Network Controllability in Transmodal Cortex Predicts Psychosis Spectrum Symptoms

Linden Parkes

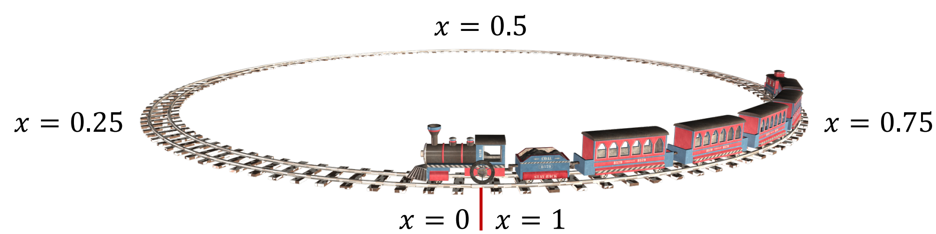
What is Network Control Theory, anyway?

- When we say “control”, we’re talking about controlling a ***dynamical system*** to ***alter*** its behavior in a desired way
- A dynamical system is a system whose states evolve forward in time in geometric space according to a function

“A dynamical system is a system whose states evolve forward in time in geometric space according to a function”

"A dynamical system is a system **whose states** evolve forward in time in geometric space according to a function"

A state is *just* a complete description of a system



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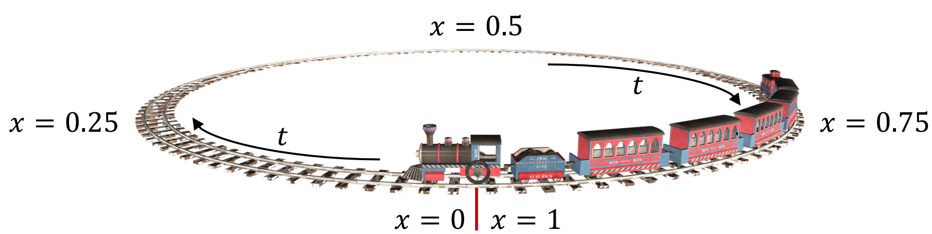

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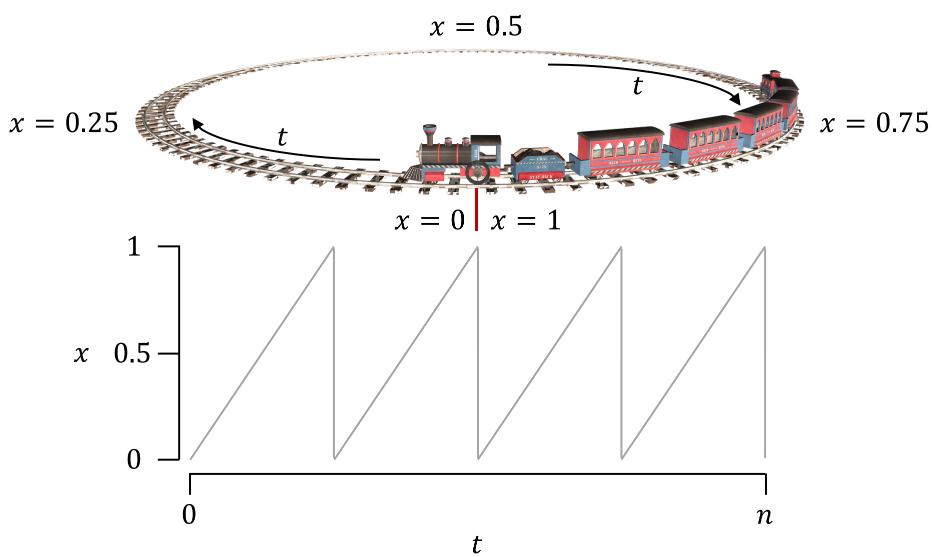
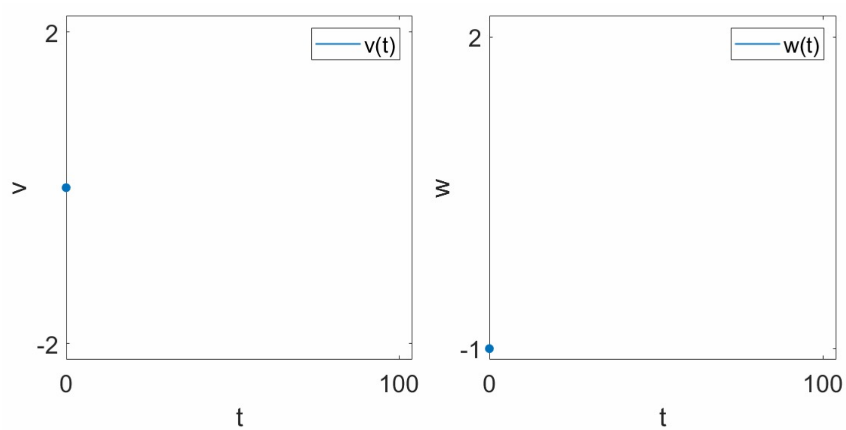


"A dynamical system is a system whose states evolve forward in time in geometric space according to a function"

FitzHugh-Nagumo model neuron. Parameterized by:

1. v, membrane voltage, excitatory input
2. w, 'inhibitory' variable that functions to diminish spikes in v



