Penn Wharton Budget Model: Dynamics

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Abstract

The federal budget is one of the most important documents produced by our government and provides a roadmap for our nation's priorities. However, there is currently a limited opportunity for policymakers to understand how a policy change will impact our nation's economy and budget while they are writing legislation. The Penn Wharton Budget Model (PWBM) seeks to inform policy discussions before bills are finalized and before lobbying and reputational stakes are put into the ground. The PWBM combines modern advances in economic modeling, big data science, cloud computing and visualization tools to provide a "sandbox" in which different policy ideas can be tested before legislation is drafted. This document describes the current version of the dynamic general equilibrium overlapping generations model used by PWBM to analyze different policies.

1 Model

1.1 Households

The economy is populated by a continuum of ex-ante households. A household is characterized by her age (j), assets (a), labor productivity (z), and average lifetime labor earnings (b_j). Let $s = (j, a, z, b_j)$ denote a household type and $\mathbf{\Phi}(s)$ denote the measure of households of type s in the economy.

Labor productivity z has four components:

$$z = z_{age} + z_{perm} + z_{trans} + z_{pers}. \tag{1}$$

When households enter the economy, they draw a permanent component of their labor productivity, z_{perm}. With probability p_{permH}, a household has a high permanent component, and with probability $(1 - p_{permH})$, a low one. There is a deterministic life cycle component of labor productivity, z_{age}, that varies with age. And, finally, there are two idiosyncratic shocks on households' labor productivity received each period: $z_{trans} \sim N(0, \sigma^2_{trans})$ is a transitory shock and z_{pers} is a persistent shock, which follows a first-order autoregression:

$$z'_{pers} = \rho z_{pers} + \eta'; \quad \eta \sim N(0, \sigma^2_{\eta}). \tag{2}$$

In every period, households are endowed one unit of time that can be allocated to work and leisure. However, at age Tr, households are forced to retire and start receiving social security benefits, $ss(b)$, that depend on their average lifetime labor earnings, b. Labor supply, n, is a continuous choice variable. In addition to leisure, households also derive utility from consumption c. The period-by-period return function is given by:

$$U(c, n) = \frac{\left(c^{\gamma} \left(1 - n\right)^{1-\gamma}\right)^{1-\sigma}}{1 - \sigma}. \tag{3}$$

At each period households aged j survive to next period with probability s_{j+1} unless $j = J$, in which case they die with probability 1. In the event of death, accidental bequests are collected by the government and uniformly distributed among the living population by means of lump-sum transfers, beq. The fraction of the total population attributable to each age cohort is fixed over time and the population grows at constant rate (for now, at least).

Households can accumulate positive assets. A unit of asset a is a portfolio that combines a share ϕ_K of physical capital and a share $(1 - \phi_K)$ of government debt. Households take ϕ_K as exogenous. This assumption is required in order to generate a positive demand for both types of assets while imposing a spread between their return rates. The sequence of government interest rates, r_G, is exogenous to the model, based on projections by the CBO. Hence, for a given share ϕ_K, the return rate on the portfolio is a weighted average of return on physical capital and government interest rate:

$$R = \phi_K \tilde{r}_K + (1 - \phi_k) r_G, \tag{4}$$

where $\tilde{r}_K$ is physical capital after tax return, including capital gains and losses. But before understanding how $\tilde{r_K}$ is determined, one must learn about capital income taxation in this economy.

Following Auerbach and Kotlikoff (1982) and Altig *et al.* (2001), let τ_{cap} denote the tax rate on physical capital income and e be the rate of investment expensing. Normalizing output price to 1, equation (5) expresses Tobin's q:

$$q = 1 - e\tau_{cap}. \tag{5}$$

For new capital the net acquisition cost is 1, the price of new capital, less the tax rebate from expensing $e\tau_{cap}$. And since old and new capital are perfect substitutes in production, their net acquisition costs must be identical in equilibrium; hence, old capital sells for $e\tau_{cap}$ less than new capital because the purchaser of new capital receives $e\tau_{cap}$ from the government, while the purchaser of old capital receives no tax rebate.

