



Statistical-thermal FIST package

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V.V., H. Stoecker, [arXiv:1901.05249](https://arxiv.org/abs/1901.05249), *Computer Physics Communications* (accepted)

Source code: <https://github.com/vlvovch/Thermal-FIST>

ALICE Physics Week 2019, Prague, Czech Republic

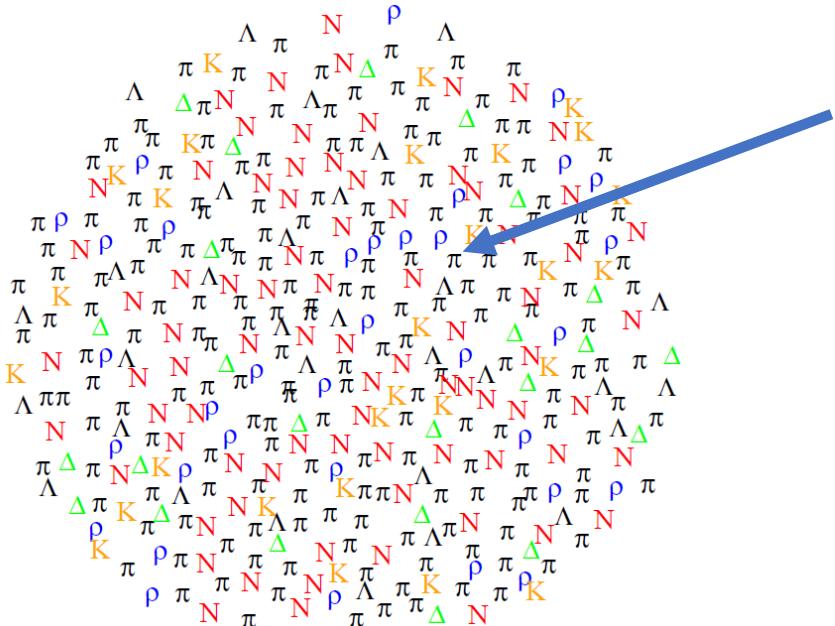
July 23, 2019

Hadron resonance gas (HRG) at freeze-out

HRG: Equation of state of hadronic matter as a multi-component (non-)interacting gas of known hadrons and resonances

$$\ln Z \approx \sum_{i \in M, B} \ln Z_i^{id} = \sum_{i \in M, B} \frac{d_i V}{2\pi^2} \int_0^\infty \pm p^2 dp \ln \left[1 \pm \exp \left(\frac{\mu_i - E_i}{T} \right) \right]$$

Grand-canonical ensemble: $\mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$ *chemical equilibrium*



Thermal model:

Equilibrated hadron resonance gas at the chemical freeze-out stage of high-energy collisions

Model parameters:

T – temperature

μ_B, μ_Q, μ_S – chemical potentials

V – system volume

Thermal model tools

Available thermal model codes:

- 1) **SHARE 3** [G. Torrieri, J. Rafelski, M. Petran, et al.] *Since 2003*
Fortran/C++. Chemical (non-)equilibrium, fluctuations, charm, nuclei
open source: <http://www.physics.arizona.edu/~gtshare/SHARE/share.html>

- 2) **THERMUS 4** [S. Wheaton, J. Cleymans, B. Hippolyte, et al.] *Since 2004*
C++/ROOT. Canonical ensemble, EV corrections, charm, nuclei
open source: <https://github.com/thermus-project/THERMUS>

New development:



Thermal-FIST* (current version: v1.2.1) [V.V., H. Stoecker]

C++. Chemical (non-)equilibrium, EV/vdW corrections, Monte Carlo, (higher-order) fluctuations, canonical ensemble, combinations of effects

open source: <https://github.com/vlvovch/Thermal-FIST>

Since 2018

physics manual: arXiv:1901.05249



Using Thermal-FIST

The package is cross-platform (Linux, Mac, Windows, Android)

Installation using **git** and **cmake**

```
# Clone the repository from GitHub
git clone https://github.com/vlvovch/Thermal-FIST.git
cd Thermal-FIST

# Create a build directory, configure the project with cmake
# and build with make
mkdir build
cd build
cmake ../
make

# Run the GUI frontend
./bin/QtThermalFIST

# Run the test calculations from the paper
./bin/examples/cpc1HRGTDep
./bin/examples/cpc2chi2
./bin/examples/cpc3chi2NEQ
./bin/examples/cpc4mcHRG
```

GUI requires free [Qt5 framework](#), the rest of the package has [no external dependencies](#)

[Quick start guide](#)

[Documentation](#)

[Physics manual](#)

Thermal-FIST



Graphical user interface for general-purpose thermal model applications

The screenshot shows the Thermal-FIST 1.0 graphical user interface. At the top, there's a menu bar with File, View, and Help. Below it, a toolbar has a button for Particle list file (set to C:/FIST/PDG2014/list-withnuclei.dat), Load particle list..., and Load decays... . The main window has tabs for Thermal model, Thermal fits, Equation of state, Event generator, and Particle list editor. The Thermal model tab is active, showing a table of data to fit. The table includes columns for Name, Fit?, Exp. value, Exp. error, Model value, Deviation, Data/Model, and Feeddown. The data rows are:

	Name	Fit?	Exp. value	Exp. error	Model value	Deviation	Data/Model	Feeddown
1	π^+	<input checked="" type="checkbox"/>	669.5	48	605.439	-1.33461	1.106 ± 0.079	Strong+EM decays
2	π^-	<input checked="" type="checkbox"/>	668	47	605.474	-1.33034	1.103 ± 0.078	Strong+EM decays
3	K^+	<input checked="" type="checkbox"/>	100	8	108.722	1.09024	0.920 ± 0.074	Strong+EM decays
4	K^-	<input checked="" type="checkbox"/>	99.5	8.5	108.657	1.07724	0.916 ± 0.078	Strong+EM decays
5	p	<input checked="" type="checkbox"/>	31.5	2.5	33.4123	0.764917	0.943 ± 0.075	Strong+EM decays
6	\bar{p}	<input checked="" type="checkbox"/>	30.5	2.5	33.2773	1.11091	0.917 ± 0.075	Strong+EM decays
7	Lambda	<input checked="" type="checkbox"/>	24	2.5	19.3002	-1.87991	1.244 ± 0.130	Strong+EM decays

Below the table are buttons for Add quantity to fit..., Remove selected quantity from fit, Load data from file..., and Save data to file... . To the right of the table is the HRG model configuration panel, which includes dropdowns for Model (Ideal), Ensemble (Grand-canonical), Statistics (Boltzmann or Quantum for All particles), Use quadratures, Resonance widths (eBW), Conservation laws..., EV/vdW interactions..., and Other options... .

The extracted parameters panel shows values for T (MeV), μ_B (MeV), γ_q , γ_s , R (fm), V (fm³), and chi2/dof. The plots tab is currently set to Yields.

The bottom right of the main window shows a plot titled "Data/Model" with red points representing experimental data and a solid blue line representing the model fit. The x-axis lists various particles: n^+ , n^- , K^+ , K^- , p , \bar{p} , Λ , $\bar{\Lambda}$, Ξ^- , Ξ^- , Ω , $\bar{\Omega}$, $K^0 S$, $\phi(1020)$, d , \bar{d} , ^3He , $^3\text{H}\bar{A}$, ^4He , $^4\text{H}\bar{A}$.

At the bottom right of the main window, there's a copyright notice: © 2014-2019 Volodymyr Vovchenko.

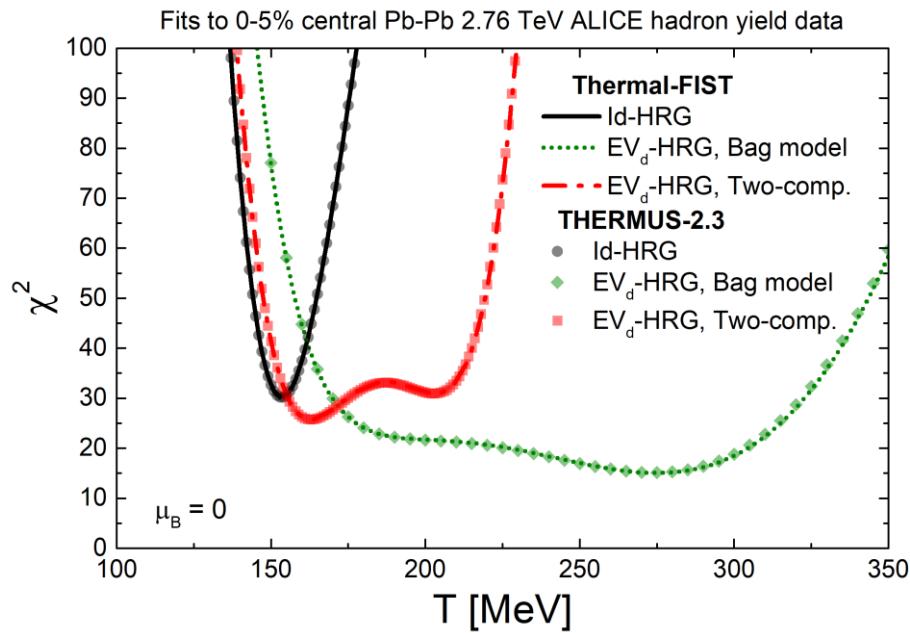
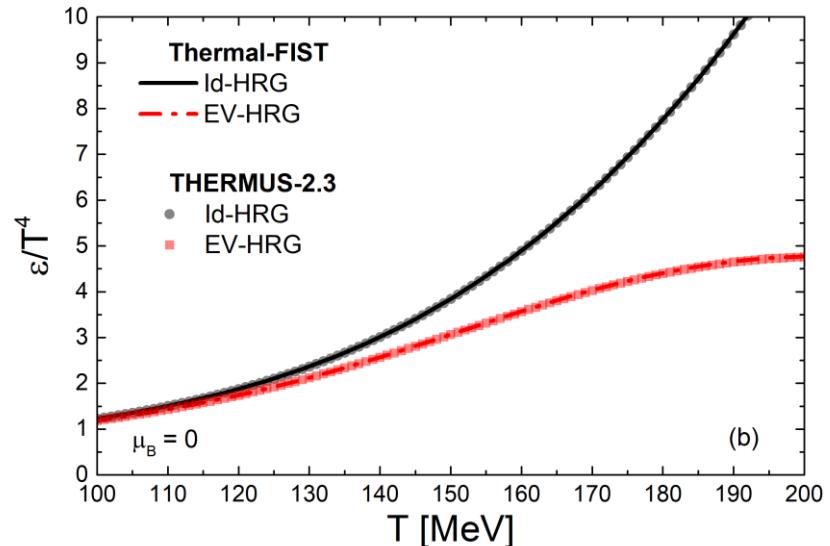
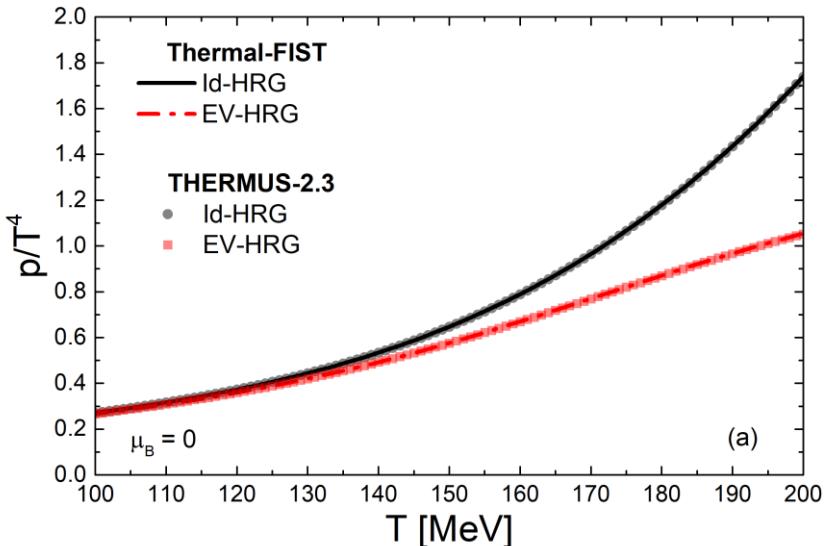
"So that's how you get your results so quickly!"

J. Cleymans

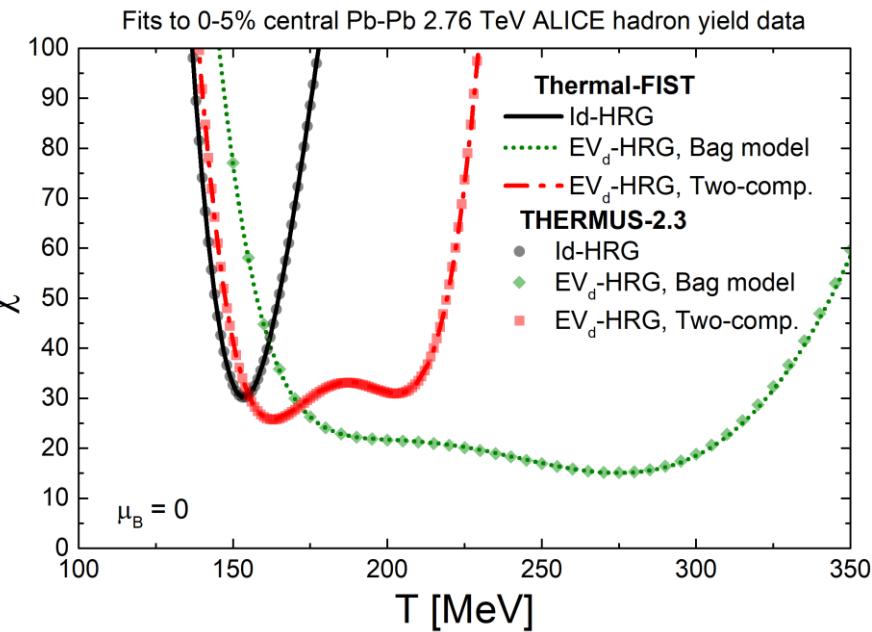
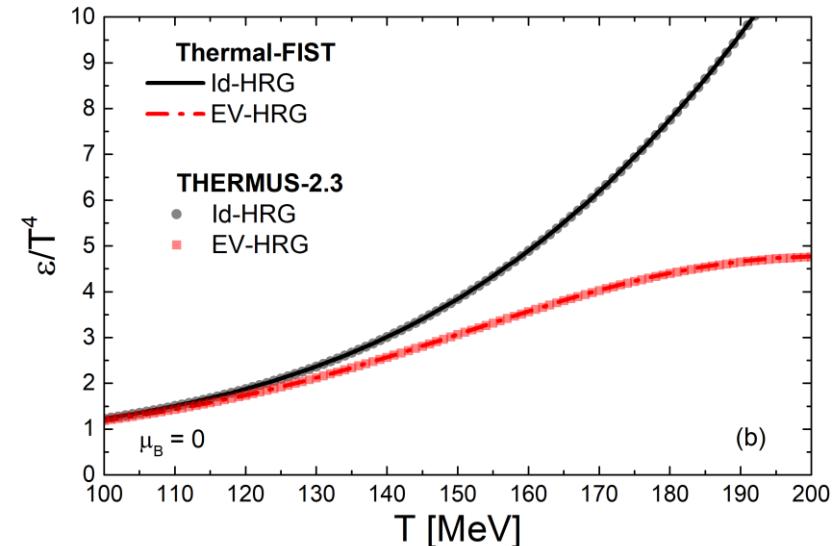
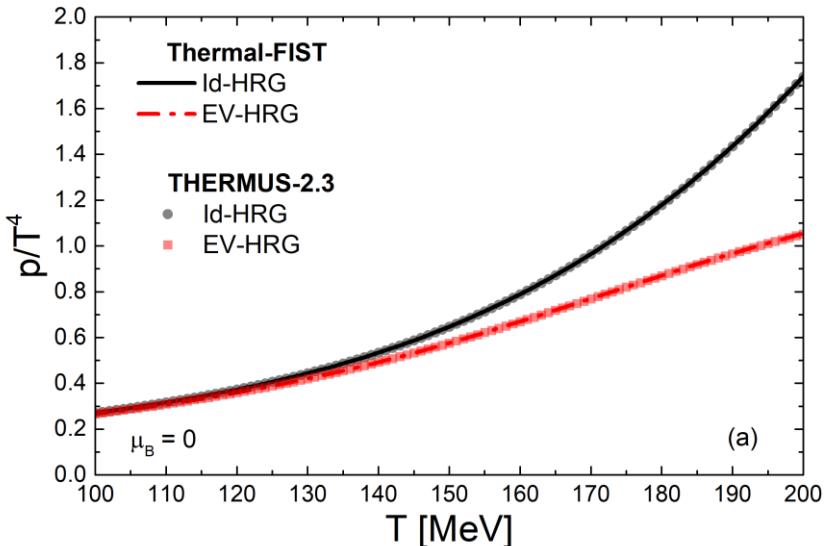
"Thanks for reproducing my results!"

