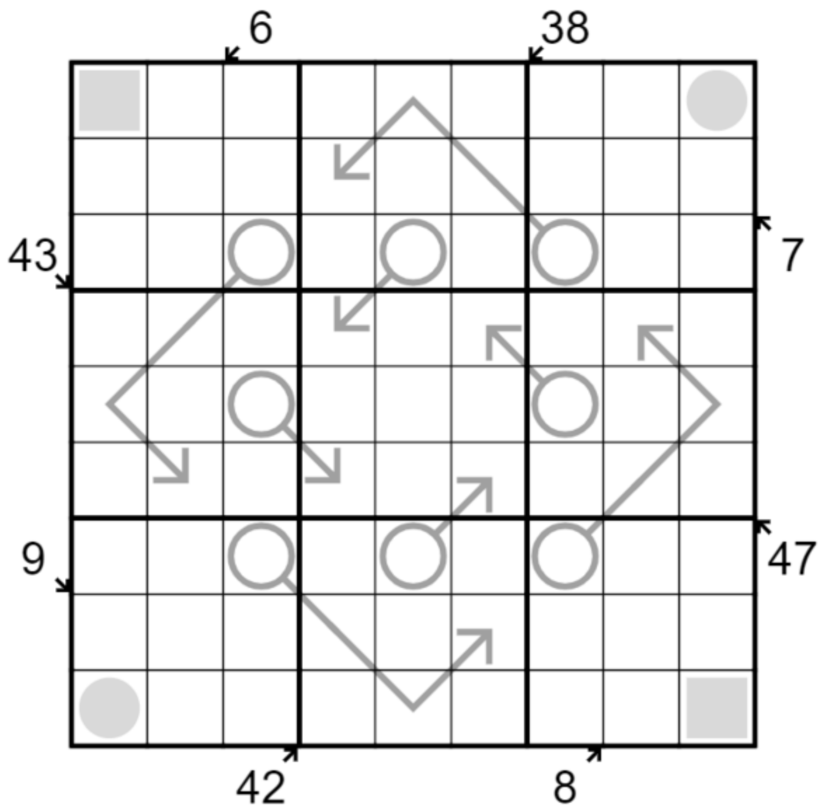
'Orbit' – Puzzle and solution

This solving path makes heavy use of color labels, and several images to visualize geometry. I apologize for any inconvenience this causes.

The puzzle



It is a normal sudoku, with a few extra rules which I assume are commonly known:

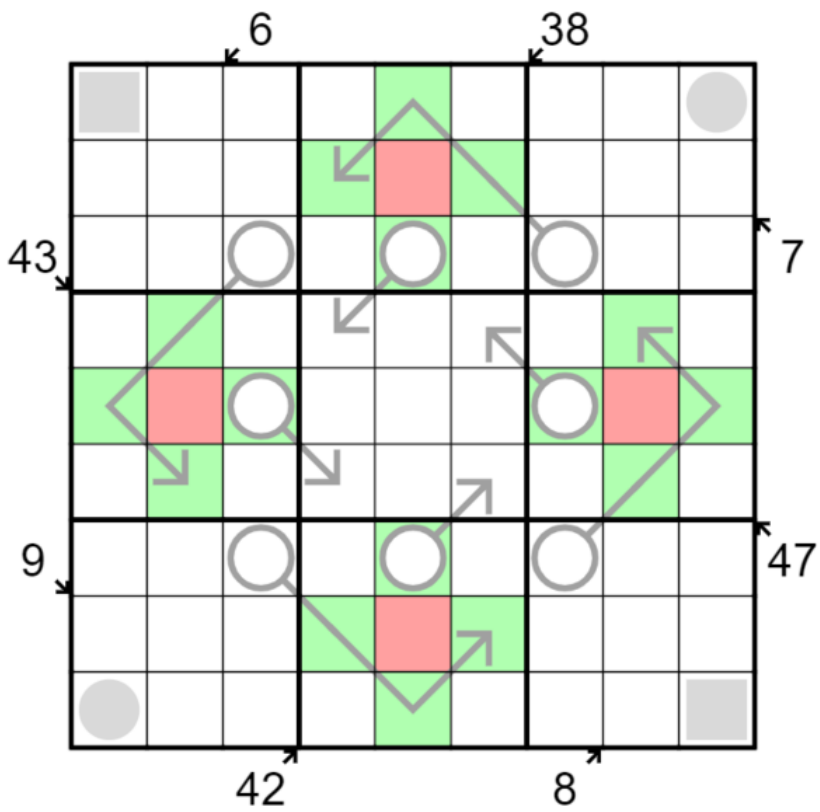
1) Normal sudoku rules apply.
2) Arrows: Digits along an arrow in the grid must sum to the digit in its circle. Digits may repeat along an arrow.
3) Little killers: Digits along diagonals indicated by arrows outside the grid must sum to the given number at that arrow. Digits may repeat along a diagonal.
4) Odd/even: Cells with a grey circle must be odd. Cells with a grey square must be even.

This is the f-puzzles link with the puzzle:

https://f-puzzles.com/?id=y5593j5q

Solution

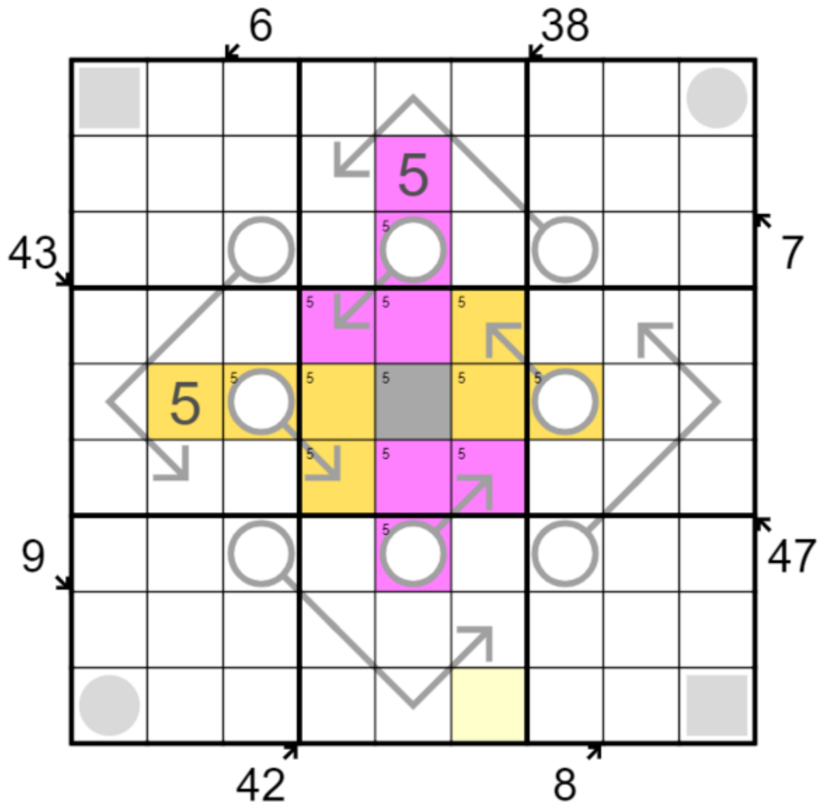
The first thing to notice is that the 4 'long' little killer clues are very high. They add up to 43+38+47+42 = 170. Now, if you take the sum of all the digits in boxes 2, 4, 6 and 8, you get 180. If you subtract all the long little killer clues you get 10. Here, the center cells of said boxes are subtracted twice. And so 10 must be the exact difference between those center cells, and the sum of all 16 cells that are orthogonally adjacent to them.



[Sum of green cells] - [sum of red cells] = 10.

The lowest I can make the green cells is 4 * (1+2+3+4) = 40. The highest I can make the red cells is 2 * (8+9) = 34, a difference of 40 - 34 = 6, lower than 10.

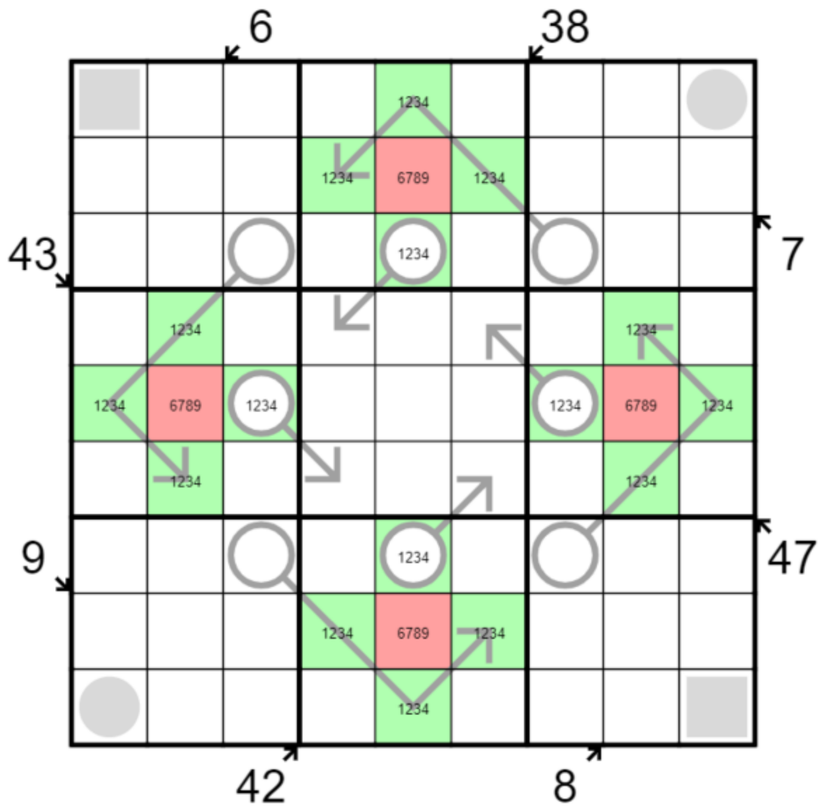
But in fact, I cannot reuse the same digit twice for any of the red cells. Look at this image:



