03_PW_Sol

November 7, 2018

0.1 Exercice 1 Classification system

0.2 a. Getting started

a) Read the training data from file ex1-data-train.csv . The first two columns are x 1 and x 2 . The last column holds the class label y

```
In [7]: import numpy as np
        import matplotlib.pyplot as plt
        import pandas as pd
        from math import pi,exp

        %matplotlib inline

        your_data_path = './ex1-data-train.csv'

        dataset = pd.read_csv(your_data_path,names=['x1','x2','y'])
        dataset.head()

        # load data into variables
        x1 = dataset['x1'].values
        x2 = dataset['x2'].values
        y = dataset['y'].values

        # and a list for each coordinate
        x1_0 = []
        x1_1 = []
        x2_0 = []
        x2_1 = []

        for i in np.arange(len(x1)):
            if y[i] == 0:
                x1_0.append(x1[i])
                x2_0.append(x2[i])
            else:
                x1_1.append(x1[i])
                x2_1.append(x2[i])
```

b) Compute the priors of both classes P(C0) and P(C1)

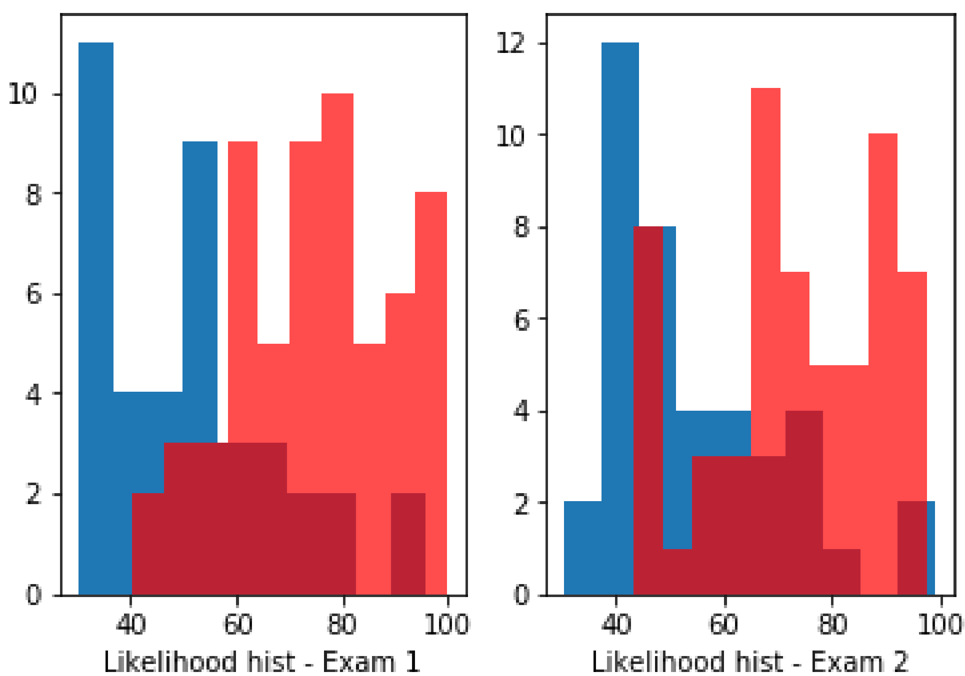
```
In [8]: N = len(y)
        prior0 = 1.0 * len(x1_0) / N
        prior1 = 1.0 * len(x1_1) / N
        print(prior0, prior1)
```

```
0.4 0.6
```

c) Compute histograms of x1 and x2 for each class (total of 4 histograms). Plot these histograms. Advice : use the numpy histogram(a,bins='auto') function.

```
In [9]: x1_0_hvals,x1_0_binedges = np.histogram(x1_0)
        x1_1_hvals,x1_1_binedges = np.histogram(x1_1)
        x2_0_hvals,x2_0_binedges = np.histogram(x2_0)
        x2_1_hvals,x2_1_binedges = np.histogram(x2_1)

        plt.subplot(1, 2, 1)
        plt.hist(x1_0)
        plt.hist(x1_1,color='red',alpha=0.7)
        plt.xlabel('Likelihood hist - Exam 1')
        plt.subplot(1, 2, 2)
        plt.hist(x2_0,)
        plt.hist(x2_1,color='red',alpha=0.7)
        plt.xlabel('Likelihood hist - Exam 2')
        plt.show()
```



d) Use the histograms to compute the likelihoods p(x1 | C0), p(x1 | C1), p(x2 | C0) and p(x2 | C1). For this define a function likelihoodHist(x,histValues,edgeValues) that returns the likelihood of x for a given histogram (defined by its values and bin edges as returned by the numpy histogram() function).

In [13]:
```
def likelihood_hist(x,histValues,edgeValues):
    for i in np.arange(len(histValues)):
        if (x>edgeValues[i]) and (x<=edgeValues[i+1]):
            return 1.0 * histValues[i]
    return 0.0000001 #out of bounds of histogram, return very small value
```

e) Implement the classification decision according to Bayes rule and compute the overall accuracy of the system on the test set ex1-data-test.csv. : — using only feature x1 — using only feature x2 — using x1 and x2 making the naive Bayes hypothesis of feature independence, i.e. p(X | Ck) = p(x1 | Ck) ů p(x2 | Ck) Which system is the best ?

In [14]:
```
# using x1
missed = 0
for x in x1_0:
    if prior0 * likelihood_hist(x,x1_0_hvals,x1_0_binedges) < prior1 * likelihood_his
        missed+=1
for x in x1_1:
    if prior0 * likelihood_hist(x,x1_0_hvals,x1_0_binedges) > prior1 * likelihood_his
        missed+=1
print('error rate using x1 = ', 100.0 * missed/N)

# using x2
missed = 0
for x in x2_0:
    if prior0 * likelihood_hist(x,x2_0_hvals,x2_0_binedges) < prior1 * likelihood_his
        missed+=1
for x in x2_1:
    if prior0 * likelihood_hist(x,x2_0_hvals,x2_0_binedges) > prior1 * likelihood_his
        missed+=1
print('error rate using x2 = ', 100.0 * missed/N)

# using x1 and x2, naive bayes
missed = 0
for i in np.arange(len(x1_0)):
    if   prior0 * likelihood_hist(x1_0[i],x1_0_hvals,x1_0_binedges) * likelihood_hist
       < prior1 * likelihood_hist(x1_0[i],x1_1_hvals,x1_1_binedges) * likelihood_hist
        missed+=1
for i in np.arange(len(x1_1)):
    if prior0 * likelihood_hist(x1_1[i],x1_0_hvals,x1_0_binedges) * likelihood_hist(x
       > prior1 * likelihood_hist(x1_1[i],x1_1_hvals,x1_1_binedges) * likelihood_hist
        missed+=1
print('error rate using x1 and x2 = ', 100.0 * missed/N)
```

```
error rate using x1 =  21.0
error rate using x2 =  22.0
```

```
error rate using x1 and x2 =  10.0
```

The best one is when using both exams x1 and x2.

