 & -2\\ -2 & -1\end{pmatrix}_{II\cdot 2-I}\sim\begin{pmatrix}2 & 1\\ 0 & 0\end{pmatrix}$

$Rg=1,\Rightarrow 1$ решение

x - базис

y - свободн.

$\begin{cases}8x+1=0\\ y=1\end{cases}\Rightarrow\quad x=-\frac{1}{2};\ y=1$

$E\begin{pmatrix}-\frac{1}{2}\\ 1\end{pmatrix}=E\begin{pmatrix}1\\ -2\end{pmatrix}$ $\qquad\|e_0\|=\sqrt{5}\Rightarrow e_0=\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{2}{-\sqrt{5}}\end{pmatrix}$

$\lambda_2=4$

$\begin{pmatrix}1 & -2\\ -2 & 4\end{pmatrix}\sim\begin{pmatrix}1 & -2\\ 0 & 0\end{pmatrix}$

$Rg=1$, 1 решение $\quad\begin{cases}y=1\\ x-2=0\end{cases}\Rightarrow\begin{cases}y=1\\ x=2\end{cases}$

x - базис. y - своб

$E\binom{2}{1} \Rightarrow e_0 = \begin{pmatrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \end{pmatrix}$

3) $P = \begin{pmatrix} \frac{1}{\sqrt5} & \frac{2}{\sqrt5} \\ -\frac{2}{\sqrt5} & \frac{1}{\sqrt5} \end{pmatrix}$

$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt5} & \frac{2}{\sqrt5} \\ -\frac{2}{\sqrt5} & \frac{1}{\sqrt5} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \quad x = 1/\sqrt5 (z_1 + 2z_2)$

$y = \frac{1}{\sqrt5}(-2z_1 + z_2)$

$\sqrt5 x = z_1 + 2z_2$

$\sqrt5 y = -2z_1 + 2z_2$

$9z_1^2 + 4z_2^2 + 16(z_1 + 2z_2) + 8(-2z_1 + z_2) - 44 = 0$

$9z_1^2 + 4(z_2^2 + 2\cdot5\cdot z_2 + 25 - 25 - 11) = 0$

$9z_1^2 + 4(z_2+5)^2 = 144$

$\frac{z_1^2}{16} + \frac{(z_2+5)^2}{36} = 1$ - эллипс

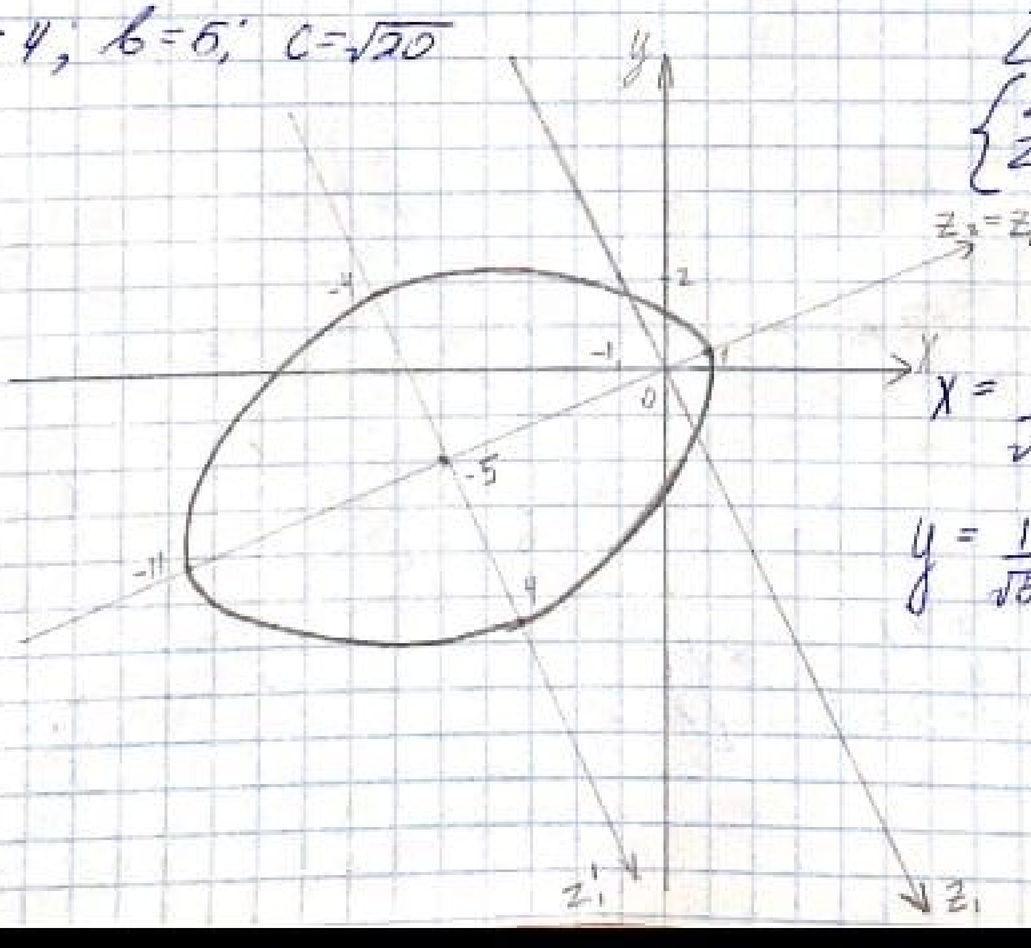
$\begin{cases} x = \frac{1}{\sqrt5}(z_1 + 2z_2) \\ y = \frac{1}{\sqrt5}(-2z_1 + z_2) \end{cases}$