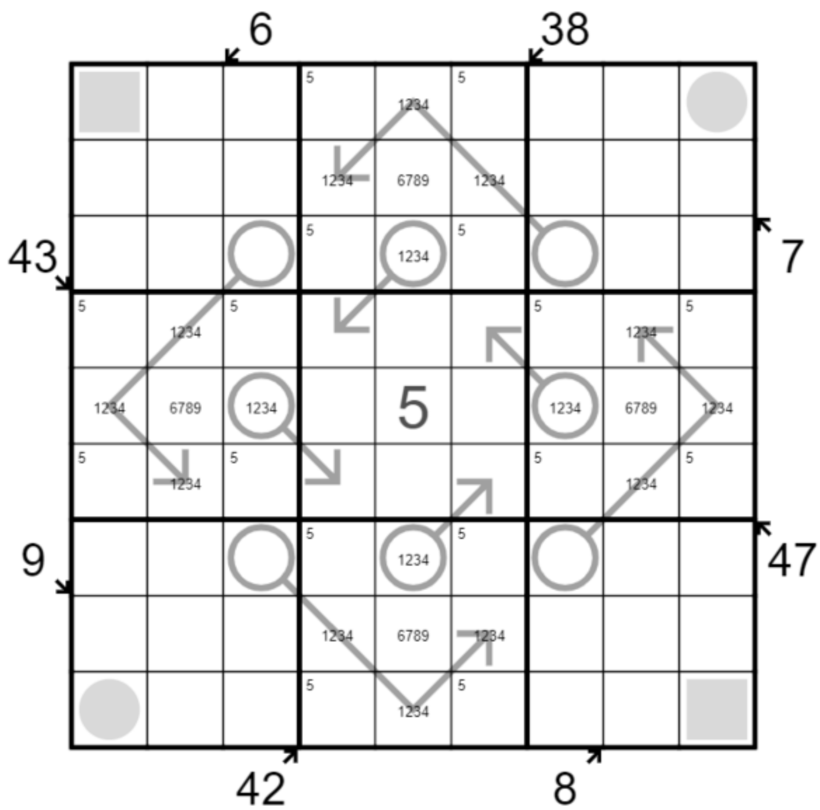
Here, I've used 5 as an example digit, but this reasoning is true for any digit, and for all combinations of cells, because of the symmetry of the 'short' arrows pointing into box 5. If I use two 5's, it breaks box 5. The 5 in box 2 rules out 5 as an option from all purple cells and the center cell, and the 5 in box 4 rules out 5 as an option from all yellow cells and the center cell, so 5 cannot go anywhere in box 5.

Conclusion: we need to use 4 <u>different</u> digits for the red cells in the first image, and the highest we can make them is 6+7+8+9 = 30. 40 - 30 = 10 exactly, and so there are no degrees of freedom at all.

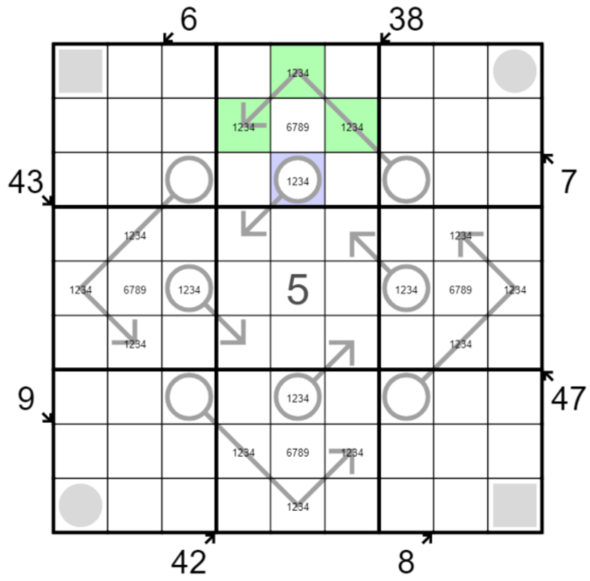
So the green cells must necessarily be 1+2+3+4, the red cells 6+7+8+9.

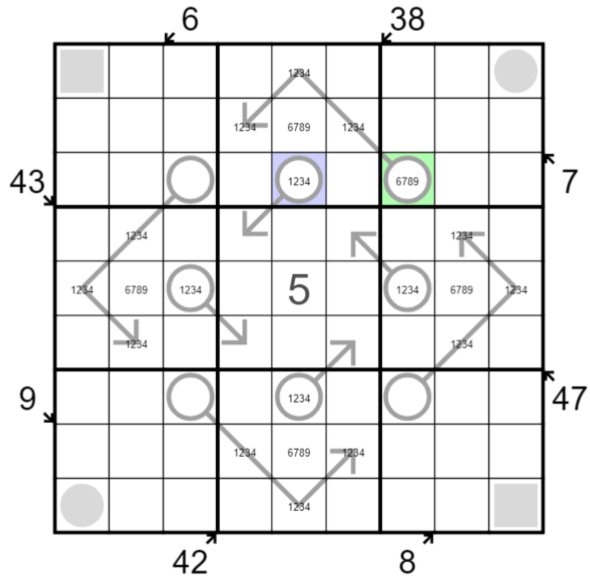


Notice now that there's a double X-wing (or whatever it's called) on 5's, which immediately places 5 in the center cell of box 5.

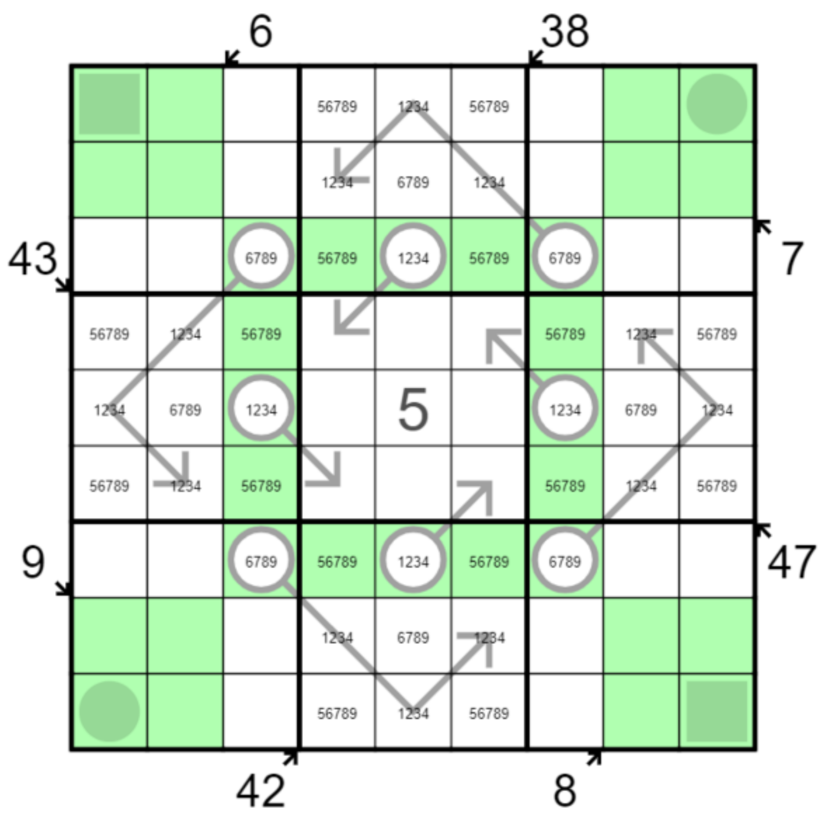


Now consider the 'long' arrows. The digit in their circle has to be the sum of three digits between 1 and 4. We also know that all 4 1+2+3+4 quadruples add up to 10. This means that the green and blue cells in below images must also add up to 10, and so for the circle cell of each long arrow we have options 10 - x for 1 <= x <= 4, which is 6,7,8, or 9.



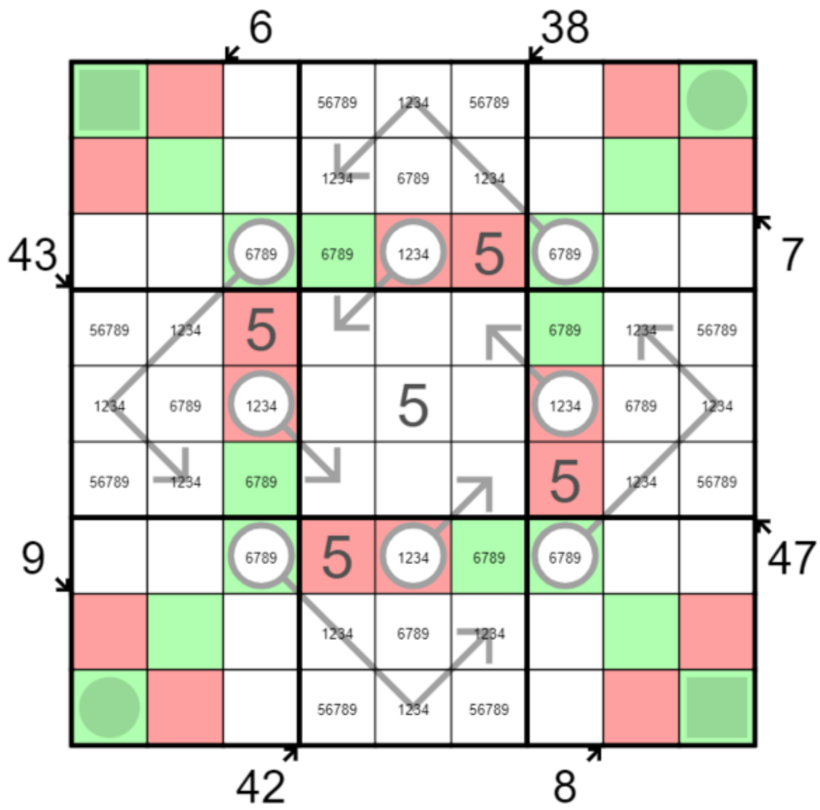


The next step involves (yet another) application of what's been called *Phistomefel's theorem*. In this instance, it makes the solve considerably easier, and I hope it's an accepted enough technique for solvers to be able to use it. Otherwise, you basically have to prove it from scratch in order to proceed with the solve, I can't see a way around it.