F. Becattini

FIST in THERMUS mode: cross-check

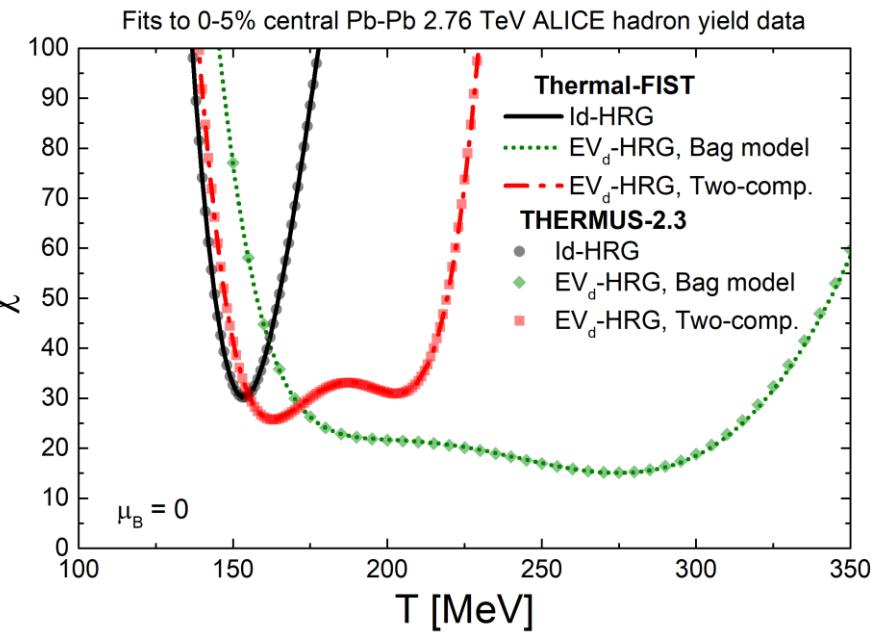
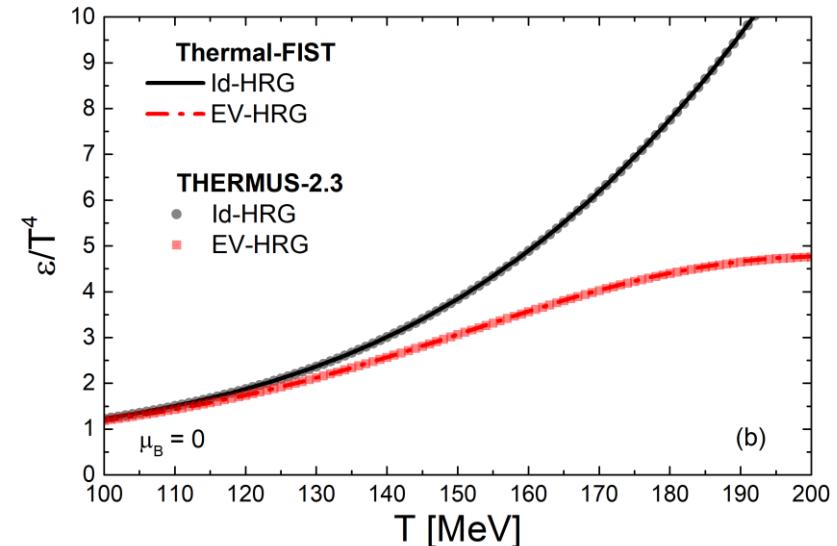
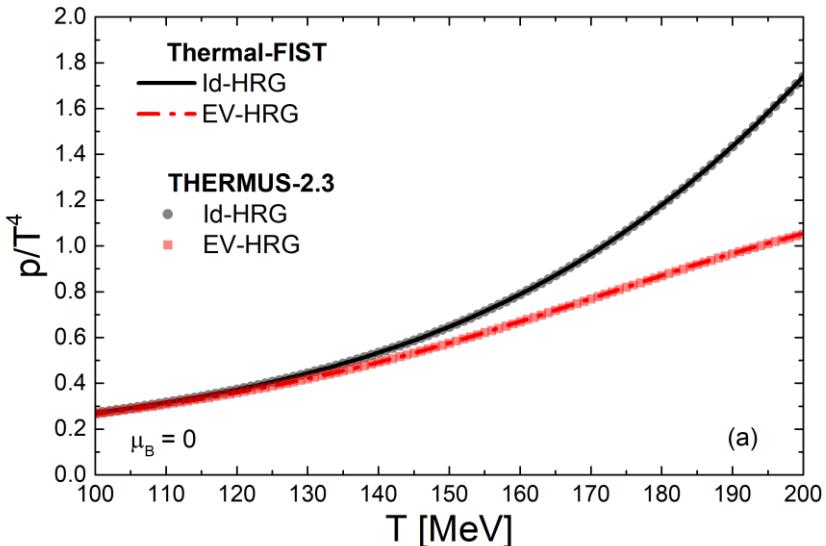


FIST in THERMUS mode: cross-check



FIST: Fist IS Thermus

FIST in THERMUS mode: cross-check



FIST: Fist IS Thermus

and more...

FIST and Jupyter notebooks



Usual usage: through GUI or compiled C++ macros

NEW: Interactive notebooks through Jupyter (xeus kernel and ROOT-cling)*

jupyter FitExample Last Checkpoint: несколько секунд назад (unsaved changes) Logout

File Edit View Insert Cell Kernel Widgets Help Trusted C++14

Initialize and run the fitter

```
In [7]: // Set chemical potentials to zero
model.SetBaryonChemicalPotential(0.0);
model.SetElectricChemicalPotential(0.0);
model.SetStrangenessChemicalPotential(0.0);
model.SetCharmChemicalPotential(0.0);
model.FillChemicalPotentials();

// Initialize the fitter
ThermalModelFit fitter(&model);

// Do not fit muB, it is zero at LHC
fitter.SetParameterFitFlag("muB", false);

// Pass the data to the fitter
fitter.SetQuantities(dataPbPb010);

// Perform the fit
ThermalModelFitParameters fitResult = fitter.PerformFit(false);
```

Print the fitted parameters and the χ^2

```
In [8]: cout << "Extracted parameters:" << endl;
cout << setw(15) << "T [MeV]" << " = " << setw(15) << 1.e3 * fitResult.T.value << " +- " << 1.e3 * fitResult.T.error <<
cout << setw(15) << "R [fm]" << " = " << setw(15) << fitResult.R.value << " +- " << fitResult.R.error << endl;
cout << setw(15) << "chi2/dof" << " = " << setw(15) << fitResult.chi2 << "/" << fitResult.ndf << endl;
```

Extracted parameters:
T [MeV] = 155.28 +- 2.78665
R [fm] = 10.3342 +- 0.545205
chi2/dof = 15.3832/6

*Since version 1.2.1, example at github.com/vlvovch/FIST-jupyter



Thermal model aspects in FIST

- Extensions of the HRG model
 - finite resonance widths
 - repulsive interactions (excluded volume)
 - van der Waals interactions (*criticality*)
 - particle number fluctuations and correlations
 - chemical non-equilibrium (γ_q , γ_s) a la Rafelski
- Equation of state
- Canonical statistical model (CSM)
 - (local) (selective) exact conservation of conserved charges
 - uniform description of hadrochemistry from small to large systems
- Monte Carlo generator (Blast-wave, canonical ensemble,...)
- Hadronic phase and dynamical freeze-out
 - Partial chemical equilibrium
 - Suppression of resonance yields
 - Evolution of light nuclei abundances via the Saha equation



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Finite resonance widths

$$n_i(T, \mu; m_i) \rightarrow \int_{m_i^{\min}}^{m_i^{\max}} dm \rho_i(m) n_i(T, \mu; m)$$

1) Zero-width approximation

Simplest and common possibility

2) Energy-(in)dependent Breit-Wigner

$$\rho_i^{\text{BW}}(m) = A_i \frac{2 m m_i \Gamma_i}{(m^2 - m_i^2)^2 + m_i^2 \Gamma_i^2}$$

$$\Gamma_{i \rightarrow j}(m) = b_{i \rightarrow j} \Gamma_i \left[1 - \left(\frac{m_{i \rightarrow j}^{\text{thr}}}{m} \right)^2 \right]^{L_{i \rightarrow j} + 1/2}$$

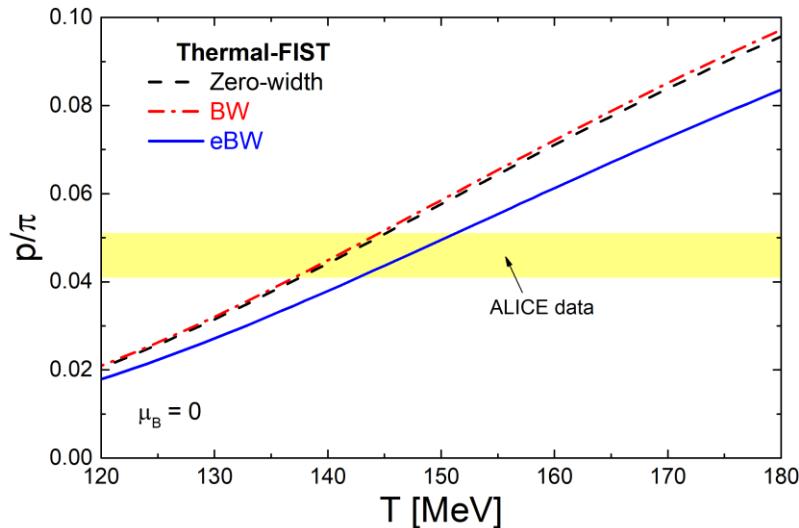
suppression of the spectral strength at the threshold

Alternative: S-matrix approach using phase shifts $\rho_i(m) \propto \frac{\partial \delta(m)}{\partial m}$

Usually based on measured scattering phase shifts

[cf. Huovinen et al., PLB '17; P.M. Lo et al, PLB '19]

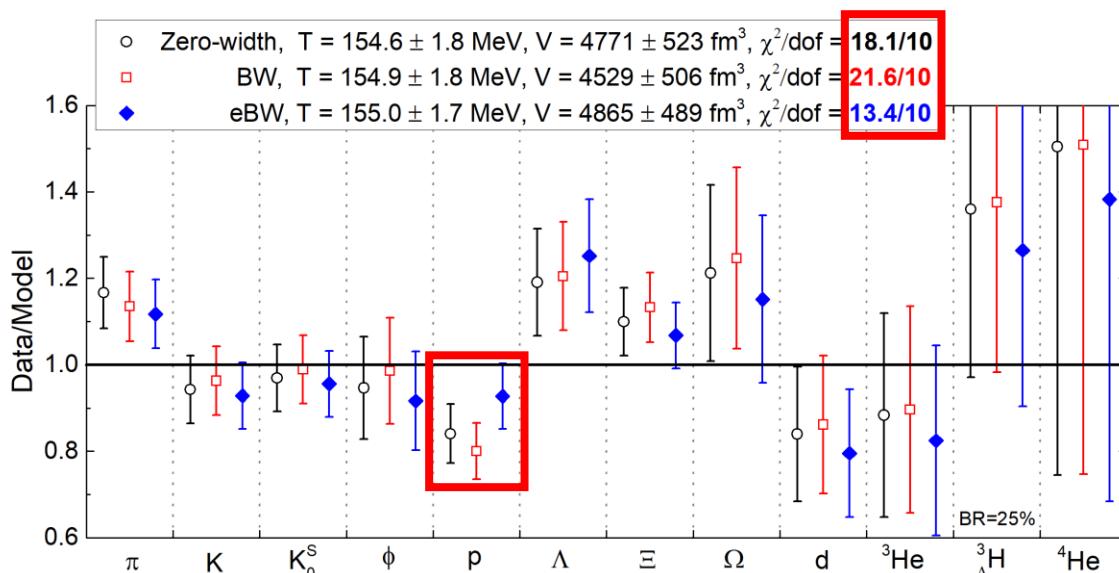
Finite resonance widths: data description



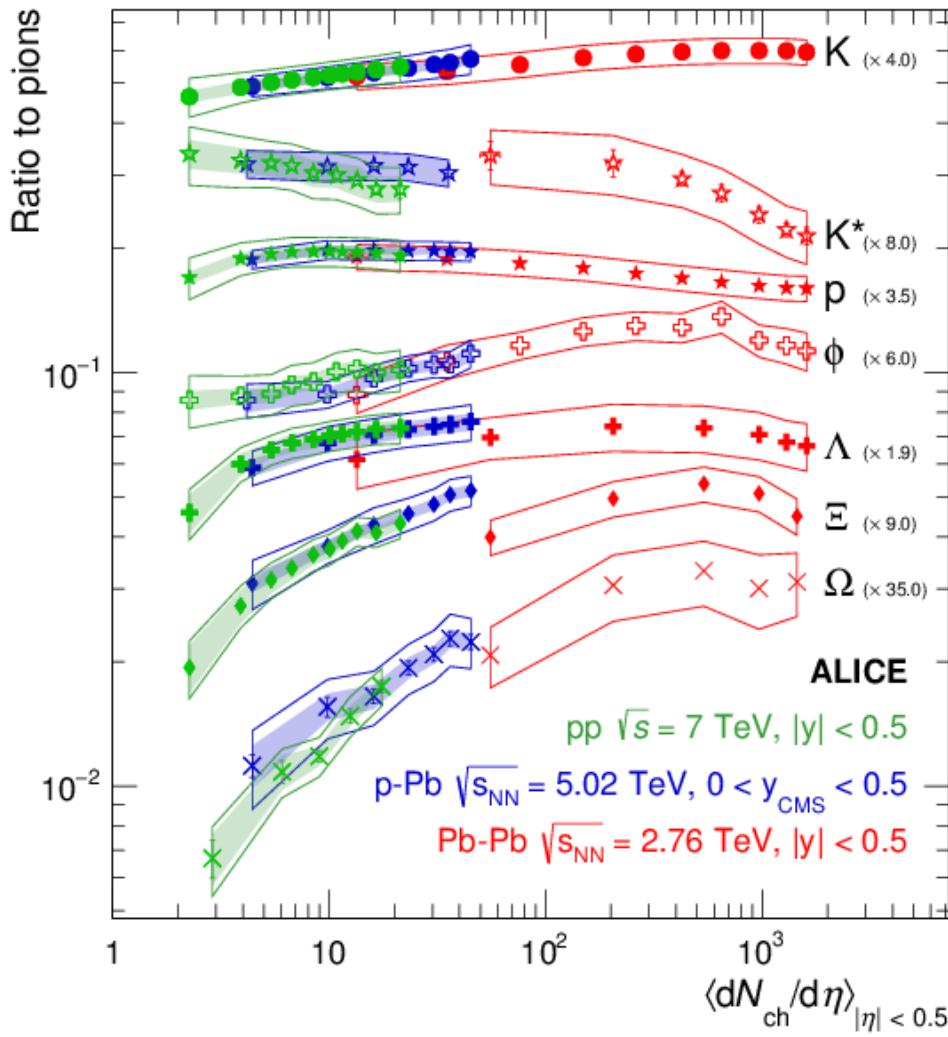
Energy-dependent Breit-Wigner leads to a **15% suppression of proton yields**
also affected: 10% decrease of Λ

This is enough to describe the
'proton yield anomaly' at 2.76 TeV*

*but beware of the 5 TeV data, see below



Multiplicity dependence of hadrochemistry



[ALICE collaboration, 1807.11321]

- Hadron yield ratios exhibit multiplicity dependence
- Grand-canonical picture predicts no multiplicity dependence
- Ratios appear to approach a plateau at high-multiplicities
→ grand-canonical plateau?
- Can multiplicity-dependence be considered in a **macroscopic** canonical statistical model?



