04_PW_Sol

November 7, 2018

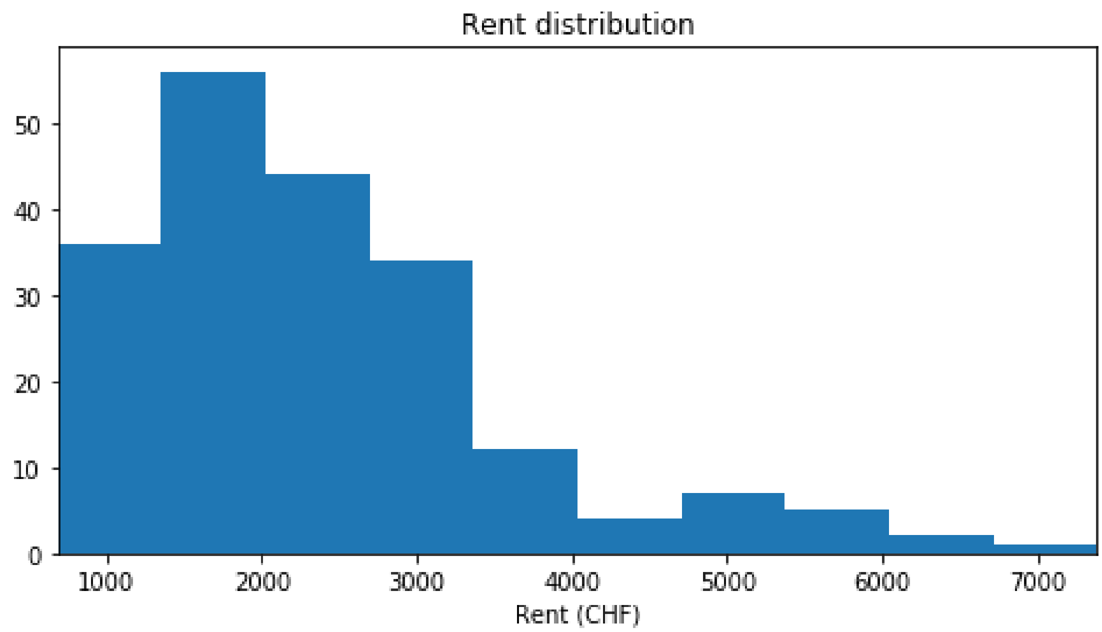
1 PW 04 : Linear Regression

In [57]:
```
from numpy.linalg import inv
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import pandas as pd
import random as rd

%matplotlib inline
```

1.1 Exercice 1 Get the data

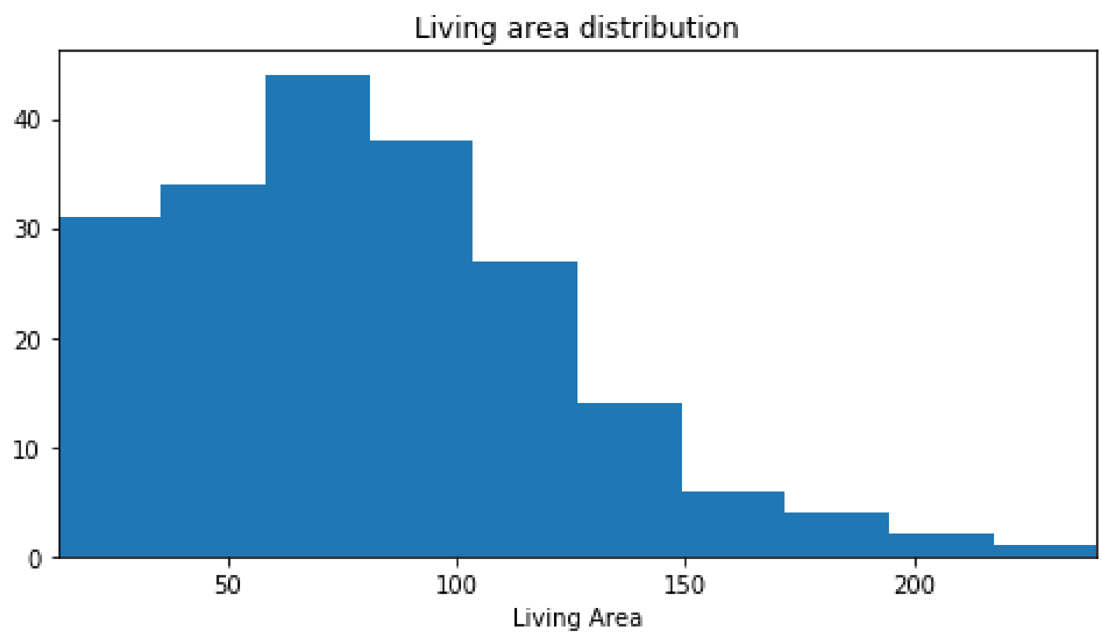
a) histogram to visualize the distribution of the renting price
b) histogram to visualize the distribution of the living area
c) scatter plot of living area as a function of renting price

