SRS_CIS-520-001 2020A Final Version A

Claudia Zhu

TOTAL POINTS

73 / 75

QUESTION 1

Problem 1 18 pts

1.1 a **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.2 b **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.3 c **2 / 2**

- ✓ **- 0 pts** Correct

1.4 d **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.5 e **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.6 f **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.7 g **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.8 h **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

1.9 i **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect

QUESTION 2

Problem 2 27 pts

2.1 a **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.2 b **3 / 3**

- ✓ **- 0 pts** Correct

2.3 c **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.4 d **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.5 e **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.6 f **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.7 g **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.8 h **3 / 3**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect

2.9 i 3 / 3

✓ **- 0 pts** **Correct**

- 3 pts Incorrect

QUESTION 3

3 Problem 3 8 / 10

- 0 pts Correct

- 1 pts Everything correct, missing explicit derivation

✓ **- 2 pts** **Correct logic, correct derivation, correct h*, incorrect diagram**

- 4 pts Correct logic, partially correct derivation, correct h*, correct diagram

- 5 pts Correct logic, partially correct derivation, correct h*, missing/incorrect diagram

- 5 pts Correct logic, partially correct derivation, missing/incorrect h*, correct diagram

- 7 pts Correct logic, everything else either missing/incorrect

- 7 pts Incorrect logic and derivation, everything else correct

- 10 pts Incorrect

QUESTION 4

Problem 4 10 pts

4.1 a 2 / 2

✓ **- 0 pts** **Correct**

- 2 pts Click here to replace this description.

4.2 b 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts incorrect result for pi

- 1 pts incorrect result for lambda_a

- 1 pts incorrect results for lambda_b

- 1 pts no derivation

- 4 pts Click here to replace this description.

4.3 c 4 / 4

✓ **- 0 pts** **Correct**

- 4 pts Click here to replace this description.

- 1 pts incorrect result for pi

QUESTION 5

Problem 5 10 pts

5.1 a 2 / 2

✓ **- 0 pts** **Correct**

- 1 pts Incorrect derivation

5.2 b 3 / 3

✓ **- 0 pts** **Correct**

- 2 pts Incorrect derivation

- 1 pts Incorrect final answer

- 1 pts no derivation

5.3 c 5 / 5

✓ **- 0 pts** **Correct**

- 1 pts No derivation

- 1 pts final answer is incorrect

- 4 pts Only some derivations are correct. Logic is mostly incorrect.

- 5 pts Incorrect.

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1a) 60

1b) 165

1c) 1

1d) h_1, h_2, h_3, h_4

1e) $O(kd)$

1f) 120000

1g) X_3

1h) $\pi^*(1) = C$, $\pi^*(2) = b$, $\pi^*(3) = C$

1j) $(-1, 2)^T$

2a) 0.2

2b) -1

2c) $\ln(4)$

2d) $17/42$

2e) 259

2f) C

2g) $4/5$

2h) 19

2i) 21

1.1 a **2 / 2**

✓ **- 0 pts** Correct

- 2 pts Incorrect

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1.2 b **2 / 2**

✓ **- 0 pts** Correct

- 2 pts Incorrect

Claudia Zhu Version A 464505 85

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1.8 h **2 / 2**

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2.1 a **3 / 3**

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2.2 b 3 / 3

✓ - 0 pts Correct

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2.3 C **3 / 3**

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2.5 e **3 / 3**

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2.8 h **3 / 3**

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- 3 pts Incorrect

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2.9 i **3 / 3**

✓ **- 0 pts** Correct

- 3 pts Incorrect

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3) $er_D[h^*] = \inf_{h: X \to \{1,2,3\}} er_D[h]$

$$= \mathop{E}_{(X,Y)\sim D}\left[\ell(Y, h(X))\right]$$

$$= E_X E_{Y|X}\Big[\mathbb{1}(h(x)=2)\,\eta_1 + 2\cdot\mathbb{1}(h(x)=3)\,\eta_1 + \mathbb{1}(h(x)=1)\,\eta_2 + \mathbb{1}(h(x)=3)\,\eta_2 + 2\cdot\mathbb{1}(h(x)=1)\,\eta_3 + \mathbb{1}(h(x)=2)\,\eta_3\Big]$$

$$= E_X E_{Y|X}\Big[\mathbb{1}(h(x)=1)(2\eta_3+\eta_2) + \mathbb{1}(h(x)=2)(\eta_1+\eta_3) + \mathbb{1}(h(x)=3)(2\eta_3+\eta_2)\Big]$$

$$= E\left[\min(2\eta_3+\eta_2,\ \eta_1+\eta_3,\ 2\eta_1+\eta_2)\right] \leftarrow \text{Minimize this}$$