Canonical statistical model (CSM)

Exact conservation of B , Q , S in a correlation volume V_C

[Rafelski, Danos, et al., PLB '80; Hagedorn, Redlich, ZPC '85]

$$Z(B, Q, S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi} e^{-i(B\phi_B + Q\phi_Q + S\phi_S)} \exp \left[\sum_j z_j^1 e^{i(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)} \right]$$

$$z_j^1 = V_c \int dm \rho_j(m) d_j \frac{m^2 T}{2\pi^2} K_2(m/T)$$

$$\langle N_j^{\text{prim}} \rangle^{\text{ce}} = \frac{Z(B - B_j, Q - Q_j, S - S_j)}{Z(B, Q, S)} \langle N_j^{\text{prim}} \rangle^{\text{gce}}$$

[Becattini et al., ZPC '95, ZPC '97]

Implemented in Thermal-FIST for a full HRG



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Implemented in Thermal-FIST for a full HRG

Exact conservation around midrapidity, $V_C = k dV/dy$. How large is k ?

Net-proton fluctuations affected by baryon number conservation

[Braun-Munzinger, Rustamov, Stachel, 1612.00702]

$$\frac{\kappa_2(p - \bar{p})}{\langle p \rangle + \langle \bar{p} \rangle} \simeq 1 - \frac{\langle p \rangle}{k dN_B/dy}$$

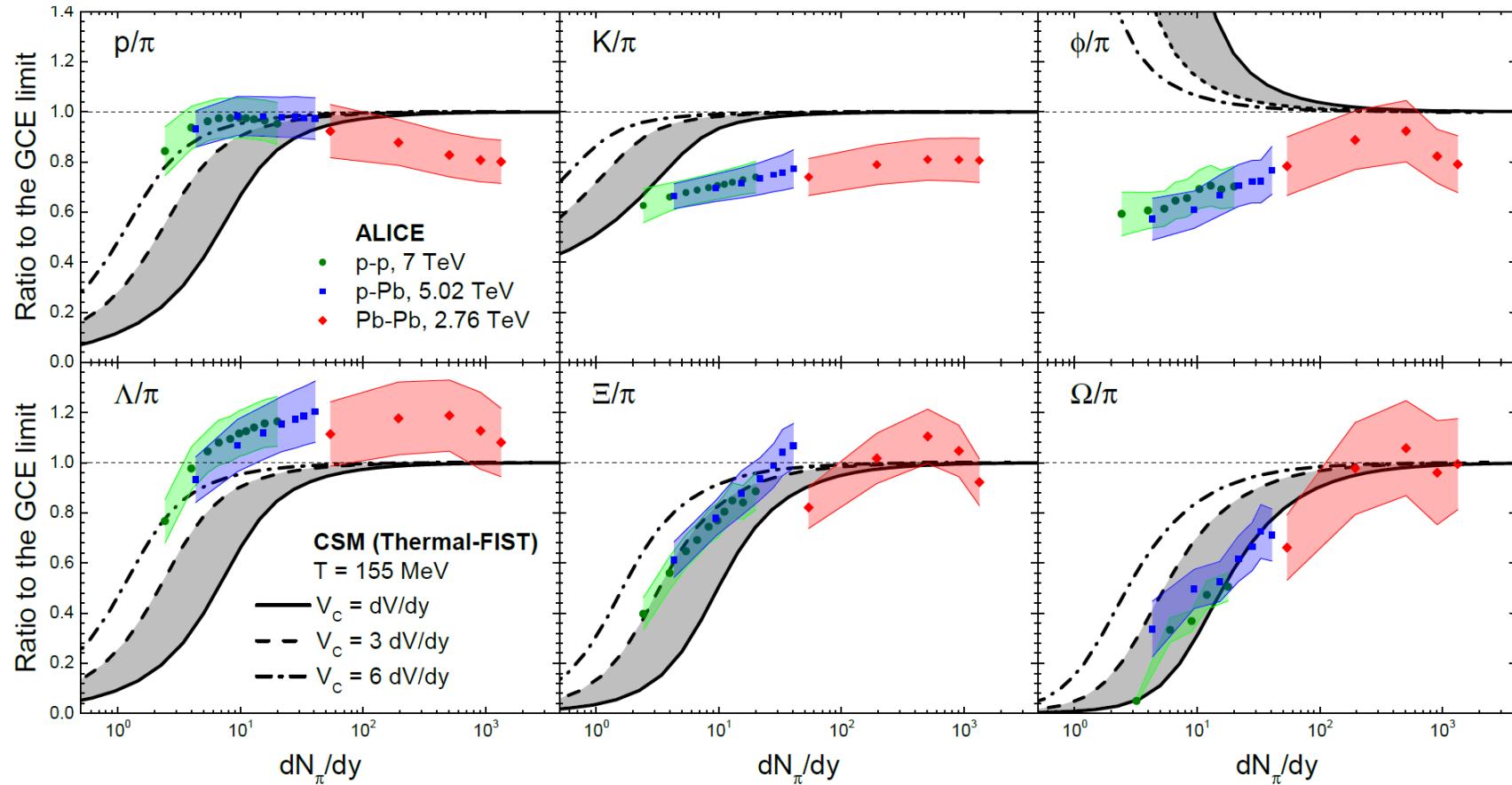
Using ALICE preliminary data for net-p fluctuations [Rustamov, 1704.05329] one obtains $k \sim 3-4$ for most centrality bins in Pb-Pb collisions

[V.V., Dönigus, Stoecker, 1906.03145]

“Vanilla” CSM at LHC



$T_{ch} = 155$ MeV, $V_C = 3dV/dy$, multiplicity dependence driven by V_C only

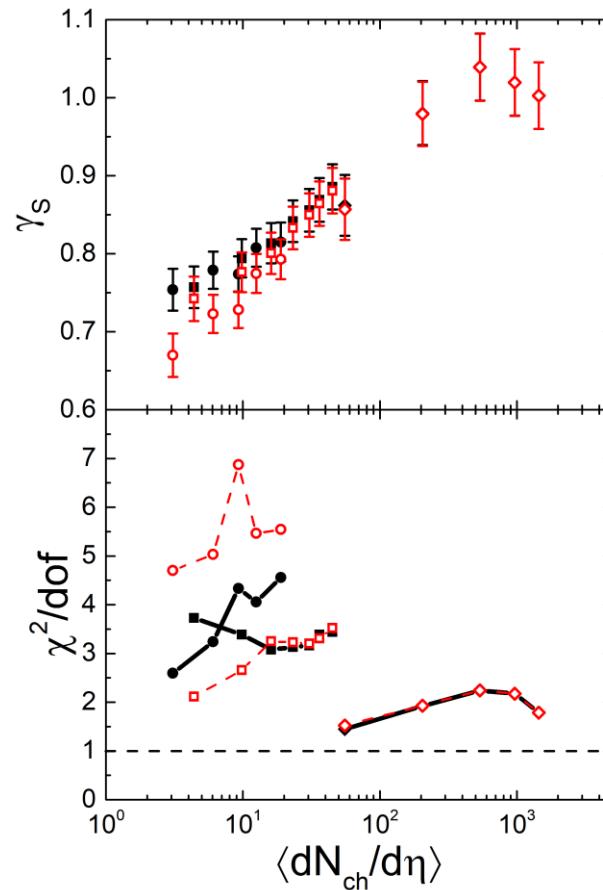
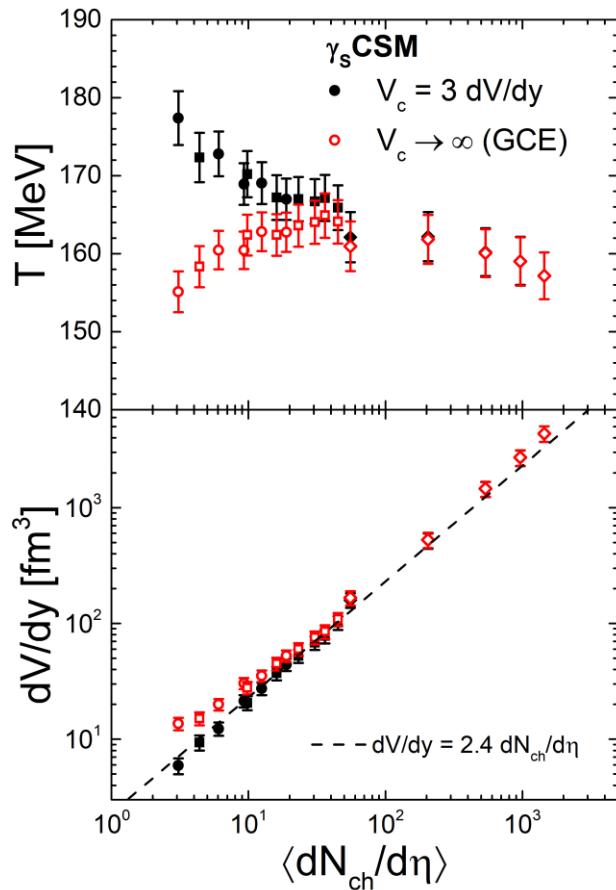


Fair for hyperons, protons and kaons worse, ϕ goes in the opposite direction



Full CSM analysis

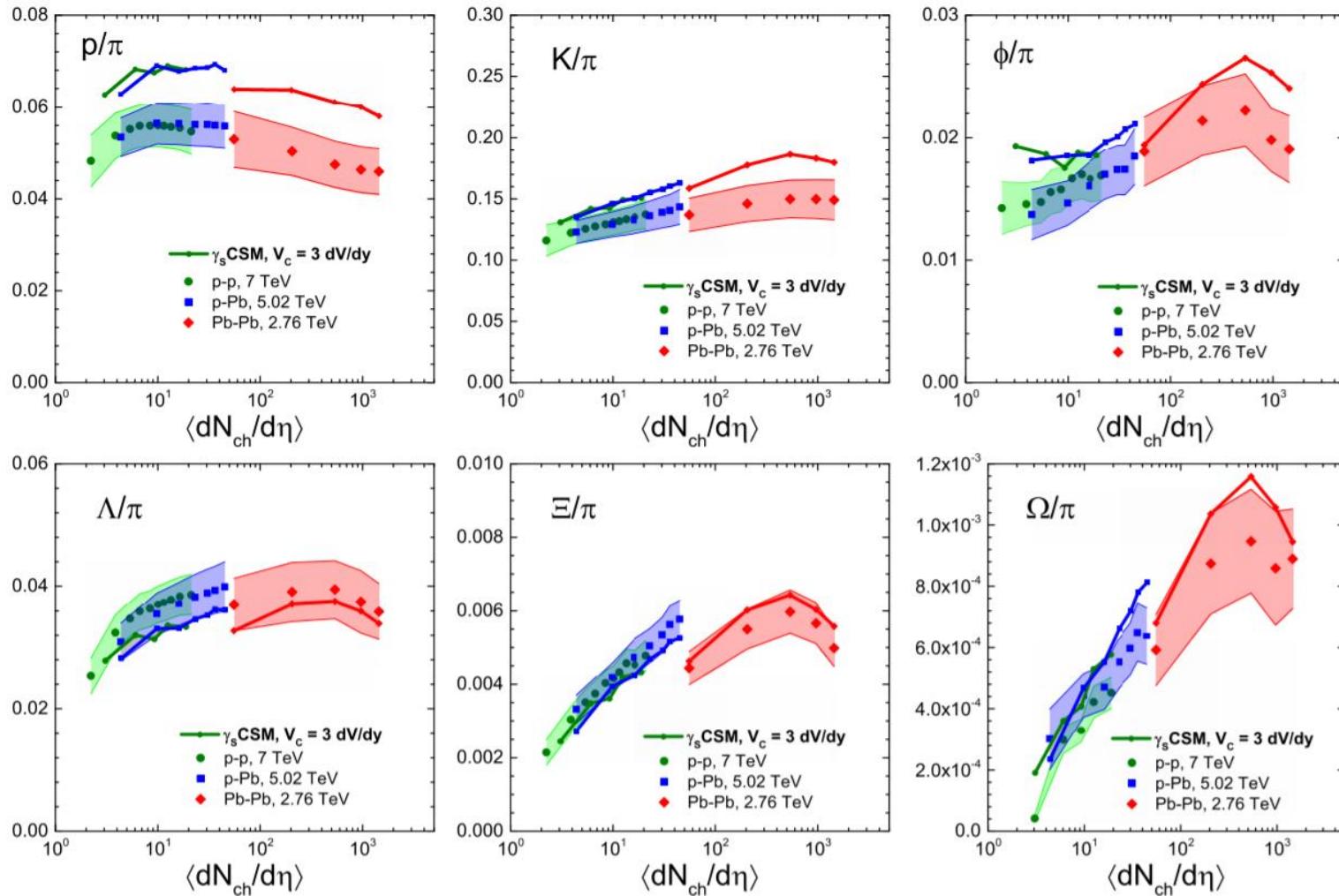
γ_s CSM: $V_C = 3dV/dy$, fit T_{ch} and γ_S at each centrality in p-p, p-Pb, Pb-Pb



Canonical suppression and strangeness saturation important below $dN_{ch}/d\eta \cong 100$



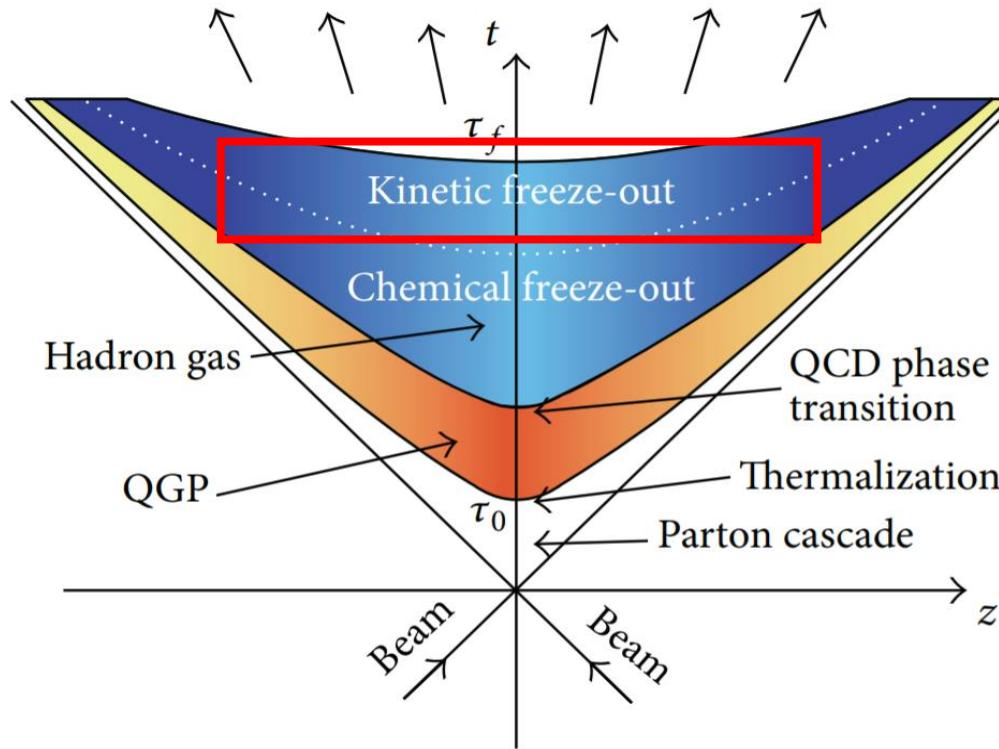
Full CSM analysis: yields



Relative accuracy of γ_s CSM is $\sim 15\%$ across all multiplicity bins



Hadronic phase



- At $T_{ch} \approx 150 - 160$ MeV inelastic collisions cease, yields of hadrons frozen
- Kinetic equilibrium maintained down to $T_{kin} \approx 100 - 120$ MeV through (pseudo-)elastic scatterings

[e.g., E. Shuryak, Rev. Mod. Phys. 89, 035001 (2017)]



Partial chemical equilibrium

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92; C.M. Hung, E. Shuryak, PRC '98]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through pseudo-elastic resonance reactions $\pi\pi \leftrightarrow \rho$, $\pi K \leftrightarrow K^*$, $\pi N \leftrightarrow \Delta$, etc.