In [59]:
```
datafile = '/Users/lorenz/Documents/ML-PW-2018/PW04/lausanne-appart.xlsx'

dataset = pd.read_excel(datafile, names=['living_area','nb_rooms','rent'])

rent = dataset['rent'].values
living_area = dataset['living_area'].values

plt.figure(1,figsize = (8, 4))
plt.hist(rent)
plt.xlabel("Rent (CHF)")
plt.title('Rent distribution')
plt.xlim(np.min(rent),np.max(rent))
plt.show()
```



b) histogram to visualize the distribution of the living area

```
In [60]: plt.figure(1,figsize = (8, 4))
         plt.hist(living_area)
         plt.xlabel("Living Area")
         plt.title('Living area distribution')
         plt.xlim(np.min(living_area),np.max(living_area))
         plt.show()
```



c) scatter plot of living area as a function of renting price

```
In [25]: plt.figure(1,figsize = (8, 4))
         plt.scatter(living_area,rent)
         plt.title('Rent as function of living area')
         plt.xlabel('living area')
         plt.ylabel('rent')
         plt.tight_layout()
         plt.show()
```

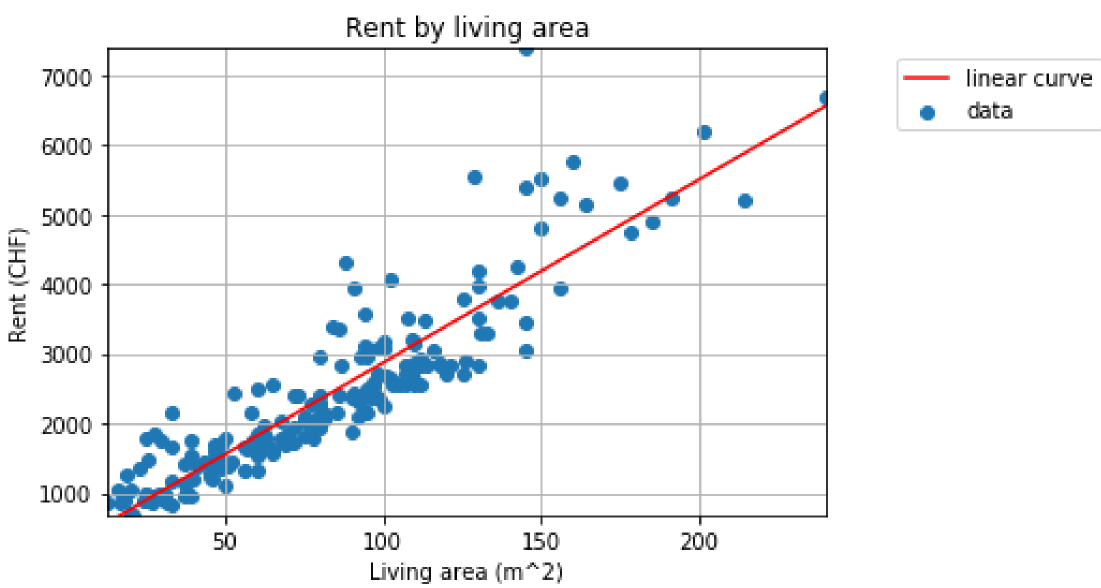
1.2 Exercice 2 Normal equations for linear regression

a) Implement Equation 3 assuming that x is the living area and y is the renting price. Use numpy for the vector operations. Plot the computed line on top of the scatter plot of exercise 1.

```
In [29]: X = living_area
         y = rent

         Xtmp = np.c_[np.ones(X.shape[0]),X]

         theta = inv(Xtmp.transpose().dot(Xtmp)).dot(Xtmp.transpose()).dot(y)
         slope = theta[1]
         intercept = theta[0]

         print('slope :',slope)
```

```
print('intercept :',intercept)

x_curve = np.array(np.linspace(np.min(X),np.max(X),200))
y_curve = x_curve * slope + intercept

plt.title('Rent by living area')
plt.xlabel('Living area (m^2)')
plt.ylabel('Rent (CHF)')
plt.scatter(X,y)
plt.plot(x_curve,y_curve,color='red')
plt.legend(['linear curve','data'],bbox_to_anchor=(1.4, 1))
plt.grid(True)
plt.xlim(np.min(X),np.max(X))
plt.ylim(np.min(y),np.max(y))
plt.show()
```

```
slope : 26.332424571995666
intercept : 240.07710726596173
```



b) Compute the overall cost value according to Equation 2.

```
In [31]: def calcualteJ(X,y,slope,intercept):
             yHat = X * slope + intercept
             diff = y - yHat
             total = sum([i ** 2 for i in diff])
             J = (total)/(2*len(X))
             return J
```

```
J = calcualteJ(X,y,slope,intercept)

print("The total value of the cost function is" , J)
```

```
The total value of the cost function is 138034.95779787414
```

1.3 Exercice 3 Batch gradient descent for linear regression

a) Plot the cost value (Equation 2) as a function of the iterations. What do you observe ?

