Color reconnection and event-by-event fluctuations of mean transverse momentum in proton-proton collisions at $\sqrt{s} = 7$ TeV in PYTHIA6.4

Qingjun Liu[1] and Wei-Qin Zhao[2]

[1] *Department of Mathematics and Physics,*
Beijing Institute of Petro-chemical Technology, Beijing 102617, People's Republic of China

[2] *Institute of High Energy Physics, Chinese Academy of Sciences,*
P. O. Box 918(4), Beijing 100049, People's Republic of China

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Based on PYTHIA6.428, event-by-event fluctuations of the mean transverse momentum in proton-proton collisions at $\sqrt{s} = 7$ TeV are studied using several measures of the fluctuations. The study compares results of the fluctuations for events where color reconnection is turned on, with those where color reconnection is turned off. The comparison reveals that some of the fluctuation measures are much more sensitive to color reconnection than others. Through the comparison it is demonstrated for the first time that for proton-proton collisions at the Large Hadron Collider, these sensitive measures of the mean transverse momentum fluctuations can serve as alternative probes to color reconnection and color reconnection is a non-trivial source of the event-by-event fluctuations.

Introduction:— Event-by-event fluctuations of the mean transverse momentum ($\langle p_t \rangle$) in proton-proton collisions have attracted lots of attention [1–4]. On one hand, it is believed that proton-proton collision may be used as a model-independent baseline system for studying non-trivial correlations and fluctuations in heavy-ion collisions. The non-trivial fluctuations or correlations in heavy-ion collisions would display as a modification of the pattern for the fluctuations or correlations in proton-proton collisions. Indeed, the modification was observed in collisions of Pb - Pb at $\sqrt{s_{NN}} = 2.76$ TeV at the Large Hadron Collider(LHC)[1, 2] and upto now satisfactory explanation for it is pending. On the other hand, the measurement of the $\langle p_t \rangle$ fluctuations may aid the study of physical mechanisms for multi-particle production in proton-proton collisions. In Refs. [1, 2], PYTHIA model[5–7] predictions for $\langle p_t \rangle$ fluctuations in proton-proton collisions were compared with data measured at the LHC. The study concluded that in proton-proton collisions at the LHC, a $\langle p_t \rangle$ fluctuation measure [8] well-known in the community of heavy-ion collisions [9–14] is of poor sensitivity to a physical mechanism for parton production called color reconnection(CR) [5, 15–17] The study also implies that in proton-proton at the LHC, CR contributes little to $\langle p_t \rangle$ fluctuations measured with that particular measure. However, several other measures [4, 18–26] which have been extensively used for studying $\langle p_t \rangle$ fluctuations in proton-proton and heavy-ion collisions at the SPS, RHIC and LHC energy regime [3, 4, 22–33], have not been applied for studying both CR and its effect on the fluctuations in proton-proton collisions yet.

Happening at the partonic stage of particle production just before hadronization in collisions with several multi-parton inter-actions (MPI)[5], CR joins partons of low and high transverse momentum according to a calculated probability to connect partons[34, 35]. Since it was first studied in the context of rearrangements of partons[36], CR has been shown later strongly suppressed at the perturbative level[37]. Data measured at LHC[38–40] has begun to witness the importance of CR in proton-proton, proton-nucleus and nucleus-nucleus collisions. However, the issue of CR in hadron collisions is extremely challenging[5], let alone in heavy-ion collisions where large amout of MPI may occur. Further theoretical and experimental studies about CR are necessary, including searching for sensitive observables and investigating CR-induced effects on the observables [41].

In this letter we demonstrate for the first time that for proton-proton collisions at the LHC, several measures of the event-by-event $\langle p_t \rangle$ fluctuations are quite sensitive to CR thus can serve as alternative probes to CR, and applying these measures CR is demonstrated to be a non-trivial source of the event-by-event $\langle p_t \rangle$ fluctuations.

Analysis, Results and Discussions:—All results presented in this paper are from analyzing proton-proton collisions at $\sqrt{s}$=7 TeV simulated by using Monte Carlo event generator PYTHIA6.428. Used for the simulations are two tunes of PYTHIA6.428, Perugia 2011 Default where CR is switched on and Perugia 2011 NOCR where CR is switched off. In the analysis, we use the charged-particles that are in the kinematic range $|\eta| < 0.8$ and $0.15 < p_t < 2$ GeV/c, where η and p_t symbolize the pseudo-rapidity and the transverse momentum of a charged particle, respectively. In this letter, event multiplicity refers to the number of charged particles in that kinematic range. Utilized in the analysis are 1000×10^6 and 1024×10^6 events generated by using Perugia 2011 Default and Perugia 2011 NOCR, respectively.