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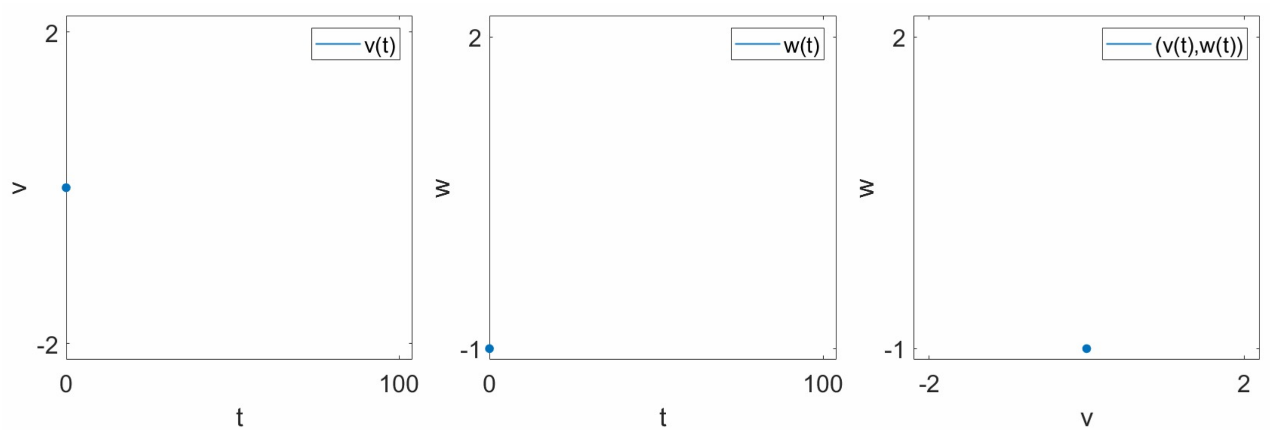


"A dynamical system is a system whose states evolve forward in time in geometric space **according to a function**"

This is where *differential equations* come into play

$$\begin{array}{r} \text{Change in } x \rightarrow \\ \text{Change in } t \rightarrow \end{array} \frac{dx}{dt} = f(x) \leftarrow \text{Function of } x \text{ defined by us!}$$

"A dynamical system is a system whose states evolve forward in time in geometric space **according to a function**"

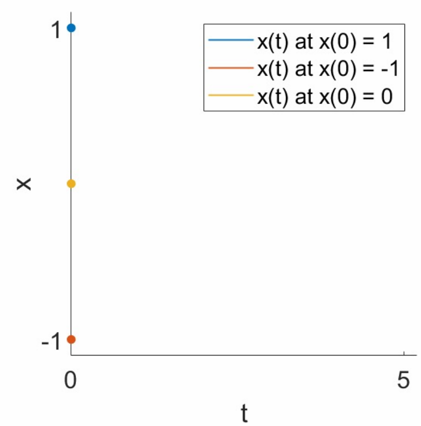
This is where *differential equations* come into play

$$\text{Change in } x \rightarrow \frac{dx}{dt} = -x \quad \leftarrow \text{Function of } x \text{ defined by us!}$$
Change in t →

If $x = 1$, then $\frac{dx}{dt} = -1$

If $x = 0$, then $\frac{dx}{dt} = 0$

If $x = -1$, then $\frac{dx}{dt} = 1$



“A dynamical system is a system whose states evolve forward in time in geometric space **according to a function**”

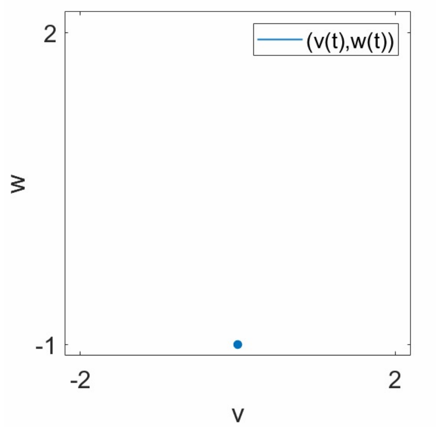
This is where ***coupled*** *differential equations* come into play

$$\underbrace{\frac{dx}{dt}}_{} = f(x)$$

Change in x → dx; Change in t → dt; ← Function of x defined by us!

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + 0.5$$

$$\frac{dw}{dt} = \frac{1}{12.5}(v + 0.8 - 0.7w)$$



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This is where ***coupled*** *differential equations* come into play

$$\underbrace{}_{\text{Change in } x \rightarrow} \frac{dx}{dt} = f(x) \leftarrow \text{Function of } x \text{ defined by us!}$$

Change in x → numerator; Change in t → denominator

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + 0.5$$

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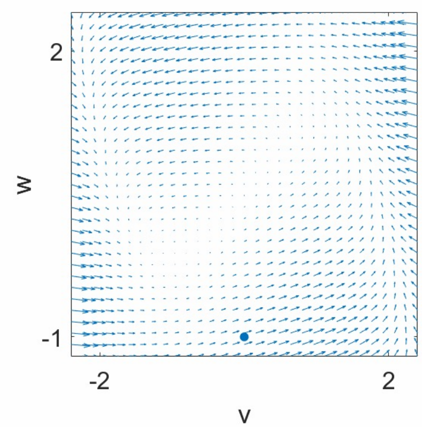