0.3 b. Bayes - Univariate Gaussian distribution

Do the same as in a. but this time using univariate Gaussian distribution to model the like- lihoods p(x1|C0), p(x1|C1), p(x2|C0) and p(x2|C1). You may use the numpy functions mean() and var() to compute the mean and variance 2 of the distribution. To model the likeli- hood of both features, you may also do the naive Bayes hypothesis of feature independence, i.e. p(X|Ck) = p(x1|Ck) ů p(x2|Ck).

```
In [17]: def likelihood_univariate_gaussian(x, mean, var):
             a = 1.0 / ((2.0 * pi * var)**0.5)
             res = np.power(x - mean,2) * -((1.0)/(2.0*var))
             res = np.exp(res)
             res = res * a
             return res

         N = len(y)
         prior0 = 1.0 * len(x1_0) / N
         prior1 = 1.0 * len(x1_1) / N
         print(prior0, prior1)

         # using x1
         missed = 0
         var_01 = np.var(x1_0)
         mean_01 = np.mean(x1_0)
         var_11 = np.var(x1_1)
         mean_11 = np.mean(x1_1)

         for x in x1_0:
             if prior0 * likelihood_univariate_gaussian(x, mean_01, var_01) < prior1 * likeliho
                 missed += 1
         for x in x1_1:
             if prior0 * likelihood_univariate_gaussian(x, mean_01, var_01) > prior1 * likeliho
                 missed += 1
         print('Error rate using x1 = ', 100.0 * missed/N)

         # using x2
         missed = 0
         var_02 = np.var(x2_0)
         mean_02 = np.mean(x2_0)
         var_12 = np.var(x2_1)
         mean_12 = np.mean(x2_1)

         for x in x2_0:
```

```
        if prior0 * likelihood_univariate_gaussian(x,mean_02,var_02) < prior1 * likelihoo
            missed+=1
    for x in x2_1:
        if prior0 * likelihood_univariate_gaussian(x,mean_02,var_02) > prior1 * likelihoo
            missed+=1
    print('Error rate using x2 = ', 100.0 * missed/N)

    # using x1 and x2, naive bayes
    missed = 0
    for i in np.arange(len(x1_0)):
        if prior0 * likelihood_univariate_gaussian(x1_0[i],mean_01,var_01) * likelihood_u
            missed += 1
    for i in np.arange(len(x1_1)):
        if prior0 * likelihood_univariate_gaussian(x1_1[i],mean_01,var_01) * likelihood_u
            missed += 1
    print('Error rate using x1 and x2 = ', 100.0 * missed/N)
```

```
0.4 0.6
error rate using x1 =  60.0
error rate using x2 =  25.0
error rate using x1 and x2 =  7.0
```

0.4 Exercice 2 System evaluation

In [82]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

%matplotlib inline


your_data_path = '/Users/lorenz/Documents/ML-PW-2018/PW03/ex2-system-a.csv'

datasetA = pd.read_csv(your_data_path, sep=';', index_col=False, names=['0','1','2','
class_names = ['0','1','2','3','4','5','6','7','8','9']
nb_classes = len(class_names)

datasetA.head()
```

Out[82]:
```
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    .dataframe tbody tr th:only-of-type {
        vertical-align: middle;
    }

    .dataframe tbody tr th {
        vertical-align: top;
```

```
    }

    .dataframe thead th {
        text-align: right;
    }
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      <th>6</th>
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      <th>8</th>
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      <th>y_true</th>
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      <td>2.330260e-08</td>
      <td>5.241270e-12</td>
      <td>9.999650e-01</td>
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      <td>3.202960e-05</td>
      <td>8.504280e-01</td>
      <td>1.669090e-03</td>
      <td>1.546460e-07</td>
      <td>2.412940e-04</td>
      <td>1.448280e-01</td>
      <td>1.122810e-11</td>
```

```
      <td>1.456330e-03</td>
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      <td>2</td>
    </tr>
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      <td>1</td>
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      <td>2.550390e-10</td>
      <td>1.112010e-05</td>
      <td>1.653200e-05</td>
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      <td>7.464220e-08</td>
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      <td>1.071610e-06</td>
      <td>0.000831</td>
      <td>4</td>
    </tr>
  </tbody>
</table>
```

</div>

a) Write a function to take classification decisions on such outputs according to Bayes'rule.

```
In [84]: def a_posteriori_probability(df):
             y_true = df['y_true'].values
             nb_classes = np.unique(y_true).size
             # counting label apparition
             label_count = np.array([(df['y_true'] == i).sum() for i in range(0,nb_classes)],d

             # a priori probability for each class
             a_priori = (label_count / label_count.sum())



             # likelihoods
             likelihoods = df[class_names].values

             a_posteriori = [l * a_priori for l in likelihoods]
             a_posteriori_label = np.argmax(a_posteriori,axis=1)
             return a_posteriori_label


         y_pred_A = a_posteriori_probability(datasetA)
         y_true = datasetA['y_true'].values
```

b) What is the overall error rate of the system ?

```
In [86]: overall_error_rate = 1 - ((y_pred_A == y_true).sum() * 1.0) / y_true.size
         print('Overall rate of the system : ', overall_error_rate)

Overall rate of the system :  0.8925
```

c) Compute and report the confusion matrix of the system.