We first present results for event-by-event $\langle p_t \rangle$ fluctu-

ation measure $C = \langle \Delta p_{\mathrm{t},i}, \Delta p_{\mathrm{t},j} \rangle$. Usually called transverse momentum correlator, through $C = \sigma^2_{\mathrm{p_t,dyn}}$ it is directly related to the dynamical component $\sigma_{\mathrm{p_t,dyn}}$ [8, 19] of the $\langle p_{\mathrm{t}} \rangle$ fluctuations. The measure has been used to study event-by-event $\langle p_{\mathrm{t}} \rangle$ fluctuations in proton-proton and nucleus-nucleus collisions at the LHC, RHIC and SPS energies [1, 24, 31, 42–47].

C_m, being C for events classified according to the event multiplicity, is defined as the mean of covariances of all pairs of particles i and j in the same event with respect to the inclusive $\hat{p}_{\mathrm{t}}$ in event class m. In order for the correlator C_m not to be influenced by multiplicity fluctuations, each event class is built to have a unique event multiplicity. This means that all events in a specific event class are of the same number of charged particles. According to Ref. [1, 8, 44], C_m is formulated as

$$C_m = \left(\sum_{k=1}^{\epsilon} \sum_{i=1}^{n_k-1} \sum_{j=i+1}^{n_k} (p_{t,i} - \hat{p}_t)(p_{t,j} - \hat{p}_t) \middle/ \left(\sum_{k=1}^{\epsilon} N_k^{\mathrm{pairs}} \right) \right). \,(1)$$

In Eq. (1), ϵ is the number of events in event class m, n_k and $N_k^{\mathrm{pairs}} = 0.5 \cdot n_k \cdot (n_k - 1)$ are the number of charged particles and charged particle pairs in the kth event of event class m, and $\hat{p}_t$ is the average transverse momentum of all particles in all events of class m. It is calculated according to

$$\hat{p}_t = \left(\sum_{k=1}^{\epsilon} \sum_{i=1}^{n_k} p_{t,i} \right) \middle/ \left(\sum_{k=1}^{\epsilon} n_k \right) = \left(\sum_{k=1}^{\epsilon} n_k \cdot \langle p_t \rangle_k \right) \middle/ \left(\sum_{k=1}^{\epsilon} n_k \right), \,(2)$$

where $\langle p_t \rangle_k = \left(\sum_{i=1}^{n_k} p_{t,i} \right) / n_k$ is the event-wise average transverse momentum $\langle p_{\mathrm{t}} \rangle$ for the kth event and $p_{t,i}$ is the transverse momentum of the ith particle in that event. By construction, C_m vanishes in the case of uncorrelated particle emission, when only statistical fluctuations are present. More details about the measure may be found in Ref. [1, 8, 44].

In Fig. 1(a), transverse momentum correlator $\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle$ is shown as a function of $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$. It can be seen from of Fig. 1(a) that due to CR, transverse momentum correlations are enhanced non-trivially. The enhancement is more evident in events of greater multiplicity because in these events there are more multi-parton hard scatterings and thus more color reconnections that induce parton correlations through a large transverse boost[7, 38]. When these correlated partons hadronize, additional transverse momentum correlations among final state charged particles appear. That boost not only enlarges $\hat{p}_t$ but also σ_{p_t}. The errors shown in Fig. 1 are statistical and are estimated by using a sub-event method. All the events falling in each of the bins defined according to N_{ch}, were divided into M sub-sets with each set containing the same number of events. We then analyzed each sub-set of the events. The mean of the results from the M sub-sets is the calculated result for all the events in that bin, and its statistical error is calculated as the standard deviation from the M sub-sets divided by
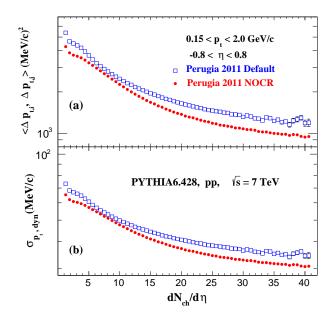

FIG. 1. (Color online) $\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle$ and $\sigma_{p_t,dyn}$ as a function of charged particle density $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$ in proton-proton collisions at $\sqrt{s}$ = TeV in PYTHIA6.428.