Let r_K denote the gross rental rate of physical capital. Net return to physical capital is given by:

$$\frac{r_K - 1}{q}, \tag{6}$$

and net return to physical capital accounting for capital gains and losses is given by:

$$\tilde{r}_K = \frac{r_K - 1}{q} + \frac{q' - q}{q}. \tag{7}$$

We can then rewrite the return rate on the portfolio of equation (4) as:

$$R = \phi_K \left(\frac{r_K - 1}{q} + \frac{q' - q}{q} \right) + (1 - \phi_k) r_G. \tag{8}$$

Finally, the amount paid on capital income taxes, $\tau_{cap}(y_{cap})$, depends on capital income, y_{cap}, as follows:

$$\tau_{cap}(y_{cap}) = \tau_{cap} y_{cap}. \tag{9}$$

We define capital income as:

$$y_{cap} = \theta \left(\frac{r_K - 1}{q} \right) \phi_K a, \tag{10}$$

where θ is the share of capital income subjected to the capital tax rate. The remaining $(1-\theta)$ share is subjected to the personal income tax rate.

In addition to capital income taxes, there are two other taxes that households must pay the government. Payroll (Social Security) tax on labor income is defined as:

$$\tau_{ss}\left(y_{ss}\right) = \tau_{ss} y_{ss}, \tag{11}$$

where y_{ss} is defined as

$$y_{ss} = \min\{wzn, y_{\text{taxmax}}\}, \tag{12}$$

and y_{taxmax} is the maximum labor income subject to payroll taxation.

Personal income tax, $\tau_{pit}(y_{pit})$, depends on personal income, y_{pit}, as follows:

$$\tau_{pit}\left(y_{pit}\right) = \int_0^{y_{pit}} \xi\left(y\right) dy, \tag{13}$$

where the tax function $\tau_{pit}\left(y_{pit}\right)$ is a cumulative tax liability from marginal tax rate function $\xi\left(y\right)$. The function $\xi\left(y\right)$ is a step-function extracted from the Tax Policy Center (TPC) which accounts for deductions and credits. Personal income is defined for each household age group in the next subsections.

1.1.1 Working-age Households

Recall that a household's type is given by $s = (j, a, z, b_j)$, where j denotes age, a denotes assets, z denotes labor productivity, and b_j denotes average lifetime labor earnings. Let $V^w(a, z, b)$ denote the value of type $s = (j, a, z, b_j)$ working-age households. The working-age household Bellman's equation is given by:

$$V^w(a, z, b_j) = \max_{c,a',n} \left\{U(c, n) + s_{j+1}\beta E_{\{z'|z\}}\left[V^w(a', z', b_{j+1})\right]\right\}$$

$$\text{s.t.} \quad c = wzn + (1+R)a - \tau_{pit}(y_{pit}) - \tau_{cap}(y_{cap}) - \tau_{ss}(y_{ss}) - a' + beq \tag{14}$$

$$b_{j+1} = \frac{1}{j}\Big((j-1)b_j + \min\{wzn, y_{\text{taxmax}}\}\Big) \tag{15}$$

$$c, a', n \geq 0, \tag{16}$$

where (14) is the budget constraint, (15) determines average earnings for Social Security benefit calculation, and (16) is the standard non-negativity constraints.

Personal income for a working-age household is defined as:

$$y_{pit} = wzn + \left(\phi_K \frac{r_K - 1}{q}(1-\theta) + (1-\phi_K)(r_G - 1)\right) a. \tag{17}$$

Notice that

$$y_{pit} \neq wzn + (1+R)\,a, \tag{18}$$

because

$$\begin{aligned}(1+R) &= \left(\phi_K\left(1+\frac{r_K-1}{q}+\frac{q'-q}{q}\right)+(1-\phi_k)r_G\right)\\ &\neq \left(\phi_K\frac{r_K-1}{q}(1-\theta)+(1-\phi_K)(r_G-1)\right).\end{aligned} \tag{19}$$

In particular, notice that only a fraction $(1-\theta)$ of the physical capital income is subjected to personal income tax. The remaining fraction is subjected to capital tax rate. Also, capital gains and losses due to changes in the price of capital are not considered capital income in our specification.

1.1.2 Retired Households

Let $V^r(a,z,b)$ denote the value of type $s=(a,z,b)$ retired households. The retired household Bellman's equation is given by:

$$V^r(a,z,b_j) = \max_{c,a'} \left\{U(c,0)+s_{j+1}\beta\left[V^r(a',z',b_{j+1})\right]\right\}$$

$$\text{s.t.}\ \ c = ss(b)+(1+R)a-\tau_{pit}(y_{pit})-\tau_{cap}(y_{cap})-a'+beq \tag{20}$$

$$b_{j+1} = b_j \tag{21}$$

$$c,\ a' \geq 0. \tag{22}$$

where (20) is the budget constraint, (21) determines average earnings for Social Security benefit calculation, and (22) is the standard non-negativity constraints.