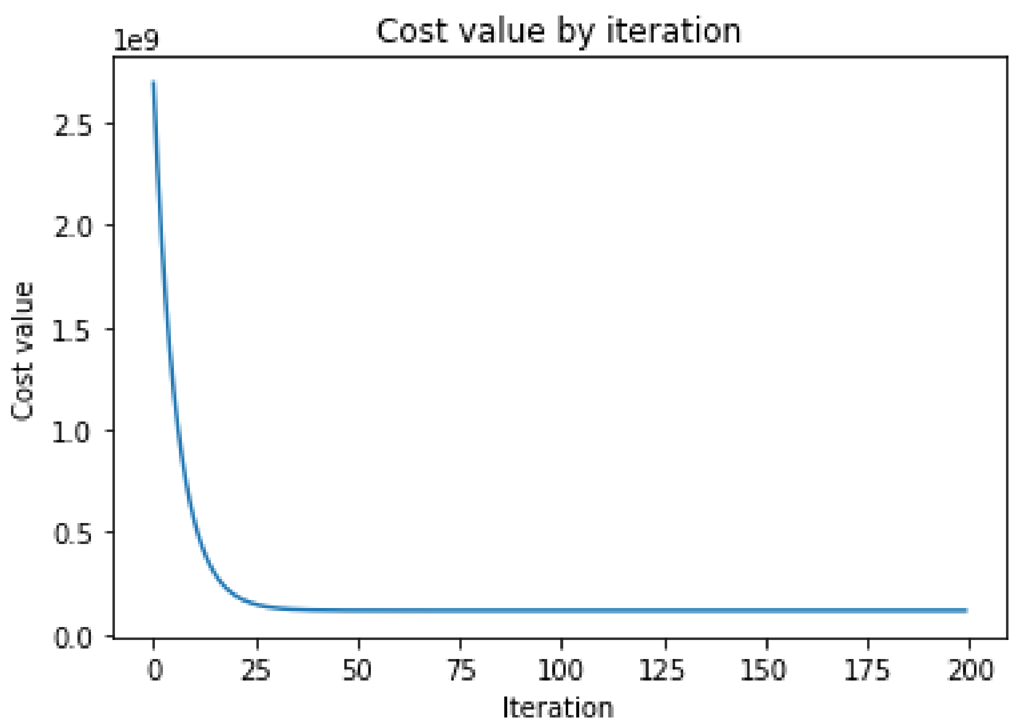
```
In [33]: # Gradient descent function
         def hypothesis(x,theta):
             return theta.transpose().dot(x)

         def gradientDescentBatch(x,y,learning_rate,num_epoch,verbose=False):
             N = x.shape[0]      # number of sample
             D = x.shape[1]      # number of dimensions
             theta = np.ones(D) # init thetas to some values
             new_theta = np.ones(D)
             cost_array = []
             for itr in range(0,num_epoch):
                 cost = 0.0
                 delta = 0.0
                 for j in range(0,D):
                     for i in range (0,N):
                         error = hypothesis(x[i],theta) - y[i]
                         delta = delta + error * x[i,j]
                         cost = cost + error**2
                     new_theta[j] = theta[j] - learning_rate * (1.0/N) * delta
                 theta = np.copy(new_theta) # update the thetas once the new values are all co
                 cost_array.append(cost)
                 if verbose:
                     print('itr : ',itr,' theta : ',theta)

             return [theta, cost_array]

         lr = 0.00001
         itr = 200

         Xtmp = np.c_[np.ones(X.shape[0]),X]
         tmp = gradientDescentBatch(Xtmp,y,lr,itr,verbose=False)
         theta = tmp[0]



         # for later
```

```
lr_batch = lr
itr_batch = itr
theta_batch = theta

slope_bgd = theta[1]
intercept_bgd = theta[0]

print('slope :',slope_bgd)
print('intercept :',intercept_bgd)
```

```
slope : 28.65186261767307
intercept : 1.3661915208241886
```

a) Plot the cost value (Equation 2) as a function of the iterations. What do you observe ?

```
In [34]: cost_array = tmp[1]

         plt.title('Cost value by iteration')
         plt.xlabel('Iteration')
         plt.ylabel('Cost value')
         plt.plot(cost_array)

         plt.show()
```



b) Imagine a stopping criterion, i.e. when do we stop iterating on the training set ?