$\sqrt{M}$. In this paper we set M to be 12. The errors shown in other figures of this paper were also obtained in this way. That $\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle$ equals to $\sigma^2_{p_t,dyn}$ [8, 24, 44] allows us to calculate $\sigma_{p_t,dyn}$ as a function of $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$. The calculated results are shown in Fig. 1(b). From Fig. 1(b) one may tell that CR contributes to $\sigma_{p_t,dyn}$ substantially. Therefore CR is a source of the dynamical $\langle p_{\mathrm{t}} \rangle$ fluctuations and it can be probed by $\langle p_{\mathrm{t}} \rangle$ fluctuation measure $\sigma_{p_t,dyn}$ or two particle transverse momentum correlator $\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle$ in proton-proton collisions at LHC.

That CR has no big effect on $\sqrt{\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle}/\hat{p}_t$ [1, 2] is understandable and allows us to learn more about $\sigma_{p_t,dyn} = \sqrt{C} = \sqrt{\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle}$ as a measure for CR. To that end, we have also repeated the calculation of $\hat{p}_t$ and the result is displayed in Fig.2(a). As shown in Fig. 1(b) and Fig. 2(a), CR causes both $\sqrt{\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle}$ and $\hat{p}_t$ to increase, which is more evident in events of greater N_{ch}. Thus one may note from a derivative of $\sqrt{\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle}/\hat{p}_t$ that when CR increases both $\sqrt{C}$ and $\hat{p}_t$ at the rate of

$$\frac{\sqrt{C}|_{Default} - \sqrt{C}|_{NOCR}}{\hat{p}_t|_{Default} - \hat{p}_t|_{NOCR}} = \sqrt{C}/\hat{p}_t, \qquad (3)$$

then the ratio $\sqrt{\langle \Delta p_{t,i}, \Delta p_{t,j} \rangle}/\hat{p}_t$ does not vary appreciably with CR. Because $\sqrt{C}/\hat{p}_t$ is quite less than one[1] as one can also see by comparing Fig. 1(b) with Fig. 2(a) for events of high multiplicity, Eq. (3) reveals that $\hat{p}_t$ is much more sensitive to CR than $\sigma_{p_t,dyn} = \sqrt{C}$ is. However, one should be reminded that $\sigma_{p_t,dyn}$ reflects the effect of CR on two-particle transverse momentum correlations while
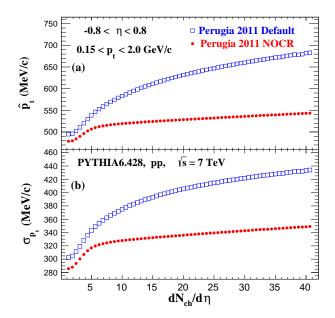
FIG. 2. (Color online) The mean $\hat{p}_t$ and the standard deviation σ_{p_t} of the charged particle inclusive transverse momentum distributions, as a function of charged particle density $dN_{\rm ch}/d\eta$ in proton-proton collisions at $\sqrt{s} = 7$ TeV in PYTHIA6.428.

$\hat{p}_t$ does not, hence together with$\langle\Delta p_{t,i}, \Delta p_{t,j}\rangle$, it may allow one to study CR in another perspective.

The measure $\sqrt{\langle\Delta p_{t,i}, \Delta p_{t,j}\rangle}/\hat{p}_t$ for the event-by-event $\langle p_t\rangle$ fluctuations relates to the specific heat of the thermalized nuclear matter produced in high energy central collisions of heavy nuclei [9–14]. According to Ref. [1], it is currently not a favored measure for studying CR in proton-proton at the LHC. However, the standard deviation σ_{p_t} of the charged particle inclusive transverse momentum distributions may be a promising one instead, as can be seen in Fig. 2(b). While $\hat{p}_t$ as a function of N_{ch} helps validate the importance of CR[1, 6] as may also be seen in Fig. 2(a), Fig.1(a) and Fig. 2(b) also demonstrate respectively that $\langle\Delta p_{t,i}, \Delta p_{t,j}\rangle$ and σ_{p_t} can be useful observables constraining theoretical models of CR in high energy proton-proton collisions at the LHC. Therefore we strongly recommend that not only $\hat{p}_t$, but also both σ_{p_t} and $\langle\Delta p_{t,i}, \Delta p_{t,j}\rangle$ for charged particles in the kinematic range $|\eta| < 0.8$ and $0.15 < p_t < 2$ GeV/c be measured all together in proton-proton collisions at the LHC.