```
In [95]: from sklearn.metrics import confusion_matrix
         verif_cm = confusion_matrix(y_true,y_pred_A)

         def confusion_matrix2(y_true, y_pred, nb_classes):
             m = [[0] * nb_classes for i in range(nb_classes)]
             for pred, exp in zip(y_pred, y_true):
                 m[pred][exp] += 1
             return np.array(m)

         cm_A = confusion_matrix2(y_pred_A,y_true,nb_classes)
         # verification for our confusion matrix function
         print((cm_A == verif_cm).sum() == nb_classes**2)
```

```
True
```

```
In [111]: def plot_confusion_matrix(cm, classes,
                                    normalize=False,
                                    title='Confusion matrix',
                                    cmap=plt.cm.Blues):
              """
              This function prints and plots the confusion matrix.
              Normalization can be applied by setting `normalize=True`.
              """
              if normalize:
                  cm = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]

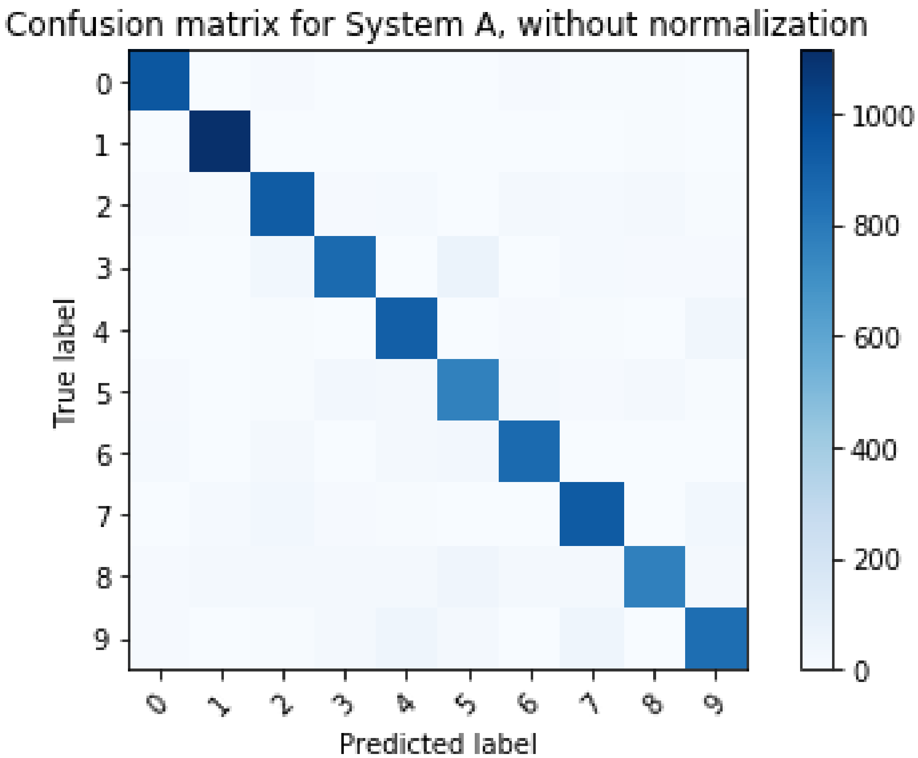
              plt.imshow(cm, interpolation='nearest', cmap=cmap)
              plt.title(title)
              plt.colorbar()
              tick_marks = np.arange(len(classes))
              plt.xticks(tick_marks, classes, rotation=45)
              plt.yticks(tick_marks, classes)

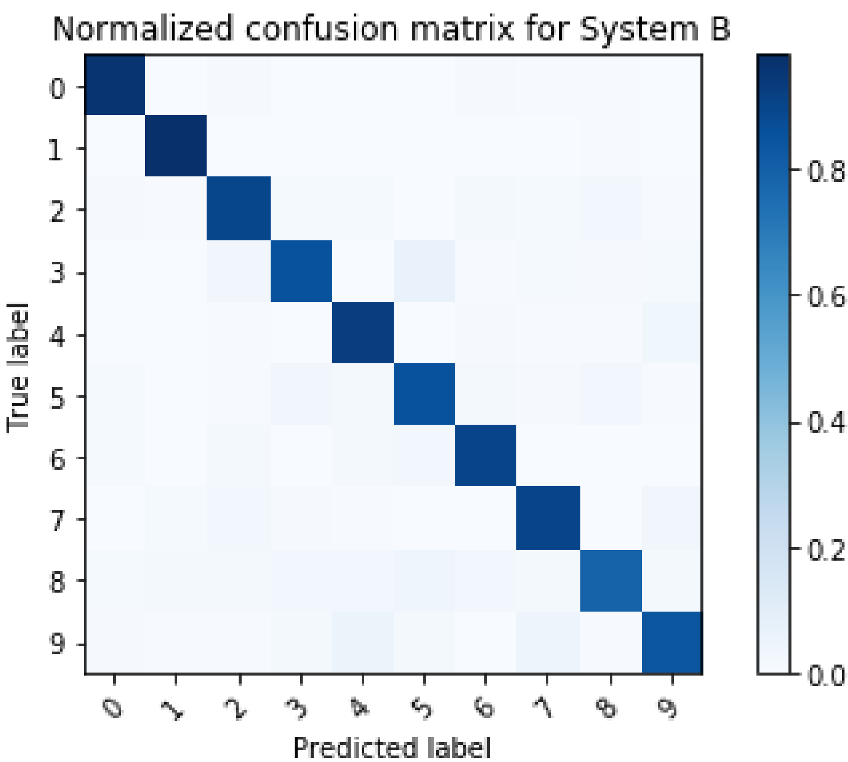
              plt.tight_layout()
              plt.ylabel('True label')
              plt.xlabel('Predicted label')

          plt.figure()
          plot_confusion_matrix(cm_A, classes=class_names,title='Confusion matrix for System A

          plt.figure()
          plot_confusion_matrix(cm_A, classes=class_names, normalize=True,
                                title='Normalized confusion matrix for System B')

          plt.show()
```


Confusion matrix for System A, without normalization
True label
Predicted label
0
1
2
3
4
5
6
7
8
9
0
200
400
600
800
1000


Normalized confusion matrix for System B
True label
Predicted label
0
1
2
3
4
5
6
7
8
9
0.0
0.2
0.4
0.6
0.8

d) What are the worst and best classes in terms of precision and recall ?

```
In [97]: def recall_per_class(cm):
             end = cm.shape[0]
             tp = np.array([cm[i,i] for i in range (0,cm.shape[0])],dtype=np.float64)
             fn = np.array([(cm[i,np.r_[0:i,(i+1):end]]).sum() for i in range(0,cm.shape[0])],d
             return tp / (tp + fn)
```

```
In [98]: recalls = recall_per_class(cm_A)

         best_class = np.argmax(recalls)
         worst_class = np.argmin(recalls)
         print('best class : ',best_class,'[',recalls[best_class],']')
         print('worst class : ',worst_class,'[',recalls[worst_class],']')
         print(recalls)
```

```
best class :  1 [ 0.9806167400881057 ]
worst class :  8 [ 0.7885010266940452 ]
[0.96326531 0.98061674 0.89437984 0.85544554 0.92566191 0.85762332
 0.90292276 0.9036965  0.78850103 0.83746283]
```

e) In file ex1-system-b.csv you find the output of a second system B. What is the best system between (a) and (b) in terms of error rate and F1.