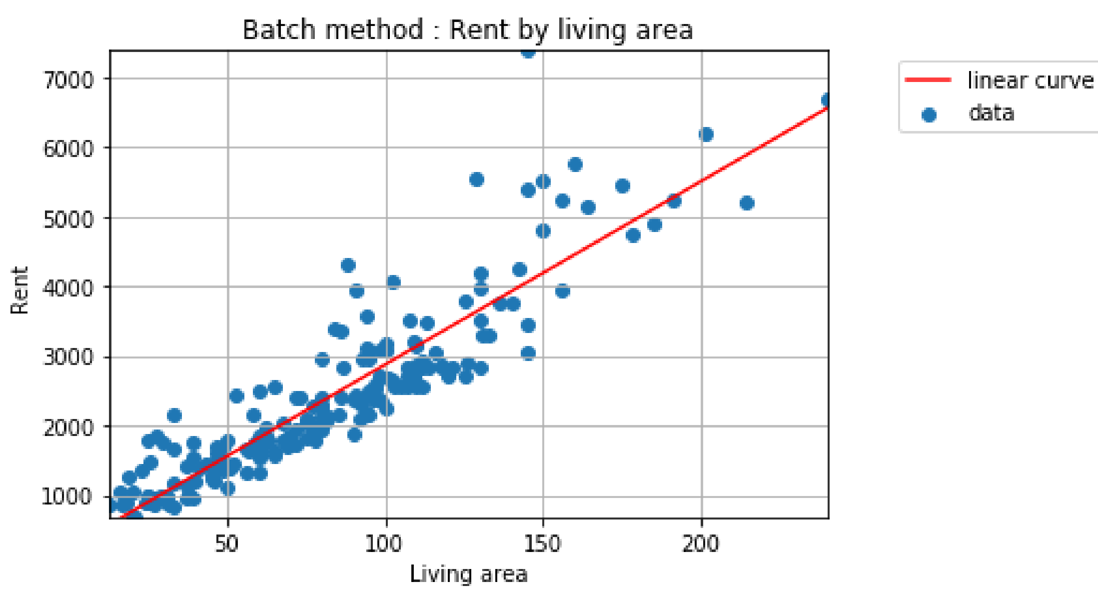
E.g. We can stop iterating, by chosing a theshold = T and check if

$$J\theta_{n-1} - J\theta_n < T.$$

We could also check if update criteria (in code delta) below a value. The reason being, that it should tend towards 0 if we approach the minimum.

c) Plot the computed line h (x) on top of the scatter plot of exercise 1

In [35]:
```
x_curve = np.array(np.linspace(np.min(X),np.max(X),200))
y_curve = x_curve * slope + intercept

plt.title('Batch method : Rent by living area')
plt.xlabel('Living area')
plt.ylabel('Rent')
plt.scatter(X,y)
plt.plot(x_curve,y_curve,color='red')
plt.legend(['linear curve','data'],bbox_to_anchor=(1.4, 1))
plt.grid(True)
plt.xlim(np.min(X),np.max(X))
plt.ylim(np.min(y),np.max(y))
plt.show()
```



d) Compute the final cost value according to Equation 2 and compare it to the one of exercise 2. What can you conclude ?

```
In [37]: J_bgd = calcualteJ(X,y,slope_bgd,intercept_bgd)


         print("The total cost value is", J_bgd, "and for closed from solution", J)
```

```
The total cost value is 144106.45423554225 and for closed from solution 138034.95779787414
```

