06_PW_Sol

November 7, 2018

```
In [163]: import pandas as pd
          import numpy as np
          import matplotlib.pyplot as plt

          %matplotlib inline
```

0.1 Exercice 1: Classification to predict student admission

0.1.1 a. Getting the training data

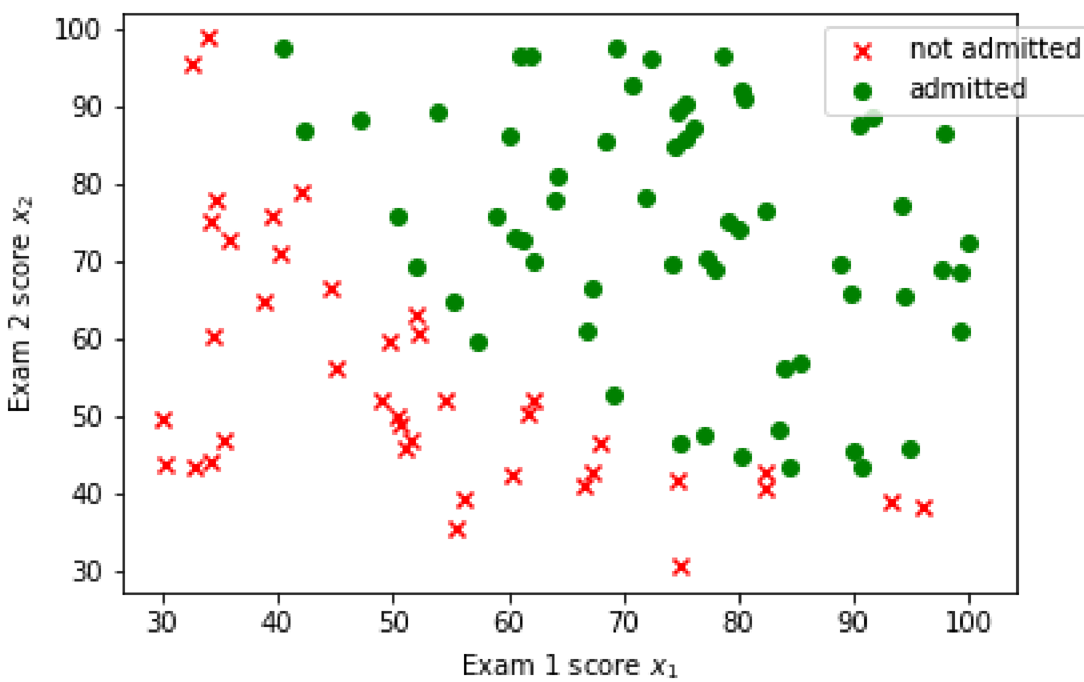
```
In [164]: path = '/Users/lorenz/Documents/ML-PW-2018/PW06/'

          dataset_train = pd.read_csv( path + 'student-dataset-train.csv',names=['x1','x2','y']
          dataset_test = pd.read_csv(path + 'student-dataset-test.csv',names=['x1','x2','y'])

          dataset_train.head()
```

```
Out[164]: <div>
          <style scoped>
              .dataframe tbody tr th:only-of-type {
                  vertical-align: middle;
              }

              .dataframe tbody tr th {
                  vertical-align: top;
              }

              .dataframe thead th {
                  text-align: right;
              }
          </style>
          <table border="1" class="dataframe">
            <thead>
              <tr style="text-align: right;">
                <th></th>
                <th>x1</th>
                <th>x2</th>
                <th>y</th>
```

```
      </tr>
    </thead>
    <tbody>
      <tr>
        <th>0</th>
        <td>34.623660</td>
        <td>78.024693</td>
        <td>0</td>
      </tr>
      <tr>
        <th>1</th>
        <td>30.286711</td>
        <td>43.894998</td>
        <td>0</td>
      </tr>
      <tr>
        <th>2</th>
        <td>35.847409</td>
        <td>72.902198</td>
        <td>0</td>
      </tr>
      <tr>
        <th>3</th>
        <td>60.182599</td>
        <td>86.308552</td>
        <td>1</td>
      </tr>
      <tr>
        <th>4</th>
        <td>79.032736</td>
        <td>75.344376</td>
        <td>1</td>
      </tr>
    </tbody>
  </table>
  </div>
```

In [94]:
```
x1_train = dataset_train['x1'].values
x2_train = dataset_train['x2'].values
x1_test = dataset_test['x1'].values
x2_test = dataset_test['x2'].values
X = np.array([np.ones(x1_train.size), dataset_train['x1'].values, dataset_train['x2']

y = dataset_train['y'].values
y_test = dataset_test['y'].values


x1_0 = x1_train[y==0]
```

```
x1_1 = x1_train[y==1]
x2_0 = x2_train[y==0]
x2_1 = x2_train[y==1]

plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
plt.xlabel('Exam 1 score $x_1$')
plt.ylabel('Exam 2 score $x_2$')
plt.legend(bbox_to_anchor=(1.1, 1))
plt.show()
```



b) Implement a z-norm normalization of the training set. You need to store the normalization values (,) for later as they will be needed to normalize the test set.

```
In [165]: def znorm(data):
              mu = np.mean(data)
              sig = np.var(data)
              return (data - mu) / sig

          x1_train_norm = znorm(x1_train)
          x2_train_norm = znorm(x2_train)
          x1_test_norm = znorm(x1_test)
          x2_test_norm = znorm(x2_test)
```

```
X_norm_train = np.array([np.ones(x1_train_norm.size), x1_train_norm, x2_train_norm])
X_norm_test = np.array([np.ones(x1_test_norm.size), x1_test_norm, x2_test_norm]).T
```