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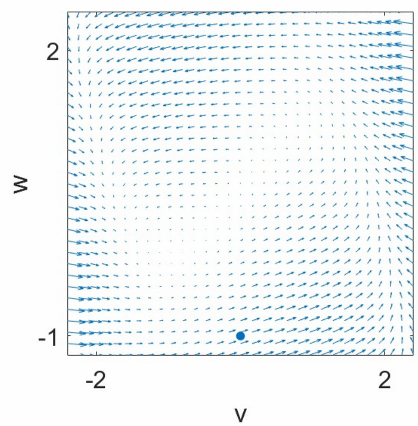

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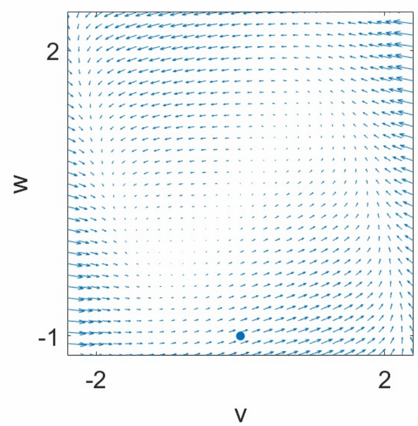

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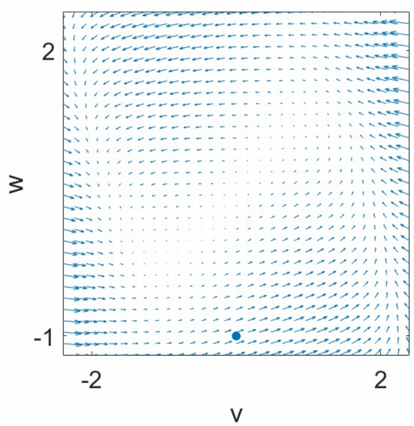

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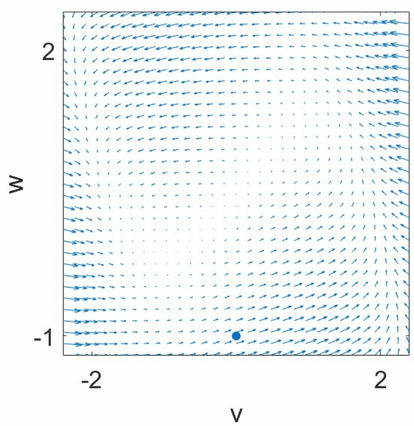

When we say “control”, we’re talking about controlling a *dynamical system* to *alter* its behavior in a desired way

We seek to *find a specific perturbation* of the system administered via some time varying external inputs that achieves a desire state transition

Change in the whole system over time

Coupling between n system elements

System state in n-dimensional space

$$\frac{dx}{dt} = f(x) \rightarrow \dot{x} = Ax$$

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$$\frac{dx}{dt} = f(x) \rightarrow \dot{x} = Ax + Bu$$

Control nodes

Time varying inputs

When we say “control”, we’re talking about controlling a *dynamical system* to *alter* its behavior in a desired way

…I’m skipping over the math of how we solve for $\boldsymbol{u}$

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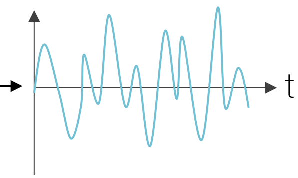
$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1 \end{bmatrix}}_{\mathbf{u}}$$



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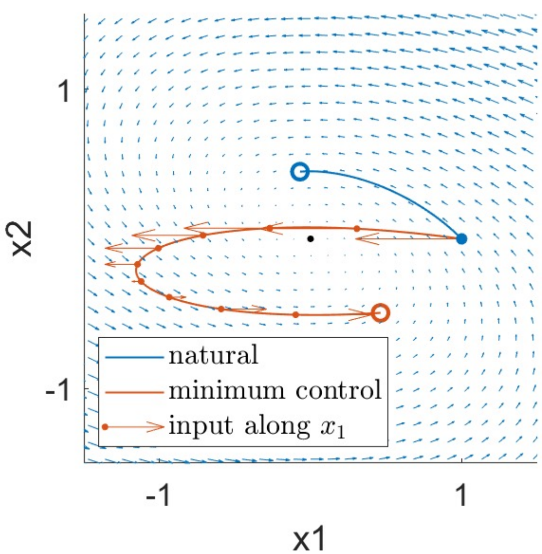

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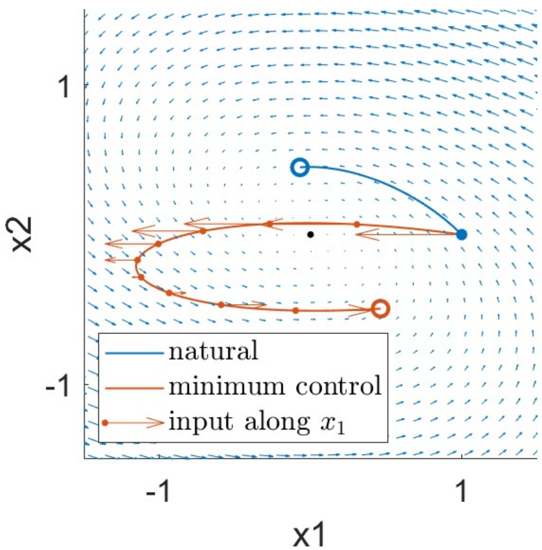