1.4 Exercice 4 Stochastic gradient descent for linear regression

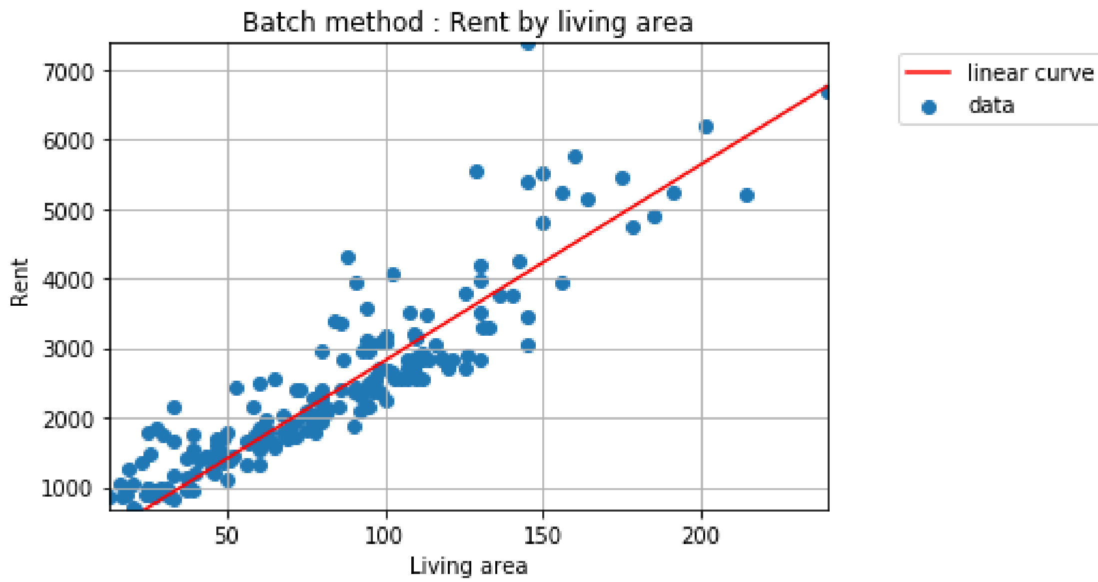
```
In [40]: def hypothesis(x,theta):
             return theta.transpose().dot(x)

         # Gradient descent function Stochastic
         def gradientDescentStochastic(x,y,learning_rate,num_iteration,verbose=False):
             N = x.shape[0]      # number of sample
             D = x.shape[1]      # number of dimensions
             theta = np.ones(D) # init the thetas to some values
             new_theta = np.ones(D)
             delta_list = []
             cost = 0
             cost_previous = cost
             no_convergence = True
             delta = 0
             counter = 0
             while no_convergence:
                 cost = 0
                 counter +=1
                 i = rd.choice(range(0,N))  # random sampling in the training set
                 for j in range(0,D):       # for all dimensions
                     error = hypothesis(x[i],theta) - y[i]
                     delta = error * x[i,j]
                     new_theta[j] = theta[j] - learning_rate * delta
                     cost = cost + error**2
                 theta = np.copy(new_theta) # update the thetas once the new values are all co
                 delta_list.append(delta)
                 if num_iteration == counter:
                     no_convergence = False
                 if counter % len(x) == 0.0:
                     if abs(cost_previous - cost) <= 500: #ă
                         no_convergence = False
                     else:
                         cost_previous = cost
                 if verbose:
                     print('itr : ',itr,' theta : ',theta,'cost : ',cost)
             return [theta,delta_list, counter]

         lr = 0.00000001
```

```
itr = 1000000

Xtmp = np.c_[np.ones(X.shape[0]),X]
tmp = gradientDescentStochastic(Xtmp,y,lr,itr)
theta = tmp[0]

# for later
lr_stochastic = lr
itr_stochastic = itr
theta_stochastic = theta

slope = theta[1]
intercept = theta[0]

print('slope :',slope)
print('intercept :',intercept)
```

```
slope : 28.191564441107403
intercept : 1.289596890447778
```

a) Plot the computed line $h_\theta(x)$ on top of the scatter plot of exercise 1.

```
In [42]: x_curve = np.array(np.linspace(np.min(X),np.max(X),200))
         y_curve = x_curve * slope + intercept

         plt.title('Batch method : Rent by living area')
         plt.xlabel('Living area')
         plt.ylabel('Rent')
         plt.scatter(X,y)
         plt.plot(x_curve,y_curve,color='red')
         plt.legend(['linear curve','data'],bbox_to_anchor=(1.4, 1))
         plt.grid(True)
         plt.xlim(np.min(X),np.max(X))
         plt.ylim(np.min(y),np.max(y))
         plt.show()
```



b) How many samples do you need to visit for reaching the convergence ?

c) What kind of stopping criterion could we use here ?

As disscused in the lecture there is not one best solution. Here I chose to calculate the cost after every epoch and compare it to the cost of the last iteration through the data set. It is very important to choose the critera in a way, that is is not impacted by the randomness of the dat point chosing. E.g.

$$\theta_{n-1} - \theta_n$$

Here, if two point are chosen, that are very close the θ does not chance alot. We stop the convergence too soon.

c) What kind of stopping criterion could we use here ?

As disscused in the lecture there is not one best solution. Here I chose to calculate the cost after every epoch and compare it to the cost of the last iteration through the data set. It is very important to choose the critera in a way, that is is not impacted by the randomness of the dat point chosing. E.g.

$$\theta_{n-1} - \theta_n$$

Here, if two point are chosen, that are very close the θ does not chance alot. We stop the convergence too soon.

d) Compute the final cost value according to Equation 2 and compare it to the one of exercise 2 and 3. What can you conclude ?

```
In [51]: J_sto = calcualteJ(X,y,slope,intercept)

         print("For the stochastic gradient desent ", J_sto,"For the badge gradient decent is "
```

```
For the stochastic gradient desent  144977.3872830564 For the badge gradient decent is  144106
```