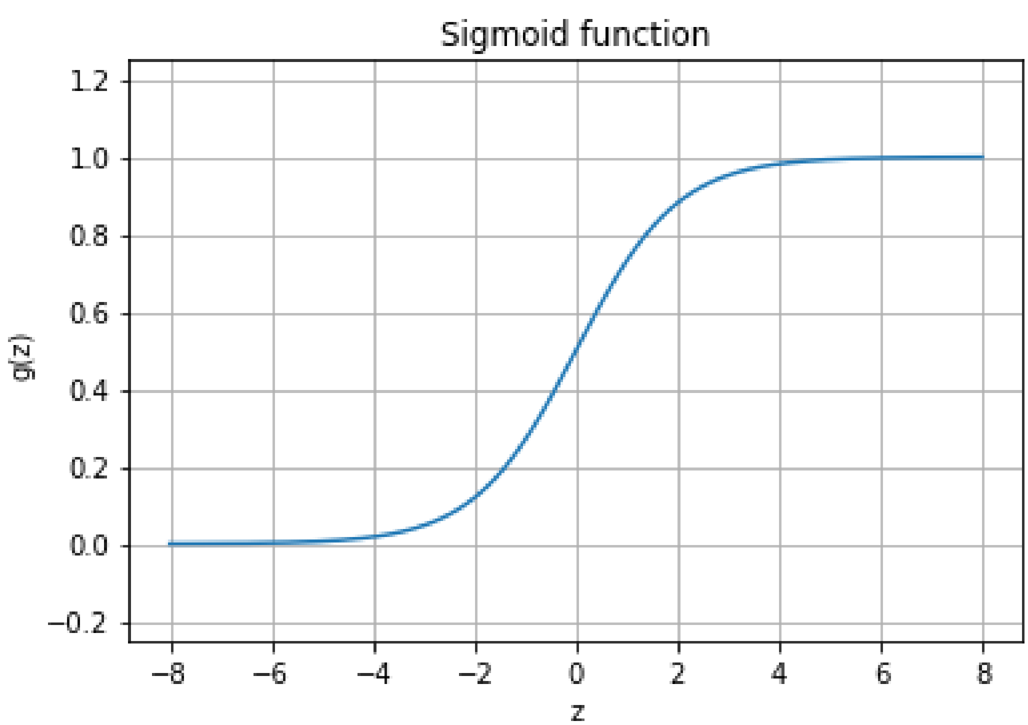
f) In a similar way as in PW02 and PW03, implement the gradient ascent with the update rule
:

c) Implement a sigmoid function

$$g(z) = \frac{1}{1 + e^z}$$

Use numpy to compute the exp so that your function can take numpy arrays as input. Plot the sigmoid function.

```
In [96]: def sigmoid_array(z):  #supporting arrays / this will serve as "g()"
             return 1 / (1 + np.exp(-z))

         z = np.linspace(-8,8,200)
         g = sigmoid_array(z)
         plt.plot(z,g)
         plt.title('Sigmoid function')
         plt.xlabel('z')
         plt.ylabel('g(z)')
         plt.ylim(-0.25,1.25)
         plt.grid()
         plt.show()
```



d) Implement the hypothesis function $h_\theta(x)$

```
In [97]: def hypothesis(X, theta):
             # X has shape (N,D) and theta has shape (D,).
             # The dot product is then broadcasted to all samples in X.
             return sigmoid_array(X.dot(theta))
```

e) Implement the objective function J(θ):

$$J(\theta) = \frac{1}{N} \sum_{n=1}^{N} y_n log(h_\theta(x_n)) + (1 y_n) log(1 h_\theta(x_n))$$

```
In [98]: def objective(X,y,theta):
             h = hypothesis(X,theta) # h has shape (N,)
             N = X.shape[0]
             tmp = y * np.log(h) + (1-y) * np.log(1-h)
             return np.sum(tmp)/N
```

f) In a similar way as in PW02 and PW03, implement the gradient ascent with the update rule
:

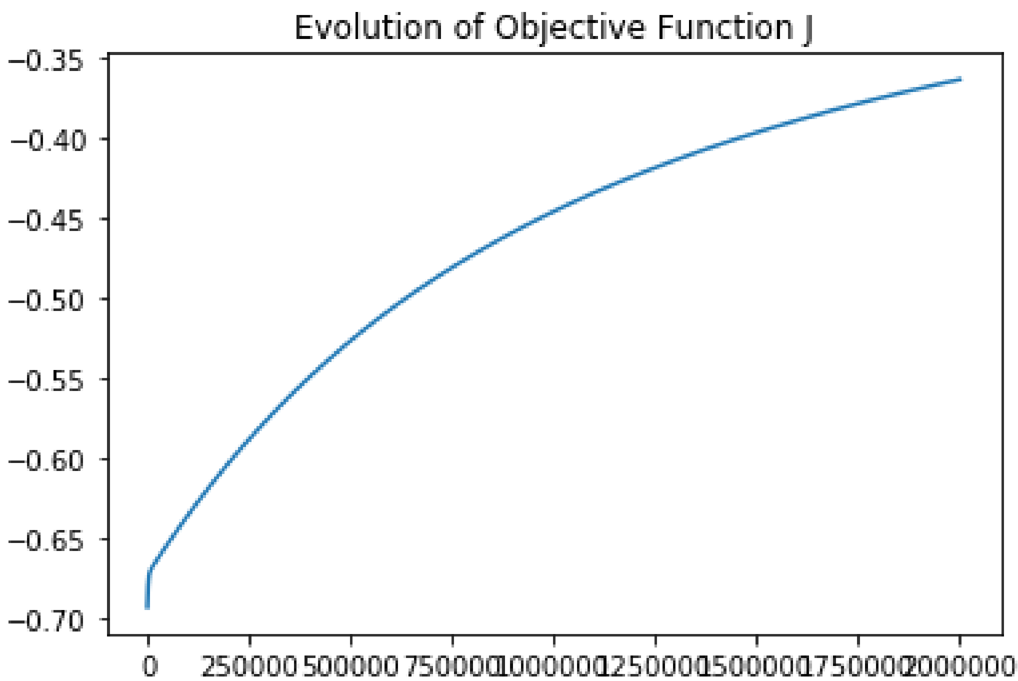
```
In [41]: def gradientAscent(X, y, learning_rate,num_epoch):
             N = X.shape[0]      # number of samples
             D = X.shape[1]      # dimensions
             theta = np.zeros(D) # init thetas to some values - in theory it can be anything
                                 # but values at zeros or close to zeros will help convergence
             J = np.zeros(num_epoch)

             for itr in range(0,num_epoch):
                 J[itr] = objective(X, y, theta)
                 h = hypothesis(X,theta)
                 loss = y - h
                 gradient = loss.dot(X)
                 theta = theta + learning_rate * (1.0 / N) * gradient
             return theta, J
```

```
In [42]: theta, J = gradientAscent(X_norm_train, y, 0.001, 2000000)

         print("theta_0 =", theta[0])
         print("theta_1 =", theta[1])
         print("theta_2 =", theta[2])
```

```
theta_0 = 0.5365153398291422
theta_1 = 17.735924642704624
theta_2 = 15.938530260988637
```

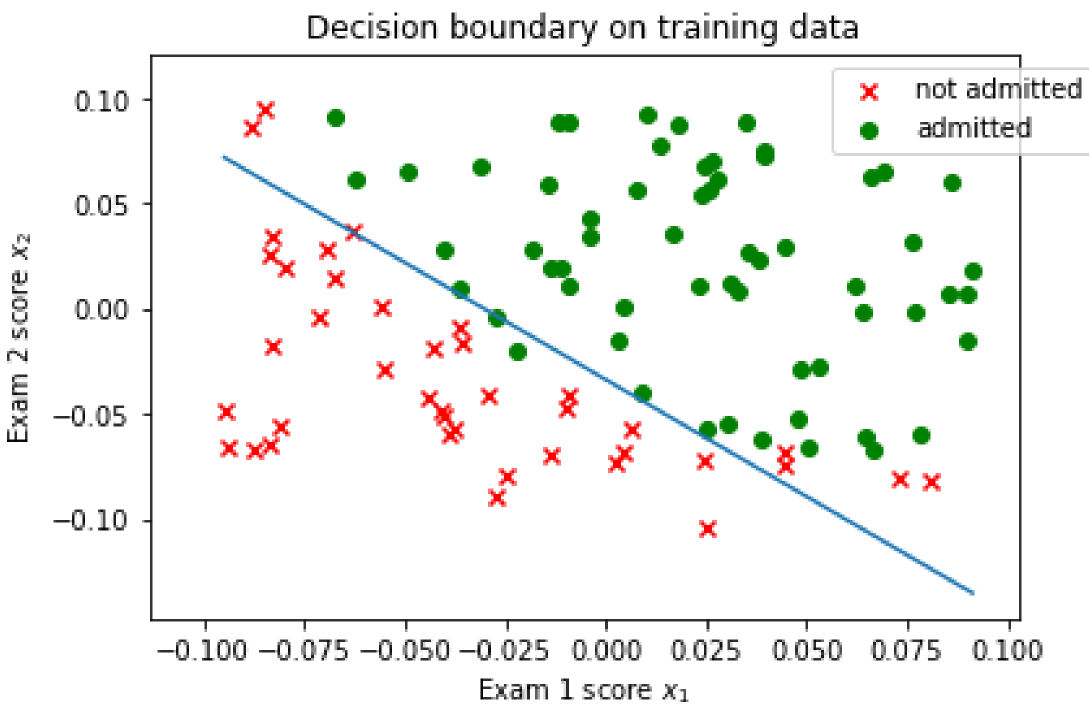
g) Test your implementation by observing the evolutions of the objective function $J(\theta)$ during the gradient ascent.

```
In [43]: plt.plot(J)
         plt.title("Evolution of Objective Function J")
         plt.xlabel("Epoch")
         plt.ylabel('J')
         plt.show()
```




```
In [44]: x1_0 = x1_train_norm[y==0]
         x1_1 = x1_train_norm[y==1]
         x2_0 = x2_train_norm[y==0]
         x2_1 = x2_train_norm[y==1]

         hy = hypothesis(X_norm_train,theta)

         plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
         plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
         plt.xlabel('Exam 1 score $x_1$')
         plt.ylabel('Exam 2 score $x_2$')
         plt.title('Decision boundary on training data')
         plt.legend(bbox_to_anchor=(1.1, 1))

         # plot the decision boundary, i.e. points where theta.x = 0
         x_1 = np.linspace(np.min(x1_train_norm), np.max(x1_train_norm), 100)
```

```
x_2 = (-theta[0] - theta[1]*x_1)/theta[2]
plt.plot(x_1, x_2)
plt.show()

num_correct = ((hy >= 0.5) == (y >= 0.5)).sum()
num_missed = y.size - num_correct
error_rate = num_missed * 1.0 / y.size

print('theta : ',theta)
print('# correct  : ', num_correct)
print('# missed   : ', num_missed)
print('error rate :  %2.2f %%'% (error_rate*100.0))
```




```
theta :   [ 0.53651534 17.73592464 15.93853026]
# correct  :  91
# missed   :  9
error rate :  9.00 %
```