```
In [103]: # load data
          your_data_path = '/Users/lorenz/Documents/ML-PW-2018/PW03/ex2-system-b.csv'

          datasetB = pd.read_csv(your_data_path, sep=';', index_col=False, names=['0','1','2',
          datasetB.head()
```

```
Out[103]: <div>
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              .dataframe tbody tr th:only-of-type {
                  vertical-align: middle;
              }

              .dataframe tbody tr th {
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              }

              .dataframe thead th {
                  text-align: right;
              }
          </style>
          <table border="1" class="dataframe">
```

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    <th>7</th>
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    <th>9</th>
    <th>y_true</th>
  </tr>
</thead>
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  </tr>
  <tr>
    <th>2</th>
    <td>5.653110e-09</td>
```

```
            <td>9.997150e-01</td>
            <td>2.231520e-04</td>
            <td>6.524200e-06</td>
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            <td>9.027630e-06</td>
            <td>8.147850e-09</td>
            <td>7.237800e-12</td>
            <td>3.341060e-09</td>
            <td>1.152500e-06</td>
            <td>3.915740e-06</td>
            <td>7.381230e-11</td>
            <td>4.782200e-08</td>
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            <th>4</th>
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            <td>9.996150e-01</td>
            <td>1.231720e-08</td>
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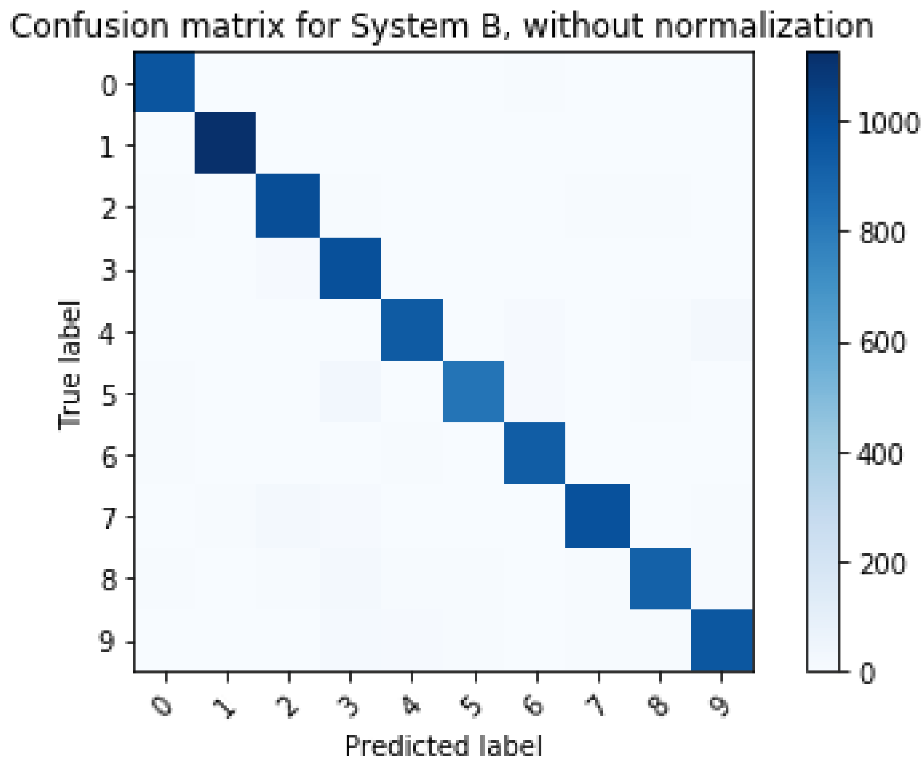
            <td>4</td>
          </tr>
        </tbody>
      </table>
      </div>
```

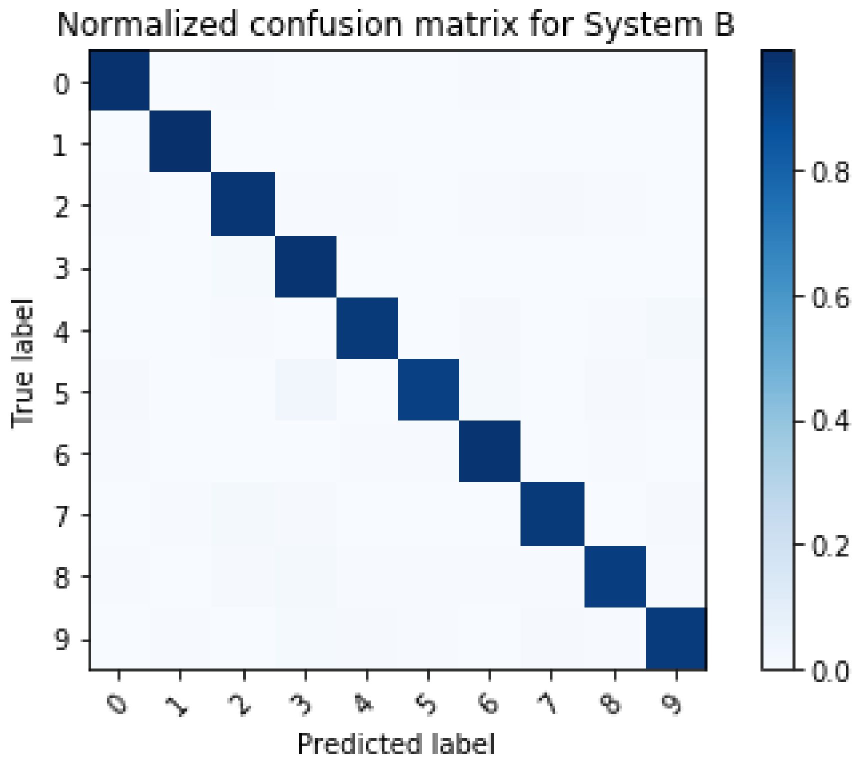
In [110]:
```
y_pred_B = a_posteriori_probability(datasetB)
cm_B = confusion_matrix2(y_pred_B,y_true,nb_classes)

plt.figure()
plot_confusion_matrix(cm_B, classes=class_names,title='Confusion matrix for System B
```

```
plt.figure()
plot_confusion_matrix(cm_B, classes=class_names, normalize=True,
                      title='Normalized confusion matrix for System B')

plt.show()
```






```
In [106]: def system_precision(cm):
              tp = np.array([cm[i,i] for i in range (0,cm.shape[0])],dtype=np.float64).sum()
              fp = np.sum([(cm[i,0:i]).sum() for i in range(0,cm.shape[0])])
              return tp / (tp + fp)


          def system_recall(cm):
              tp = np.array([cm[i,i] for i in range (0,cm.shape[0])],dtype=np.float64).sum()
              fn = np.sum([(cm[i,i+1:cm.shape[0]]).sum() for i in range(0,cm.shape[0])])
              return tp / (tp + fn)


          def system_accuracy(cm):
              # sum of the diagonal
              tp = np.array([cm[i,i] for i in range (0,cm.shape[0])],dtype=np.float64).sum()
              return tp / cm.sum()


          def f1_score(cm):
              precision = system_precision(cm)
              recall = system_recall(cm)
              return 2 * ((precision * recall)/(precision + recall))
```

```
In [107]: print('System A\n\trecall : ',system_recall(cm_A),'\n\tprecision : ',system_precisio
          print('System B\n\trecall : ',system_recall(cm_B),'\n\tprecision : ',system_precisio
```

```
System A
        recall :  0.9536275243081526
        precision :  0.932991846121681
        accuracy :  0.8925
        f1-score :  0.9431968295904888
System B
        recall :  0.984533442589368
        precision :  0.9759366433140421
        accuracy :  0.9612
        f1-score :  0.9802161941668367
```