$O(0;-5)$

$a=4;\ b=6;\ c=\sqrt{20}$

$\begin{cases} z_1 = z_1' \\ z_2 + 5 = z_2' \end{cases}$

$\begin{cases} z_1 = z_1' \\ z_2 = z_2' - 5 \end{cases}$



$x = \frac{1}{\sqrt5}(z_1' + 2z_2' - 10)$

$y = \frac{1}{\sqrt5}(-2z_1' + z_2' - 5)$

б) $4y^2+x^2+4xy-2\sqrt{5}x+6\sqrt{5}y-5=0$

$F(x,y)=4y^2+x^2+4xy$

$A=\begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix}$

1) $|A-\lambda E|=0$

$\begin{vmatrix}1-\lambda & 2\\ 2 & 4-\lambda\end{vmatrix}=\lambda^2-5\lambda+4-4=\lambda^2-5\lambda$

$\left[\begin{array}{l}\lambda_1=0\\ \lambda_2=5\end{array}\right.$

2) $\lambda_1=0$

$\begin{pmatrix}4 & 2\\ 2 & 1\end{pmatrix}\begin{array}{l}I/2\\ II-\frac{1}{2}I\end{array}\sim\begin{pmatrix}2 & 1\\ 0 & 0\end{pmatrix}$

$Rg=1$, $\Rightarrow$ 1 решение

x - базис. y - своб.

$\begin{cases}y=1\\ 2x+1=0\end{cases}$ $\begin{cases}y=1\\ x=-\frac{1}{2}\end{cases}$

$E\begin{pmatrix}-\frac{1}{2}\\ 1\end{pmatrix}=E\begin{pmatrix}1\\ -2\end{pmatrix}$ $\quad \|e_0\|=\sqrt{5}\Rightarrow e_0=\begin{pmatrix}1/\sqrt{5}\\ -\frac{2}{\sqrt{5}}\end{pmatrix}$

$\lambda_2=5$

$\begin{pmatrix}-1 & 2\\ 2 & -4\end{pmatrix}\sim\begin{pmatrix}1 & -2\\ 0 & 0\end{pmatrix}$

$Rg=1$, $\Rightarrow$ 1 решение

x - базис. y - своб.

$\begin{cases}y=1\\ x-2=0\end{cases}\Rightarrow\begin{cases}y=1\\ x=2\end{cases}$

$E\begin{pmatrix}2\\ 1\end{pmatrix}$ $\quad \|e_0\|=\sqrt{5}\Rightarrow e_0=\begin{pmatrix}\frac{2}{\sqrt{5}}\\ 1/\sqrt{5}\end{pmatrix}$

3) $P = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$

$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = P \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{cases} x = \frac{1}{\sqrt{5}}(z_1 + 2z_2) \\ y = \frac{1}{\sqrt{5}}(-2z_1 + z_2) \end{cases}$

$\begin{cases} \sqrt{5}x = (z_1 + 2z_2) \\ \sqrt{5}y = (-2z_1 + z_2) \end{cases}$