Choose $h(x)=1$ when $2\eta_3+\eta_2 < \eta_1+\eta_3$, $2\eta_3+\eta_2 < 2\eta_1+\eta_2$

Choose $h(x)=2$ when $\eta_1+\eta_3 < 2\eta_3+\eta_2$, $\eta_1+\eta_3 < 2\eta_1+\eta_2$

Choose $h(x)=3$ when $2\eta_1+\eta_2 < 2\eta_3+\eta_2$, $2\eta_1+\eta_2 < \eta_1+\eta_3$

3 Problem 3 **8 / 10**

- **- 0 pts** Correct
- **- 1 pts** Everything correct, missing explicit derivation
- ✓ **- 2 pts Correct logic, correct derivation, correct h*, incorrect diagram**
- **- 4 pts** Correct logic, partially correct derivation, correct h*, correct diagram
- **- 5 pts** Correct logic, partially correct derivation, correct h*, missing/incorrect diagram
- **- 5 pts** Correct logic, partially correct derivation, missing/incorrect h*, correct diagram
- **- 7 pts** Correct logic, everything else either missing/incorrect
- **- 7 pts** Incorrect logic and derivation, everything else correct
- **- 10 pts** Incorrect

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4a) $\ln \mathcal{L}_c(\theta) = \sum_{i=1}^{m} \ln(p(x_i, z_i, \theta))$

$$= \sum_{i=1}^{m} \ln\left(\left(\pi \frac{\lambda_A^{x_i} e^{-\lambda_A}}{x_i!}\right)^{z_i} \left((1-\pi) \frac{\lambda_B^{x_i} e^{-\lambda_B}}{x_i!}\right)^{1-z_i} \right)$$

$$= \sum_{i=1}^{m} \left(\ln\left(\left(\pi \frac{\lambda_A^{x_i} e^{-\lambda_A}}{x_i!}\right)^{z_i}\right) + \ln\left(\left((1-\pi) \frac{\lambda_B^{x_i} e^{-\lambda_B}}{x_i!}\right)^{1-z_i}\right) \right)$$

$$= \sum_{i=1}^{m} \Big(z_i \ln(\pi) + x_i z_i \ln(\lambda_A) + (-z_i)\lambda_A \cancelto{1}{\ln(e)} - z_i \ln(x_i!)$$
$$+ (1-z_i)\ln(1-\pi) + (1-z_i)x_i \ln(\lambda_B) - (1-z_i)\lambda_B \cancelto{1}{\ln(e)}$$
$$- (1-z_i)\ln(x_i!) \Big)$$

4.1 a **2 / 2**

✓ **- 0 pts** Correct

- 2 pts Click here to replace this description.

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4b) $\hat{\theta}$ ← to find, we will take partials wrt π, λ_A, λ_B

Solve for $\hat{\pi}$: $$\frac{\partial \ln \mathcal{L}_c}{\partial \pi} = \frac{\partial}{\partial \pi}\left(\sum_{i=1}^{m} z_i \ln(\pi) + (1-z_i)\ln(1-\pi)\right)$$

$$\Rightarrow \left[\sum_{i=1}^{m}\left(\frac{z_i}{\pi} - \frac{1-z_i}{1-\pi}\right) = 0\right]$$ Solve for this.

$$\Rightarrow \sum_{i=1}^{m} \frac{z_i}{\pi} = \sum_{i=1}^{m} \frac{1-z_i}{1-\pi}$$

$$\Rightarrow \frac{1}{\pi}\sum^{m} z_i = \frac{1}{1-\pi}\sum 1-z_i$$

$$\Rightarrow (1-\pi)\sum^{m} z_i = \pi \sum^{m} 1-z_i$$

$$\Rightarrow \sum^{m} z_i - \cancel{\pi\sum^{m} z_i} = m\pi - \cancel{\pi\sum^{m} z_i}$$

$$\Rightarrow \sum^{m} z_i = m\pi \quad \therefore \boxed{\hat{\pi} = \frac{1}{m}\sum_{i=1}^{m} z_i}$$

2nd Deriv test:

$$\frac{\partial^2 \ln \mathcal{L}_c}{\partial \pi^2} = -\sum_{i=1}^{m} \frac{z_i}{\pi^2} + \frac{1-z_i}{(1-\pi)^2}$$

z_i always non-negative, so $\frac{\partial^2 \ln \mathcal{L}_c}{\partial \pi^2}$ always negative

$\Rightarrow \hat{\pi}$ is maximum

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4b)

$$* \frac{\partial^2 \ln L_c}{\partial \lambda_A^2} = -\sum_{i=1}^{m} \frac{x_i z_i}{\lambda_A^2}$$

$\lambda_A, x_i, z_i \geq 0$

$\Rightarrow$ 2nd deriv always neg.