Effective chemical potentials:

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \mu_i, \quad \langle n_i \rangle_j - \text{mean number of hadron } i \text{ from decays of hadron } j, \quad j \in \text{HRG}$$

Conservation laws:

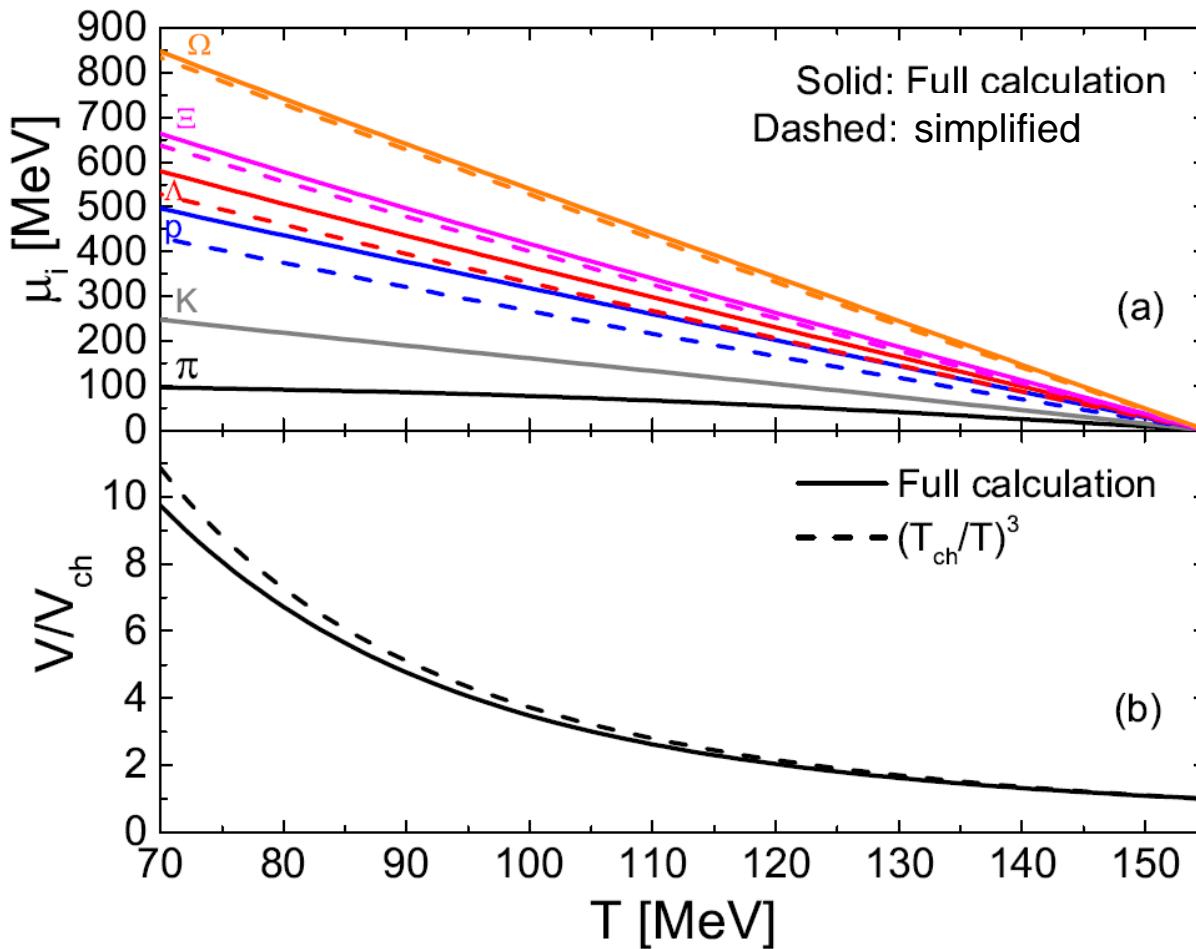
$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j n_j(T, \tilde{\mu}_j) V = N_i(T_{ch}), \quad i \in \text{stable} \quad \xrightarrow{\text{numerical solution}} \quad \{\mu_i(T)\}, V(T)$$

$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) V = S(T_{ch})$$

Numerical implementation is in Thermal-FIST development branch

<https://github.com/vlvovch/Thermal-FIST/tree/pce>

Partial chemical equilibrium at the LHC



[V.V., K. Gallmeister, J. Schaffner-Bielich, C. Greiner, 1903.10024]

“Initial conditions” from thermal fits with Thermal-FIST to 0-10% ALICE hadron yields
 $T_{ch} = 155$ MeV, $V_{ch} = 4700$ fm 3



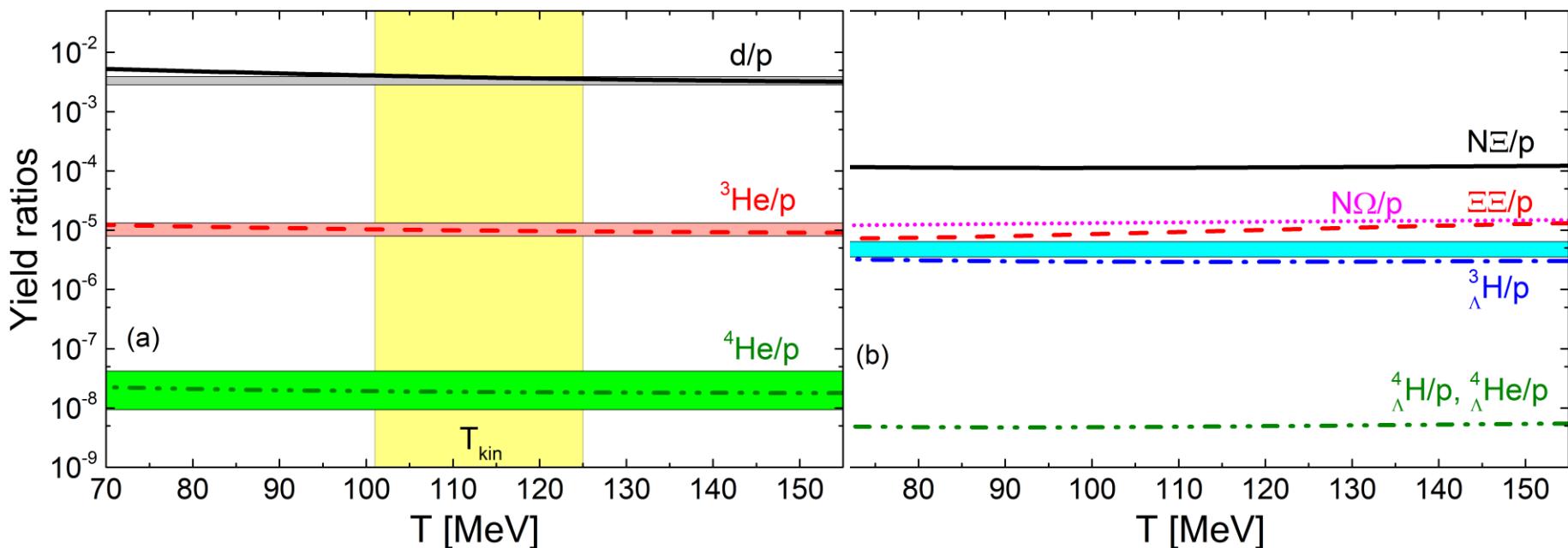
Light nuclei in the hadronic phase

Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\text{eq}}}{\prod_i n_{A_i}^{\text{eq}}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \quad \text{e.g. } \mu_d = \mu_p + \mu_n, \mu_{^3\text{He}} = 2\mu_p + \mu_n, \dots$$

Saha equation

[V.V., K. Gallmeister, J. Schaffner-Bielich, C. Greiner, 1903.10024]

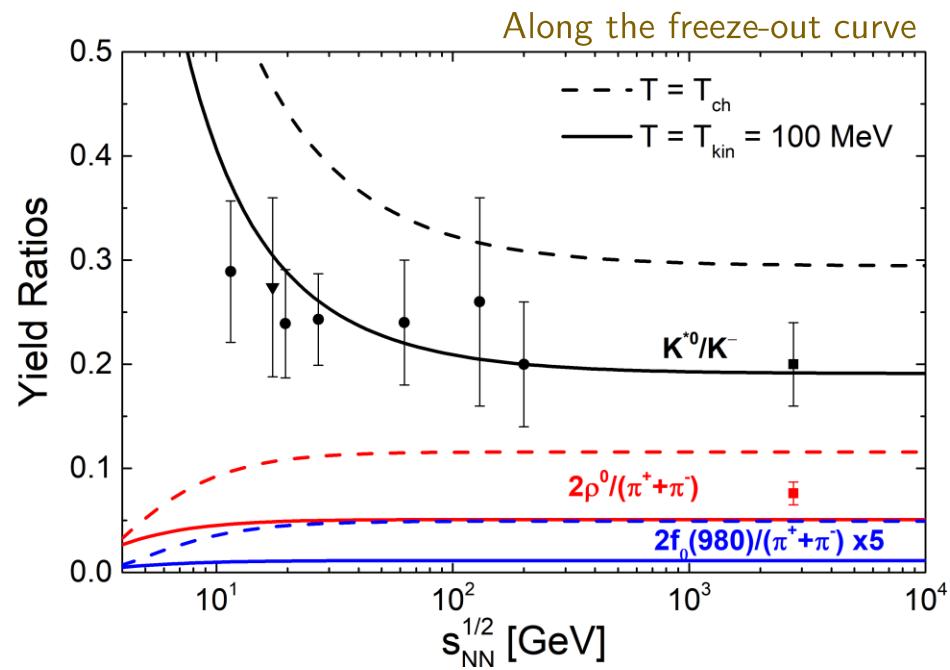
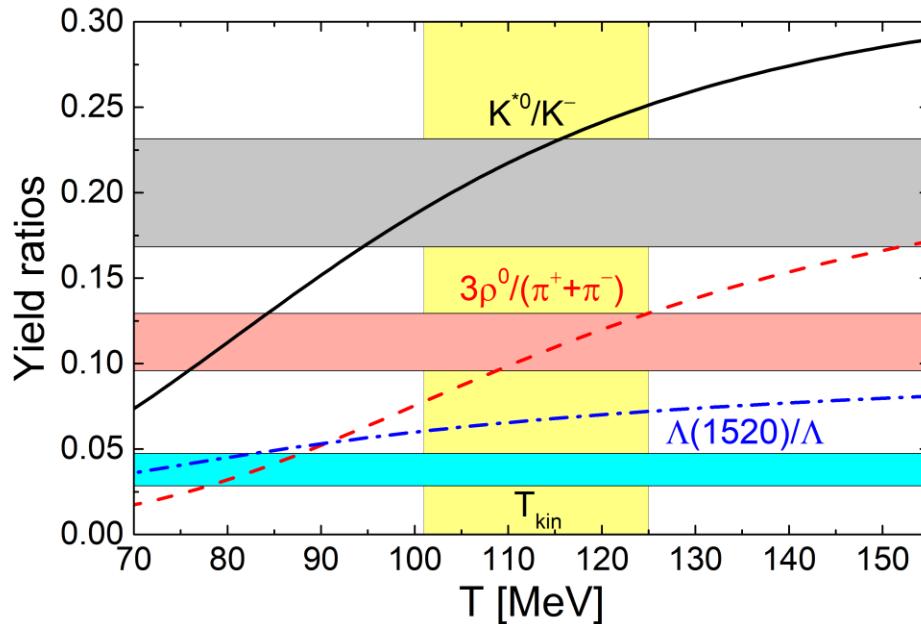


Deviations from thermal model predictions are moderate despite significant cooling and dilution. Is this the reason for why thermal model works so well?



Resonance suppression in hadronic phase

Yields of **resonances** are *not* conserved in partial chemical equilibrium
E.g. K^* yield dilutes during the cooling through reactions $\pi K \leftrightarrow K^*$

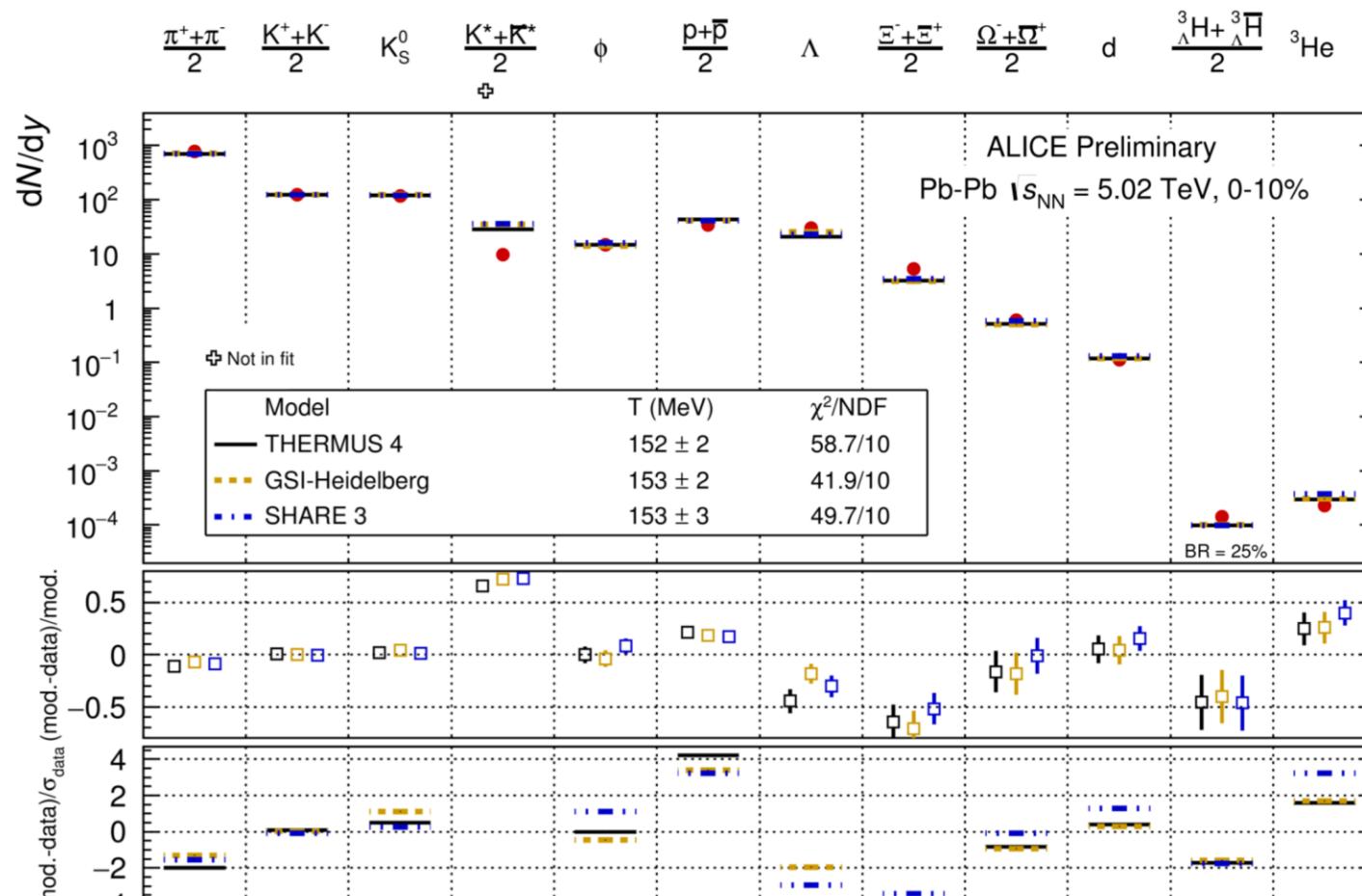


with A. Motornenko

At $T \approx T_{kin}$ the suppressed resonance yields agree quite well with ALICE/RHIC/SPS data for central A+A collisions

