Chapter 1

中微子振荡

平面波本征态

中微子质量本征态用平面波近似，具体表达式为

$$|\nu_i(t)\rangle = e^{-\frac{i}{\hbar}P^a X_a}|\nu_i\rangle, \tag{1.1}$$

四动量为$P^a = (E, \vec{p})$；四坐标为$X^a = (t, \vec{r})$；于是内积为

$$P^a X_a = Et - pL. \tag{1.2}$$

由于中微子质量非常小，所以可以做进一步近似

$$E = \sqrt{m^2c^4 + p^2c^2} = pc\sqrt{1 + \frac{m^2c^2}{p^2}} \approx pc\left[1 + \frac{m^2c^3}{2p^2c} + \mathcal{O}\left(\frac{m^4c^4}{p^4}\right)\right], \tag{1.3}$$

利用$E \approx pc, t = L/c$，最后中微子本征态为

$$|\mu_i(t)\rangle \approx e^{-i\frac{m_i^2c^3}{2\hbar E}L}|\nu_i\rangle. \tag{1.4}$$

二味混合

虑两味的混合，混合矩阵为

$$U = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}, \tag{1.5}$$

味道本征态和质量本征态的关系为

$$|\nu_\alpha\rangle \sum_i U_{\alpha i}|\nu_i\rangle. \tag{1.6}$$

那$\nu_e \to \nu_e$的振幅为

$$A_{\nu_e\to\nu_e} = \langle\nu_e|\nu_e(t)\rangle = \sum_{i,j} U_{ie}^\dagger U_{ei} e^{-i\frac{\triangle_{ij}^2c^3}{2\hbar E}L}\langle\nu_i|\nu_j\rangle = \sum_i U_{ei}U_{ei}^* e^{-i\frac{\triangle_{ij}^2c^3}{2\hbar E}L}. \tag{1.7}$$

最后几率为

$$\begin{aligned} P_{\nu_e\to\nu_e} &= A_{\nu_e\to\nu_e}A_{\nu_e\to\nu_e}^* = |U_{e1}|^4 + |U_{e2}|^4 + |U_{e1}|^2|U_{e2}|^2\left(e^{-i\frac{\triangle_{12}^2c^3}{2\hbar E}L} + e^{i\frac{\triangle_{12}^2c^3}{2\hbar E}L}\right) \\ &= \cos^4\theta_{12} + \sin^4\theta_{12} + 2\cos^2\theta_{12}\sin^2\theta_{12}\cos\left(\frac{\triangle_{12}^2c^3}{2\hbar E}L\right) \\ &= (1-\sin^2\theta_{12})^2 + \sin^4\theta_{12} + \frac{1}{2}\sin^2 2\theta_{12}\cos\left(\frac{\triangle_{12}^2c^3}{2\hbar E}L\right) \\ &= 1 + 2\sin^4\theta_{12} - 2\sin^2\theta_{12} + \frac{1}{2}\sin^2 2\theta_{12}\left[1 - 2\sin^2\left(\frac{\triangle_{12}^2c^3}{2\hbar E}L\right)\right] \\ &= 1 + 2\sin^2\theta_{12}\left(\sin^2\theta_{12} - 1\right) + \frac{1}{2}\sin^2 2\theta_{12} - \sin^2 2\theta_{12}\sin^2\left(\frac{\triangle_{12}^2c^3}{2\hbar E}L\right) \\ &= 1 - \sin^2 2\theta_{12}\sin^2\left(\frac{\triangle_{12}^2c^3}{2\hbar E}L\right). \end{aligned} \tag{1.8}$$

三味混合

PMNS矩阵为

$$
\begin{aligned}
U &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{23} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{12}s_{13}s_{23}e^{i\delta} - s_{12}c_{23} & -s_{12}s_{13}s_{23}e^{i\delta} + c_{12}c_{23} & c_{12}s_{23} \\ -c_{12}s_{13}c_{23}e^{i\delta} + s_{12}s_{23} & -s_{12}s_{13}c_{23}e^{i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}
\end{aligned} \tag{1.9}
$$