$\Rightarrow \hat{\lambda}_A$ is max.

Solve for $\hat{\lambda}_A$:

$$\frac{\partial \ln L_c}{\partial \lambda_A} = \frac{\partial}{\partial \lambda_A}\left[\sum_{i=1}^{m} x_i z_i \ln(\lambda_A) - z_i \lambda_A\right]$$

$$\Rightarrow \left[\sum_{i=1}^{m}\left(\frac{x_i z_i}{\lambda_A} - z_i\right) = 0\right]$$ solve this

$$\Rightarrow \sum^{m} \frac{x_i z_i}{\lambda_A} = \sum^{m} z_i \Rightarrow \frac{1}{\lambda_A}\sum x_i z_i = \sum z_i$$

$$\Rightarrow \lambda_A = \frac{\sum x_i z_i}{\sum z_i} \therefore \boxed{\hat{\lambda}_A = \frac{\sum_{i=1}^{m} x_i z_i}{\sum_{i=1}^{m} z_i}}$$

Solve for $\hat{\lambda}_B$:

$$\frac{\partial \ln L_c}{\partial \lambda_B} = \frac{\partial}{\partial \lambda_B}\left[\sum_{i=1}^{m} x_i(1-z_i)\lambda_B - (1-z_i)\lambda_B\right]$$

$$\Rightarrow \left[\sum_{i=1}^{m}\left(\frac{x_i(1-z_i)}{\lambda_B} - (1-z_i)\right) = 0\right]$$ Solve.

$$\Rightarrow \boxed{\hat{\lambda}_B = \frac{\sum_{i=1}^{m} x_i(1-z_i)}{\sum_{i=1}^{m}(1-z_i)}}$$

$$\frac{\partial^2 \ln L_c}{\partial \lambda_B^2} = -\sum_{i=1}^{m}\frac{(x_i(1-z_i))}{\lambda_B^2}$$

$x_i, (1-z_i), \lambda_B^2 \geq 0$

$$\therefore \frac{\partial^2 \ln L_c}{\partial \lambda_B^2} \leq 0$$

$\Rightarrow \hat{\lambda}_B$ is local max.

4.2 b 4 / 4

- ✓ **- 0 pts** Correct
- **- 1 pts** incorrect result for pi
- **- 1 pts** incorrect result for lambda_a
- **- 1 pts** incorrect results for lambda_b
- **- 1 pts** no derivation
- **- 4 pts** Click here to replace this description.

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4c) Sorry, I ran out of time, but we go through the steps in part b using γ_i^t instead of z_i.

↑ As per EM Algorithm.

So we obtain that

$$\pi^{t+1} = \frac{1}{m}\sum_{i=1}^{m} \gamma_i^t$$

$$\lambda_A^{t+1} = \frac{\sum_{i=1}^{n} \gamma_i^t x_i}{\sum_{i=1}^{n} \gamma_i^t}$$

$$\lambda_B^{t+1} = \frac{\sum_{i=1}^{n} (1-\gamma_i^t) x_i}{\sum_{i=1}^{n} (-\gamma_i^t + 1)}$$

4.3 C **4 / 4**

- ✓ **- 0 pts** Correct
- **- 4 pts** Click here to replace this description.
- **- 1 pts** incorrect result for pi