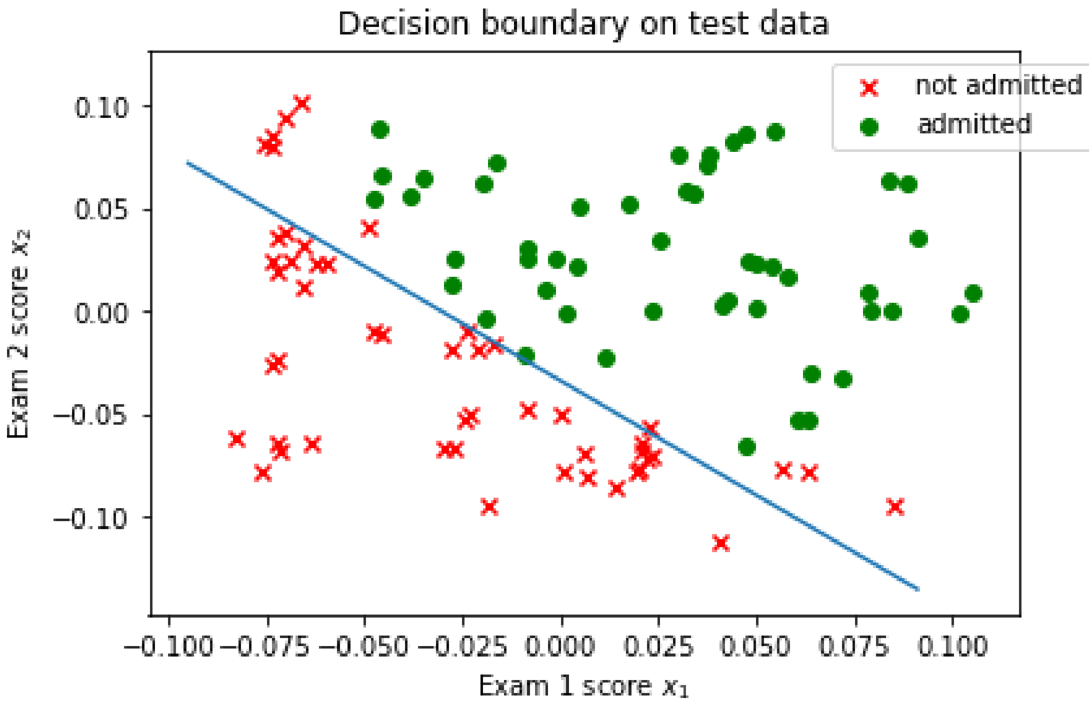
h) Compute the correct classification rate on "ex2-data-test.csv" after convergence assuming you have an estimator of the posterior probabilities with

$$P(y_n = 1 \mid x_n; \theta) = h\theta(x_n)$$

$$P(y_n = 0 \mid x_n; \theta) = 1h_\theta(x_n)$$

i) Draw the decision boundary of your system on top of the scatter plot of the testing data

In [46]:
```
x1_0 = x1_test_norm[y_test==0]
x1_1 = x1_test_norm[y_test==1]
x2_0 = x2_test_norm[y_test==0]
x2_1 = x2_test_norm[y_test==1]

plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
plt.xlabel('Exam 1 score $x_1$')
plt.ylabel('Exam 2 score $x_2$')
plt.title('Decision boundary on test data')
plt.legend(bbox_to_anchor=(1.1, 1))

plt.plot(x_1, x_2) # decision boundary (computed above)

plt.show()

hy = hypothesis(X_norm_test, theta)
num_correct = ((hy >= 0.5) == (y_test >= 0.5)).sum()
num_missed = y.size - num_correct
error_rate = num_missed / y.size


print('theta : ',theta)
print('# correct  : ', num_correct)
print('# missed   : ', num_missed)
print('error rate :  %2.2f %%'% (error_rate*100.0))
```




```
theta :   [ 0.53651534 17.73592464 15.93853026]
# correct   :  90
# missed    :  10
error rate :  10.00 %
```

j) Compare the performance of the logistic regression system with the ones of previous's week.