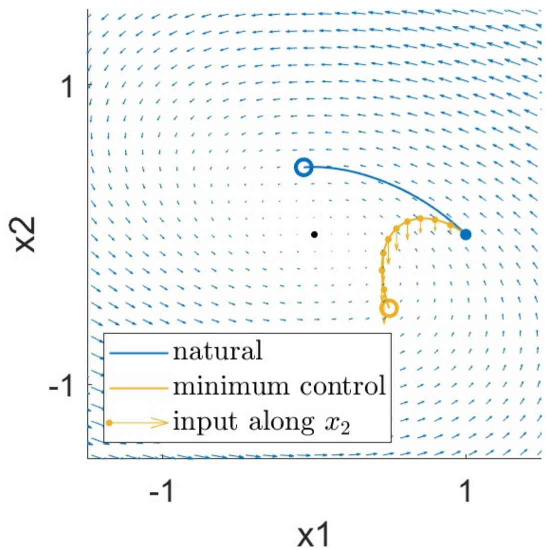

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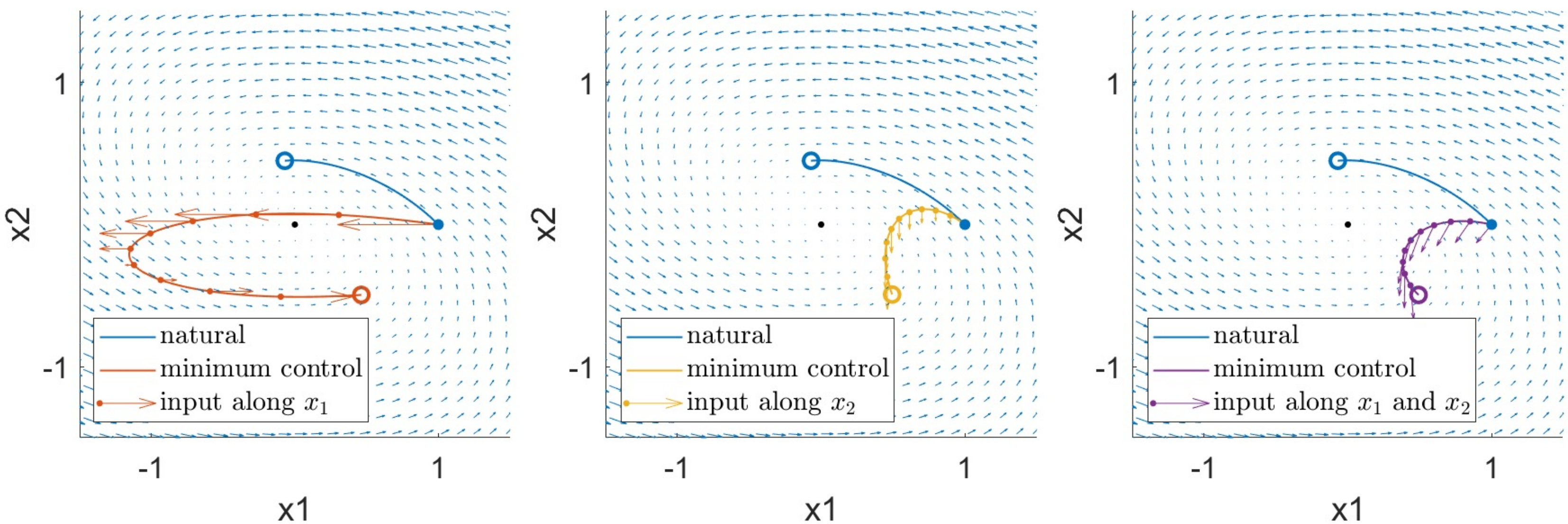
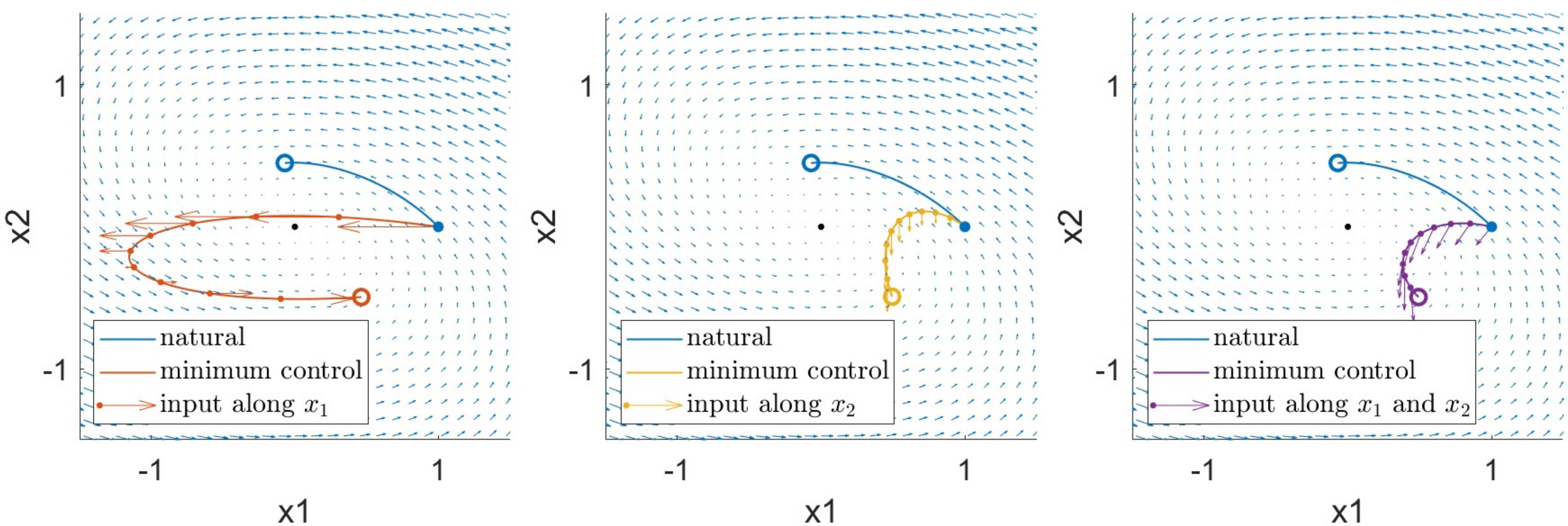