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5a) Let S be sequence $(\overset{s_1}{\text{Calm}}, \overset{s_2}{\text{excited}}, \overset{s_3}{\text{Calm}})$ we want to find

$$P(S) = P(s_1 = \text{calm}, s_2 = \text{excited}, s_3 = \text{calm})$$

$$= P(s_1 = \text{calm})\, P(s_2 = \text{excited} \mid s_1 = \text{calm})\, P(s_3 = \text{calm} \mid s_2 = \text{excited})$$

$$= \pi_C \times A_{CE} \times A_{EC} = 0.7 \times 0.4 \times 0.2$$

$$\boxed{= 0.056}$$

5b) Let s be initial state, o be initial observation

$$P(s = \text{calm} \mid O = \text{Swim}) = \frac{P(O = \text{swim} \mid s = \text{calm})\, P(s = \text{calm})}{P(o = \text{swim})} \quad \text{By Bayes}$$

$$= \frac{P(o = \text{swim} \mid s = \text{calm})\, P(s = \text{calm})}{P(o = \text{swim} \mid s = \text{calm})\, P(s = \text{calm}) + P(o = \text{swim} \mid s = \text{excited})\, P(s = \text{excited})}$$

$$= \frac{\phi_{CS}\, \pi_C}{\phi_{CS}\, \pi_C + \phi_{ES}\, \pi_E} = \frac{0.1 \times 0.7}{0.7 \times 0.1 + 0.5 \times 0.3} = \frac{0.07}{0.22} = \frac{7}{22} = \boxed{0.318}$$

5c) Let S be sequence (rest, swim). We want to find

$$P(O) = P(O, Q_1) + P(O, Q_2) + P(O, Q_3) + P(O, Q_4)$$

$Q_1 = (\text{calm}, \text{calm})$ $Q_2 = (\text{calm}, \text{excited})$ $Q_3 = (\text{excited}, \text{calm})$ $Q_4 = (\text{excited}, \text{excited})$

$$P(O, Q_i) = p(q_1)\, p(q_2 \mid q_1)\, p(o_1 \mid q_1)\, p(o_2 \mid q_2) \quad \Big|\ (o_1, o_2) = O \quad (q_1, q_2) = Q_i$$

$$P(O, Q_1) = \pi_C + \phi_{CR} + A_{CC} + \phi_{CS} = 0.7 \times 0.3 \times 0.6 \times 0.1$$

$$P(O, Q_2) = \pi_C + \phi_{CR} + A_{CE} + \phi_{ES} = 0.7 \times 0.3 \times 0.4 + 0.5$$

$$P(O, Q_3) = \pi_E + \phi_{ER} + A_{EC} + \phi_{CS} = 0.3 \times 0.1 \times 0.2 \times 0.1$$

$$P(O, Q_4) = \pi_E + \phi_{ER} + A_{EE} + \phi_{ES} = 0.3 \times 0.1 \times 0.8 \times 0.5$$

$$\boxed{P(O) = 0.0672}$$

5.1 a **2 / 2**

✓ **- 0 pts** Correct

- 1 pts Incorrect derivation

Claudia Zhu Version A ID: 46450585

5a) Let S be sequence $(\overset{s_1}{\text{Calm}}, \overset{s_2}{\text{excited}}, \overset{s_3}{\text{Calm}})$ we want to find

$$P(S) = P(s_1 = \text{calm}, s_2 = \text{excited}, s_3 = \text{calm})$$

$$= P(s_1 = \text{calm})\, P(s_2 = \text{excited} \mid s_1 = \text{calm})\, P(s_3 = \text{calm} \mid s_2 = \text{excited})$$

$$= \pi_C \times A_{CE} \times A_{EC} = 0.7 \times 0.4 \times 0.2$$

$$\boxed{= 0.056}$$

5b) Let s be initial state, o be initial observation

$$P(s = \text{calm} \mid O = \text{Swim}) = \frac{P(O = \text{swim} \mid s = \text{calm})\, P(s = \text{calm})}{P(o = \text{swim})}$$ By Bayes

$$= \frac{P(o = \text{swim} \mid s = \text{calm})\, P(s = \text{calm})}{P(o = \text{swim} \mid s = \text{calm})\, P(s = \text{calm}) + P(o = \text{swim} \mid s = \text{excited})\, P(s = \text{excited})}$$

$$= \frac{\phi_{CS}\, \pi_C}{\phi_{CS}\, \pi_C + \phi_{ES}\, \pi_E} = \frac{0.1 \times 0.7}{0.7 \times 0.1 + 0.5 \times 0.3} = \frac{0.07}{0.22} = \frac{7}{22} = \boxed{0.318}$$