See previous solutions

0.1.2 b. Optional - Stochastic gradient ascent

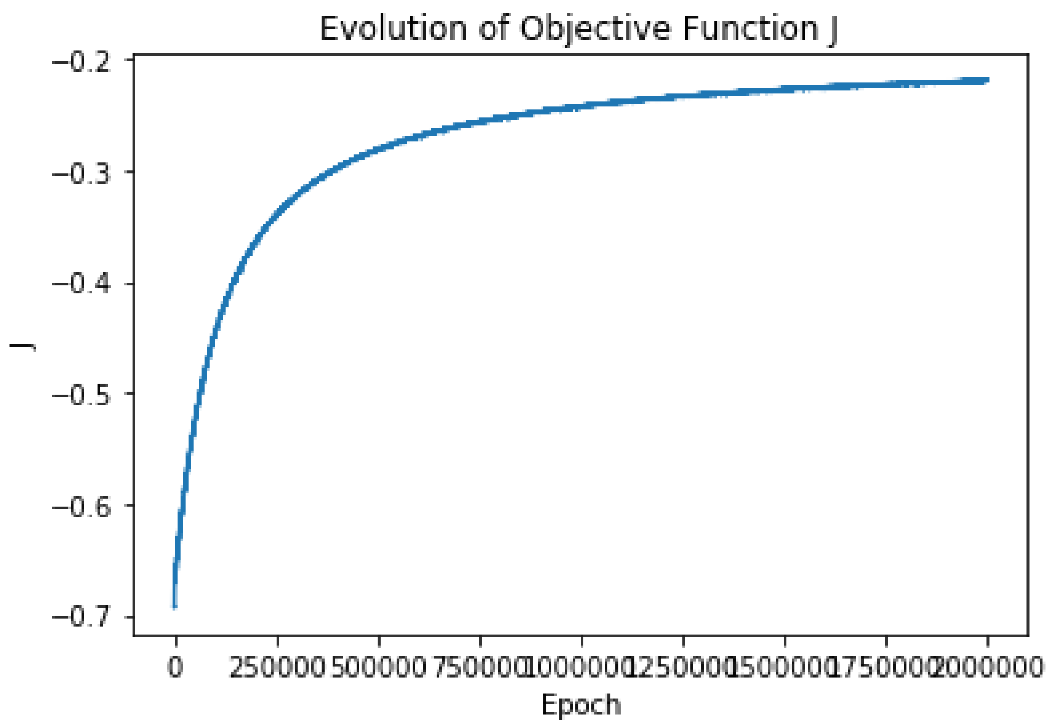
```
In [168]: import random as rd


          def gradient_ascent_stochastic(X,y,learning_rate,num_epoch):
              N, D = X.shape     # number of samples and dimensions
              theta = np.zeros(D)
              J = []
              for itr in range(0,num_epoch):
                  i = rd.randint(0, N-1)
                  error = y[i] - hypothesis(X[i], theta)
                  J.append(objective(X, y, theta))
                  for j in range(0,D):
```

```
                delta = error * X[i,j]
                theta[j] = theta[j] + learning_rate * delta / N
        return theta, J
```

In [169]:
```
theta_stoch, J = gradient_ascent_stochastic(X_norm_train, y, 1, 2000000)

print(theta_stoch)
```