System B is has the higher scores and accuracy.

0.5 Exercice 3 System evaluation

a) How would you build a Bayesian classification for the same task ? Comment on the prior probabilities and on the likelihood estimators. More specifically, what kind of estimator could we use in this case ?

I will try a naive bayes, where all prixels are independend. This is a strong assumbtion and probably not true for a picture. Prior probabilities should not matter as long as I do not choose 0 or 1 (never do that!).

b) Optional : implement it and report performance !

```
In [94]: import pandas as pd
         import numpy as np
         import os
         from scipy.stats import multivariate_normal as MVN

         your_data_path = '/Users/lorenz/Documents/ML-PW-2018/PW03/mnist/'

         def load_MNIST(ROOT):
           '''load all of mnist
           training set first'''
           Xtr = []
           train = pd.read_csv(os.path.join(ROOT, 'mnist_train.csv'))
           X = np.array(train.drop('label', axis=1))
           Ytr = np.array(train['label'])
           # With this for-loop we give the data a shape of the acctual image (28x28)
           # instead of the shape in file (1x784)
           for row in X:
               Xtr.append(row.reshape(28,28))
           # load test set second
           Xte = []
```

```
    test = pd.read_csv(os.path.join(ROOT, 'mnist_test.csv'))
    X = np.array(test.drop('label', axis=1))
    Yte = np.array(test['label'])
    # same reshaping
    for row in X:
        Xte.append(row.reshape(28,28))

    return np.array(Xtr), np.array(Ytr), np.array(Xte), np.array(Yte)
```

In [95]:
```
X_train, y_train, X_test, y_test = load_MNIST(your_data_path)
```

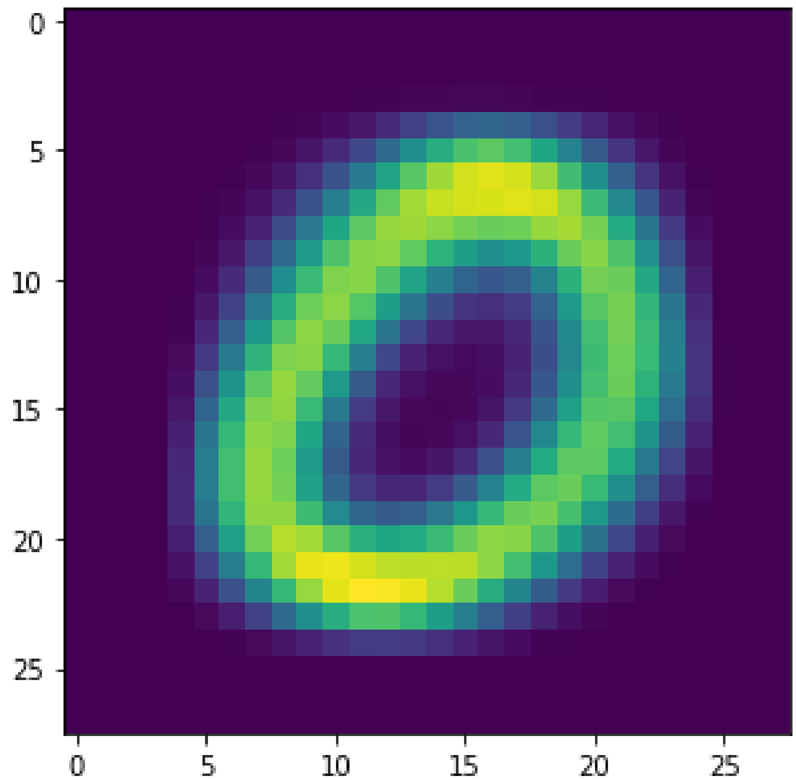
In [207]:
```
def calculate_mean_and_var(X, Y, s = 0.0001):
    labels = set(Y)
    gaussians = {}
    prior = {}
    for label in labels:
        x = X[Y == label]
        gaussians[label] = {'mean': x.mean(axis=0),
                            'var': x.var(axis=0) + s,
                             }
        prior[label] = float(len(Y[Y == key]) / len(Y))

    return gaussians, prior


class_gaussians, count_prior = calculate_mean_and_var(X_train, y_train)
```