味道本征态和质量本征态的关系为

$$
|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle. \tag{1.10}
$$

振荡概率为

$$
\begin{aligned}
P_{\alpha\to\beta} = |\langle\nu_\beta|\nu_\alpha\rangle|^2 &= \left|\sum_{ij} U^\dagger_{i\beta} U_{\alpha i} e^{-i\frac{\triangle_{ij}^2 c^3}{2\hbar E}L} \langle\nu_i|\nu_j\rangle\right|^2 = \left|\sum_i U_{\alpha i} U^*_{\beta i} e^{-i\frac{\triangle_{ij}^2 c^3}{2\hbar E}L}\right|^2 \\
&= \sum_i |U_{\alpha i}U^*_{\beta i}|^2 + 2\sum_{i<j} Re[U_{\alpha i}U^*_{\beta i}U_{\alpha j}U_{\beta j}] \cos\left(\frac{\triangle_{ij}^2 c^3}{2\hbar E}L\right) + 2\sum_{i<j} Im[U_{\alpha i}U^*_{\beta i}U_{\alpha j}U_{\beta j}] \sin\left(\frac{\triangle_{ij}^2 c^3}{2\hbar E}L\right) \\
&= \left|\sum_i U_{\alpha i}U^*_{\beta i}\right|^2 - 4\sum_{i<j} Re[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^2\left(\frac{\triangle_{ij}^2 c^3}{4\hbar E}L\right) - 2J\sum_{k\gamma} \varepsilon_{ijk}\varepsilon_{\alpha\beta\gamma} \sin\left(\frac{\triangle_{ij}^2 c^3}{2\hbar E}L\right) \\
&= \delta_{\alpha\beta} - 4\sum_{i<j} Re[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^2\left(\frac{\triangle_{ij}^2 c^3}{4\hbar E}L\right) + 8J\sum_{\gamma} \varepsilon_{\alpha\beta\gamma} \sin\left(\frac{\triangle_{13}^2 c^3}{4\hbar E}L\right) \textbf{TBDDDDDDDDDD}
\end{aligned} \tag{1.11}
$$

于是电子中微子到电子中微子的振荡概率为

$$
\begin{aligned}
&P_{\nu_e\to\nu_e} \\
&= 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\left(\frac{\triangle_{12}^2 c^3}{4\hbar E}L\right) - 4|U_{e1}|^2|U_{e3}|^2 \sin^2\left(\frac{\triangle_{13}^2 c^3}{4\hbar E}L\right) - 4|U_{e2}|^2|U_{e3}|^2 \sin^2\left(\frac{\triangle_{23}^2 c^3}{4\hbar E}L\right) \\
&= 1 - 4\cos^2\theta_{12}\cos^4\theta_{13}\sin^2\theta_{12}\sin^2\left(\frac{\triangle_{12}^2 c^3}{4\hbar E}L\right) - 4\cos^2\theta_{12}\cos^2\theta_{13}\sin^2\theta_{13}\sin^2\left(\frac{\triangle_{13}^2 c^3}{4\hbar E}L\right) \\
&- \sin^2\theta_{12}\cos^2\theta_{13}\sin^2\theta_{13}\sin^2\left(\frac{\triangle_{23}^2 c^3}{4\hbar E}L\right) \\
&= 1 - \sin^2 2\theta_{13}\cos^4\theta_{13}\sin^2\left(\frac{\triangle_{12}^2 c^3}{4\hbar E}L\right) - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2\left(\frac{\triangle_{13}^2 c^3}{4\hbar E}L\right) + \sin^2\theta_{12}\sin^2\left(\frac{\triangle_{23}^2 c^3}{4\hbar E}L\right)\right]
\end{aligned} \tag{1.12}
$$