

12.1.23 פועל 100 שעות

הכנסה ~ שעות - h  
 עלות ~ שעות - m

$$\frac{m}{h} \cdot \frac{m-1}{h-1} = \frac{1}{2}$$

$$\frac{m^2 - m}{h^2 - h} = \frac{1}{2} \quad | \cdot 2(h^2 - h)$$

$$(1) \quad 2m^2 - 2m = h^2 - h \Leftrightarrow 0 = h^2 - h + 2m - 2m^2$$

$$h = \frac{-(-1) \pm \sqrt{1 - 4 \cdot 1 \cdot (2m - 2m^2)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1 - 8m + 8m^2}}{2}$$

$$h = \frac{1 \pm \sqrt{8m^2 - 8m + 1}}{2} \Rightarrow 8m^2 - 8m + 1 = (2a - 1)^2$$

$$(2) \quad 8m^2 - 8m + 1 = 4a^2 - 4a + 1 \quad | -4a^2 + 4a$$

$$8m^2 - 8m - 4a^2 + 4a = 0$$

$$m = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 8 \cdot (-4a^2 + 4a)}}{2 \cdot 8} = \frac{8 \pm \sqrt{64 - 32(4a^2 - 4a)}}{16}$$

$$m = \frac{8 \pm \sqrt{64 - 128a + 128a^2}}{16} = \frac{8 \pm \sqrt{64(1 - 2a + 2a^2)}}{16}$$

$$= \frac{8 \pm 8\sqrt{2a^2 - 2a + 1}}{16} = \frac{1 \pm \sqrt{2a^2 - 2a + 1}}{2}$$

הצבה: המשוואה (3)  $2a^2 - 2a + 1 = b^2$

$$2a^2 - 2a + 1 - b^2 = 0$$

$$a = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot (1 - b^2)}}{2 \cdot 2}$$

$$= \frac{2 \pm \sqrt{4 - 4(2 - 2b^2)}}{4} = \frac{2 \pm \sqrt{4(1 - 2 + 2b^2)}}{4}$$

$$= \frac{2 \pm 2\sqrt{2b^2 - 1}}{4} = \frac{1 \pm \sqrt{2b^2 - 1}}{2}$$

pell's equation

(4)  $2b^2 - 1 = c^2 \Leftrightarrow c^2 - 2b^2 = -1$

$(b_1, c_1)$  נקודה ראשונה

$b=1 \quad c=1 \quad \checkmark \Rightarrow (b_1, c_1) = (1, 1)$

$$c_h = \frac{(c_1 + b_1\sqrt{2})^{2h-1} + (c_1 - b_1\sqrt{2})^{2h-1}}{2}$$

$$= \frac{(1 + \sqrt{2})^{2h-1} + (1 - \sqrt{2})^{2h-1}}{2}$$

$$b_h = \frac{(c_1 + b_1\sqrt{2})^{2h-1} - (c_1 - b_1\sqrt{2})^{2h-1}}{2\sqrt{2}}$$

$$\boxed{= \frac{(1 + \sqrt{2})^{2h-1} - (1 - \sqrt{2})^{2h-1}}{2\sqrt{2}}}$$