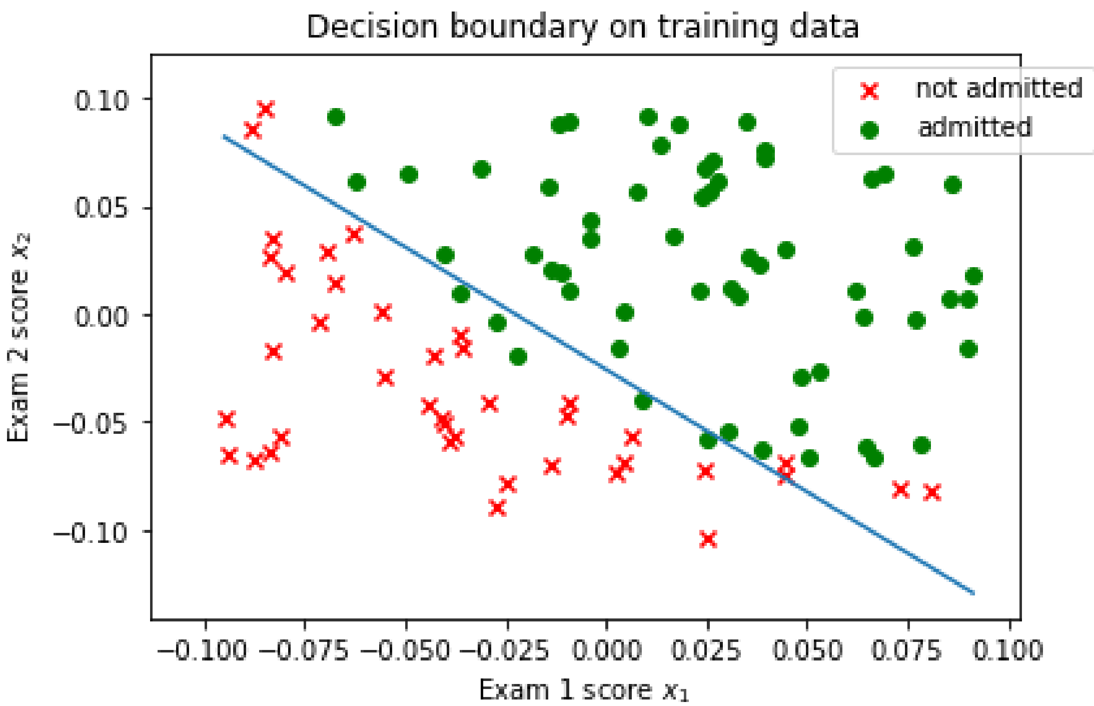
```
[ 1.15478345 51.00418624 44.94000293]
```

In [170]:
```
plt.plot(J)
plt.title("Evolution of Objective Function J")
plt.xlabel("Epoch")
plt.ylabel('J')
plt.show()
```



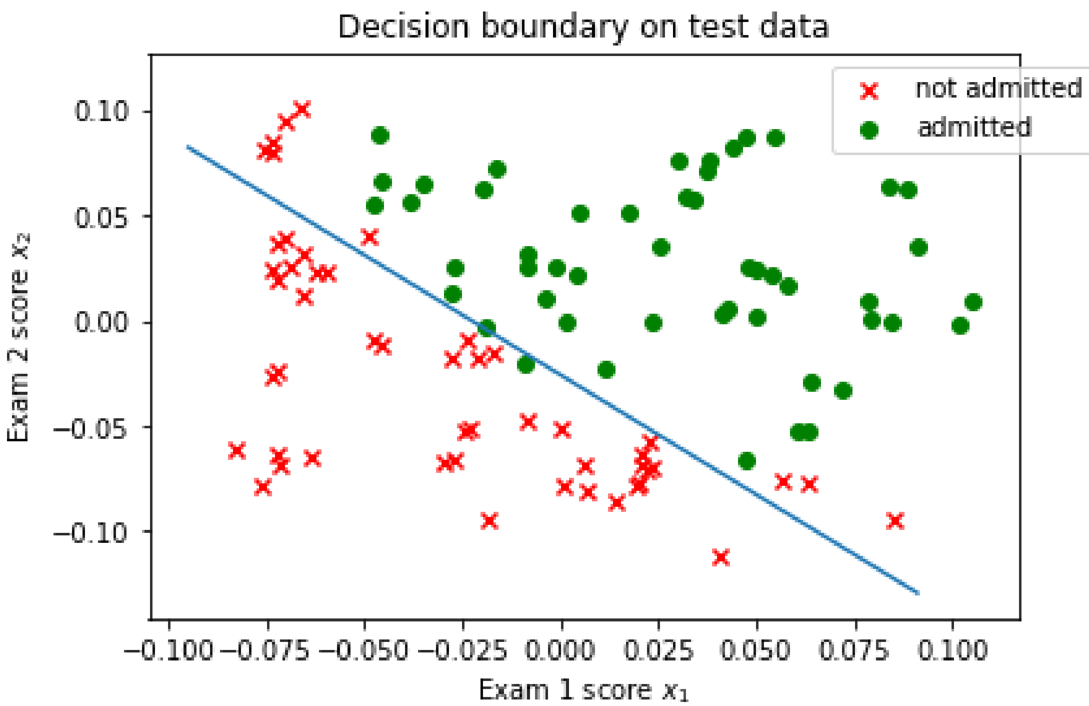
In [171]:
```
x1_0 = x1_train_norm[y==0]
x1_1 = x1_train_norm[y==1]
x2_0 = x2_train_norm[y==0]
x2_1 = x2_train_norm[y==1]

plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
```

```
plt.xlabel('Exam 1 score $x_1$')
plt.ylabel('Exam 2 score $x_2$')
plt.title('Decision boundary on training data')
plt.legend(bbox_to_anchor=(1.1, 1))

x_1 = np.linspace(np.min(x1_train_norm), np.max(x1_train_norm), 100)
x_2 = (-theta_stoch[0] - theta_stoch[1]*x_1)/theta_stoch[2]

plt.plot(x_1, x_2) # decision boundary (computed above)

plt.show()
```




```
In [173]: x1_0 = x1_test_norm[y_test==0]
          x1_1 = x1_test_norm[y_test==1]
          x2_0 = x2_test_norm[y_test==0]
          x2_1 = x2_test_norm[y_test==1]

          plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
          plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
          plt.xlabel('Exam 1 score $x_1$')
          plt.ylabel('Exam 2 score $x_2$')
          plt.title('Decision boundary on test data')
          plt.legend(bbox_to_anchor=(1.1, 1))
```

```python
plt.plot(x_1, x_2) # decision boundary (computed above)

plt.show()

hy = hypothesis(X_norm_test, theta_stoch)
num_correct = ((hy >= 0.5) == (y_test >= 0.5)).sum()
num_missed = y.size - num_correct
error_rate = num_missed / y.size


print('theta : ', theta_stoch)
print('# correct  : ', num_correct)
print('# missed   : ', num_missed)
print('error rate :  %2.2f %%'% (error_rate*100.0))
```




```
theta :   [ 1.15478345 51.00418624 44.94000293]
# correct  :  90
# missed   :  10
error rate :  10.00 %
```