When we say "control", we're talking about controlling a *dynamical system* to *alter* its behavior in a desired way

Transition energy = integrate over time varying inputs ($\boldsymbol{u}$)

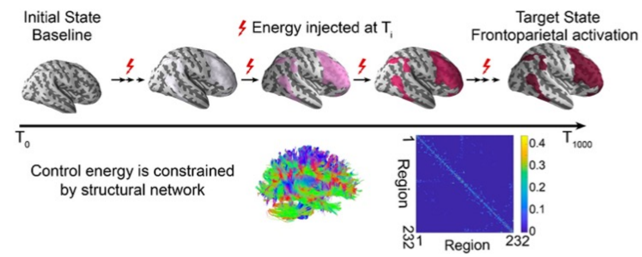
It's not the energy the brain exerts to do a thing, but rather the *energy the model must exert to complete a transition that is unnatural given the dynamics enabled by the system's coupling*

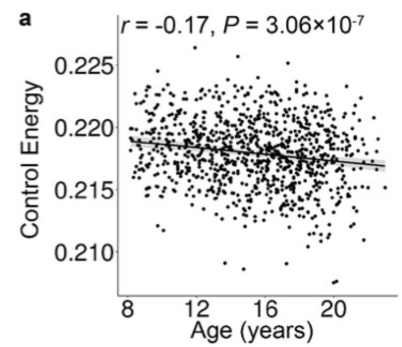


How do we use NCT?

Typically, we use it one of two ways:

1. Analyse transition energies associated with specific state transitions defined *a priori*
   - e.g., Zaixu Cui's eLife paper
2. Analyse controllability statistics
   - We do this if don't care so much about specific state transitions





Archival Report

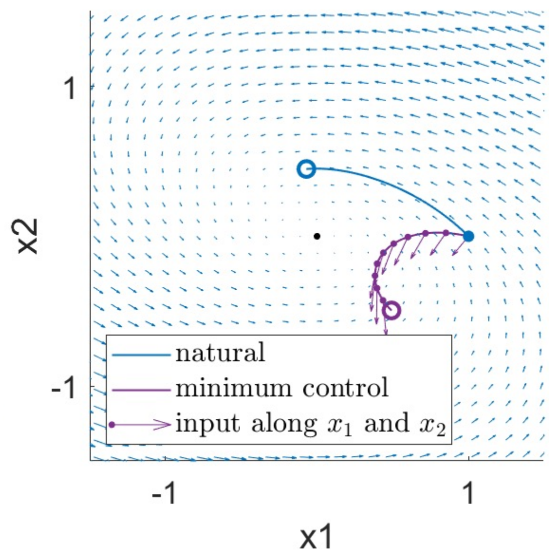
Biological Psychiatry

Network Controllability in Transmodal Cortex Predicts Positive Psychosis Spectrum Symptoms

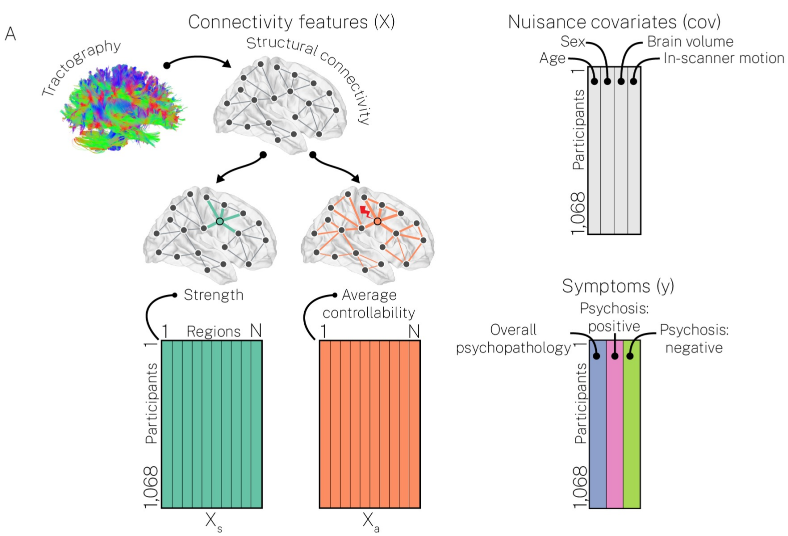
Linden Parkes, Tyler M. Moore, Monica E. Calkins, Matthew Cieslak, David R. Roalf, Daniel H. Wolf, Ruben C. Gur, Raquel E. Gur, Theodore D. Satterthwaite, and Danielle S. Bassett

Average controllability

Average controllability is a regional summary of control that estimates the extent to which each region can leverage the dynamical system to facilitate changes between most/all low energy states (or ‘easy to each’).