In [196]:
```
import matplotlib.pyplot as plt
%matplotlib inline
plt.rcParams['figure.figsize'] = (5.0, 5.0)

# lets have a look at what the "mean" of the numbers look like
for key in class_gaussians:
    plt.imshow(class_gaussians[key]['mean'].astype('uint8'))
    plt.show()
```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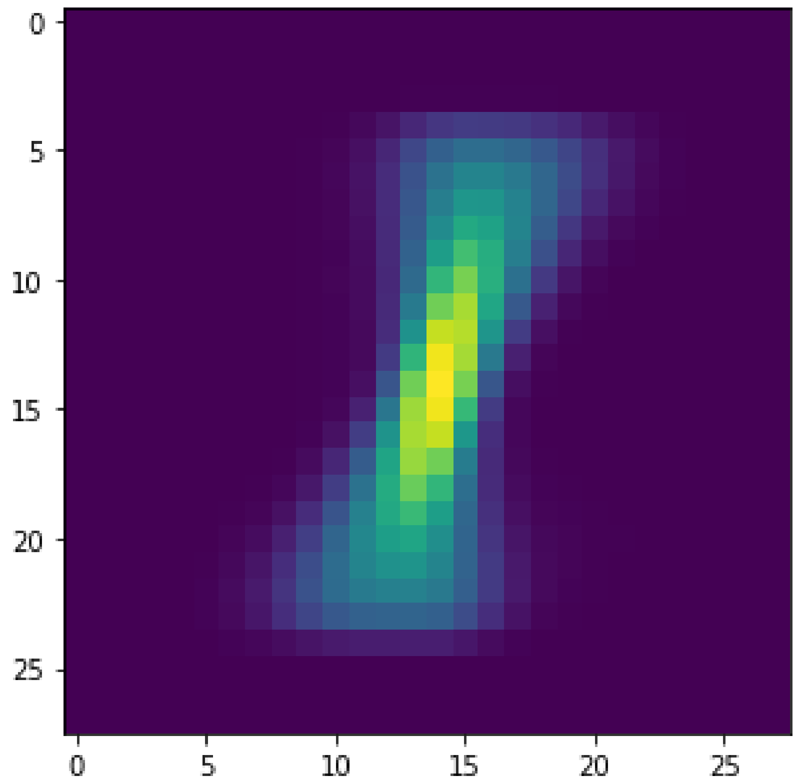

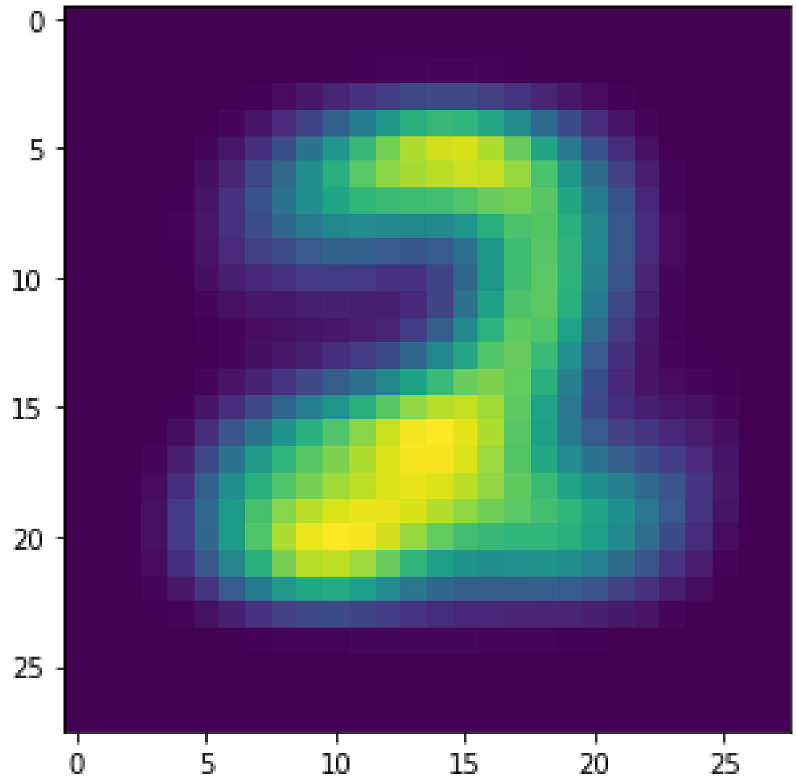
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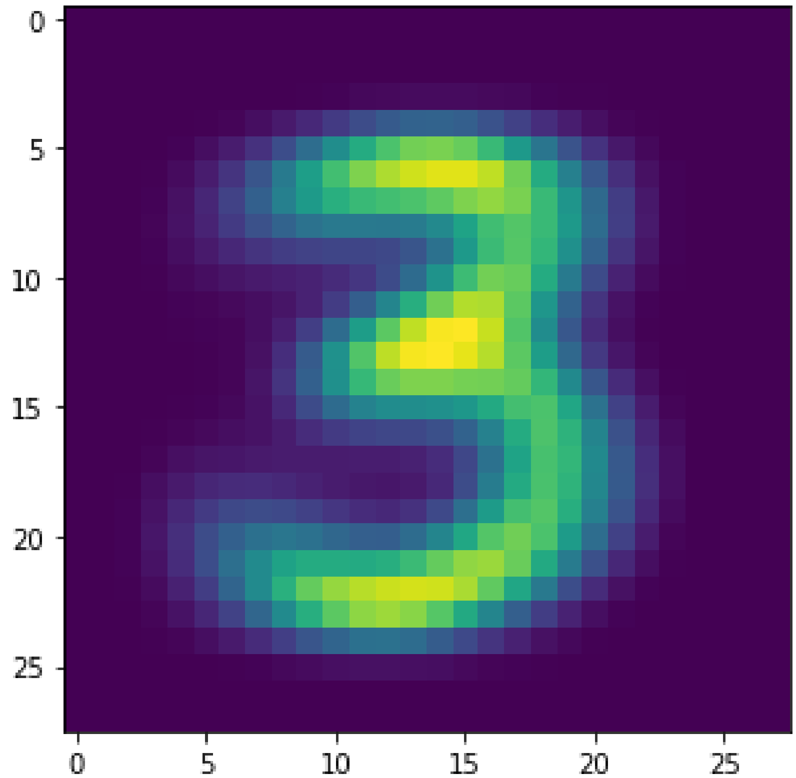
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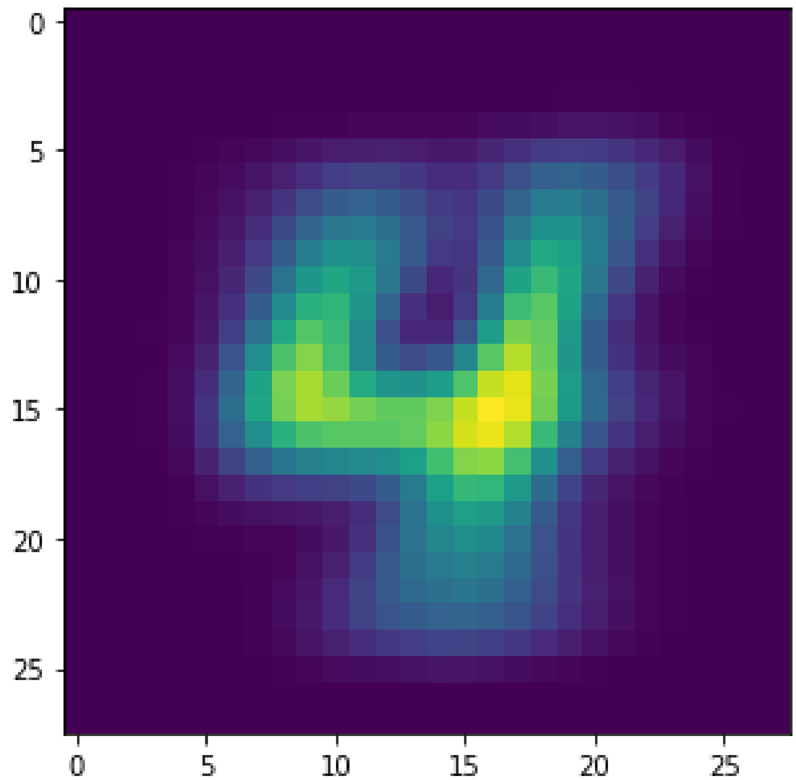
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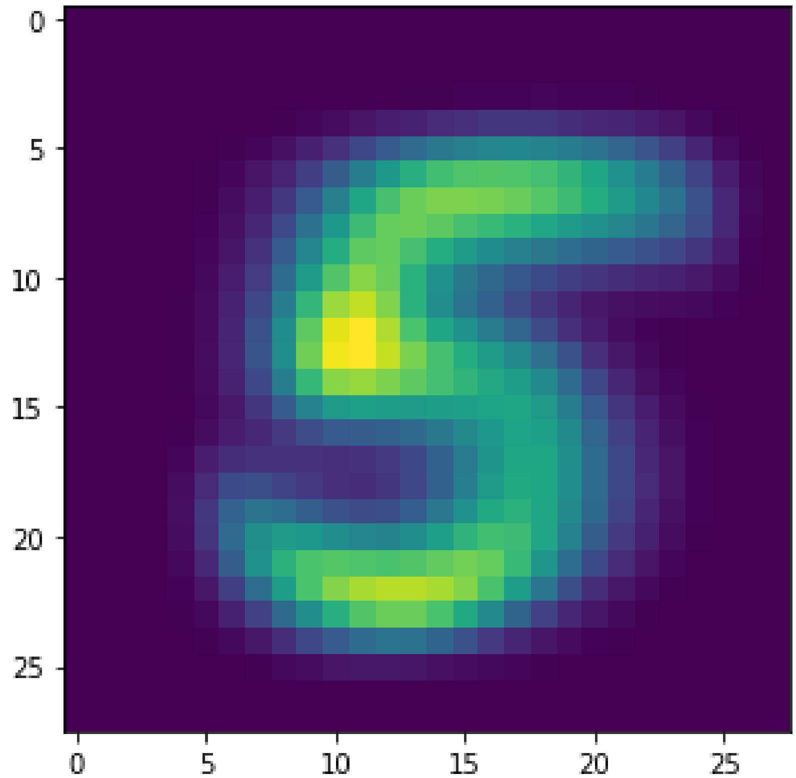