1.5 Exercice 5 : Mini-batch gradient descent for linear regression

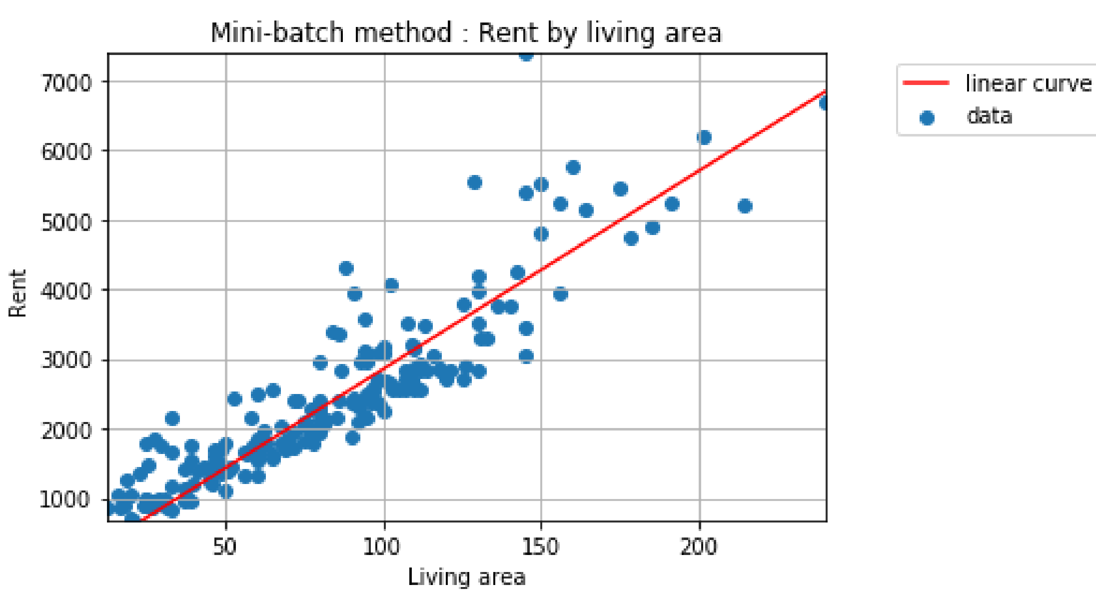
Implement the mini-batch gradient descent algorithm for the previous problem, adding a parameter B defining the size of the mini-batch. Check that when B=N, you fall back on the batch gradient descent solution, and when B=1, you get the behaviour of stochastic gradient descent.

In [53]:
```python
def hypothesis(x,theta):
    return theta.transpose().dot(x)

# Gradient descent function "Mini-Batch"
# when batch_size == 1 --> Stochastic
def gradientDescentMiniBatch(x,y,learning_rate,num_batch,batch_size=1,verbose=False):
    N = x.shape[0]      # number of sample
    D = x.shape[1]      # number of dimensions
    theta = np.ones(D) # init the thetas to some values
    new_theta = np.ones(D)

    for itr in range(0,num_batch):
        batch = rd.sample(range(0,N),batch_size) # pick b values randomly in set of i
        for j in range(0,D):
            cost = 0.0
            for i in batch:
                error = hypothesis(x[i],theta) - y[i]
                cost = cost + error * x[i,j]
            new_theta[j] = theta[j] - learning_rate * (1.0/batch_size) * cost
        theta = np.copy(new_theta) # update the thetas once the new values are all co
        if verbose:
            print('itr : ',itr,'theta : ',theta)
    return theta

lr = 0.000001
itr = 10000

Xtmp = np.c_[np.ones(X.shape[0]),X]
theta = gradientDescentMiniBatch(Xtmp,y,lr,itr,batch_size=5)

slope = theta[1]
intercept = theta[0]

print('slope :',slope)
print('intercept :',intercept)
```

```
slope : 28.49703518399393
intercept : 1.7743758015269415
```

In [61]:
```python
x_curve = np.array(np.linspace(np.min(X),np.max(X),200))
y_curve = x_curve * slope + intercept
```

```
plt.title('Mini-batch method : Rent by living area')
plt.xlabel('Living area')
plt.ylabel('Rent')
plt.scatter(X,y)
plt.plot(x_curve,y_curve,color='red')
plt.legend(['linear curve','data'],bbox_to_anchor=(1.4, 1))
plt.grid(True)
plt.xlim(np.min(X),np.max(X))
plt.ylim(np.min(y),np.max(y))
plt.show()

lr = lr_batch
itr = itr_batch

Xtmp = np.c_[np.ones(X.shape[0]),X]
theta = gradientDescentMiniBatch(Xtmp,y,lr,itr,batch_size=X.shape[0])

print('theta :',theta,'\ntheta_batch:',theta_batch)

lr = lr_stochastic
itr = itr_stochastic

Xtmp = np.c_[np.ones(X.shape[0]),X]
theta = gradientDescentMiniBatch(Xtmp,y,lr,itr,batch_size=1)

print('theta :',theta,'\ntheta_stochastic:',theta_stochastic)
```



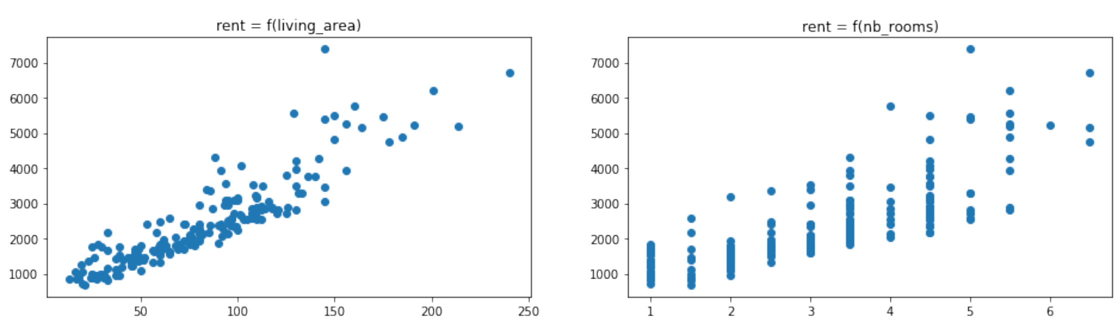

```
theta : [ 1.36968058 28.64581688]
```