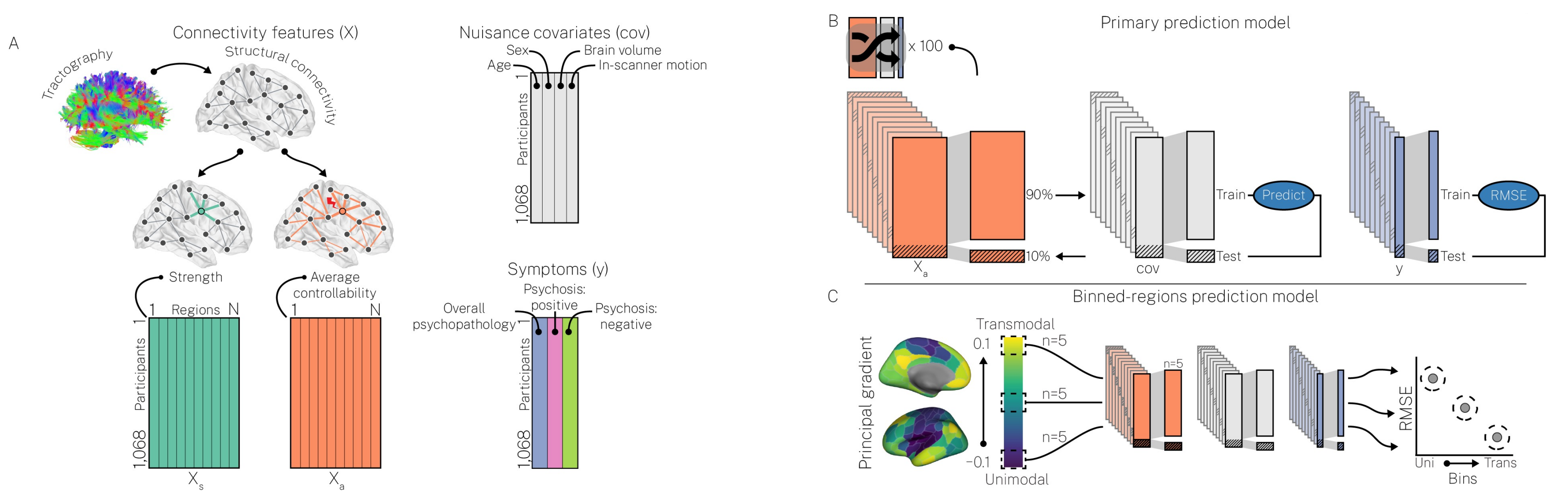


Methods



A
Connectivity features (X)
Tractography
Structural connectivity
Strength
Average controllability
1
Regions
N
1
N
1
Participants
1,068
X_s
X_a
Nuisance covariates (cov)
Sex
Age
Brain volume
In-scanner motion
1
Participants
1,068
Symptoms (y)
Overall psychopathology
Psychosis: positive
Psychosis: negative
1
Participants
1,068