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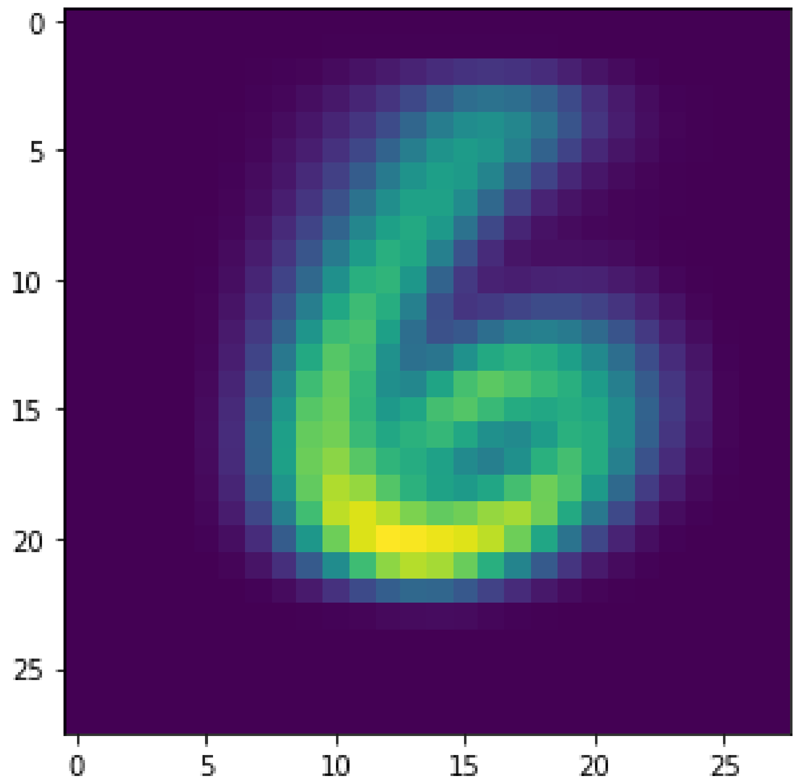
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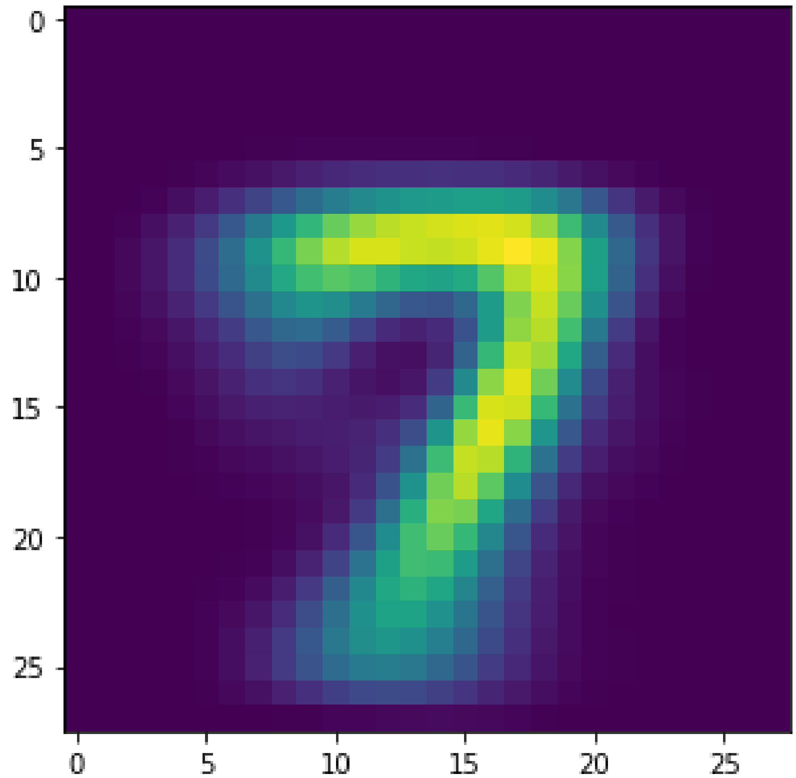
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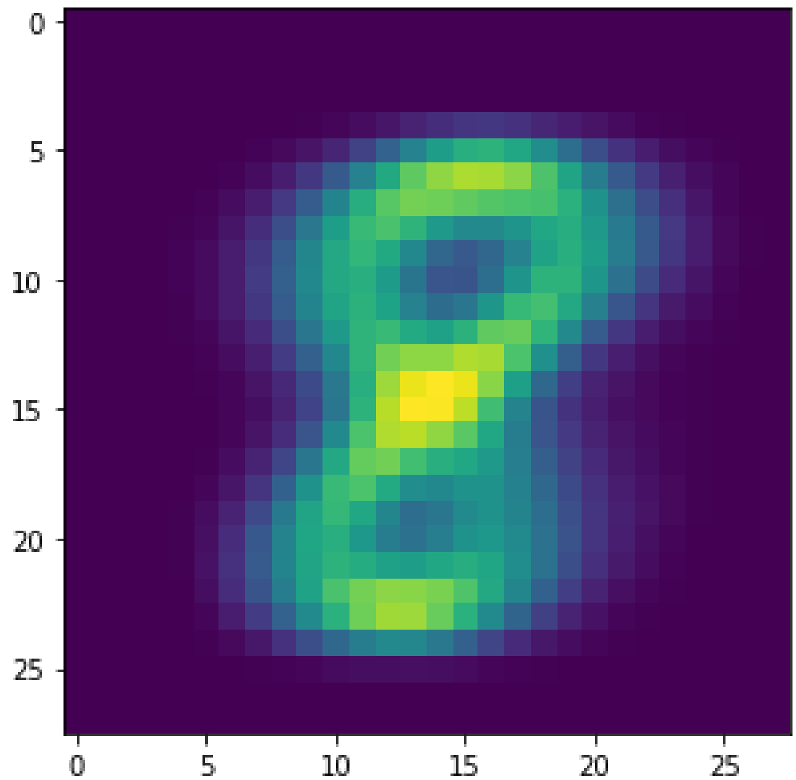
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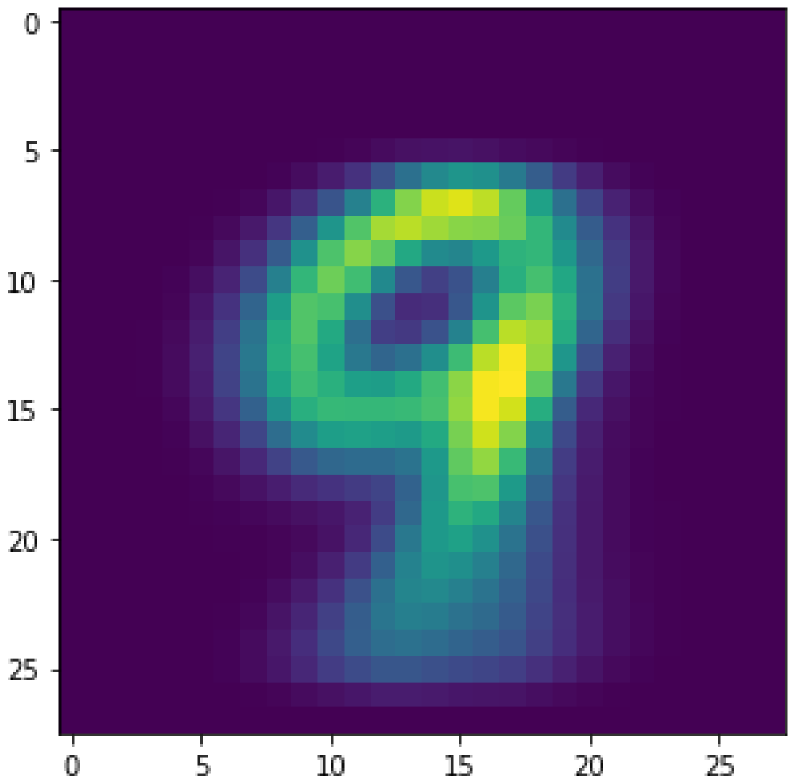