Personal income for a retired household is defined as:

$$y_{pit} = (1-\phi_{ss})ss(b)+\left(\phi_K\frac{r_K-1}{q}(1-\theta)+(1-\phi_K)(r_G-1)\right)a, \tag{23}$$

where ϕ_{ss} is the fraction of Social Security benefits deductible from federal income taxation. Again, notice that only a fraction $(1-\theta)$ of the physical capital income is subjected to personal income tax and that capital gains and losses due to changes in the price of capital are not considered capital income in our specification.

1.2 Production

There is a representative firm that demands labor services and rents physical capital. The firm also bears the depreciation costs of capital. Total physical capital in the economy, K, equals aggregate savings, A, less government debt, D, in capital terms (as opposed to dollar units because the variables in the model are denominated in dollars):

$$K = \frac{A - D}{q}. \tag{24}$$

The problem of the representative firm is:

$$\max_{K,L} \Big\{ K^{\alpha} L^{1-\alpha} - (\delta + r_K) K - wL \Big\}, \tag{25}$$

where δ is capital depreciation rate and r_K is the rental rate of capital faced by firms (or the gross rental rate of physical capital). Capital and labor demand are determined according to FOC:

$$r_K = \alpha K^{\alpha-1} L^{1-\alpha} - \delta \tag{26}$$

$$w = (1 - \alpha) K^{\alpha} L^{-\alpha}. \tag{27}$$

1.2.1 Closed Economy

In the closed economy model, the rental rate of capital r_K is determined in equilibrium.

1.2.2 Open Economy

In the open economy model, there is trade in international capital flows, which implies that foreigners hold some of the total physical capital of the economy, that is:

$$K = K_d + K_f, \tag{28}$$

where K_d denotes physical capital held by domestic households and K_f denotes physical capital held by foreigners. Notice that $K_f > 0$ implies foreigners hold domestic capital, while $K_f < 0$ implies domestic households hold foreign capital abroad in net terms.

Given the territorial capital tax assumption, capital tax on the fraction θ of total assets in the economy also applies to foreign capital. However foreigners' personal income cannot be taxed, in other words, a fraction $(1 - \theta)$ of K_f income is not taxed in the US.

We assume that in the open economy model the rental rate of capital r_K is such that the after-tax net capital return equals the international after-tax net capital return, λ, that is:

$$r_K (1 - \tau_{cap}) = \lambda. \tag{29}$$

Rearranging terms in (29) gives:

$$r_K = \frac{\lambda}{(1 - \tau_{cap})}. \tag{30}$$

Under the small open economy assumption, λ is set to a constant by the international capital market. Assuming a time invariant τ_{cap}, equations (26) and (30) imply a unique capital-to-labor ratio $\kappa = \frac{K}{L}$ given in the firm's FOC:

$$r_K = \alpha \kappa^{\alpha-1} - \delta, \tag{31}$$

Then, equations (30) and (31) give:

$$\kappa = \left[\frac{1}{\alpha} \left(\frac{\lambda}{1 - \tau_{cap}} + \delta \right) \right]^{\frac{1}{\alpha-1}}. \tag{32}$$

Hence, although labor is immobile in our open economy model, (27) and (32) imply fixed wages:

$$w = (1 - \alpha)\kappa^{\alpha}. \tag{33}$$

1.3 Government

Government issues debt, D, that pays a return r_G. Government debt evolves according to:

$$D' + R = r_G D + G, \tag{34}$$

where R is government revenue and G is government expenditures.