What this theorem says is that the 16 digits in the green ring around box 5 are the exact same 16 digits that appear in the 2-by-2 boxes in the corners of boxes 1, 3, 7 and 9.

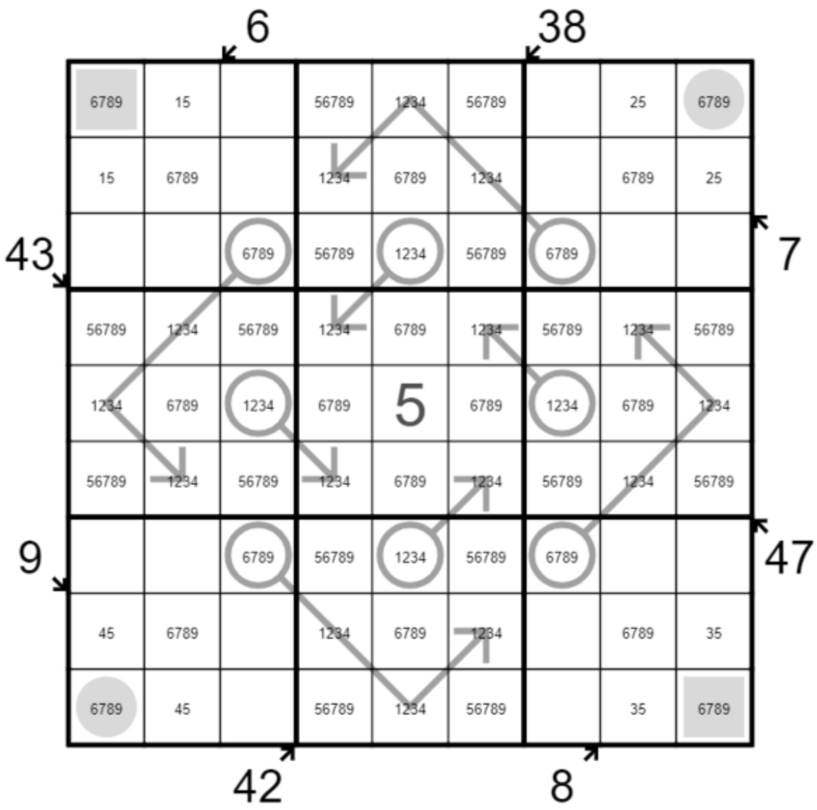
We know a lot about the digits in the ring surrounding box 5. They contain exactly one instance each of the digits 1, 2, 3 and 4. The short arrows force them all to be different, and we have exactly 4 cells with those 4 options in the ring. We also know that there are at most 4 of each digit in the ring. This is true in particular for the digit 5. Now, if you select the 8 lowest possible digits from the ring and add them up, you get 1+2+3+4 + 4*5 = 30. If you add up the 'short' little killer clues, which are all fully contained in the 4x4 corner boxes, you also get 6+7+8+9 = 30.



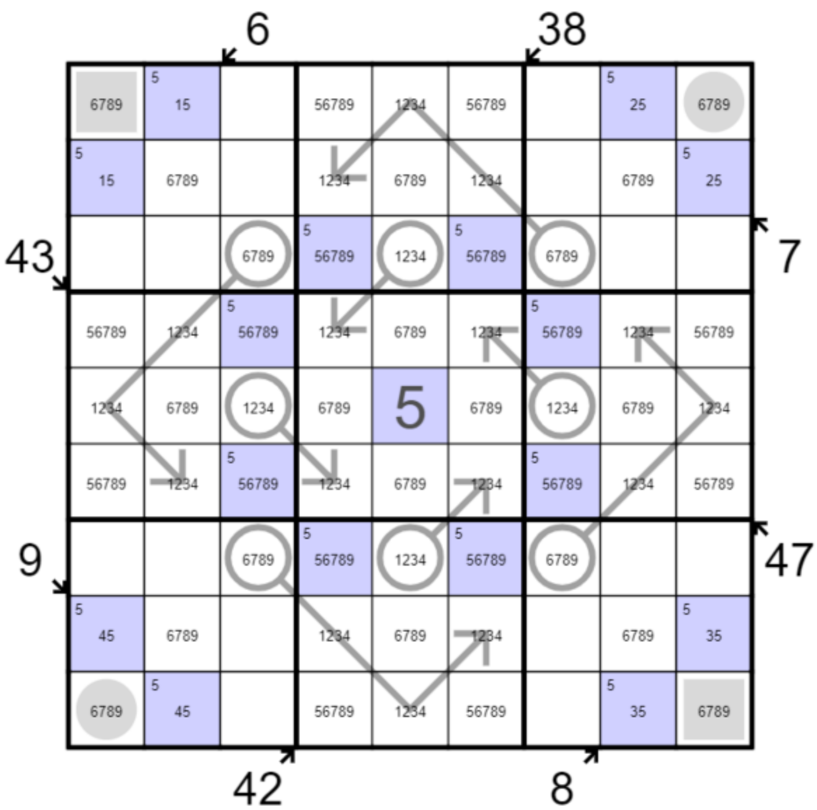
In above image the 5's have example locations, they are there to illustrate that the totals of the red cells in the ring as well as in the corner boxes are the minimum of 30, and therefore their digits have to be the same as well.

So the digits used for those corner little killer clues have to be 1, 2, 3, 4, 5, 5, 5, 5. There is only one way in which they can be divided among the corner little killer clues: 1+5 = 6, 2+5 = 7, 3+5 = 8 and 4+5 = 9.

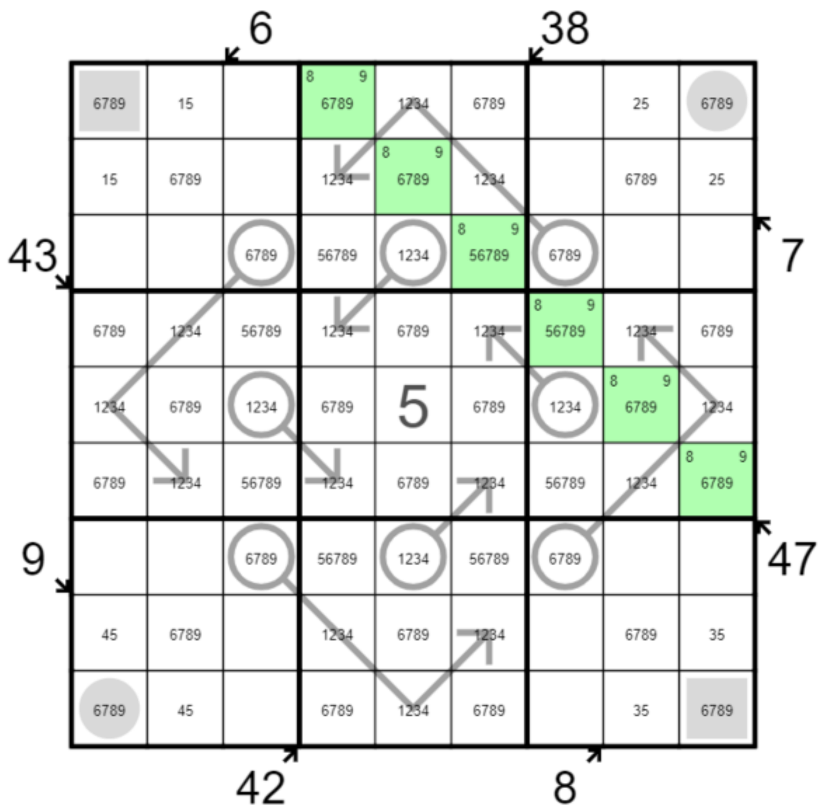
Pencil-marking everything in yields the image on the next page.



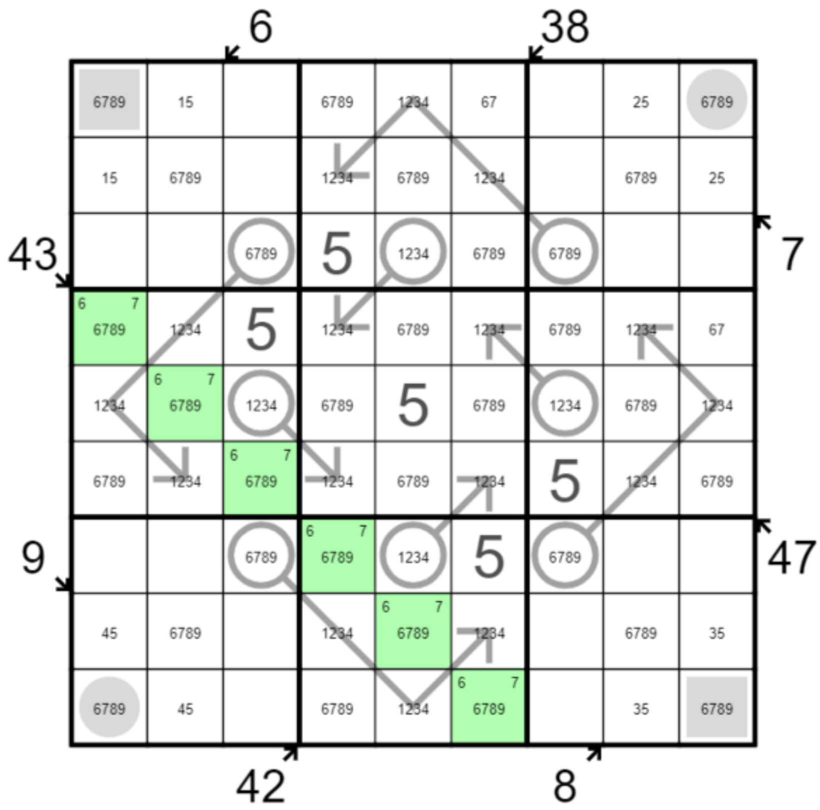
Another conclusion is that the ring indeed must contain no less than four 5's. If this is missed, there's still X-wings to rely on to remove 5 as an option from 8 cells.