In [204]:
```
def predict_class(X, gaussians, prior):
    X = np.reshape(X, (X.shape[0], -1))
    N = len(X)
    K = len(gaussians)
    predict = np.zeros((N, K))
    for key, value in gaussians.items():
        mean, var = value['mean'], value['var']
        # we flatten the "mean" and "var" for MVN
        mean = [i for x in mean for i in x]
        var = [i for x in var for i in x]
        if isinstance(prior, float):
            predict[:,key] = MVN.logpdf(X, mean, var) + prior
        else:
            predict[:,key] = MVN.logpdf(X, mean, var) + prior[key]
    return np.argmax(predict, axis=1)
```

In [220]:
```
prior_mnist = 0.1
# I assume that random numbers from 0 - 9 are equally probably
#ăWe learned in class, that the prior should be overcome by the data.

test_predict = predict_class(X_test, class_gaussians, prior_mnist)
test_score = np.mean(test_predict == y_test)
```

```
print('We get an accuracy of %s in the test set' % (test_score))
```

```
We get an accuracy of 0.571 in the test set
```

```
In [219]: #ăHere I try with the prior that was counted
          test_predict = predict_class(X_test, class_gaussians, count_prior)
          test_score = np.mean(test_predict == y_test)

          print('We get an accuracy of %s in the test set. A different prior does not help (wha
```

```
We get an accuracy of 0.571 in the test set. A different prior does not help (what we expect)
```

We can see that the classification is not very good. This can be due to a mistake in my implementation, that I can not find. Or if could mean, that the naive gaussian approach is not very good.