$0 \cdot z_1^2 + 5z_2^2 - 2(-2z_1 + z_2) + 6(z_1 + 2z_2) - 5 = 0$

$5z_2^2 + 4z_1 - 2z_2 + 6z_1 + 12z_2 - 5 = 0$

$5z_2^2 + 10z_1 + 10z_2 - 5 = 0$

$5(z_2^2 + 2z_1 + 2z_2 - 1) = 0$

$5((z_2+1)^2 + 2z_1 - 2) = 0$

$5(z_2+1)^2 = -10z_1 + 10$

$(z_2+1)^2 = -2z_1 + 2$ – парабола

$(z_2+1)^2 = 2(-z_1+1) = y$

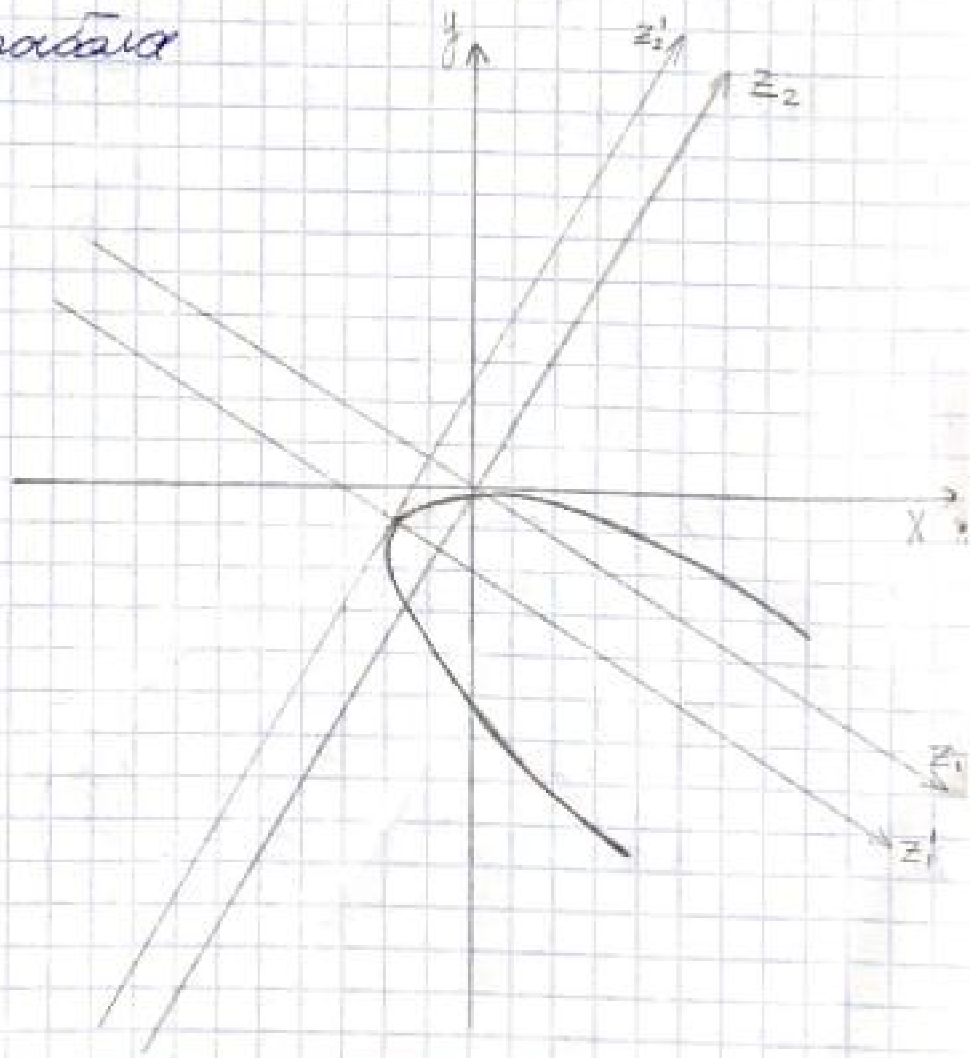
$\begin{cases} z_2' = z_2 + 1 \\ z_1' = z_1 - 1 \end{cases}$

$(z_2')^2 = 2z_1'$

$\begin{cases} z_1 = -z_1' + 1 \\ z_2 = z_2' - 1 \end{cases}$

$\begin{cases} x = \frac{1}{\sqrt{5}}(-z_1' + 1 + 2z_2' - 2) \\ y = \frac{1}{\sqrt{5}}(2z_1' - 2 + z_2' - 1) \end{cases}$