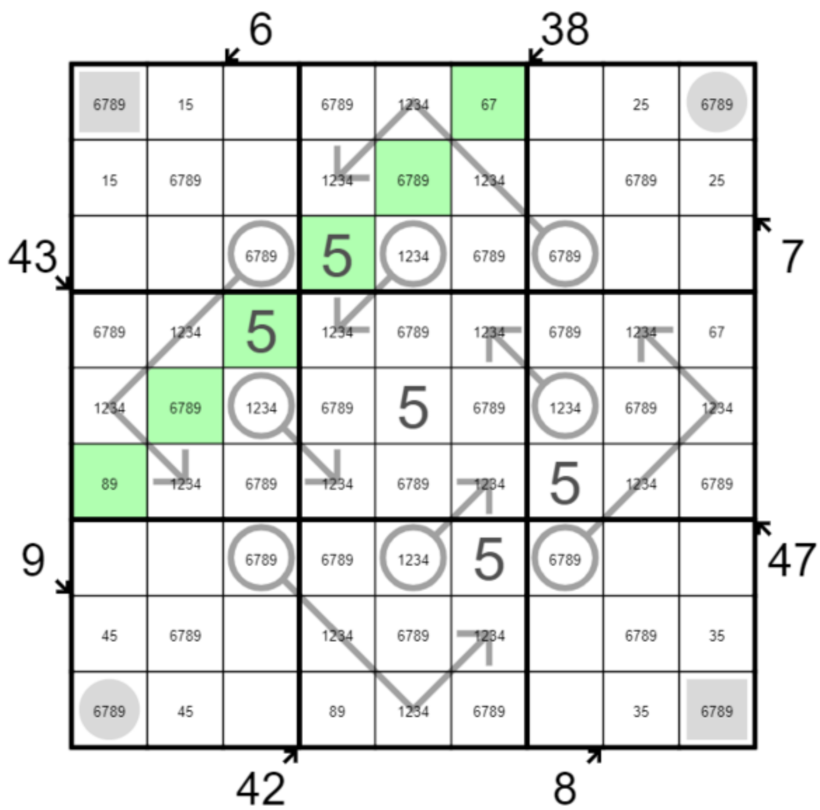
5 can only go into blue squares.



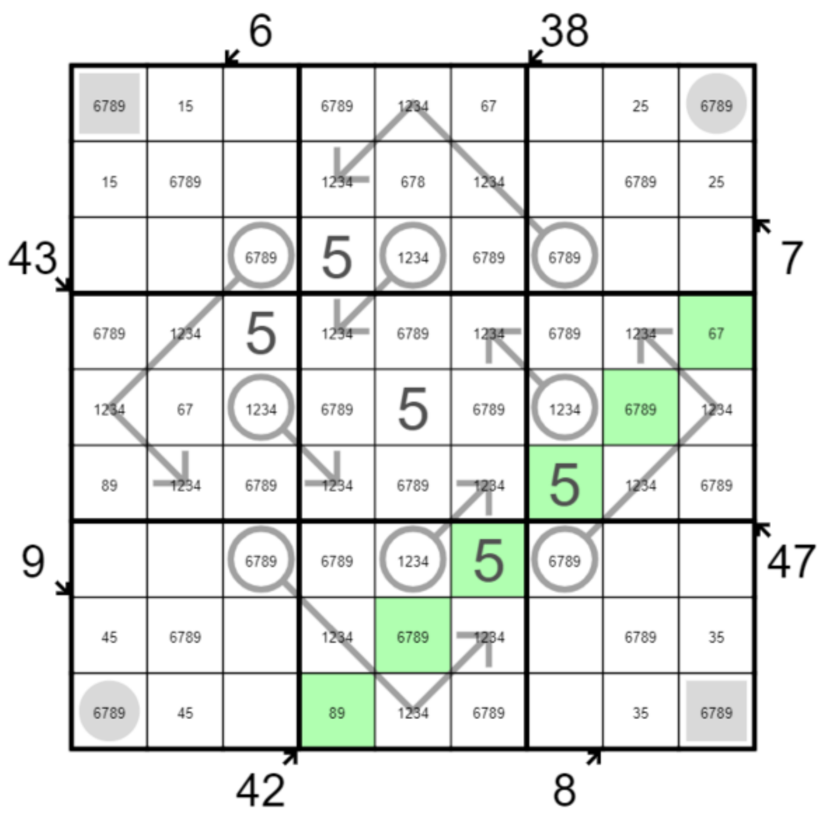
The next step is to consider the 47 clue. That can only be 23 + 24 for any combination of 3 digits in boxes 2 and 6. As 23 must be 6+8+9 and 24 must be 7+8+9, both 8 and 9 in both boxes can only go onto the 47 diagonal. Also, 5 cannot be on that diagonal either, which places four 5's in the grid.



Then consider the 43 clue, which now cannot contain a 5 anymore. Using the lowest possible digits, 43 must be 6+7+8 = 21 in one box, and 6+7+9 = 22 in the other box. This rules out 6 and 7 from cells in boxes 4 and 8 not on the diagonal.

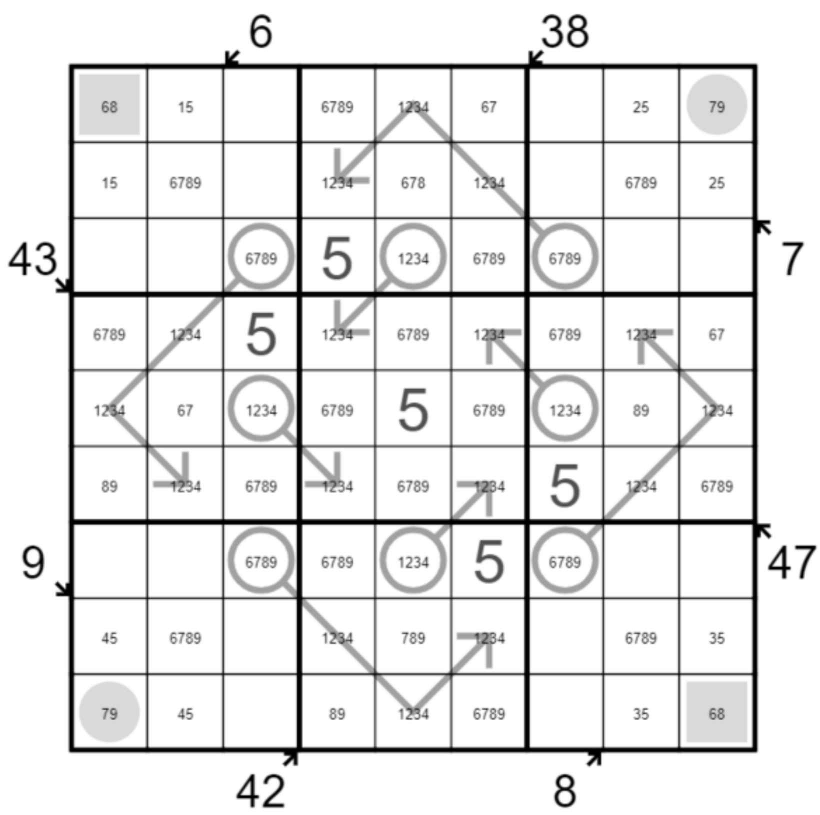


The 38 clue now becomes important. As it contains two 5's, its remaining total is 28. With the available digits, the lowest value for two cells in the same box is 6+7 = 13, the highest is 8+9 = 17. To make 28, the only options we have are 13+15 and 14+14. In box 4 therefore, as r6c1 can only be 8 or 9, 8 and 9 are already ruled out from center cell r5c2 because that would create a total of 17. The only remaining options are 6+8 = 14, 6+9 = 15 and 7+8 = 15. Notice that the 13 total is not available here. This in turn rules out 15 as a possible total for the two cells on the 38 diagonal in box 2. And so center cell r2c5 cannot be 9 anymore as 6+9 = 15 > 14.

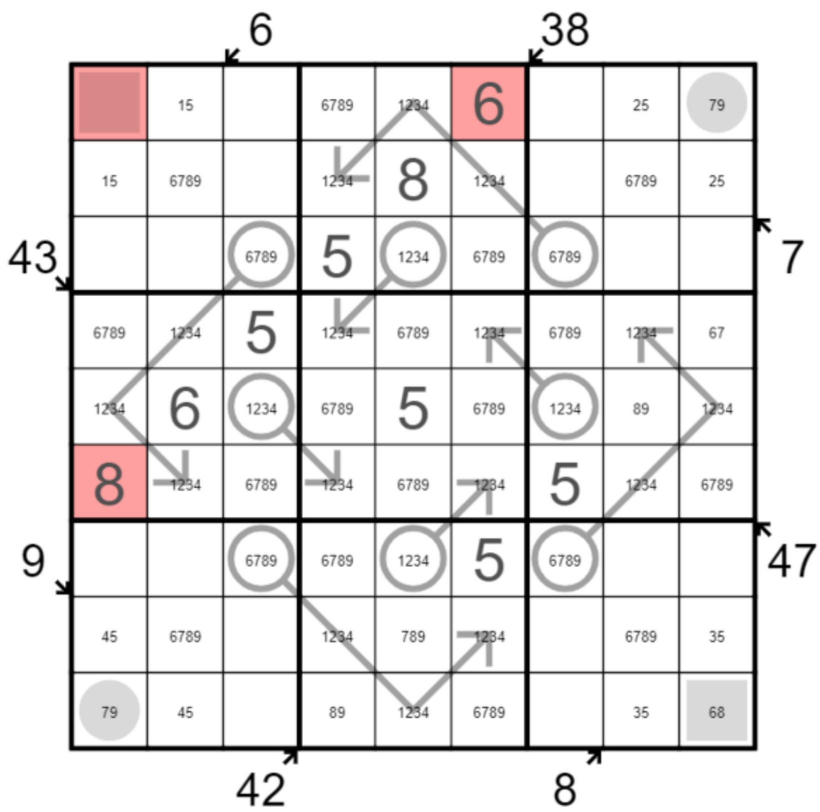


Analogous reasoning must be applied for the 42 clue. To make 42 - (5+5) = 32, we can only use 15+17 or 16+16. This removes options 6 and 7 from the center cell of box 6 (r5c8), and removes 6 as an option from the center cell of box 8 (r8c5).