We furthermore explore the effect of CR on $\langle p_t\rangle$ fluctuations by using other measures of the fluctuations. Those measures include the Φ_{p_t}[18], $\Delta\sigma_{p_t:n}$ or $\Delta\sigma_{p_t}$[19, 20](now named $\Delta\sigma_{P_t|n}$[21]), $\Delta\sigma^2_{P_t|n}$ and $\bar{B}$ [21], F_{p_t}[4, 22], Σ_{p_t}[23, 24],$\Delta[P_t, n]$ and $\Sigma[P_t, n]$[25, 26], where n and P_t represent the event multiplicity and the total transverse momentum of charged particles in an event. One may note that $n_k = N_{ch} = n$ and $P_t = N_{ch}\langle p_t\rangle$. According to Refs. [8, 19, 21, 23, 26], for a sample of events with event

multiplicity n, we formulated the interrelations among the $\langle p_t\rangle$ fluctuation measures as

$$\Delta\sigma^2_{P_t|n} = (n-1)C = \bar{B}/n$$
$$= 2\sigma_{p_t}\Delta\sigma_{P_t|n} \approx 2\sigma_{p_t}\Phi_{p_t} \approx 2\sigma^2_{p_t}F_{p_t} . \quad (4)$$

$$\sqrt{\Sigma[P_t, n]} = \sqrt{\Delta[P_t, n]} = \Phi_{p_t}/\sigma_{p_t} + 1 \approx F_{p_t} + 1 . \quad (5)$$

$$\Sigma_{p_t} = \sqrt{(n-1)/n}\sqrt{C}/\hat{p}_t. \quad (6)$$

Shown in Fig. 3 are $\Delta\sigma_{P_t|n}$, $\Delta\sigma^2_{P_t|n}$ and F_{p_t}, which were calculated applying both Eq. (4) and the results already obtained for C and σ_{p_t}.

It can be seen from Fig. 3 that $\Delta\sigma_{P_t|n}$ is of no sensitivity to CR in the whole explored range of event multiplicity while $\Delta\sigma^2_{P_t|n}$ and F_{p_t} are quite sensitive to CR in high multiplicity events. Eq. (6) reveals that Σ_{p_t} is no more sensitive to CR than $\sqrt{\langle\Delta p_{t,i}, \Delta p_{t,j}\rangle}/\hat{p}_t$ is. Henceforth Σ_{p_t} and $\Delta\sigma_{P_t|n}$, together with Φ_{p_t} being approximately equivalent to $\Delta\sigma_{P_t|n}$ as pointed out by Eq. (4), are not suitable for studying CR , and $\langle p_t\rangle$ fluctuations studied by using these three measures are not influenced by CR. Because $\Delta\sigma_{P_t|n}$ does not vary with CR and is defined as
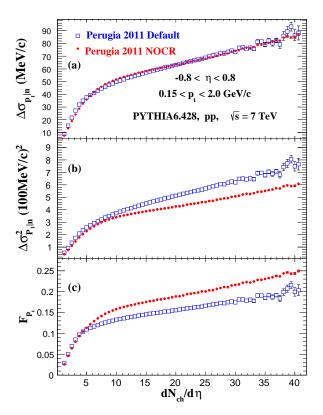

FIG. 3. (Color online) $\Delta\sigma_{P_t|n}$, $\Delta\sigma^2_{P_t|n}$, and F_{p_t}, as a function of charged particle density $dN_{\rm ch}/d\eta$ in proton-proton collisions at $\sqrt{s} = 7$ TeV in PYTHIA6.428.

$\Delta\sigma^2_{P_t|n}/2\sigma_{p_t}$ as shown in Eq. (4), one may get

$$\frac{\Delta\sigma^2_{P_t|n}|_{Default} - \Delta\sigma^2_{P_t|n}|_{NOCR}}{2(\sigma_{p_t}|_{Default} - \sigma_{p_t}|_{NOCR})} = \Delta\sigma_{P_t|n}. \qquad (7)$$