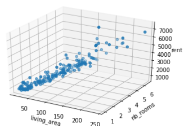
```
theta_batch: [ 1.36619152 28.65186262]
theta : [ 1.77441631 28.66076418]
theta_stochastic: [ 1.28959689 28.19156444]
```

1.6 Exercice 6a : Multi-variable linear regression

a) Implement one of the gradient descent algorithm (ex. 3-5) for the multi-variable linear regression assuming x 1 being the living area and x 2 the square of the living area. Plot the computed curve (second order) on top of the scatter plot of exercise 1.

```
In [70]: rent = dataset['rent'].values
         living_area = dataset['living_area'].values
         nb_rooms = dataset['nb_rooms'].values

         A1 = dataset.loc[:,['living_area']].values.flatten()
         A2 = dataset.loc[:,['nb_rooms']].values.flatten()
         X = dataset.loc[:,['living_area','nb_rooms']].values
         B = dataset.loc[:,['rent']].values.flatten()

         plt.figure(1,figsize = (16, 4))
         plt.subplot(1, 2, 1)
         plt.title('rent = f(living_area)')
         plt.scatter(living_area,rent)
         plt.subplot(1,2,2)
         plt.title('rent = f(nb_rooms)')
         plt.scatter(nb_rooms,rent)
         fig = plt.figure()
         ax = fig.gca(projection='3d')
         ax.scatter(living_area,nb_rooms,rent)
         ax.set_xlabel('living_area')
         ax.set_ylabel('nb_rooms')
         ax.set_zlabel('rent')
         plt.show()
```





In [71]:
```
# For verification
from sklearn import linear_model

regr = linear_model.LinearRegression()
res = regr.fit(X,rent)

theta = [regr.intercept_,regr.coef_[0],regr.coef_[1],]

slope_1 = theta[1]
slope_2 = theta[2]
intercept = theta[0]
print(theta)
```

```
[286.93086825718365, 28.661188757120193, -76.06313748941866]
```

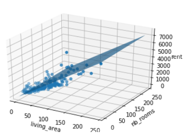
In [72]:
```
def hypothesis(theta,X):  #theta = 1xD, X = DxN, output 1xN
    """
    Compute the actual value of the h function using x
    """
    return np.dot(theta,X)

# Gradient descent function using matricial calculus
def gradientDescent(X,y,learning_rate,num_epoch,verbose=False):
    N = X.shape[0]      # number of sample
    D = X.shape[1]      # number of dimensions
    theta = np.zeros(D) # init thetas to some values
    X_trans = X.transpose() # X_trans is DxN
    for i in range(0,num_epoch):
        h = hypothesis(theta,X_trans)  #N dimension
        loss = h-y                     #N dimension
        if verbose :
            J = np.sum(loss ** 2) / (2 * N)
            print('itr :',i,' cost : ',J)
        gradient = X_trans.dot(loss) / N
        theta = theta - learning_rate * (1.0/N) * gradient
    return theta
```

In [73]:
```
Xtmp = np.c_[np.ones(X.shape[0]),X]
theta = np.zeros(Xtmp.shape[1])
```

```
lr = 0.001
itr = 1000

theta = gradientDescent(Xtmp,rent,lr,itr,verbose=False)
print(theta)

slope_1 = theta[1]
slope_2 = theta[2]
intercept = theta[0]
```

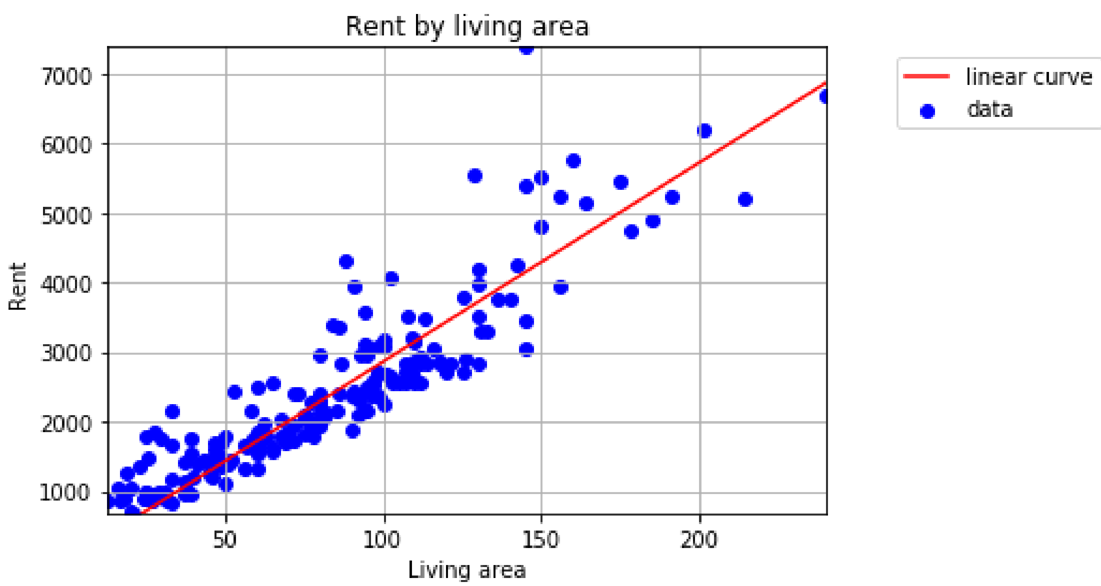
```
[ 0.53074014 28.61513173  1.06130166]
```