Now remove odd digits as options from corner cells with a square, and even digits as options from corner cells with a circle to prepare for the next deduction.

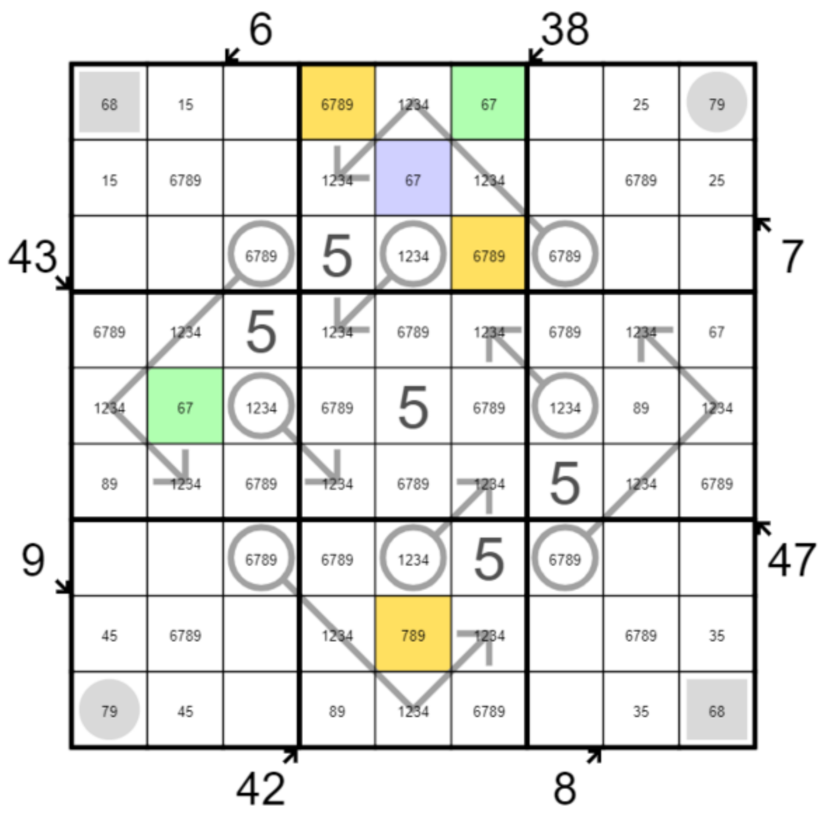


Can the remaining 28 total of the 38 clue be 14+14? No, see this image:

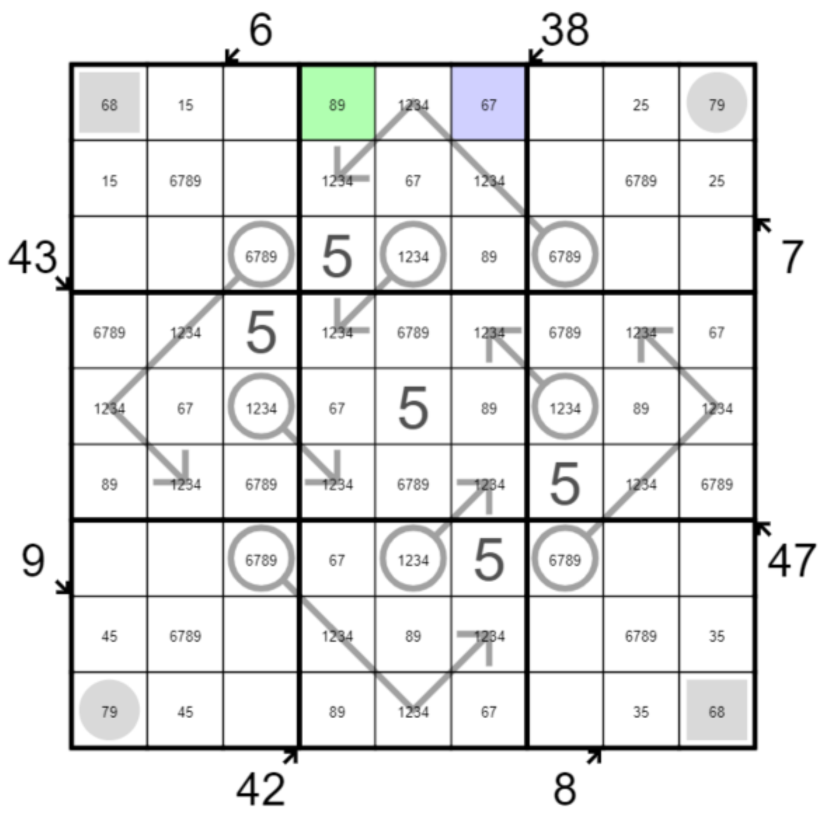


The only way to make 14 in both boxes 2 and 4 is to use 6+8 + 6+8. But that breaks the corner cell r1c1, it has no options left.

This removes 8 as an option from the center cell of box 2, which creates a 6+7 pair in box 2.



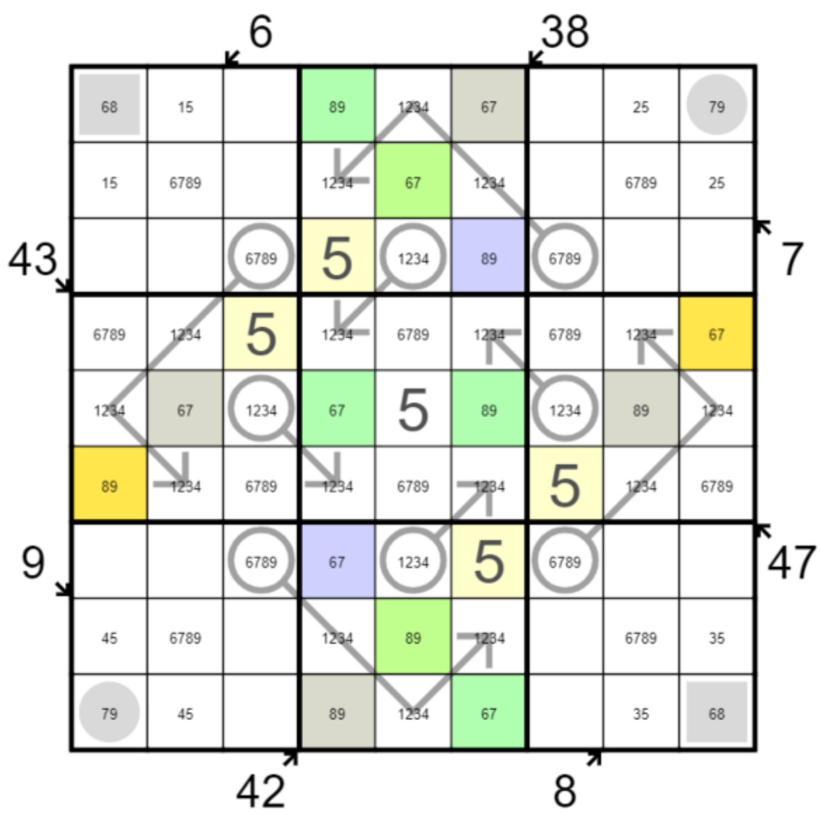
Recall that all digits in the center cells of boxes 2, 4, 6 and 8 have to be different as well. Not only are 6 and 7 removed as options from the yellow cells in box 2, but also from the yellow cell r8c5 in box 8. This creates more sets of pairs in boxes 5 and 8, which removes several options from those cells.



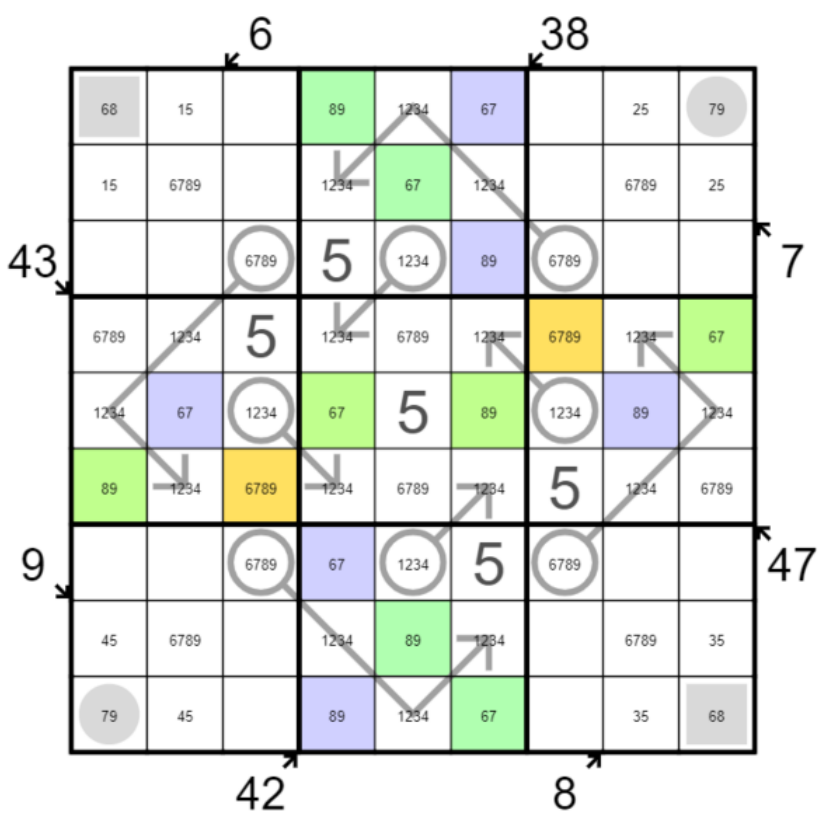
The way forward here is to use the even/odd clues. We know that in the digits 6, 7, 8, 9, two are odd and two are even. As the corner cells contain alternating even/odd cells, this means that in column 1, row 1, column 9 and row 9 the two remaining 6789 cells have opposite parity, i.e. one is odd, the other even.