This implies significant **resonance regeneration** in the hadronic phase

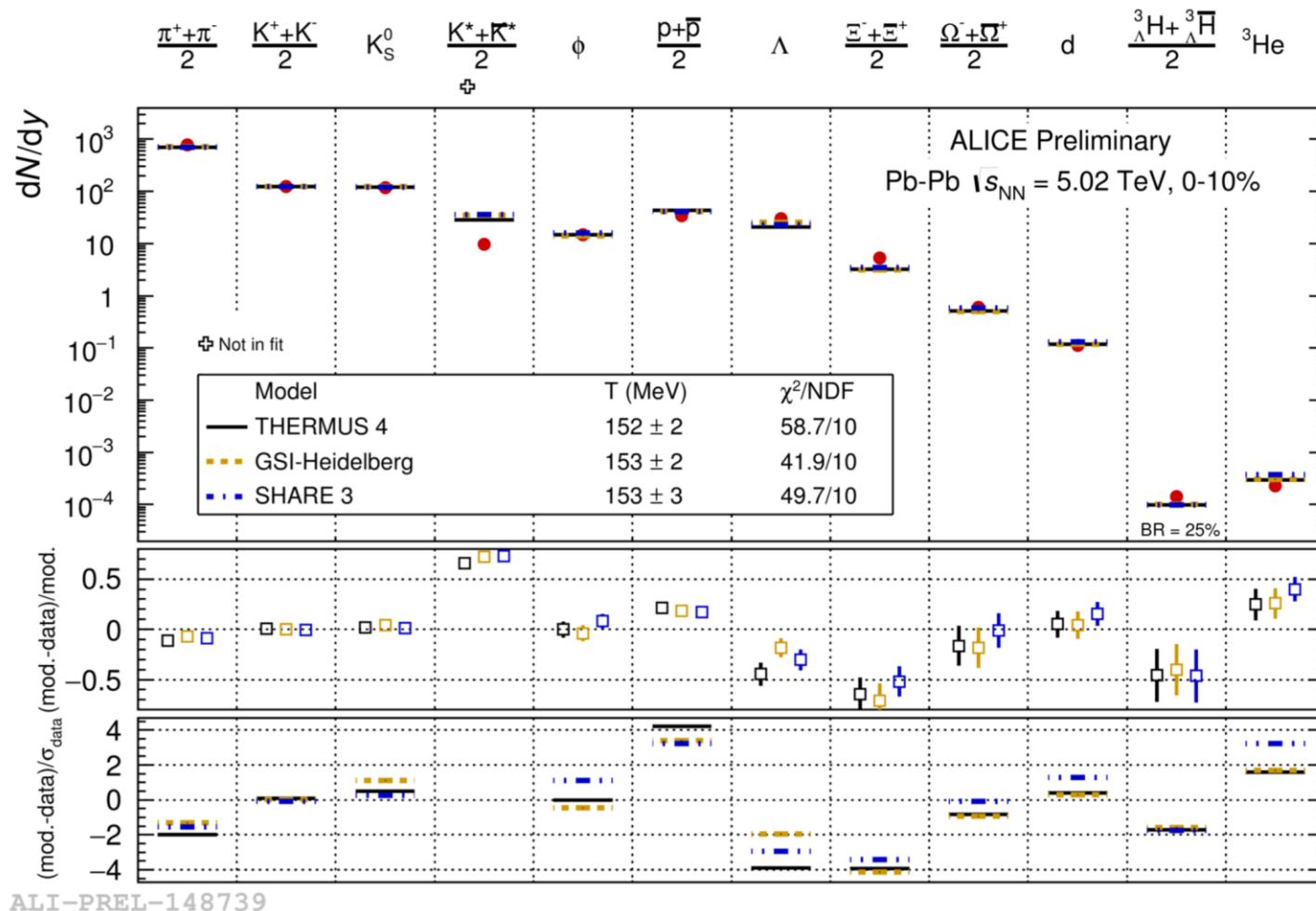
5 TeV Pb-Pb data



ALI-PREL-148739



5 TeV Pb-Pb data



FIST v1.2 (zero-width): $T_{ch} = 151 \pm 2$ MeV, $\chi^2/NDF = 47.4/9$

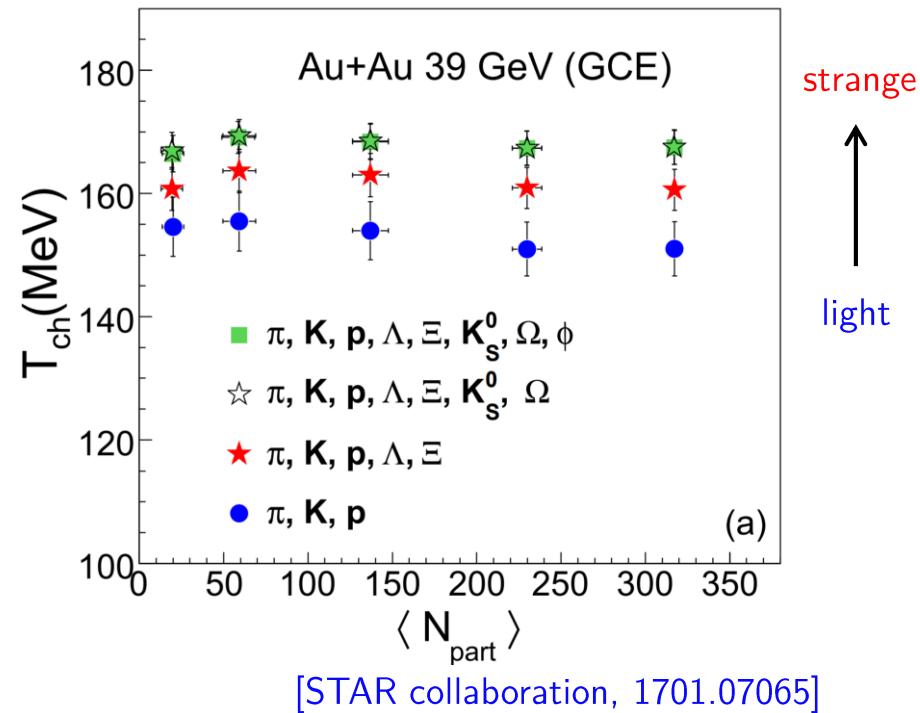
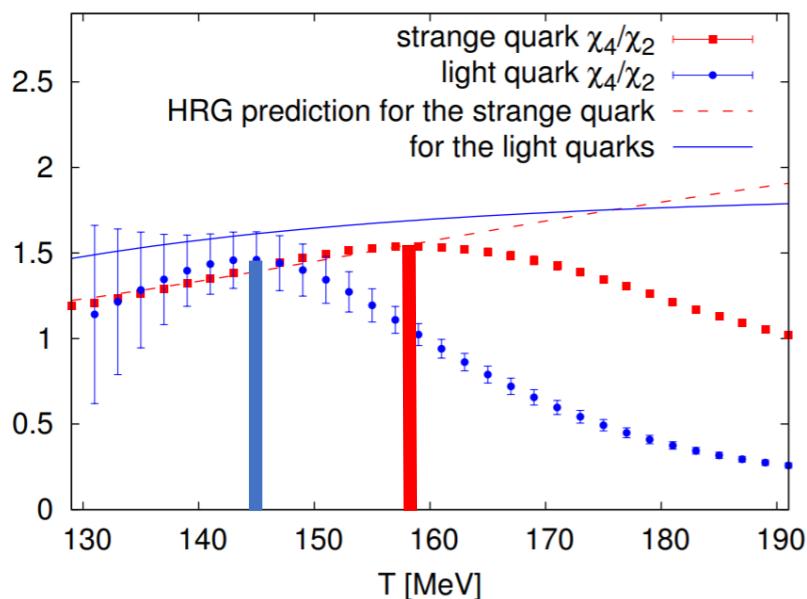
FIST v1.2 (eBW): $T_{ch} = 152 \pm 2$ MeV, $\chi^2/NDF = 41.9/9$



Flavor hierarchy at freeze-out

QCD transition is a broad crossover \Rightarrow different “ T_c ” for different observables?

strange vs light number susceptibility

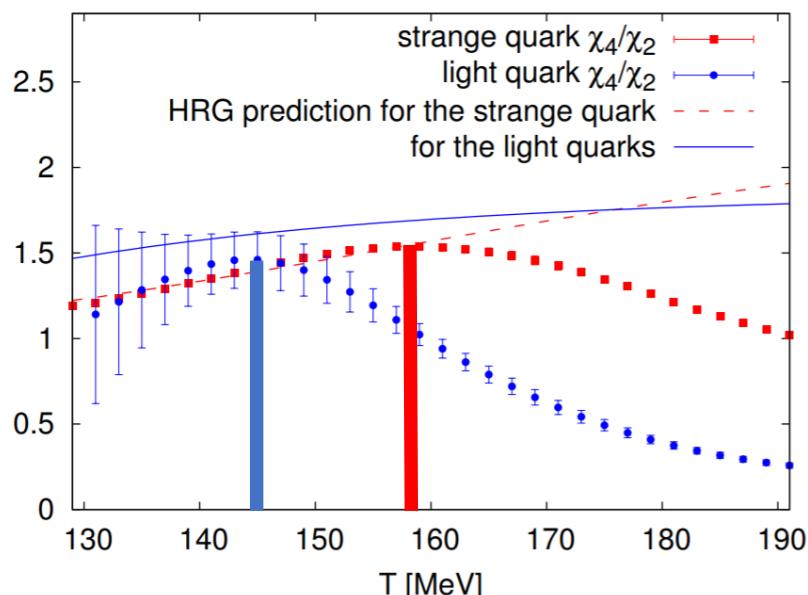


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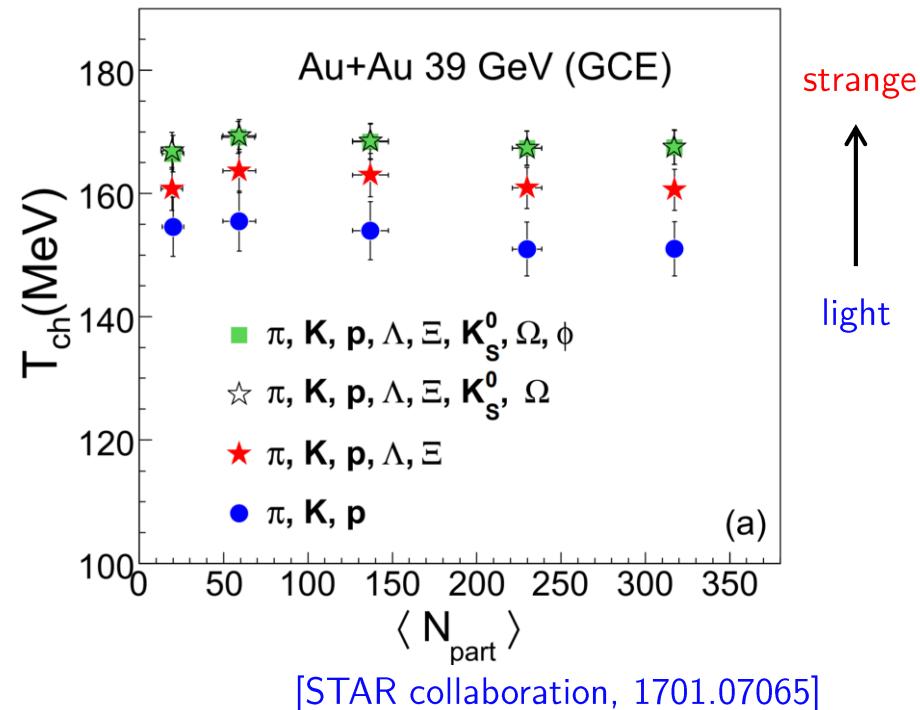


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[R. Bellwied et al., 1305.6297]



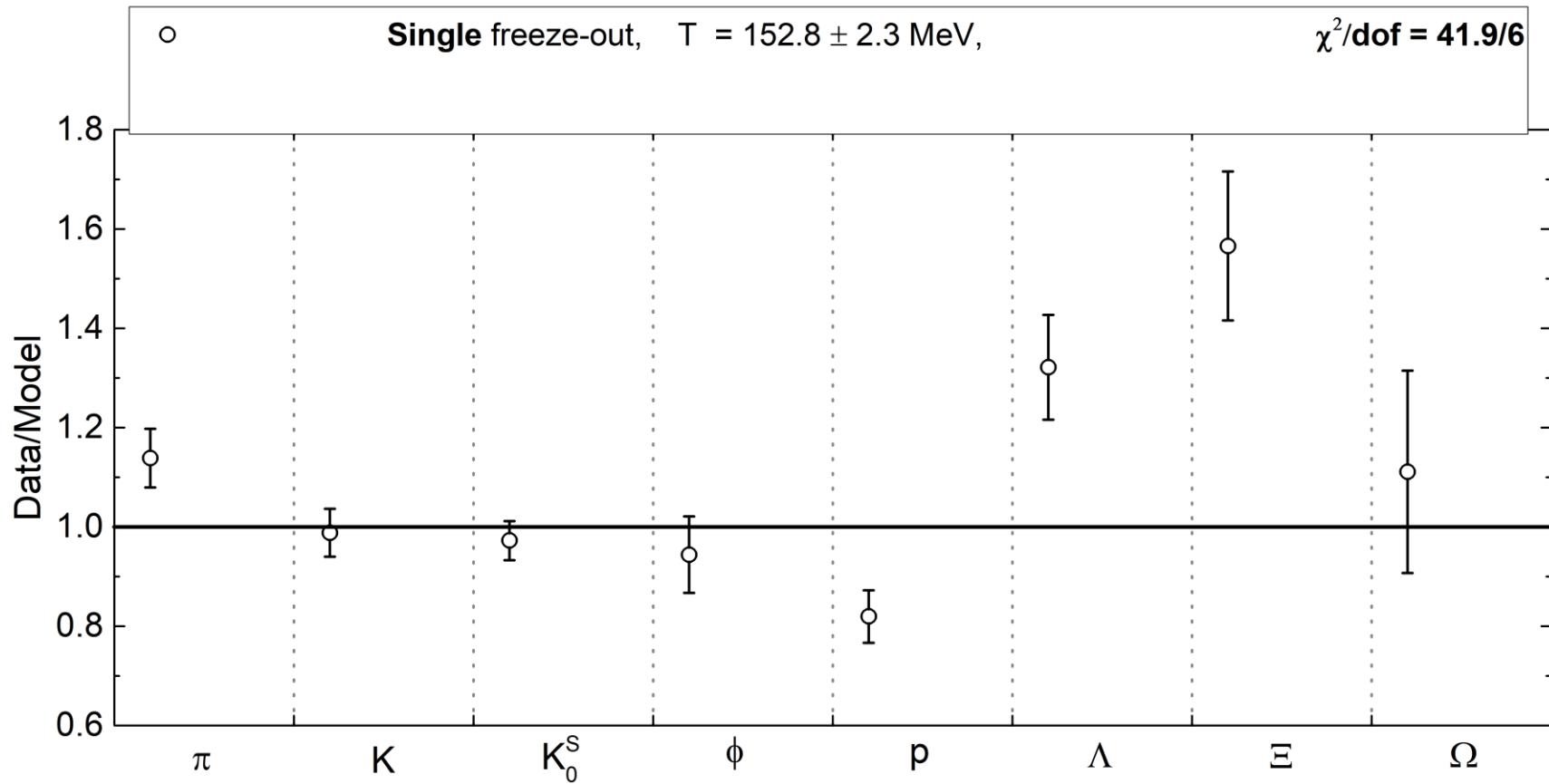
Flavor hierarchy implementation in FIST:

- At $T = T_S$ inelastic strangeness reactions freeze, yields of strange hadrons frozen
 - At $T_{NS} < T_S$ yields of all light-flavored stable hadrons freeze out
 - From T_S to T_{NS} evolution in partial chemical equilibrium of strangeness, strange hadrons, and resonances decaying into them, attain fugacity factors