Methods



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Connectivity features (X)
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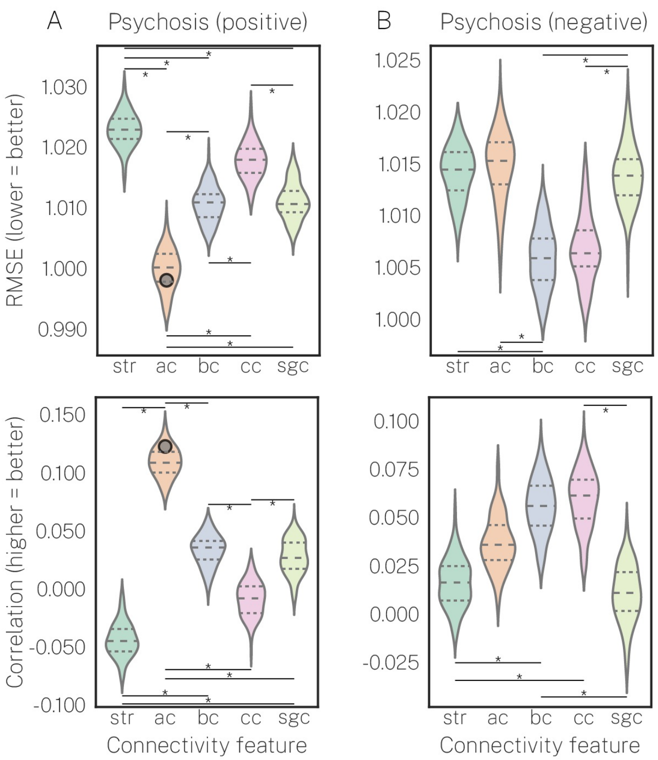
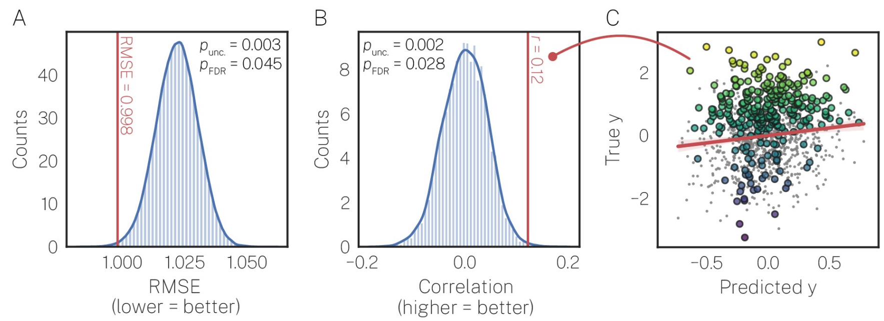
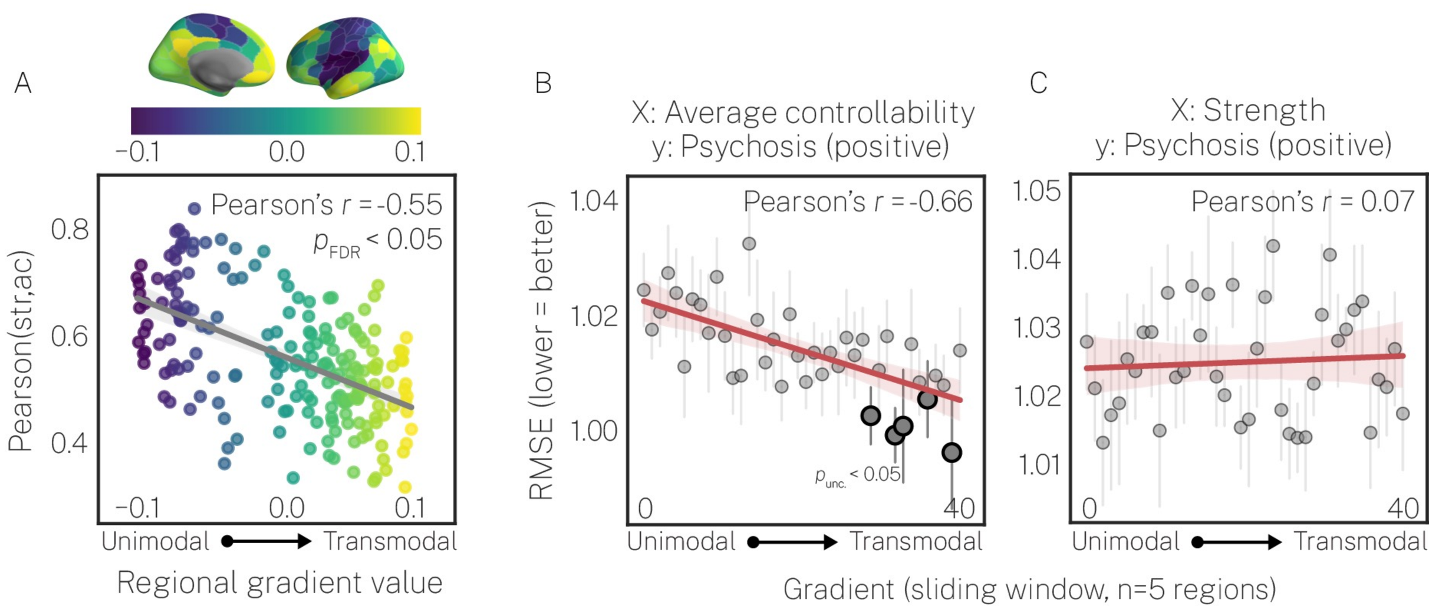
X_s
X_a
Nuisance covariates (cov)
Sex
Age
Brain volume
In-scanner motion
Symptoms (y)
Overall psychopathology
Psychosis: positive
Psychosis: negative
B
Primary prediction model
x 100
90%
10%
Train
Test
Predict
RMSE
cov
y
C
Binned-regions prediction model
Principal gradient
Transmodal
Unimodal
0.1
-0.1
n=5
RMSE
Uni
Trans
Bins

Average controllability was the best predictor of positive psychosis spectrum symptoms and was the only connectivity feature to predict beyond chance levels.





Average controllability and strength were less correlated in transmodal cortex compared to unimodal cortex, and prediction performance for average controllability improved as regions traversed up the gradient.