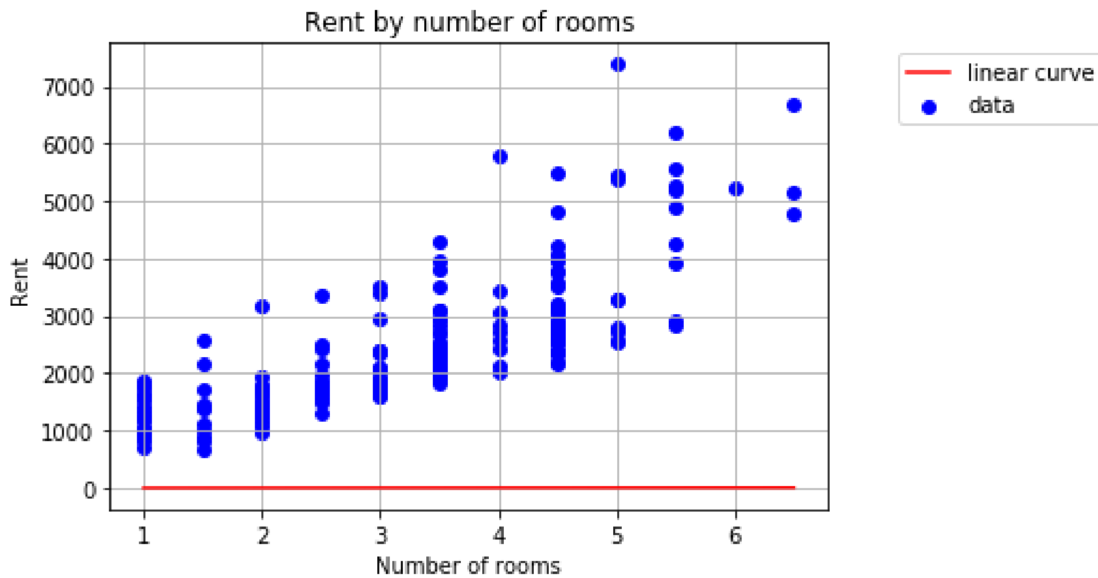
```
In [74]: x = np.array(np.linspace(np.min(X),np.max(X),200))
         y = np.array(np.linspace(np.min(X),np.max(X),200))

         x,y = np.meshgrid(x,y)

         z = intercept + (x * slope_1) + (y * slope_2)

         fig = plt.figure()
         ax = fig.gca(projection='3d')

         ax.scatter(living_area,nb_rooms,rent)
         ax.plot_surface(x,y,z)
         ax.set_xlabel('living_area')
         ax.set_ylabel('nb_rooms')
         ax.set_zlabel('rent')

         plt.show()
```



```
In [75]: % matplotlib inline
         x_curve = np.array(np.linspace(np.min(living_area),np.max(living_area),200))
         y_curve = x_curve * slope_1 + intercept

         plt.title('Rent by living area')
         plt.xlabel('Living area')
         plt.ylabel('Rent')
         plt.scatter(A1,B,color='blue')
         plt.plot(x_curve,y_curve,color='red')
         plt.legend(['linear curve','data'],bbox_to_anchor=(1.4, 1))
```

```
plt.grid(True)
plt.xlim(np.min(living_area),np.max(living_area))
plt.ylim(np.min(rent),np.max(rent))
plt.show()

x_curve = np.array(np.linspace(np.min(nb_rooms),np.max(nb_rooms),200))
y_curve = x_curve * slope_2 + intercept

plt.title('Rent by number of rooms')
plt.xlabel('Number of rooms')
plt.ylabel('Rent')
plt.scatter(nb_rooms,B,color='blue')
plt.plot(x_curve,y_curve,color='red')
plt.legend(['linear curve','data'],bbox_to_anchor=(1.4, 1))
plt.grid(True)
plt.show()
```





b) Implement one of the gradient descent algorithm (ex. 3-5) for the multi-variable linear regression assuming x 1 being the living area and x 2 the number of bedrooms.