Flavor hierarchy at freeze-out: Fits



Three global fit parameters: T_S , V_S , and T_{NS}

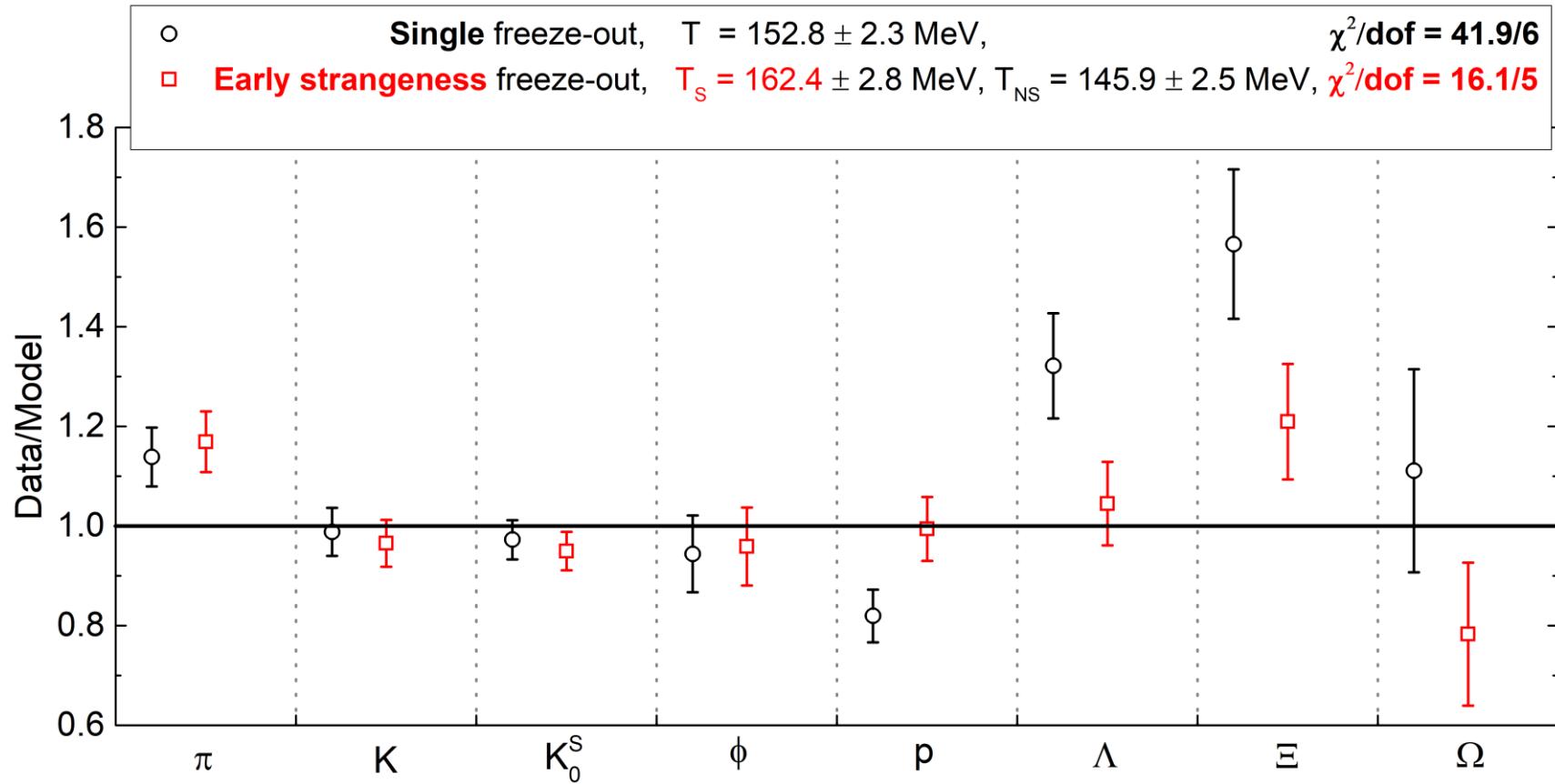


*no light nuclei here

Flavor hierarchy at freeze-out: Fits



Three global fit parameters: T_S , V_S , and T_{NS}

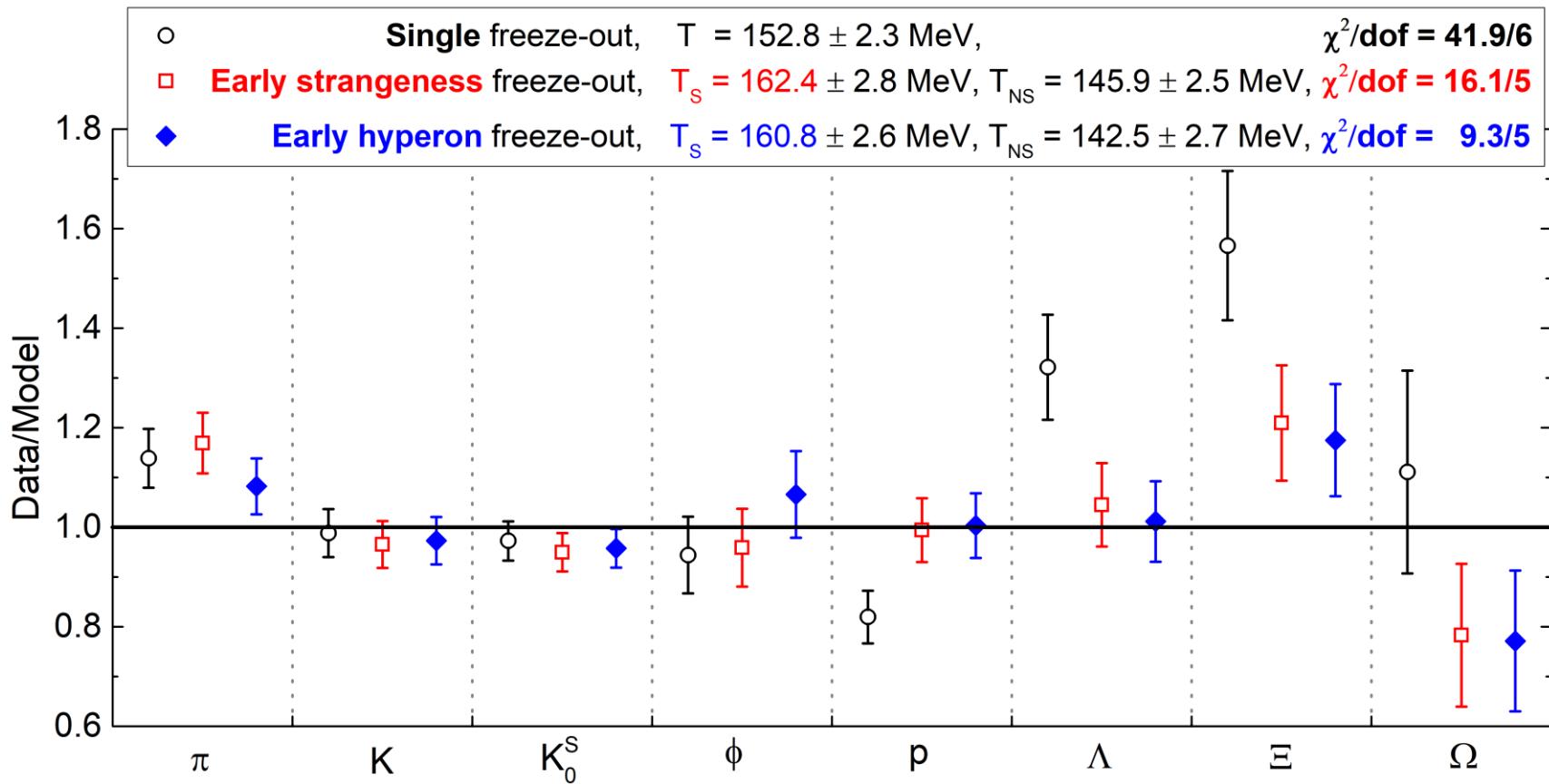


*no light nuclei here



Flavor hierarchy at freeze-out: Fits

Three global fit parameters: T_S , V_S , and T_{NS}



Precision data paves the way to test new scenarios

*no light nuclei here



Summary and outlook

- **Thermal-FIST** is a user-friendly open source package for general purpose thermal model applications with a modular structure.
- **Usage:**
 - Graphical user interface
 - C++ scripts
 - Jupyter notebooks
- **Highlights of FIST results for ALICE**
 - Resonance widths as a possible solution to ‘proton yield anomaly’
 - Canonical statistical model description of p-p, p-Pb, and Pb-Pb collisions
 - Formation of light nuclei deep in the hadronic phase via the Saha equation
 - Suppression of resonance yields as a consequence of partial chemical equilibrium
 - A unified description of the flavor hierarchy in the chemical freeze-out



Summary and outlook

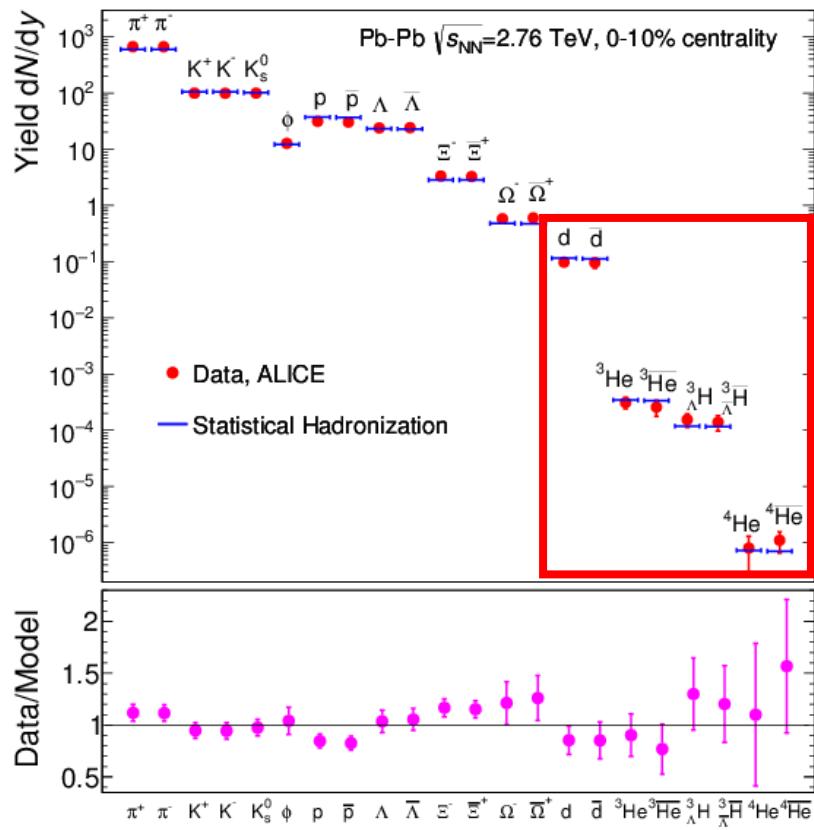
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Thanks for your attention!

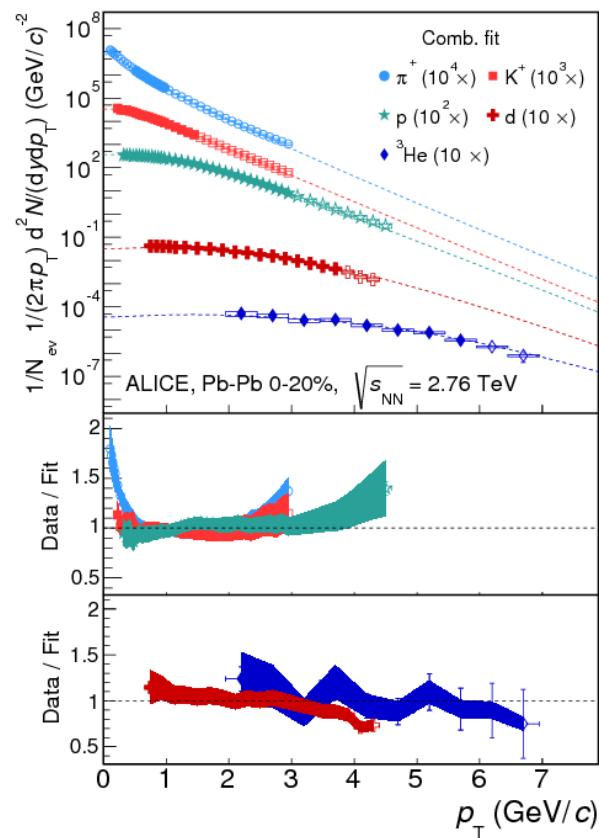
Backup slides

Two experimental observations at the LHC

1. Measured yields are described by thermal model at $T_{ch} \approx 155$ MeV*



2. Spectra described by blast-wave model at $T_{kin} \approx 100 - 120$ MeV*

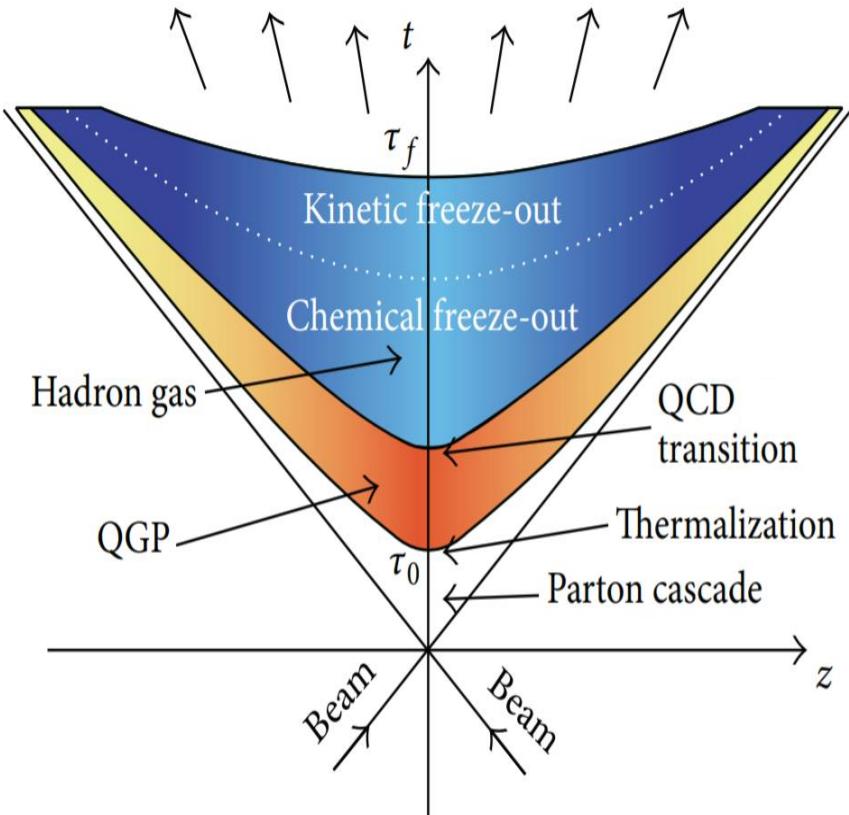
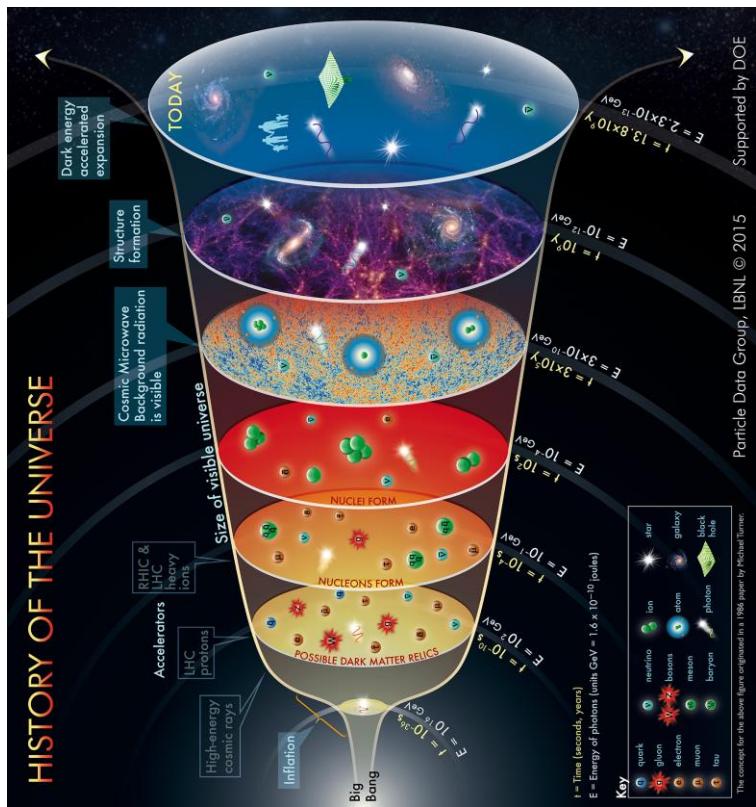


[A. Andronic et al., Nature 561, 321 (2018)]

[ALICE collaboration, PRC 93, 024917 (2016)]

What happens between T_{ch} and T_{kin} ?