I needed to set the learning rate (alpha) very high. Then we can reach a good result.

0.1.3 c. Logistic regression classifier with non-linear decision boundary

Redo the experiments of 2.a by increasing the complexity of the model in order to have a non-linear decision boundary :

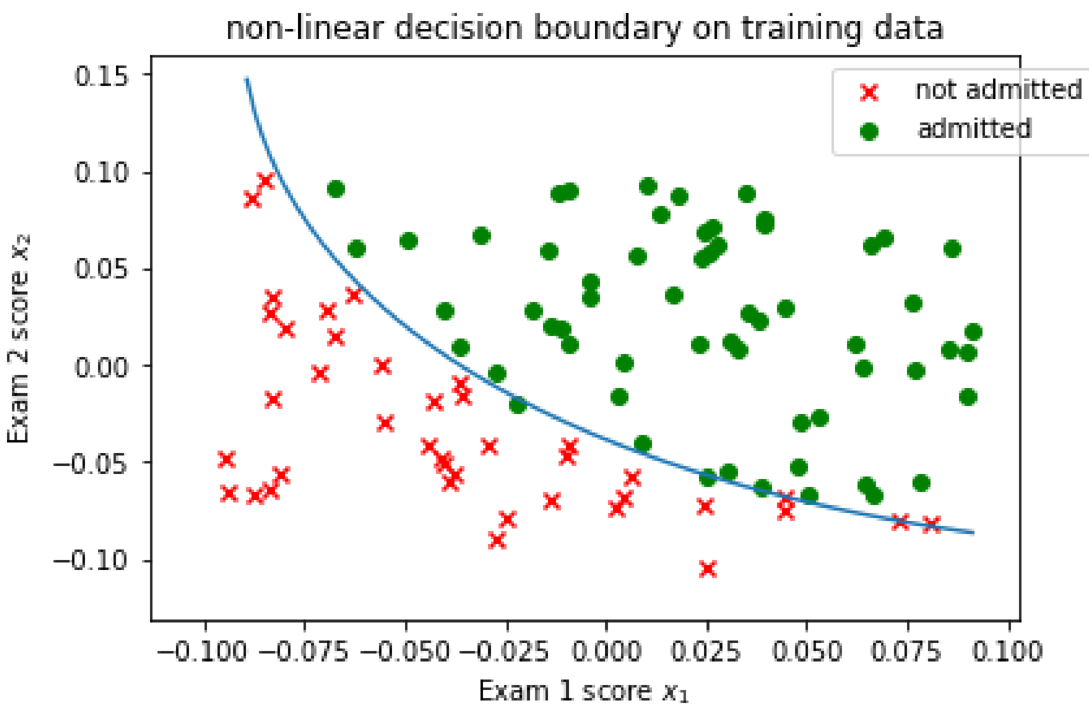
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2 + \theta_5 x_1 x_2 + ...)$$

In [180]:
```
X_train_norm_multi = np.array([np.ones(x1_train_norm.size),x1_train_norm,x2_train_no
                                x2_train_norm**2, x1_train_norm*x2_train_norm]).T

tmp_result = gradientAscent(X_train_norm_multi, y, 1, 2000000)

theta = tmp_result[0]
accJ = tmp_result[1]
```