Government expenditures have two components:

$$G = SSEXP + \tilde{G}, \tag{35}$$

where $SSEXP$ denotes Social Security expenditures and $\tilde{G}$ is an exogenous variable that denotes the non-interest government budget surplus not accounted by the explicit model revenue and expenditure components. In the model, Social Security expenditures are given by:

$$SSEXP = \int ss(b_j)(s)\Phi(\{Tr, \ldots, J\} \times da \times dz \times db_j). \tag{36}$$

Government revenue is composed of all tax revenues:

$$R = PIT + CT + SSREV, \tag{37}$$

where PIT denotes personal income taxes, CT denotes capital taxes, and $SSEXP$ denotes payroll taxes. Those tax revenues are respectively defined as:

$$\begin{aligned}
PIT &= \int \tau_{pit} y_{pit}(s)\Phi(\{1,\ldots,J\} \times da \times dz \times db_j) \\
CT &= \int \tau_{cap} y_{cap}(s)\Phi(\{1,\ldots,J\} \times da \times dz \times db_j) \\
&= \tau_{cap}\phi_K \left(\frac{r_K - 1}{q}\right)\theta K \qquad (38) \\
SSREV &= \int \tau_{ss} y_{ss}(s)\Phi(\{1,\ldots,Tr\} \times da \times dz \times db_j). \qquad (39)
\end{aligned}$$

In some counter-factual experiments, we include an exogenous component $\tilde{T}$ to denote a non-explicit tax revenue. If not explicitly mentioned, this component is absent.

2 Calibration

This section describes how we map the model to the data. We start by presenting the demographic data and the idiosyncratic earnings process used in this paper. Next, we present calibration of the preference and technology parameters. Then we discuss the government parameters, that is, taxes, debt, government debt interest rate, $\tilde{G}$, and Social Security structure. And lastly, we summarize all the parameters in tables with their corresponding sources and/or targets.

2.1 Demographic Parameters

One period in our model is associated with 1 year of calendar time. Households enter the model at age 21. We set T_r to 47 so that households retire at age 67. The maximum life span is set to 100 years.

Our demographic parameters come from the output of our microsim model. Mortality rates are chosen to match those of the U.S. population in 2016. We set annual birth rate of national population to 1.9% to match the growth rate of the 21-year-olds cohort in 2016. Lastly, we use the distribution of immigrants by age and the annual growth rates of legal and illegal immigration to build the measures of immigrants by age. Legal immigration rate equals 0.0016, while the illegal one equals 0.0024.

2.2 Labor Earnings Process

Our labor productivity structure mimics Storesletten *et al.* (2004) labor earnings process. We use numbers from Barro and Barnes (2016) to estimate the deterministic lifecycle labor productivity profiles, z_{age}. In our estimates, the peak occurs at age 51, roughly 30% higher than at age 21. For the idiosyncratic shock process we use some point estimates from Storesletten *et al.* (2004) and recalibrate others. Specifically, we use their numbers for the permanent and persistent variances, $\sigma^2_{perm} = 0.2105$, $\sigma^2_{trans} = 0.0630$, respectively. We apply Tauchen's discretization method and obtain a first order markov process with realizations $\{\pm 0.3661\}$ for the permanent shock. For the transitory shocks, we use an i.i.d. two-state Markov chain, with realizations $\{\pm 0.2003\}$. Innovations to the persistent component are assumed to be i.i.d. with realizations $\{\pm 0.4638\}$ derived from a Tauchen discretization process where $\sigma_\eta = 0.018$ and the persistence parameter is set to 0.990.

2.3 Preference and Technology Parameters

Preferences parameters are jointly calibrated within the model. In particular, we target a capital-output ratio equal to 3, a Frisch labor supply elasticity equal to 0.5, and an elasticity of intertemporal substitution equal to 0.5 to determine discount factor (β), consumption share in intratemporal utility (γ), and risk aversion parameter (σ). Also, we set model dollars so that the value of GDP per adult is 79,800 dollars, as observed by our microsim model.

We normalize total factor productivity to one. Capital's share of income is set to 0.34 get source from Alex). Finally, we consider two rates of depreciation, 0.056 and 0.08. We set the depreciation rate to 0.056 because that yields a depreciated capital-output ratio of 25.5%, as observed in reference or database. However, in acknowledging that our model does not account for aggregate uncertainty, we experiment with the higher 0.08 depreciation rate as an *ad hoc* way to reduce the capital return rate to a level closer to the risk-free rate. Hence, we have two distinct economies, a high return to capital economy and a low return to capital one.

2.4 Government Parameters

We use Tax Policy Center's data for setting our model's tax structure. For each tax plan proposal currently under dicussion, there are multiple combinations of changes in the taxes considered in our model. Each combination enables us to identify the effect of a particular set of change in taxes. TPC provides data for all of these tax plans, from which we generate as many sets of parameters as combinations of tax changes. We will then feed our model with parameters for a given combiantion in order to evaluate the consequences of that specific combination.