We can use colors to label cells with equal or opposite parity. In the image, I've used green and blue. It means for example that the only two valid combinations for the two colored cells in box 2 are 8+7 and 6+9.

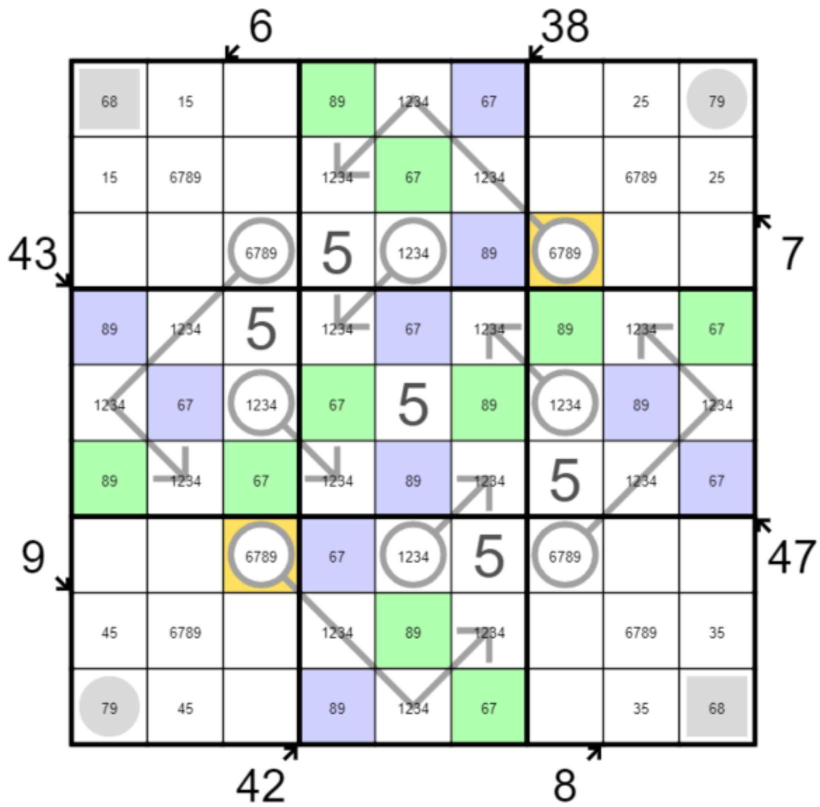
There are several deductions we can make to determine whether cells are green or blue. One thing is that if a 67-pair exists in the same column, row or box, they must have opposite parity. Same goes for 89-pairs. Another thing is that in each column, row and box we must have exactly two green cells and two blue cells.



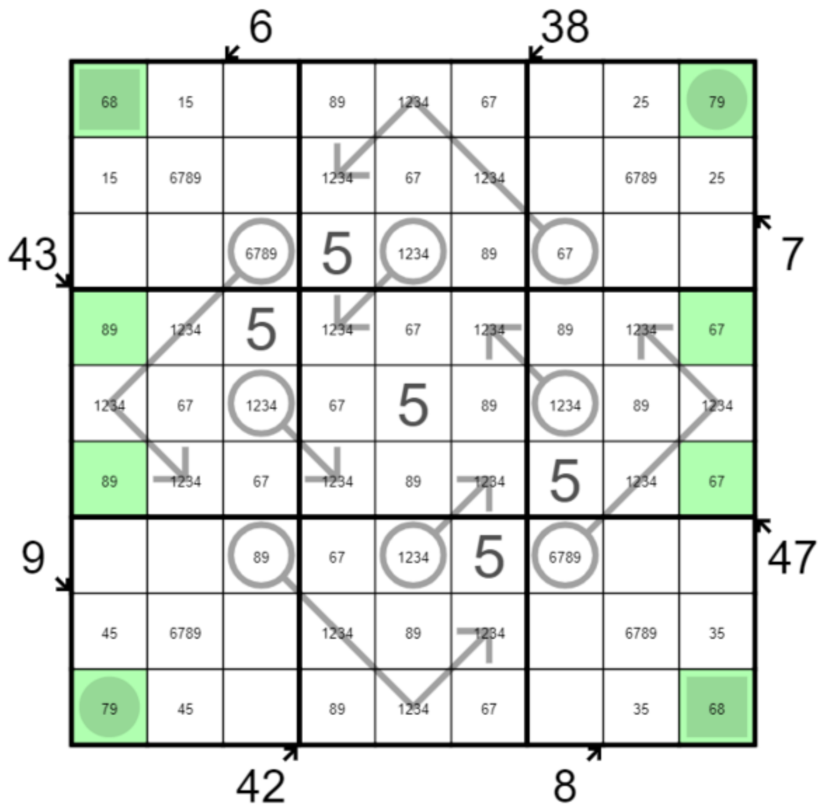
Consider the 38 and 42 clues again. Both are even. With 4 cells, this means either 0 cells are odd, 2 cells are odd, or 4 cells are odd. This leaves only one option for the yellow cells: both must be green.



It is now important to notice that the value of all green cells with a 67-pair must be the same, as they have equal parity. I.e. they are all 6, or all 7. The same is true for green cells with a 89-pair, they are all 8, or all 9. Now where can the remaining green 67-cell go in box 4, and the remaining green 89-cell in box 6? Both have only one option, r6c3 is a green 67-pair, r4c7 is a green 89-pair. With pairs thus created in boxes 4 and 6, and in rows 4 and 6, we can remove several options from 6789-cells, and finish the parity coloring as well.

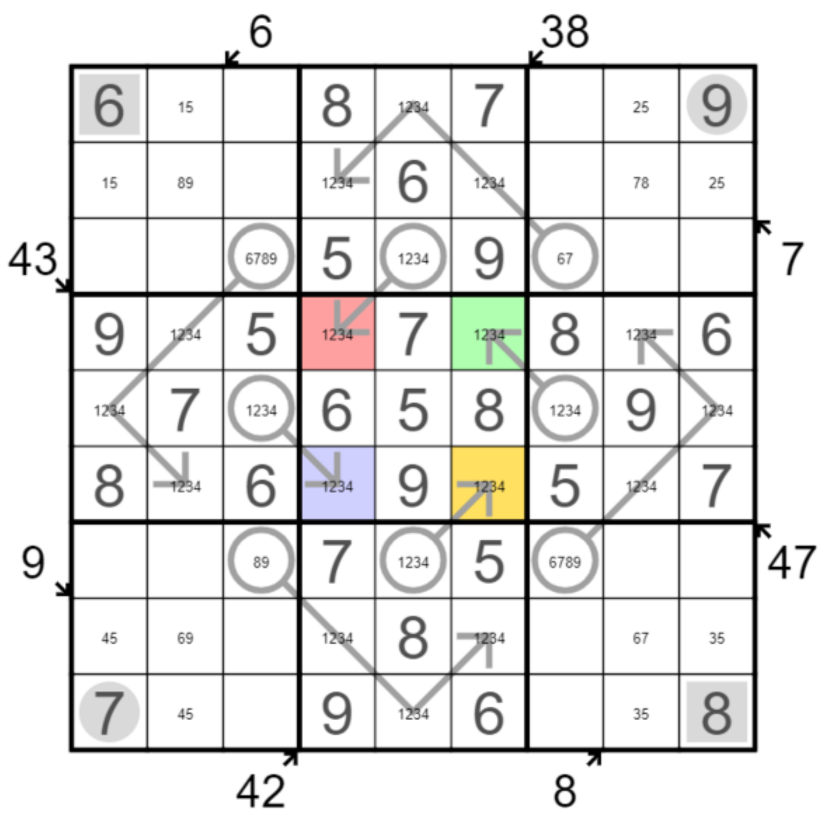


Look at the yellow cells now (r7c3, r3c7). r7c3 sees a blue 67 as well as a green 67 directly next to it. So r7c3 cannot be 6 or 7, it can only be 8 or 9. Similarly, r3c7 sees a green as well as a blue 89 next to it, so it can't be 8 or 9, it can only be 6 or 7. *[Edit: I see now that this step is easier if the following step is taken before it, as it places the 6, 7, 8 and 9 in the neighbors of both r7c3, r3c7.]*

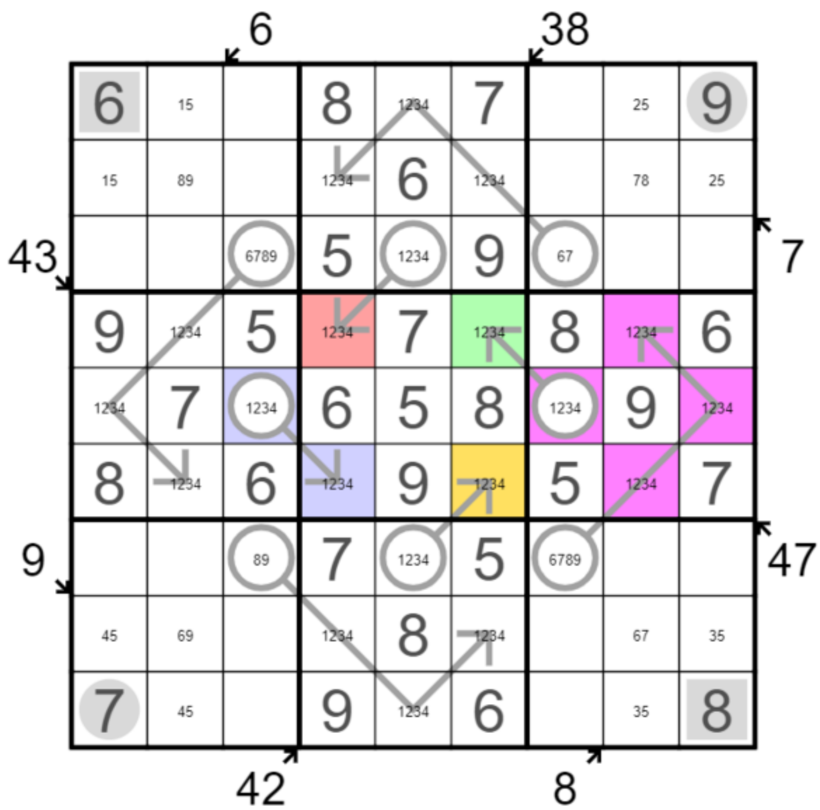


Pairs in columns 1 and 9 now fix the values of the corner cells r1c1, r1c9, r9c1, r9c9. This allows us to resolve several 67 and 89 pairs, and place all 6's, 7's, 8's and 9's in boxes 2, 4, 6 and 8. The long little killer clues are now fully used.