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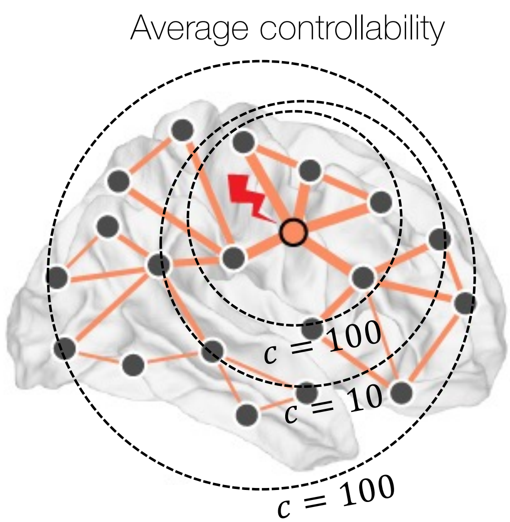
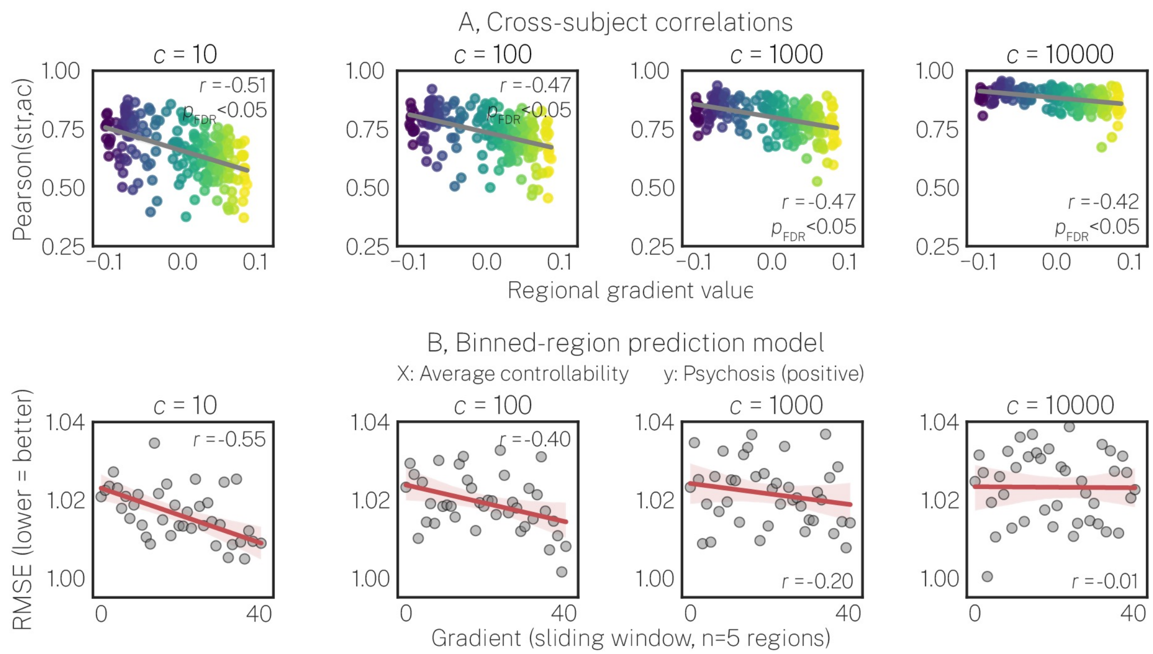




Summary

- Network controllability yields improved prediction of psychosis spectrum symptoms compared to common network measures from graph theory
- Network controllability is sensitive to transmodal regions' position along the (functional) cortical hierarchy

Python package

https://github.com/BassettLab/control_package

README.rst

network_control: A toolbox for implementing Network Control Theory analyses in python

Overview

DOI 10.5281/zenodo.4973760 · docs passing · license MIT

Network Control Theory (NCT) is a branch of physical and engineering sciences that treats a network as a dynamical system. Generally, the system is controlled through signals that originate at a control point (or control points) and move through the network. In the brain, NCT models each region's activity as a time-dependent internal state that is predicted from a combination of three factors: (i) its previous state, (ii) whole-brain structural connectivity, and (iii) external inputs. NCT enables asking a broad range of questions of a networked system that are highly relevant to network neuroscientists, such as: which regions are positioned such that they can efficiently distribute activity throughout the brain to drive changes in brain states? Do different brain regions control system dynamics in different ways? Given a set of control nodes, how can the system be driven to specific target state, or switch between a pair of states, by means of internal or external control input?

`network_control` is a Python toolbox that provides researchers with a set of tools to conduct some of the common NCT analyses reported in the literature. Below, we list select publications that serve as a primer for these tools and their use cases:

Contributors 3

- jastiso Jennifer Stiso
- lindenmp Linden Parkes
- jk6294

Languages

- Jupyter Notebook 98.0%
- Python 2.0%

The Potential of Linear Controlled Response ¶

So now we reach the final question: **how do we design the controlled response, $\mathbf{u}(t)$, that brings our system from an initial state $\mathbf{x}(0)$ to a desired target state $\mathbf{x}(T)$?** And the great thing about this question is that we already know how to do it because the controlled response is *linear*. By linear, we again mean that for some input $\mathbf{u}_1(t)$ that yields an output $\mathbf{y}_1 = \mathcal{L}(\mathbf{u}_1(t))$, and another input $\mathbf{u}_2(t)$ that yields an output $\mathbf{y}_2 = \mathcal{L}(\mathbf{u}_2(t))$, we have that

$$a\mathbf{y}_1 + b\mathbf{y}_2 = \mathcal{L}(a\mathbf{u}_1 + b\mathbf{u}_2)$$

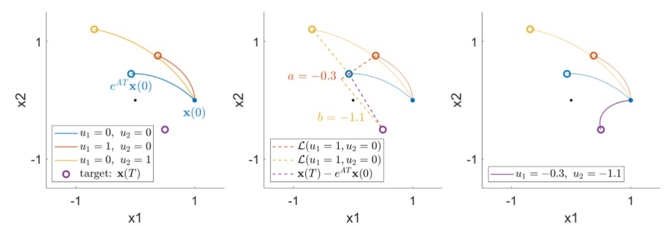
This fact comes from the fact that the convolution operator is *linear*.

a simple 2-state example

So let's try to derive some intuition with the same 2-state example as before, but now our system will have a controlled input such that

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\mathbf{u}}.$$

The natural trajectory of the system is shown as the blue curve, while the first controlled trajectory when $u_1 = 1$ is shown in the red curve, and the second controlled trajectory when $u_2 = 1$ is shown in the yellow curve (left).



Other resources

Steve Brunton: https://www.youtube.com/watch?v=Pi7l8mMjYVE

3Blue1Brown: https://www.youtube.com/c/3blue1brown/featured

1. Essence of linear algebra
2. Essence of calculus
3. Differential equations

Acknowledgments

- Dani S. Bassett
- Theodore D. Satterthwaite
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- *Jennifer Stiso*
- *Eli Cornblath*
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- Matthew Cieslak
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