Capital share in total assets, ϕ_K, is determined by the difference between the endogenous model object asset holdings and the exogeneous value of government debt. The parameters related to capital income tax are the capital tax rate, τ_{cap}, the capital tax share, θ, and the expensing share, e. In our baseline, we set τ_{cap} to 18.6%, θ to 100%, and e to 58%.

Payroll (Social Security) tax τ_{ss} is set to 12.4% and it falls upon a maximum annual labor income of 118,500 dollars. Given an average lifetime labor earning b, we set Social Security benefits as follows. First, we identify in which earnings bracket b falls in. There are 3 monthly earnings brackets: below 856 dollars, in between 856 and 5157 dollars, and above 5157 dollars. We then use bracket-specific discounting constants and replacement rates as follows:

$$ss(b) = (b - \text{discounting}_b) * \text{replacement}_b, \tag{40}$$

where

$$\text{discounting} = [0,\ 856,\ 5157]\,, \text{ and} \tag{41}$$

$$\text{replacement} = [0.9,\ 0.32,\ 0.15]\,. \tag{42}$$

Personal income taxation has a cumulative tax liability structure with 14 income brackets, each with a corresponding tax rate and tax burden. For instance, if a household's personal income falls within the tenth bracket, her personal income tax equals the sum of the tax burden of the first ninth brackets plus the product between the marginal rate for bracket number ten and her income subtracted by the income level starting bracket number ten. We set the fraction of deductible Social Security earnings, ϕ_{ss} to 0.85.

We get series for government debt interest rate, government expenditures, government revenue, and debt from the CBO. We construct the exogenous non-interest government budget surplus $\tilde{G}$ by multiplying the total tax revenue net of federal government interest surplus as a percentage of GDP by GDP and then subtracting Social Security benefits. Similarly, we compute $\tilde{T}$ by multiplying the total tax revenue as a percentage of GDP by GDP and subtracting the revenue sources already accounted for by the model, that is, personal income taxes, Social Security taxes, and capital income taxes.

2.5 Exogenously Determined Parameters

Table 1: Exogenous Parameters

Parameter	Description	Values	Source
Demographics			
`T_life`	Maximum life span (100 years)	80	
`T_work`	Retirement age (67 years)	47	
`birth_rate`	Annual birth rate	0.0200	
`legal_rate`	Annual legal immigration rate	0.0016	
`illegal_rate`	Annual illegal immigration rate	0.0024	
Labor productivity			
p_{permH}	High permanent prod. probability	0.500	
σ_{perm}	Permanent productivity variance	0.2105	Storesletten *et al.*
σ_{trans}	Transitory productivity variance	0.063	Storesletten *et al.*
σ_{pers}	Persistent productivity variance	0.018	Storesletten *et al.*
`pep`	Persistent prod. autocorrelation	0.990	Storesletten *et al.*
Output			
`A`	Total factor productivity	1	Normalization
`alpha`	Capital share of output	0.45	Paper/reference
`d`	Depreciation rate	0.056	
Capital income tax			
`captaxshare`	Capital tax share	1	TPC
`expshare`	Expensing share	0.58	TPC
`taucap`	Capital tax rate	0.186	TPC
Social Security tax			
`ssthresholds`	SS thresholds for earnings brackets	[856, 5157]*12	
`ssrates`	SS mg benefit rates for earnings brackets	[0.9, 0.32, 0.15]	
`ss_scale`	SS benefit scaling factor	1.6	
`sstax`	SS tax rate	0.124	
`ssincmaxs`	SS maximum taxable earnings	1.185e5	
`sstaxcredit`	SS benefit tax credit percentage	0.15	
Personal income tax			
`income_thresholds`	Income brackets	14-dimensional	TPC
`tax_rates`	Income brackets tax rates	14-dimensional	TPC

Table 2: Exogenous Series

Parameter	Description	CSV File
Age-dependent demographic parameters		
`surv`	Survival probability	SIMSurvivalProbability
`imm_age`	Immigrants by age	SIMImmigrantAgeDistribution
Income-dependent taxes parameters		
`tax_rates`	Personal income marginal tax rate	TPCPIT_taxplan
`income_thresholds`	Personal income tax brackets	TPCPIT_taxplan

Note: The first capitalized letters of the csv file correspond to the data source.

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