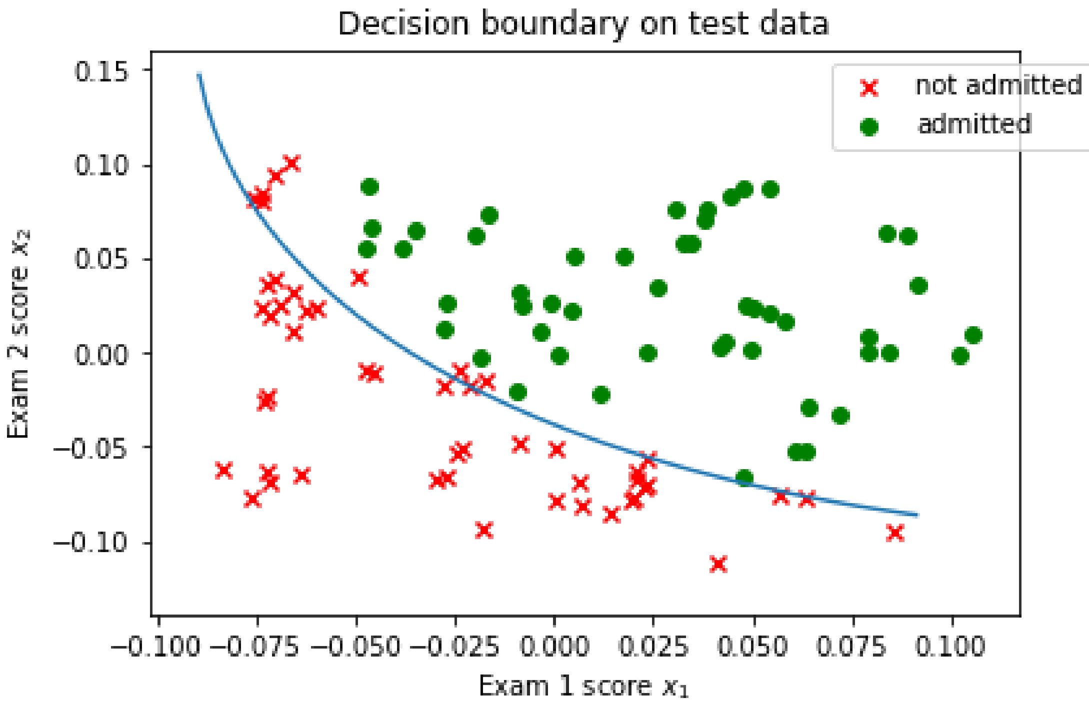
In [187]:
```
print(theta)

x1_0 = x1_train_norm[y==0]
x1_1 = x1_train_norm[y==1]
x2_0 = x2_train_norm[y==0]
x2_1 = x2_train_norm[y==1]

plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
plt.xlabel('Exam 1 score $x_1$')
plt.ylabel('Exam 2 score $x_2$')
plt.title('non-linear decision boundary on training data')
plt.legend(bbox_to_anchor=(1.1, 1))

x_2_non = np.linspace(np.min(x1_train_norm), np.max(x1_train_norm), 100)
a = theta[4]
b = theta[2] + theta[5]*x_2_non
c = theta[0] + theta[1]*x_2_non + theta[3]*(x_2_non**2)
delta = b**2 - 4*a*c
y_2_non = (-b + delta**0.5) / (2*a)


plt.show()
```

```
[   4.76577877  121.51843795  114.39759918 -262.68202506 -253.92422367
  239.99247591]
```

```
/Users/lorenz/.local/share/virtualenvs/lorenz-VPug_12e/lib/python3.6/site-packages/ipykernel_la
```



In [193]:
```
x1_0 = x1_test_norm[y_test==0]
x1_1 = x1_test_norm[y_test==1]
x2_0 = x2_test_norm[y_test==0]
x2_1 = x2_test_norm[y_test==1]

plt.scatter(x1_0,x2_0,marker='x',color='red',label='not admitted')
plt.scatter(x1_1,x2_1,marker='o',color='green',label='admitted')
plt.xlabel('Exam 1 score $x_1$')
plt.ylabel('Exam 2 score $x_2$')
plt.title('Decision boundary on test data')
plt.legend(bbox_to_anchor=(1.1, 1))

plt.plot(x_2_non, y_2_non) # decision boundary (computed above)
plt.show()

hy = hypothesis(X, theta)

classification = ((hy >= 0.5) == (y >= 0.5)).sum()
error_rate = classification * 1.0 / y.size


print('theta : ', theta)

print('error rate :  %2.2f %%'% (error_rate*100.0))
```




```
theta :  [   4.76577877  121.51843795  114.39759918 -262.68202506 -253.92422367
  239.99247591]
error rate :  96.00 %
```

0.1.4 d. Using SciKit Learn

```
In [164]: from sklearn.linear_model import SGDClassifier

          path = '/Users/lorenz/Documents/ML-PW-2018/PW06/'

          dataset = pd.read_csv( path + 'student-dataset-train.csv',names=['x1','x2','y'])

          x1 = dataset['x1'].values
          x2 = dataset['x2'].values
          y = dataset['y'].values

          X_train = [] #ăbring the data in a form sklearn can read
          for i in range(len(x1)):
              X_train.append([x1[i],x2[i]])
```

```
In [171]: clf = SGDClassifier(loss='log', max_iter=200000)
          clf.fit(X_train, y)
```

```python
X_test = [] #ăbring the data in a form sklearn can read
for i in range(len(x1)):
    X_test.append([x1[i],x2[i]])

prediction = clf.predict(X_test)

errors = 0

for i in range(len(prediction)):
    if prediction[i] != y[i]:
        errors += 1

print('Total erors: ' + str(errors))
print('Error rate: ' + str(errors/len(X_test)))
```

```
Total erors: 10
error rate: 0.1
```

We see, that this reaches 10% and our implementation 11%. So we need indeed a nice job. We can assume sklearn did it correct :D