Big Bang vs “Little Bangs”



- Hadrons (nucleons) form and “freeze-out” chemically before nuclei
- Bosons (photons or pions) catalyse nucleosynthesis

e.g. $p + n \leftrightarrow d + \gamma$ vs $p + n + \pi \leftrightarrow d + \pi$

LHC nucleosynthesis: simplified setup

- Chemical equilibrium lost at $T_{ch} = 155$ MeV, abundances of nucleons are frozen and acquire effective fugacity factors: $n_i = n_i^{eq} e^{\mu_N/T}$
- Isentropic expansion driven by effectively massless mesonic d.o.f.

$$\frac{V}{V_{ch}} = \left(\frac{T_{ch}}{T}\right)^3, \quad \mu_N \simeq \frac{3}{2} T \ln\left(\frac{T}{T_{ch}}\right) + m_N \left(1 - \frac{T}{T_{ch}}\right)$$

- Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{eq}}{\prod_i n_{A_i}^{eq}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \quad \text{e.g. } \mu_d = \mu_p + \mu_n, \mu_{^3\text{He}} = 2\mu_p + \mu_n, \dots$$

Saha equation



$$X_A = d_A \left[(d_M)^{A-1} \zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{3+A}{2}} \right] A^{5/2} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03$$

$$\text{BBN: } X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}} \right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

(Simplified) Saha equation vs thermal model

Saha equation:

$$\frac{N_A(T)}{N_A(T_{\text{ch}})} \simeq \left(\frac{T}{T_{\text{ch}}} \right)^{\frac{3}{2}(A-1)} \exp \left[B_A \left(\frac{1}{T} - \frac{1}{T_{\text{ch}}} \right) \right]$$

$B_A \ll T$

Thermal model:

$$\left[\frac{N_A(T)}{N_A(T_{\text{ch}})} \right]_{\text{eq.}} \simeq \left(\frac{T}{T_{\text{ch}}} \right)^{\frac{3}{2}} \exp \left[-m_A \left(\frac{1}{T} - \frac{1}{T_{\text{ch}}} \right) \right]$$

$m_A \gg T$

Strong exponential dependence on the temperature is eliminated in the Saha equation approach

Further, quantitative applications require numerical treatment of full spectrum of *massive* mesonic and baryonic resonances

LHC deuteron-synthesis

PHYSICAL REVIEW C 99, 044907 (2019)

Editors' Suggestion

Featured in Physics

Microscopic study of deuteron production in PbPb collisions at $\sqrt{s} = 2.76$ TeV via hydrodynamics and a hadronic afterburner

Dmytro Oliinchenko,¹ Long-Gang Pang,^{1,2} Hannah Elfnner,^{3,4,5} and Volker Koch¹

¹Lawrence Berkeley National Laboratory, 1 Cyclotron Rd, Berkeley, California 94720, USA

²Physics Department, University of California, Berkeley, California 94720, USA

³Frankfurt Institute for Advanced Studies, Ruth-Moufang-Strasse 1, 60438 Frankfurt am Main, Germany

⁴Institute for Theoretical Physics, Goethe University, Max-von-Laue-Strasse 1, 60438 Frankfurt am Main, Germany

⁵GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany

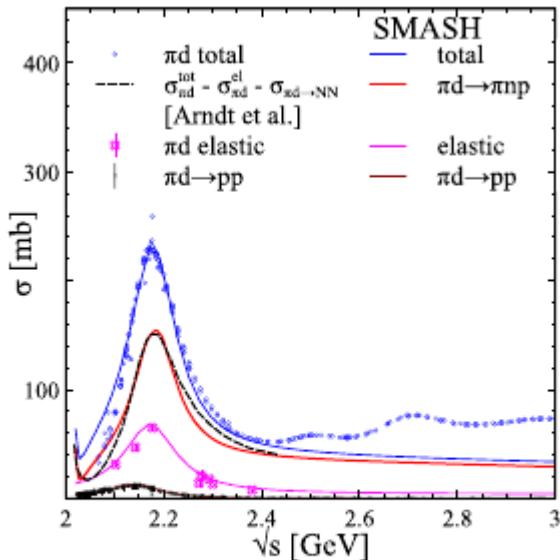


FIG. 1. Deuteron-pion interaction cross sections from SAID database [40] and partial wave analysis [41] are compared to our parametrizations (Tables II and III in the Appendix). Inelastic $d\pi \leftrightarrow$

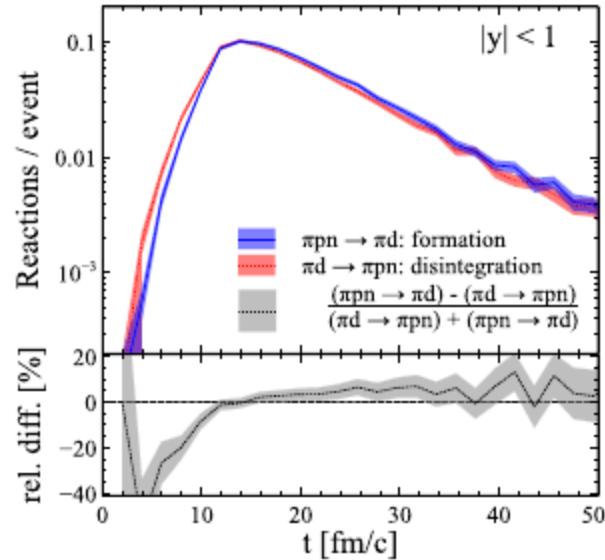


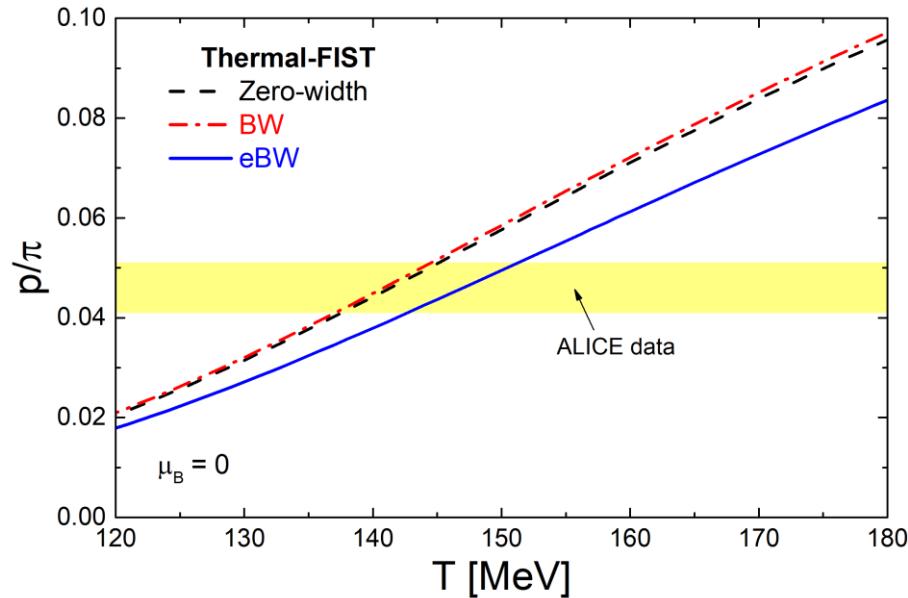
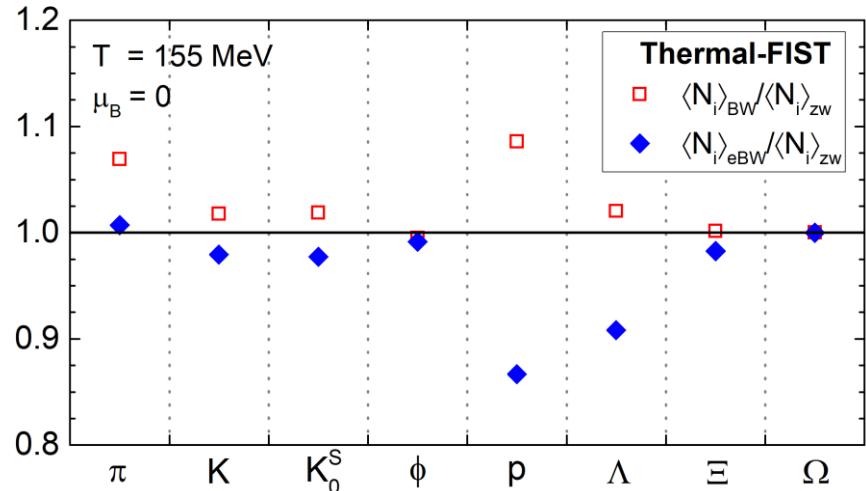
FIG. 5. Reaction rates of the most important $\pi d \leftrightarrow \pi pn$ reaction in forward and reverse direction.

■ Law of mass action at work

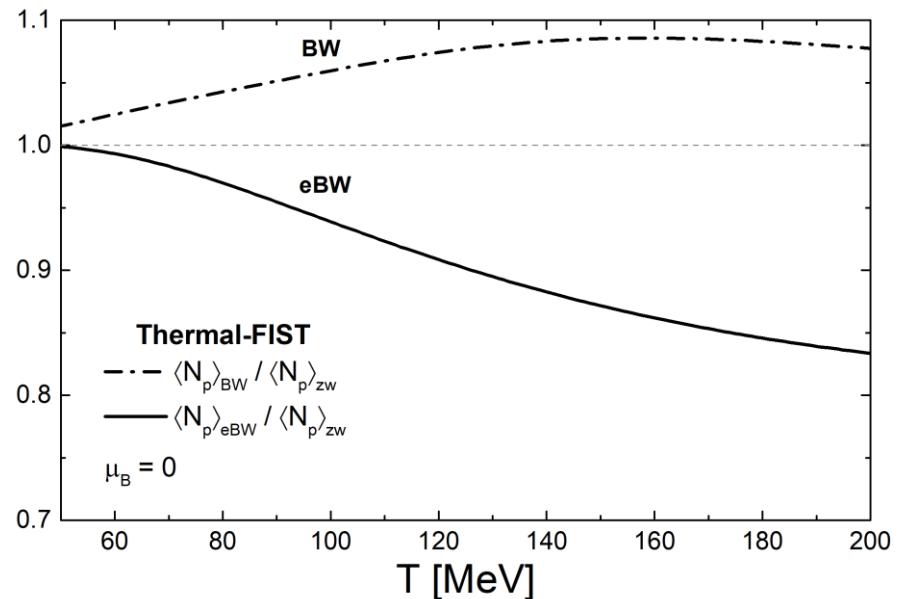
Modeling widths: Effect on hadron yields



Modification of final hadron yields



protons



- BW enhances, eBW suppresses feeddown
- Strongest effect for protons & Λ
- p/π ratio suppressed in eBW

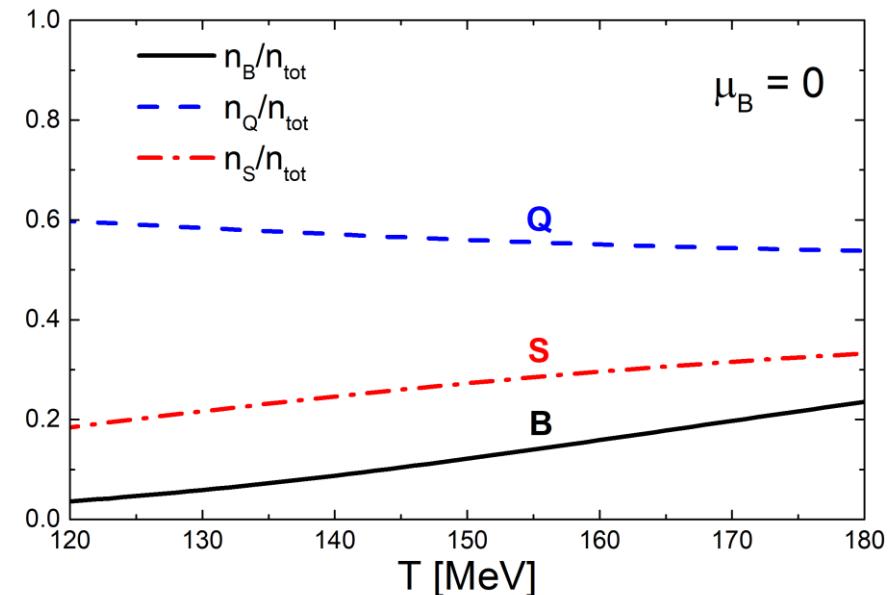
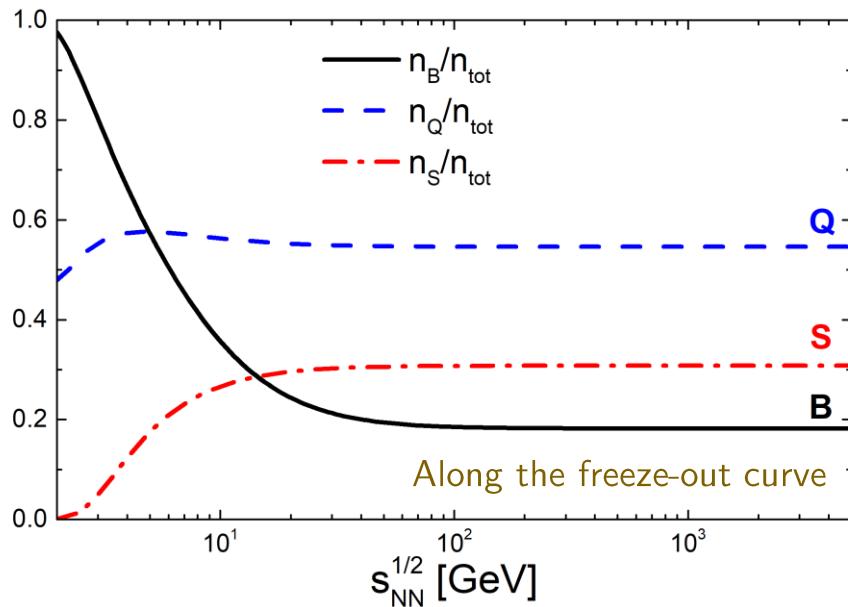
When is the canonical treatment necessary?



Normally, when the total number of particles carrying a conserved charge is **smaller or of the order of unity**

The canonical treatment is often restricted to strangeness only (**SCE**)

[STAR collaboration, 1701.07065; ALICE collaboration, 1807.11321]



- **Strangeness** conservation is most important at low energies (**HADES, CBM**)
- *Small systems at RHIC and LHC*: exact **baryon** conservation at least as important as **strangeness**

CSM at LHC



Enforce exact conservation of charges, $B = Q = S = 0$, in a *correlation volume* V_C around midrapidity

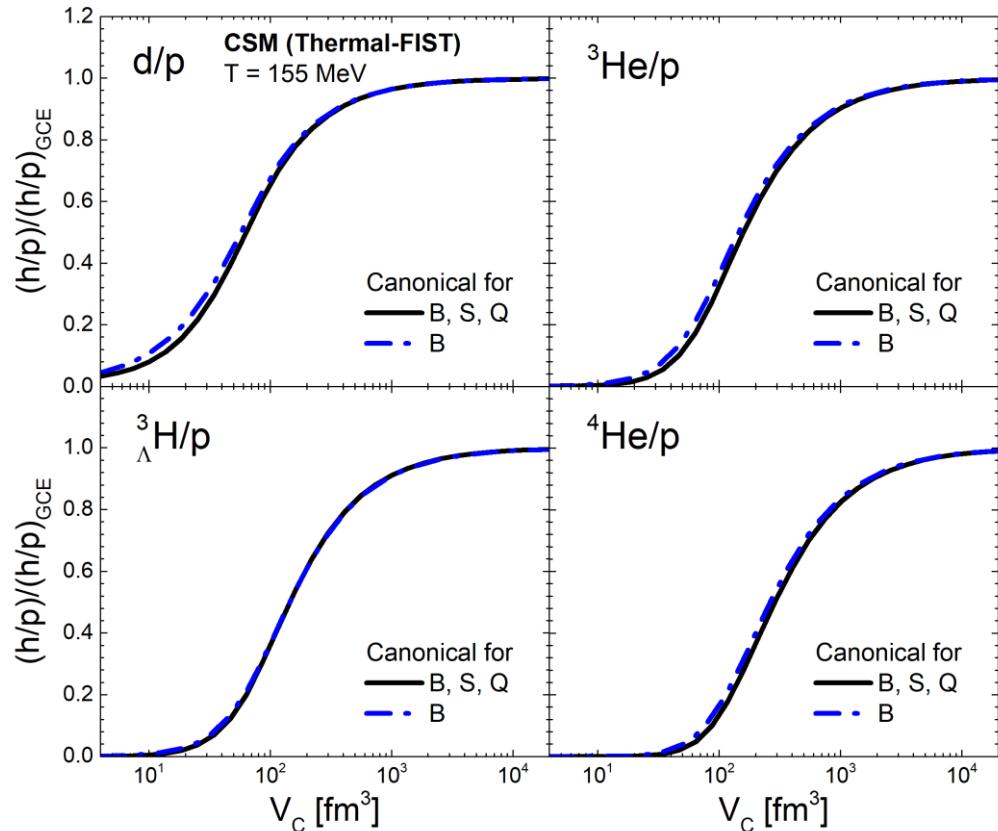
In general, $V_C \neq dV/dy$

Causality argument: exact conservation across a few units of rapidity?