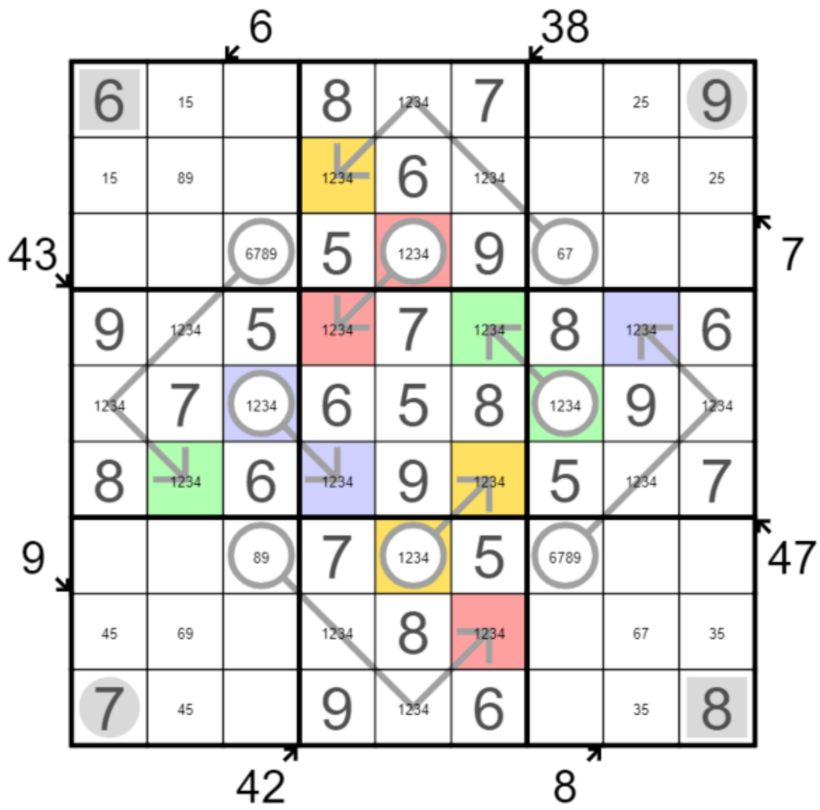
A good deduction to make here is to reuse the correspondence between circle cells of long arrow clues and circle cells of short arrow clues. This removes 3 and 4 as an option from r7c5 for example. But, the idea in the next step is more powerfully illustrated if this step is postponed until after.



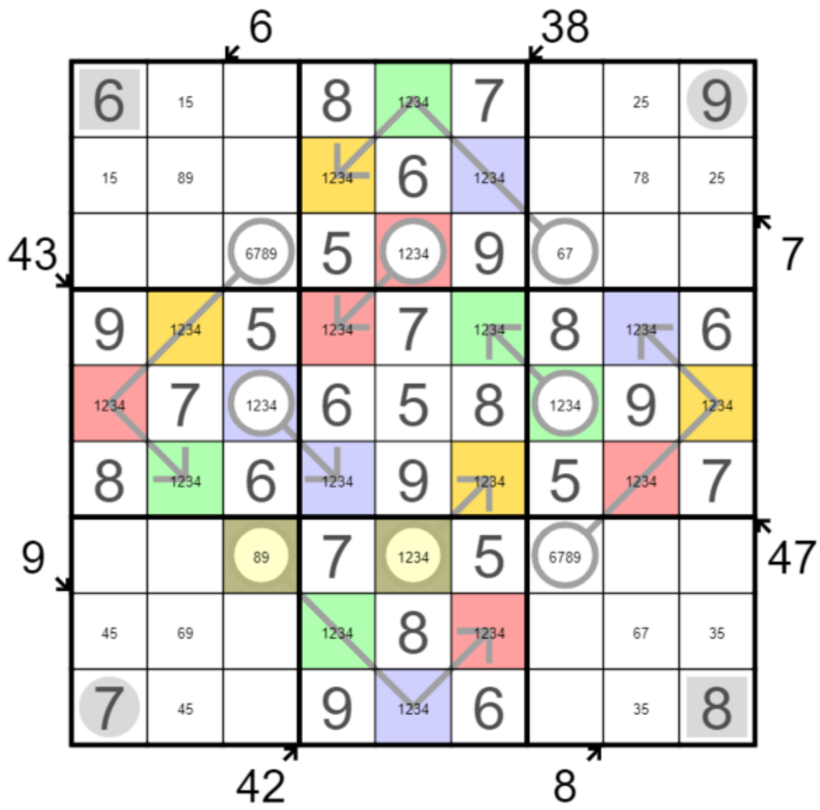
Here it is important to notice that the 1,2,3,4 digits are heavily constraining each other by their lay-out in the grid. It helps therefore to use color labels again, 4 colors this time to label 1234 digits - equal colors for equal digits, different colors for different digits.



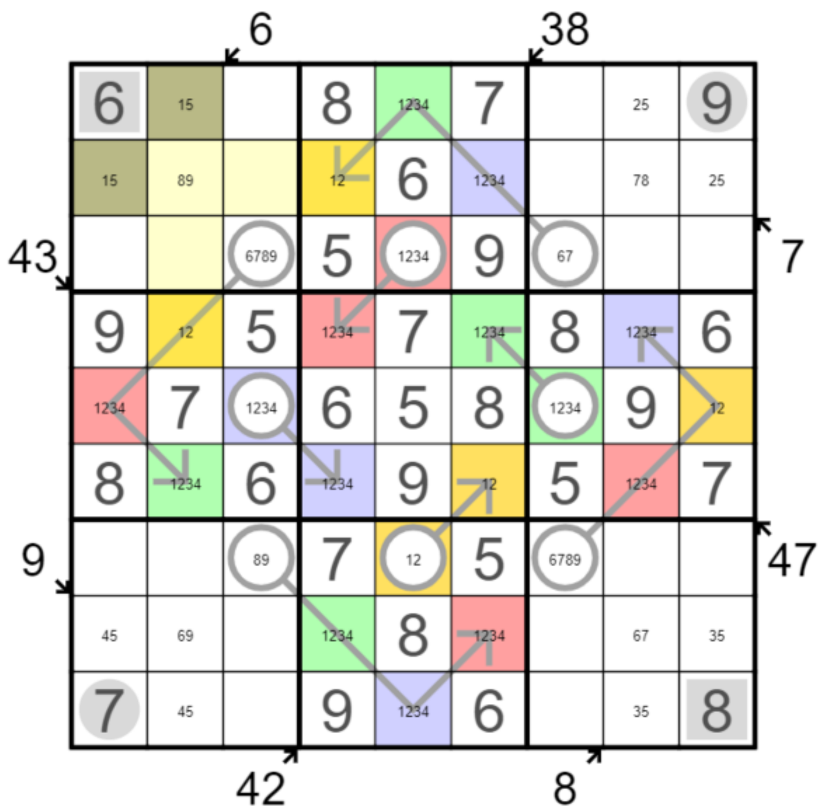
The first deduction we can make is that the short arrow clues 'export' one color each into boxes 2, 4, 6 and 8. Now where does the blue cell (r5c3, r6c4) go into box 6? It must go into one of the purple cells (r4c8, r5c7, r5c9, r6c8) but it can only go into r4c8 by sudoku. A symmetric deduction exists for the other three colors.



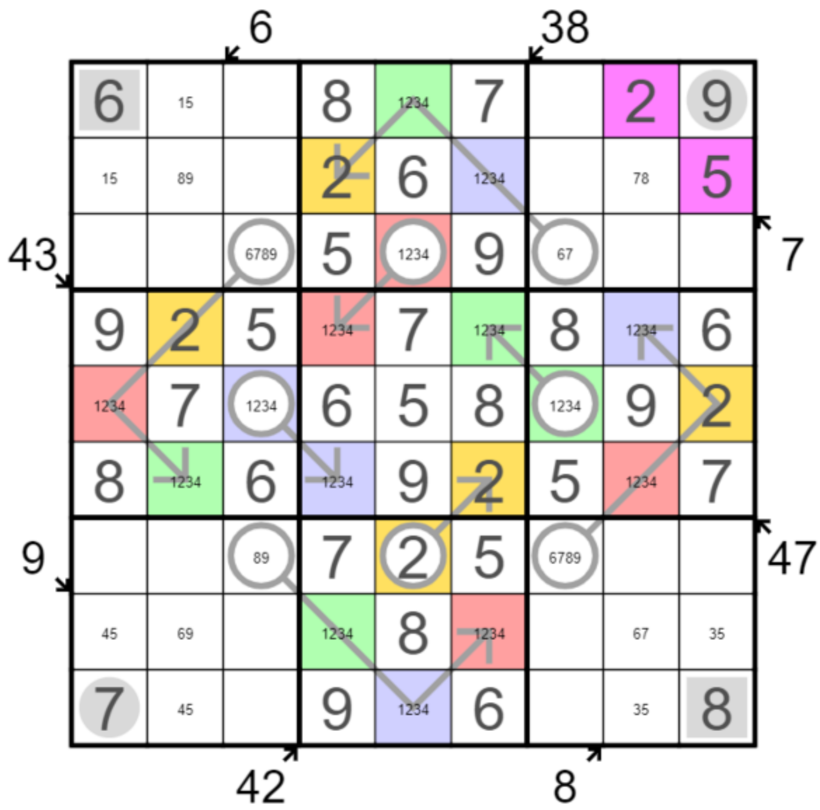
Columns 4 and 6, and rows 4 and 6 have only one uncolored 1234 cell remaining, and it must be different from the 3 colors it sees. Then, boxes 2, 4, 6, 8 each have only one uncolored 1234 cell remaining. Again, it has only one option for a color.