$\begin{cases} x = \frac{1}{\sqrt{5}}(-z_1' + 2z_2' - 1) \\ y = \frac{1}{\sqrt{5}}(2z_1' + z_2' - 3) \end{cases}$



c) $5x^2+5y^2-2z^2+4xy+6x-6y+4z+46=0$

$F(x,y,z)=5x^2+5y^2-2z^2+4xy+0yz+0xz$

$$A=\begin{pmatrix}5&2&0\\2&5&0\\0&0&-2\end{pmatrix}$$

1) $|A-\lambda E|=0$

$$\begin{vmatrix}5-\lambda&2&0\\2&5-\lambda&0\\0&0&-2-\lambda\end{vmatrix}=\lambda^3-8\lambda^2+\lambda+42=0 \Rightarrow \left[\begin{array}{l}\lambda_1=-2\\ \lambda_2=3\\ \lambda_3=7\end{array}\right.$$

2) $\lambda_1=-2$ $\begin{pmatrix}7&2&0\\2&7&0\\0&0&0\end{pmatrix}\sim\begin{pmatrix}7&2&0\\0&7&0\\0&0&0\end{pmatrix}$ $Rg=2$, 1 решение

x, y - базисные; z - свободный

$$\begin{cases}7x+2y=0\\2y=0\\z=1\end{cases}\Rightarrow\begin{cases}x=0\\y=0\\z=1\end{cases}\Rightarrow E\begin{pmatrix}0\\0\\1\end{pmatrix};\ \|e_{01}\|=1;\ e_{01}=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$\lambda_2=3$ $\begin{pmatrix}2&2&0\\2&2&0\\0&0&-5\end{pmatrix}\sim\begin{pmatrix}1&1&0\\0&0&1\\0&0&0\end{pmatrix}$ $Rg=2$, 1 решение

x, z - базисные; y - свободный

$$\begin{cases}y=1\\x+1=0\\z=0\end{cases}\Rightarrow\begin{cases}x=-1\\y=1\\z=0\end{cases}\Rightarrow E\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

$\|e_{02}\|=\sqrt{2}; \Rightarrow e_{02}=\begin{pmatrix}-1/\sqrt{2}\\1/\sqrt{2}\\0\end{pmatrix}$

$\lambda_3=7.$ $\begin{pmatrix}-2&2&0\\2&-2&0\\0&0&-9\end{pmatrix}\sim\ \sim\begin{pmatrix}1&-1&0\\0&0&1\\0&0&0\end{pmatrix}$ $Rg=2$, 1 решение

x, y - базисные; y - свободная

$$\begin{cases}x-1=0\\z=0\\y=1\end{cases}\Rightarrow\begin{cases}x=1\\y=1\\z=0\end{cases}\Rightarrow E\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$\|e_{03}\|=\sqrt{2}\Rightarrow e_{03}=\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\\0\end{pmatrix}$

3) $P = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{2}}(z_2 + z_3) \\ y = \frac{1}{\sqrt{2}}(-z_2 + z_3) \\ z = z_1 \end{cases}$$

$$-2z_1^2 + 3z_2^2 + 7z_3^2 + \frac{6}{\sqrt{2}}(-z_2 + z_3) - \frac{6}{\sqrt{2}}(z_2 + z_3) + 4z_1 + 46 = 0$$