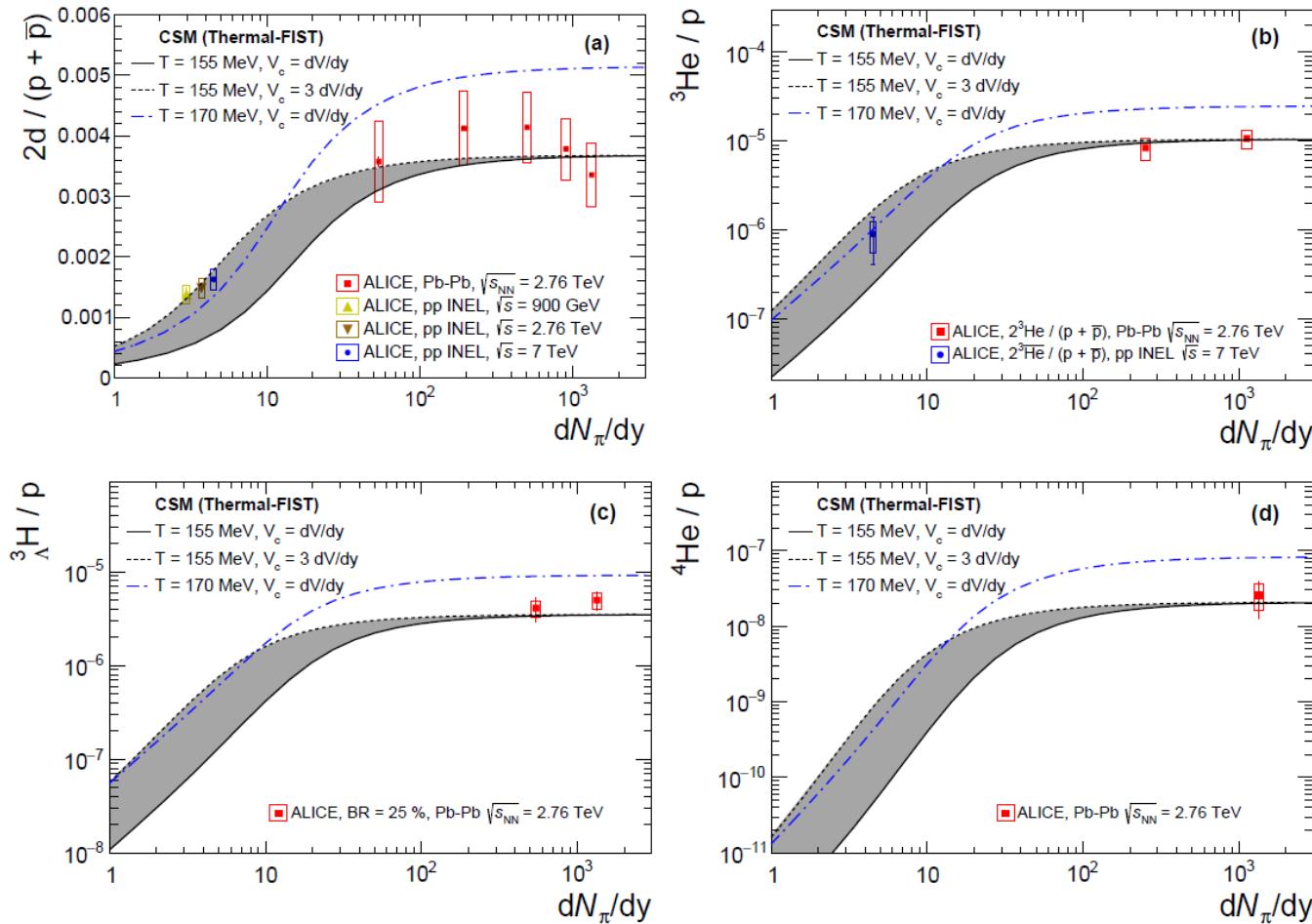
[Castorina, Satz, 1310.6932]

New application: CSM for light nuclei

- Suppression of nuclei-to-proton ratios at low multiplicities
- For these observables sufficient to enforce exact baryon conservation only



CSM at LHC: light nuclei



[V.V., B. Doenigus, H. Stoecker, 1808.05245]

- **CSM** qualitatively captures the behavior seen in the data
- Data prefers $V_C > dV/dy$ and/or $T_{p+p} > T_{Pb+Pb}$



Excluded volume corrections

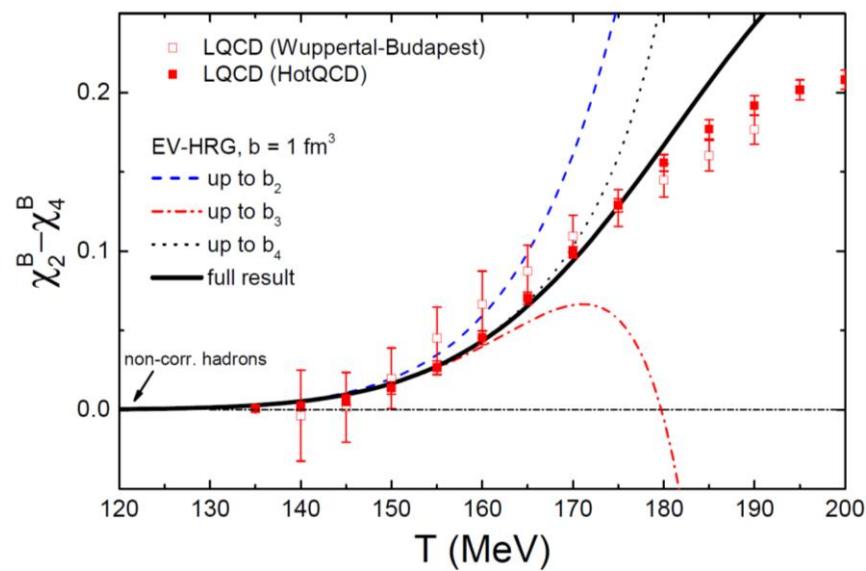
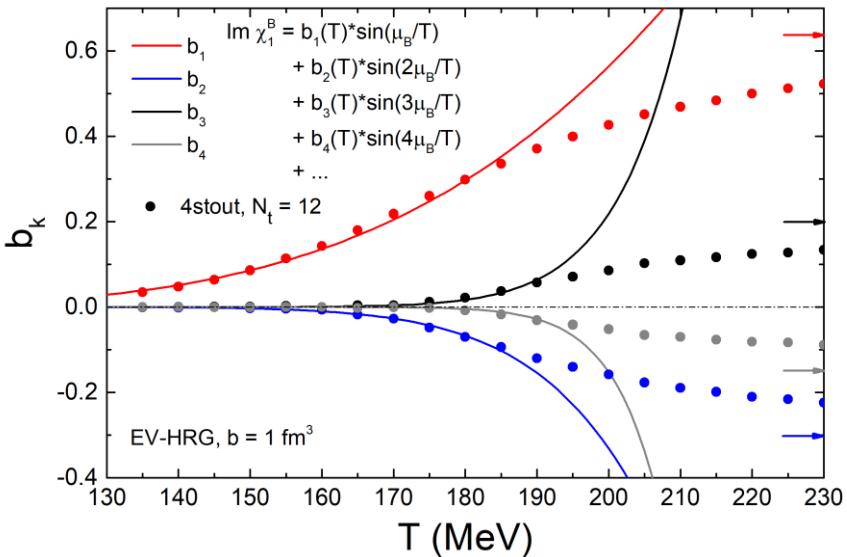
Notion that hadrons have finite eigenvolume suggested a while ago

[R. Hagedorn, J. Rafelski, PLB '80]

Excluded volume model: $V \rightarrow V - bN$ \Rightarrow repulsive interactions
[D. Rischke et al., Z. Phys. C '91]

Whether EV corrections are needed at all has been debated...

Recent lattice data favor EV-like effects in baryonic interactions



V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

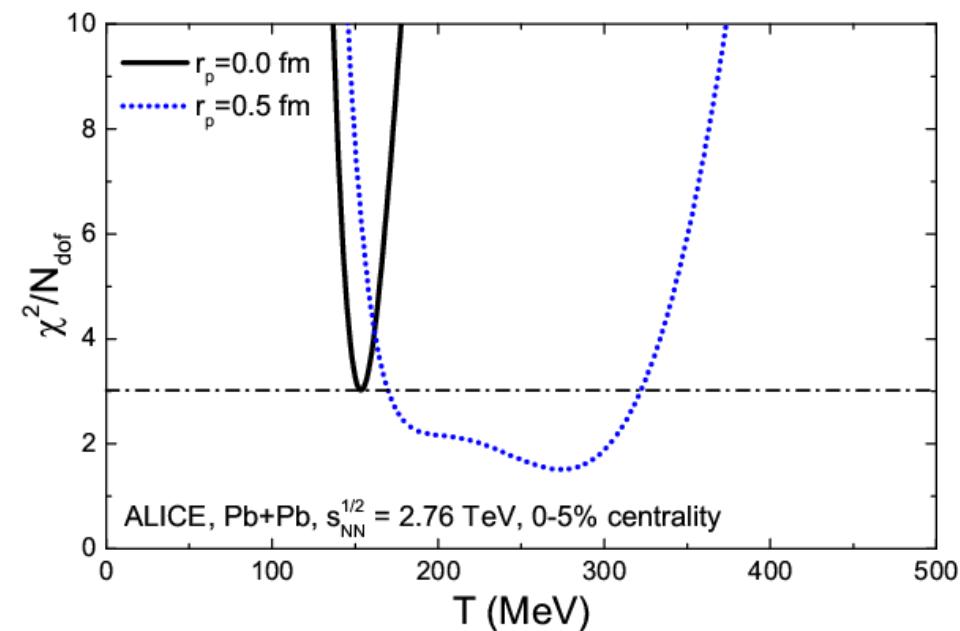
but not much info regarding (non-)existence of EV effects for mesons



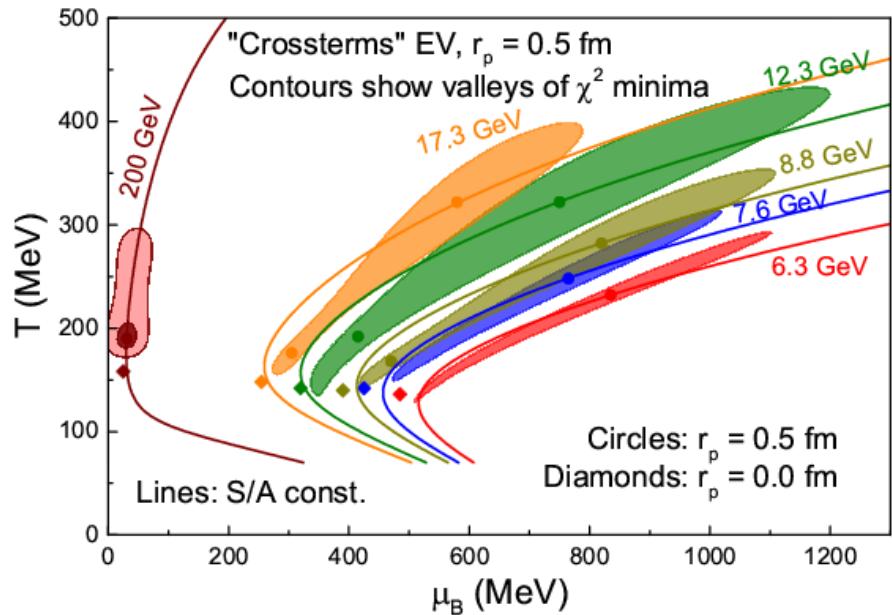
Another extreme: bag model scaling

Bag model: $v_i \propto m_i$

[Chodos et al., PRD '74; Kapusta et al., NPA '83, PRC '15]



[V.V., H. Stoecker, 1512.08046]



[V.V., H. Stoecker, 1606.06218]

Extraction of T and μ can be quite sensitive w.r.t EV corrections,
but entropy per baryon, S/A, is a robust observable

NB: This calculation disregards Hagedorn states needed to model the crossover transition



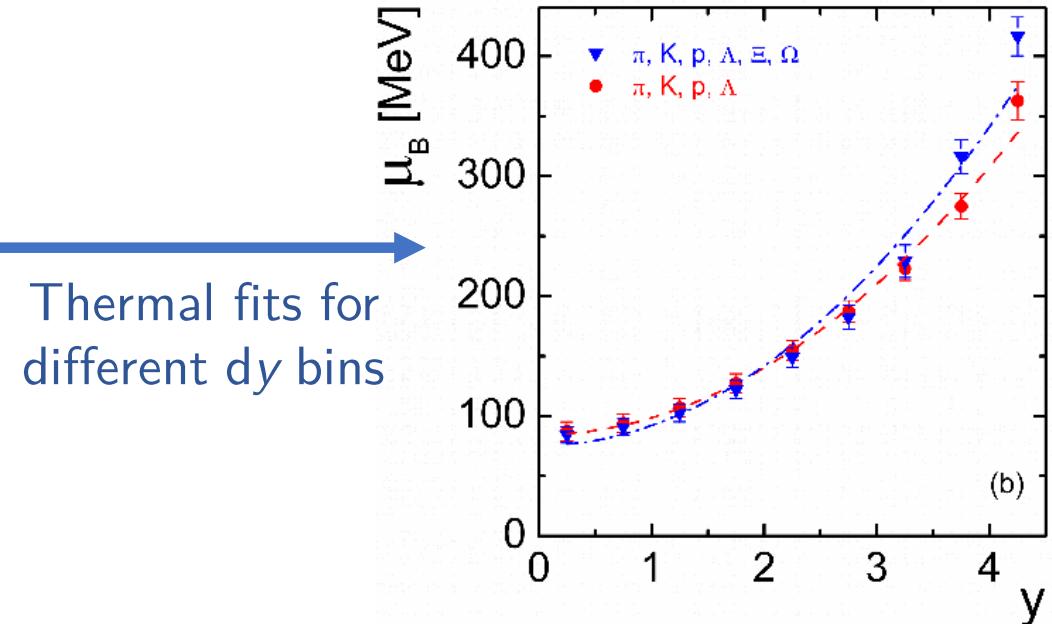
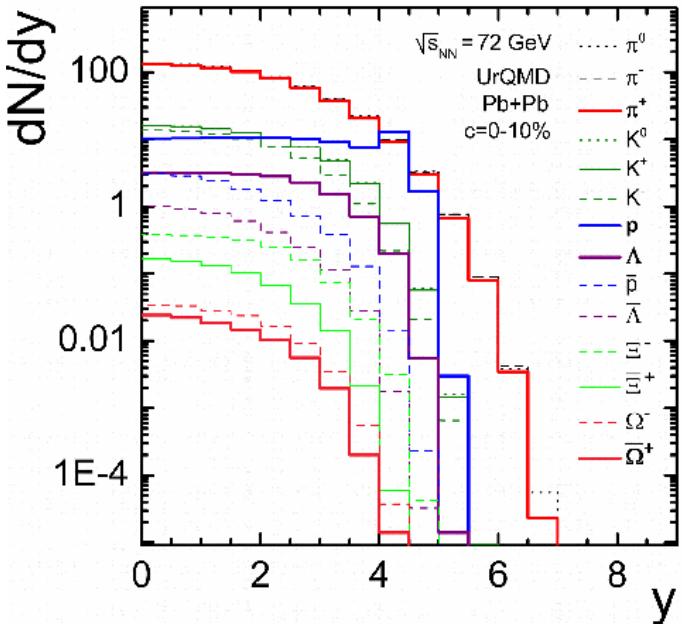
Rapidity scan

Fireballs at midrapidity: $\mu_B(y_s) \approx \mu_B(0) + b y_s^2$

RHIC @ $\sqrt{s_{NN}} = 200$ GeV: $\mu_B(y_s) \approx 25 + 11y_s^2$ [MeV]

[Becattini et al., 0709.2599]

Example: AFTER@LHC project: Pb+Pb collisions @ $\sqrt{s_{NN}} = 72$ GeV



[Begun, Kikola, V.V., Wielanek, 1806.01303]

Rapidity scan: complementary approach to scan QCD phase diagram

see also Li, Kapusta, 1604.08525; Brewer, Mukherjee, Rajagopal, Yin, 1804.10215