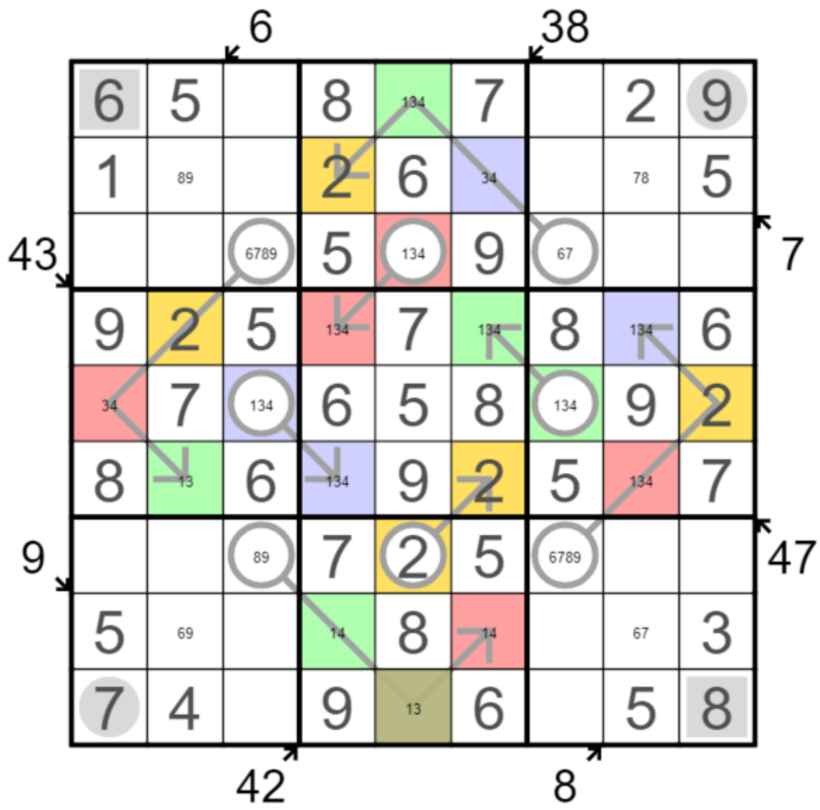
Here then is the deduction which removes 3 and 4 as options from r7c5, the grey cells r7c3 and r7c5 always add up to 10. But not only does this remove 3 and 4 from that single yellow cell, it removes 3 and 4 from all the yellow cells simultaneously.



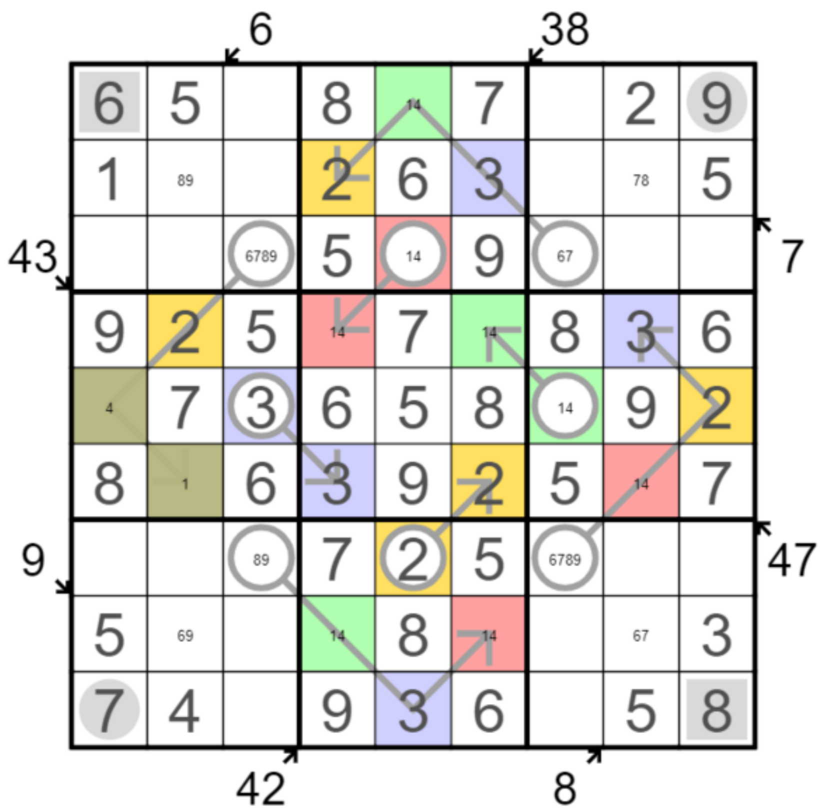
Now look at what this does to box 1. If we choose 1 for the yellow cells, it removes 1 as an option of <u>both</u> grey cells of the 6 clue. The conclusion must be that all yellow cells contain a 2. This immediately places the 2 and 5 of the 7-clue (purple cells in below image).



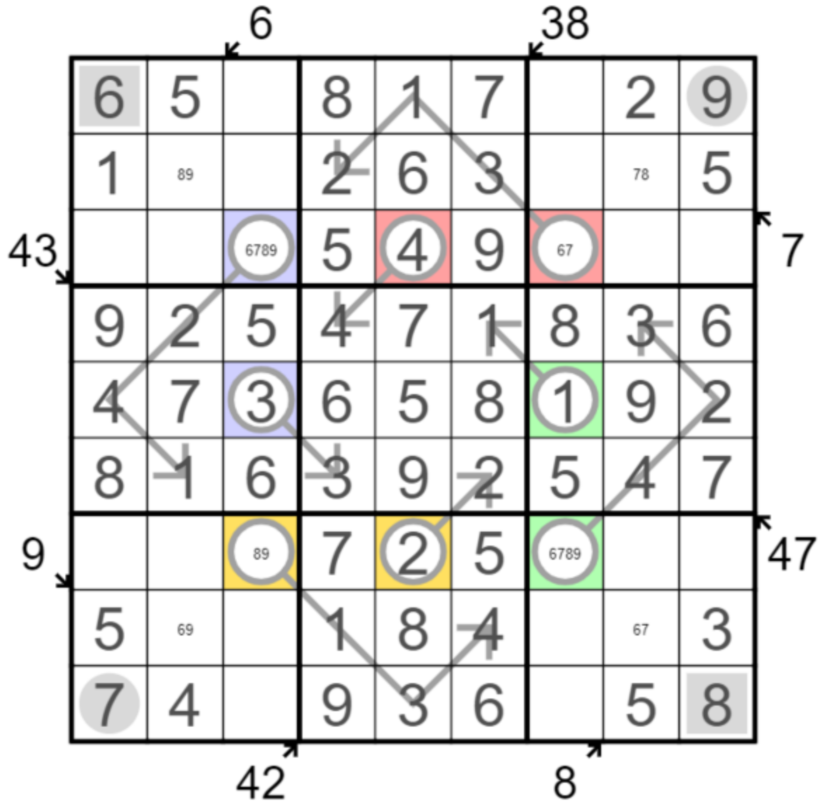
By sudoku, we can now remove options from several color-labeled 1234 cells.



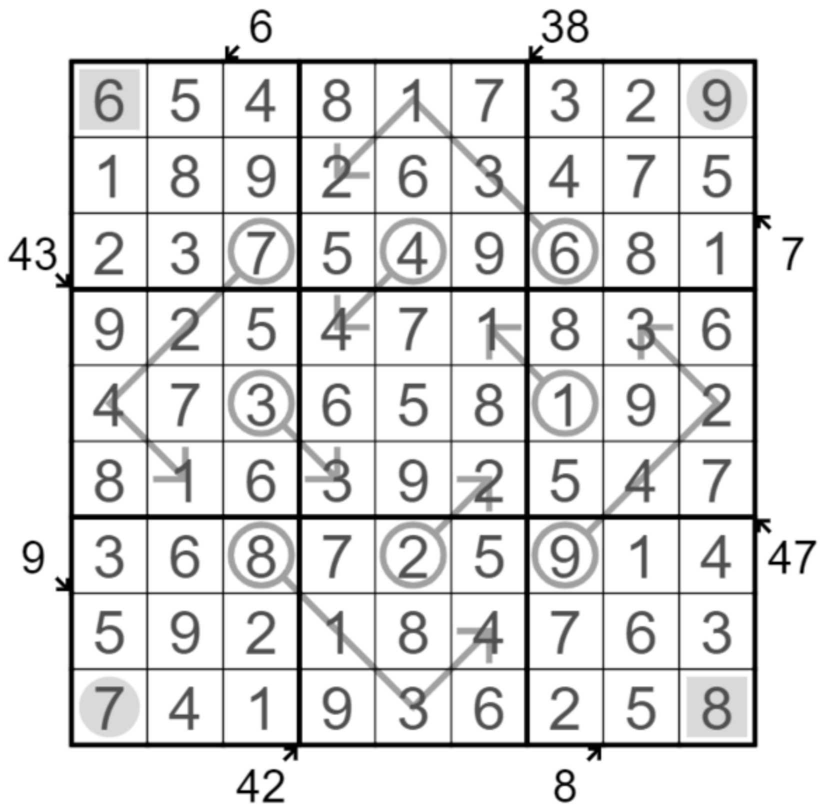
We can use box 8 for example, which now contains a hidden single. Its 3 must go to the bottom cell, r9c5. This means that all blue cells become 3.



Box 4 now contains two naked singles. All red cells are 4, all green cells are 1.



The last step is to finally resolve the long arrow clues. This places a 7 in r3c3, a 6 in r3c7, an 8 in r7c3, and a 9 in r7c7. The remaining 20 cells are all easy hidden and naked singles.



And that is how to solve the orbit puzzle. I hope you enjoyed it as much as I did.