Graph Theory
Chair of Operations Research
School of Computation, Information & Technology

Graph Theory (Einführung in die Graphentheorie)

Contents

Basics

1. Basic definitions and Operations on graphs
   (a) Incident, joins, adjacent, homomorphism
   (b) Graph property
      i. Containing a graph
      ii. Graph invariant
   (c) Operations on graphs
      i. disjoint
      ii. subgraph
      iii. proper subgraph
      iv. induced subgraph
      v. induces
      vi. graph sum
      vii. graph product
      viii. complement
2. Degree of Vertices
   (a) neigbors
   (b) degree (minimum degree, maximum degree, average degree, edge-vertex degree)
3. Paths & Cycles
   (a) end vertices, inner vertices, length, independent, connected
   (b) cycle, girth, circumference, chord, distance, central, radius
   (c) walk
4. Connectivity
   (a) separator, bridge
   (b) (vertex) connectivity
   (c) complete
   (d) edge-connectivity
5. Trees and forests (vertices of degree are in a tree are called leaves)
   (a) spanning tree, rooted tree
   (b) parent (predecessor), child, height and depth of a vertex
   (c) independent or stable set, independence number

   (d) weighted graph, minimum spanning tree (Kruskal algorithm $O(m^2)$)

6. Bipartite graphs
7. Graph minors (deleting an edge, contracting an edge, deleting a vertex)
   (a) subdivision,sepalogical minor
   (b) Euler tours (Eulerian)

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Matchings

1. definition, h-regular
2. Matchings in Bipartite games
   (a) alternating path
   (b) augmenting path
   (c) vertex cover
   (d) maximum cardinality
3. Theorems
   (a) König's theorem (maximum cardinality)
   (b) Hall's theorem
4. Chinese Postman problem (see ADO)

Connectivity

1. Menge's theorem (minimum number of vertices)

Planar graphs

1. Topological prerequisites
   (a) homomorphism
   (b) straigh-line-segment
   (c) polgon
   (d) polgonal arc
   (e) plane graph
   (f) regions
   (g) frontier
2. Jordan curve theorem for Polygons
3. Plane graphs (faces, outer face, inner faces, boundary, incident)
4. A graph $G = (V, E)$ with $|E| \geq 2$ is 2-connected iff it has an open ear decomposition

Colorings

Ramsey theory for graphs