Since $\Delta\sigma_{P_t|n}$ becomes much greater than 1 in events of greater event multiplicity as shown in Fig. 3, Eq. (7) tells that $\Delta\sigma^2_{P_t|n}$ is a lot more sensitive to CR than σ_{p_t} is and the sensitivity increases with the increase of N_{ch}. Furthermore from Eq. (4) and Eq. (5), one may get $\Sigma[P_t, n] = \Delta[P_t, n] = (1 + F_{p_t})^2$ and $\bar{B} = n\Delta\sigma^2_{P_t|n}$, which reveal that $\Sigma[P_t, n]$ and $\Delta[P_t, n]$ are even more sensitive to CR than F_{p_t} is, and $\bar{B}$ has a better sensitivity to CR than $\Delta\sigma^2_{P_t|n}$ does. That both $\bar{B}$ and $\Sigma[P_t, n] = \Delta[P_t, n]$ are quite sensitive to CR may be clearly seen in Fig. 4. Additionally from Eq. (4) one gets $C = \Delta\sigma^2_{P_t|n}/(n-1)$, which indicates that with the increasing of event multiplicity C becomes being less sensitive to CR than $\Delta\sigma^2_{P_t|n}$ is. Furthermore, from Eq. (4), one may get $F_{p_t} \approx \Delta\sigma_{P_t|n}/\sigma_{p_t}$ and $\Delta\sigma^2_{P_t|n} = 2\sigma_{p_t}\Delta\sigma_{P_t|n}$. Taking into considerations again the fact that $\Delta\sigma_{P_t|n}$ does not vary with CR, these two expressions reveal that the sensitivity of both F_{p_t} and $\Delta\sigma^2_{P_t|n}$ to CR is not all but partly attributed to σ_{p_t} being sensitive to CR, which has already been demonstrated by Fig. 2(b). This also holds true for $\bar{B}$ and $\Sigma[P_t, n] = \Delta[P_t, n]$ because $\bar{B} = 2n\sigma_{p_t}\Delta\sigma_{P_t|n}$ and $\sqrt{\Sigma[P_t, n]} = \sqrt{\Delta[P_t, n]} \approx \Delta\sigma_{P_t|n}/\sigma_{p_t} + 1$ according to Eq. (4) and Eq. (5). Therefore F_{p_t}, both $\Sigma[P_t, n]$ and $\Delta[P_t, n]$, $\bar{B}$, $\Delta\sigma^2_{P_t|n}$ and C are preferred for studying CR through measuring $\langle p_t \rangle$ fluctuations in proton-proton collisions at the LHC and CR adds significantly to the $\langle p_t \rangle$ fluctuations in the collisions of high event multiplicity when the fluctuations are analyzed using these measures.

Summary:— We present an analysis of event-by-event fluctuations of the mean transverse momentum for PYTHIA6.428-simulated proton-proton collisions at $\sqrt{s} = 7$ TeV by using various measures of the event-by-event fluctuations. Two tunes of PYTHIA6.428, Perugia 2011 Default and Perugia 2011 NOCR, were used for simulating the collisions. The results from the Perugia 2011 Default where color reconnection is turned on have been compared with those from Perugia 2011 NOCR where color reconnection is turned off. The comparison has clearly indicated that while measures F_{p_t}, both $\Sigma[P_t, n]$ and $\Delta[P_t, n]$, $\Delta\sigma^2_{P_t|n}$, $\bar{B}$ and C are quite sensitive to color reconnection in collisions of high event multiplicity, Σ_{p_t}, $\Delta\sigma_{P_t|n}$ and Φ_{p_t} are not at all in our analysis. Therefore these sensitive measures are promising probes to color reconnection and color reconnection is a non-trivial source of the event-by-event fluctuations of the mean transverse momentum in proton-proton and heavy-ion collisions at the LHC when the event-by-event fluctuations are analyzed with these sensitive measures. Furthermore σ_{p_t} has been shown to be quite sensitive to color reconnection, hence it is strongly recommen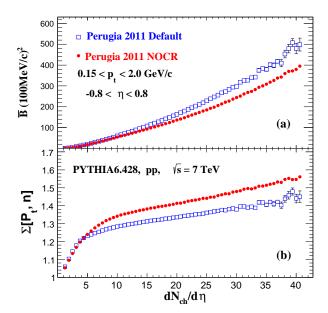ded that σ_{p_t} be measured that may also help constrain color reconnection model in proton-proton collisions at the LHC.

FIG. 4. (Color online) $\bar{B}$ and $\Sigma[P_t, n]$ as a function of charged particle density $dN_{ch}/d\eta$ in proton-proton collisions at $\sqrt{s} = 7$ TeV in PYTHIA6.428.

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