5c) Let S be sequence (rest, swim). We want to find

$$P(O) = P(O, Q_1) + P(O, Q_2) + P(O, Q_3) + P(O, Q_4)$$

$Q_1 = (\text{calm}, \text{calm})$ $Q_2 = (\text{calm}, \text{excited})$ $Q_3 = (\text{excited}, \text{calm})$ $Q_4 = (\text{excited}, \text{excited})$

$$P(O, Q_i) = p(q_1)\, p(q_2 \mid q_1)\, p(o_1 \mid q_1)\, p(o_2 \mid q_2) \quad \Big| \; (o_1, o_2) = O \quad (q_1, q_2) = Q_i$$

$$P(O, Q_1) = \pi_C + \phi_{CR} + A_{CC} + \phi_{CS} = 0.7 \times 0.3 \times 0.6 \times 0.1$$

$$P(O, Q_2) = \pi_C + \phi_{CR} + A_{CE} + \phi_{ES} = 0.7 \times 0.3 \times 0.4 + 0.5$$

$$P(O, Q_3) = \pi_E + \phi_{ER} + A_{EC} + \phi_{CS} = 0.3 \times 0.1 \times 0.2 \times 0.1$$

$$P(O, Q_4) = \pi_E + \phi_{ER} + A_{EE} + \phi_{ES} = 0.3 \times 0.1 \times 0.8 \times 0.5$$

$$\boxed{P(O) = 0.0672}$$

5.2 b **3 / 3**

- ✓ **- 0 pts** Correct
- **- 2 pts** Incorrect derivation
- **- 1 pts** Incorrect final answer
- **- 1 pts** no derivation

Claudia Zhu Version A ID: 46450585

5a) Let S be sequence $(\overset{s_1}{\text{Calm}}, \overset{s_2}{\text{excited}}, \overset{s_3}{\text{Calm}})$ we want to find

$$P(S) = P(s_1 = \text{calm}, s_2 = \text{excited}, s_3 = \text{calm})$$

$$= P(s_1 = \text{calm})\, P(s_2 = \text{excited} \mid s_1 = \text{calm})\, P(s_3 = \text{calm} \mid s_2 = \text{excited})$$

$$= \pi_C \times A_{CE} \times A_{EC} = 0.7 \times 0.4 \times 0.2$$

$$\boxed{= 0.056}$$

5b) Let s be initial state, o be initial observation

$$P(s = \text{calm} \mid O = \text{Swim}) = \frac{P(O = \text{swim} \mid s = \text{calm})\, P(s = \text{calm})}{P(o = \text{swim})} \quad \text{By Bayes}$$

$$= \frac{P(o = \text{swim} \mid s = \text{calm})\, P(s = \text{calm})}{P(o = \text{swim} \mid s = \text{calm})\, P(s = \text{calm}) + P(o = \text{swim} \mid s = \text{excited})\, P(s = \text{excited})}$$

$$= \frac{\phi_{CS}\, \pi_C}{\phi_{CS}\, \pi_C + \phi_{ES}\, \pi_E} = \frac{0.1 \times 0.7}{0.7 \times 0.1 + 0.5 \times 0.3} = \frac{0.07}{0.22} = \frac{7}{22} = \boxed{0.318}$$

5c) Let S be sequence (rest, swim). We want to find

$$P(O) = P(O, Q_1) + P(O, Q_2) + P(O, Q_3) + P(O, Q_4)$$

$Q_1 = (\text{calm}, \text{calm})$ $Q_2 = (\text{Calm}, \text{excited})$ $Q_3 = (\text{excited}, \text{calm})$ $Q_4 = (\text{excited}, \text{excited})$

$P(O, Q_i) = p(q_1)\, p(q_2 \mid q_1)\, p(o_1 \mid q_1)\, p(o_2 \mid q_2)$ $\mid (o_1, o_2) = O$ $(q_1, q_2) = Q_i$

$$P(O, Q_1) = \pi_C + \phi_{CR} + A_{CC} + \phi_{CS} = 0.7 \times 0.3 \times 0.6 \times 0.1$$

$$P(O, Q_2) = \pi_C + \phi_{CR} + A_{CE} + \phi_{ES} = 0.7 \times 0.3 \times 0.4 + 0.5$$

$$P(O, Q_3) = \pi_E + \phi_{ER} + A_{EC} + \phi_{CS} = 0.3 \times 0.1 \times 0.2 \times 0.1$$

$$P(O, Q_4) = \pi_E + \phi_{ER} + A_{EE} + \phi_{ES} = 0.3 \times 0.1 \times 0.8 \times 0.5$$

$$\boxed{P(O) = 0.0672}$$

5.3 C **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** No derivation
- **- 1 pts** final answer is incorrect
- **- 4 pts** Only some derivations are correct. Logic is mostly incorrect.
- **- 5 pts** Incorrect.