$$-2z_1^2 + 3z_2^2 + 7z_3^2 - 6\sqrt{2}z_2 + 4z_1 + 46 = 0$$

$$-2(z_1 - 1)^2 + 2 + 3(z_2 - \sqrt{2})^2 - 6 + 7z_3^2 + 46 = 0$$

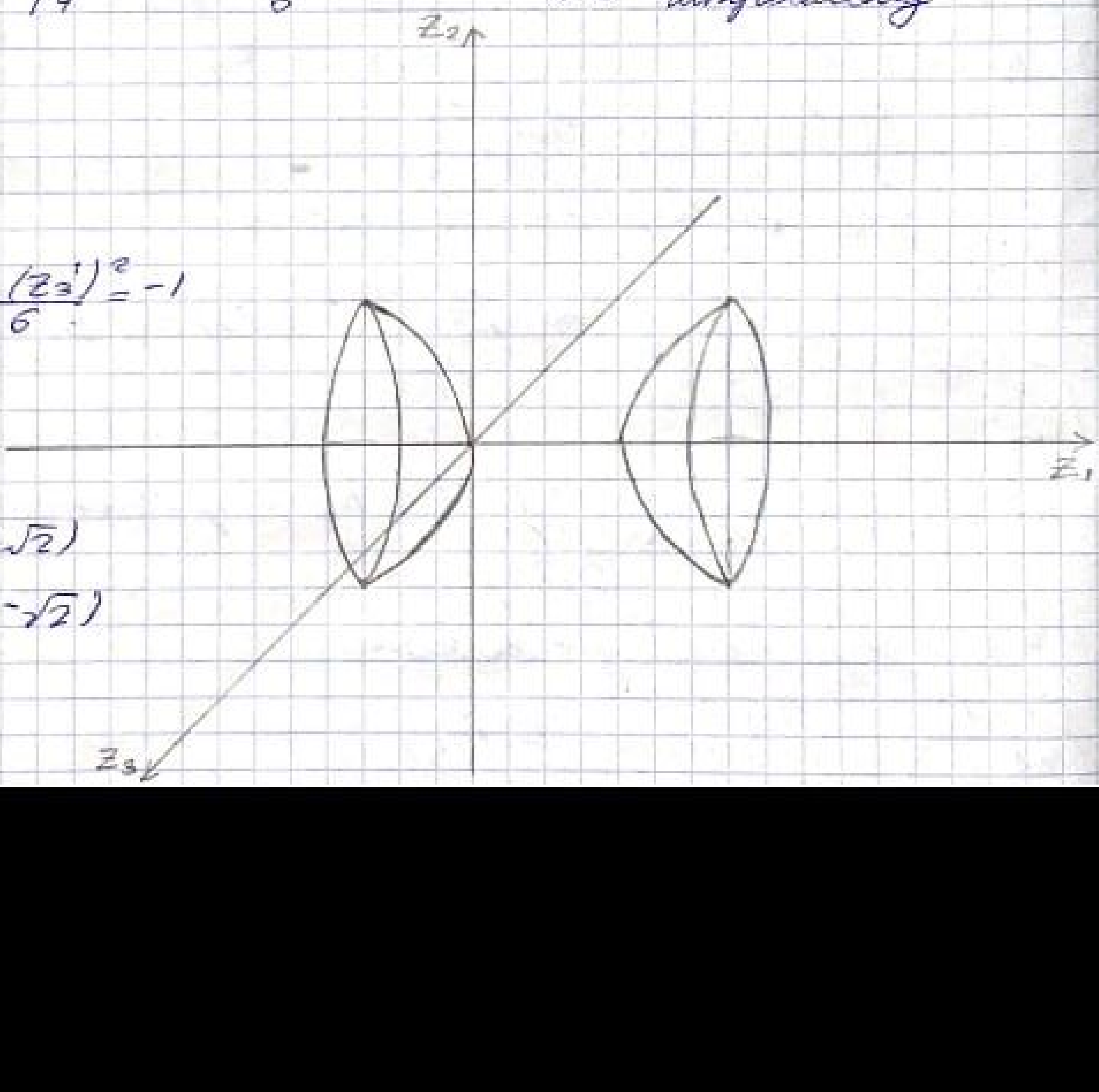
$$-\frac{(z_1 - 1)^2}{21} + \frac{(z_2 - \sqrt{2})^2}{14} + \frac{z_3^2}{6} = -1$$ – двуполостный гиперболоид

$$\begin{cases} z_1' = z_1 - 1 \\ z_2' = z_2 - \sqrt{2} \\ z_3' = z_1 \end{cases}$$

$$-\frac{(z_1')^2}{21} + \frac{(z_2')^2}{14} + \frac{(z_3')^2}{6} = -1$$

$$\begin{cases} z_1 = z_1' + 1 \\ z_2 = z_2' + \sqrt{2} \\ z_3 = z_3' \end{cases}$$

$$\begin{cases} x = \frac{1}{\sqrt{2}}(z_2' + z_3' + \sqrt{2}) \\ y = \frac{1}{\sqrt{2}}(-z_2' + z_3' - \sqrt{2}) \\ z = z_1' + 1 \end{cases}$$



d) $2x^2+2xy-2xz-2yz-6\sqrt{3}y+6\sqrt{3}z+18=0$

$F(x,y,z)=2x^2+2xy-2xz-2yz$

$A=\begin{pmatrix}2&1&-1\\1&0&-1\\-1&-1&0\end{pmatrix}$

1) $|A-\lambda E|=0$

$\begin{vmatrix}2-\lambda&1&-1\\1&-\lambda&-1\\-1&-1&-\lambda\end{vmatrix}=\lambda^3-2\lambda^2-3\lambda=0$

$\left[\begin{array}{l}\lambda_1=-1\\\lambda_2=0\\\lambda_3=3\end{array}\right.$

2) $\lambda_1=-1$

$\begin{pmatrix}3&1&-1\\1&1&-1\\-1&-1&1\end{pmatrix}\sim\begin{pmatrix}3&1&-1\\0&1&-1\\0&0&0\end{pmatrix}$ $Rg=2\Rightarrow$ 1 решение; x,y - базисные, z - свободная

$\begin{cases}z=1\\y-1=0\\3x+y-1=0\end{cases}\Rightarrow\begin{cases}x=0\\y=1\\z=1\end{cases}$

$E\begin{pmatrix}0\\1\\1\end{pmatrix}$; $\|e_{01}\|=\sqrt{2}\Rightarrow e_{01}=\begin{pmatrix}0\\1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$

$\lambda_2=0$

$\begin{pmatrix}2&1&-1\\1&0&-1\\-1&-1&0\end{pmatrix}\sim\begin{pmatrix}2&1&-1\\0&1&1\\0&0&0\end{pmatrix}$ $Rg=2\Rightarrow$ 1 решение; x,y - базисные, z - свобод.

$\begin{cases}2x+y-1=0\\y+1=0\\z=1\end{cases}\Rightarrow\begin{cases}x=\frac{1-y}{2}\\y=-1\\z=1\end{cases}\Rightarrow\begin{cases}x=1\\y=-1\\z=1\end{cases}$ $\|e_{02}\|=\sqrt{3}\Rightarrow e_{02}=\begin{pmatrix}1/\sqrt{3}\\-1/\sqrt{3}\\1/\sqrt{3}\end{pmatrix}$

$\lambda_3=3$; $\begin{pmatrix}1&1&-1\\1&-3&-1\\-1&-1&-3\end{pmatrix}\sim\begin{pmatrix}1&-1&1\\0&1&1\\0&0&0\end{pmatrix}$ $Rg=2\Rightarrow$ 1 решение x,y - базисные; z - свободн.

$\begin{cases}x-y+1=0\\y+z=0\\z=1\end{cases}\Rightarrow\begin{cases}x=y-1\\y=-1\\z=1\end{cases}\Rightarrow\begin{cases}x=-2\\y=-1\\z=1.\end{cases}$ $E\begin{pmatrix}-2\\-1\\1\end{pmatrix}\sim E\begin{pmatrix}2\\1\\-1\end{pmatrix}\Rightarrow\|e_{03}\|=\sqrt{6}$

$e_{03}=\begin{pmatrix}2/\sqrt{6}\\1/\sqrt{6}\\-1/\sqrt{6}\end{pmatrix}$

3) $P = \begin{pmatrix} 0 & 1/\sqrt{3} & \frac{2}{\sqrt{6}} \\ 1/\sqrt{2} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{pmatrix}$

$$\begin{cases} x = \frac{1}{\sqrt{3}} z_2 + 2/\sqrt{6}\, z_3 \\ y = 1/\sqrt{2}\, z_1 - 1/\sqrt{3}\, z_2 + 1/\sqrt{6}\, z_3 \\ z = 1/\sqrt{2}\, z_1 + 1/\sqrt{3}\, z_2 - 1/\sqrt{6}\, z_3 \end{cases}$$

$$-z_1^2 + 3z_3^2 - \frac{6\sqrt{3}}{\sqrt{6}}(\sqrt{3}z_1 - \sqrt{2}z_2 + z_3) + \frac{6\sqrt{3}}{\sqrt{6}}(\sqrt{3}z_1 + \sqrt{2}z_2 - z_3) + 18 = 0$$

$$-z_1^2 + 12z_2 + 3(z_3^2 - 2\sqrt{2}z_3 + 2 - 2) + 18 = 0$$

$$z_1^2 - 12z_2 - 3(z_3 - \sqrt{2})^2 = 12$$

$$z_1^2 - 3(z_3 - \sqrt{2})^2 = 12(z_2 + 1)$$

$\frac{z_1^2}{12} - \frac{(z_3 - \sqrt{2})^2}{4} = (z_2 + 1)$ – гиперболический параболоид