

# Opportunistic underinvestment in debt renegotiation and capital structure\*

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This paper models debt renegotiation as a bargaining game between debtholders and shareholder-oriented management, in which management credibly threatens to run down firm assets to force concessions from the creditors. Creditors anticipate this opportunistic behavior by management, creating an upper bound on debt capacity that is less than the value of the firm. If an advantage to debt is introduced, such as favorable tax treatment, an interior optimal capital structure obtains even in the absence of realized bankruptcy costs. Our model also explains variations in debt–equity ratios and the use of certain puzzling debt covenants.

## 1. Introduction

Myers (1977) suggests that the existence of debt in the capital structure of the firm leads to an underinvestment problem because a firm with outstanding debt will have an incentive to forego future investment opportunities if the benefits accrue to bondholders rather than shareholders. Aivazian and Callen (1980) argue, on the other hand, that debt renegotiations will ensue in such cases, and efficiency will be restored. The purpose of this paper is to offer a reconciliation and synthesis of these two approaches, and to obtain new insights by explicitly modeling the strategic behavior of the agents within an equilibrium framework. We assume throughout that management acts

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strictly in the shareholders' interest in order to focus on the agency problems associated with the bondholder–shareholder conflict, and we write interchangeably about the shareholders' actions and management's actions on the shareholder's behalf. [For the classical treatment of the shareholder–management agency issue, see Jensen and Meckling (1976).]

The prospects of a firm are usually uncertain at the inception of the debt contract, and firm value at debt maturity may barely suffice to repay the promised amount. We submit that, if this happens, management will optimally use its discretion over the investment decisions of the firm to wrest concessions from the firm's creditors by threatening to sap firm value through suboptimal investment policies.

We find that when the value of the firm falls below a certain cutoff level, such opportunistic behavior becomes optimal even if the firm is solvent, and shareholders stand to lose from carrying out their threats. As long as the potential loss is 'small enough', threats will be communicated and creditors will accede to 'reasonable' demands. We suggest that, in many instances, this type of exchange underlies debt renegotiations between owners and creditors.<sup>1</sup>

Creditors rationally expect this opportunistic behavior on the part of debtors. They deduce that, when the firm increases the debt face-value while maintaining the same investment plan, there is an increased probability that the debt contract will ultimately be renegotiated to the creditor's disadvantage, which creates an upper bound on the amount of debt in the financial structure of the firm. This upper bound – or debt capacity – is *less* than the value of the firm.

It follows that, in an environment otherwise free from imperfections, capital structure does not matter as long as the debt level is less than the debt capacity; debt levels beyond it are simply unattainable. However, if an advantage to debt is introduced – tax-deductibility of interest expense plays the required role in this paper – then an *interior* optimal capital structure (positive optimal debt and equity values) ensues, in which the debt level is pushed all the way up to the debt-capacity barrier.

Some theories of capital structure [e.g., Kraus and Litzenberger (1973), Scott (1976), Kim (1978)] have suggested that bankruptcy costs prevent pure debt financing when debt is advantageous. However, to serve that purpose, *realized* bankruptcy costs must be rather large so that, coupled with an

<sup>1</sup>Two spectacular debt-renegotiation processes are currently under way in Israel. One involves the Kibootsim (ideologically-motivated communes) whose population of about 100,000 collectively owes Israeli banks an amount of close to four billion dollars. Their negotiating team effectively uses the term 'burning of repayment power' to convey an explicit threat. The other involves Coor, the Israeli Labor Union-owned conglomerate, which has \$24 million in owners' equity and which owes Israeli and American banks and bondholders about 2 *billion* dollars. Interestingly, the 'project' that Coor threatens to destroy is a government aid program designed to save it from bankruptcy.

empirically small bankruptcy probability, *expected* bankruptcy costs would still be significant. But in fact, the significance of realized bankruptcy costs has been called into question both on empirical [Warner (1977)] and on theoretical [Haugen and Senbet (1978, 1988)] grounds. In contrast, our model implies an interior capital structure even in the absence of realized bankruptcy costs.

Our model also explains variations in capital structure across industries. It is well known that firms with a large proportion of intangible assets tend to have rather low debt-to-equity ratios [Brealey and Myers (1988, p. 407)]. We suggest that the management of such a firm is in a better position to successfully renegotiate away value from its debtholders and, consequently, can borrow less in equilibrium.

A related issue is the existence of certain debt covenants that seem superfluous, because they require the firm to refrain from actions that hurt the shareholders as well as the bondholders. Our model suggests that the purpose of these covenants is to place an *ex ante* limit on management's bargaining power in debt renegotiations in order to increase the debt capacity of the firm, which in turn increases firm value because of the benefits of debt.

In section 2 we discuss the underlying assumptions of the model. Section 3 presents the model as a noncooperative game in which an entrepreneur incorporates a project and sells claims on the firm's after-tax cash flows in the form of equity and debt. When the firm falls on bad times, shareholders and bondholders renegotiate the terms of the debt to the shareholders' advantage. Section 4 presents the (unique subgame-perfect) equilibrium of the game. Section 5 discusses the empirical implications of the model for corporate debt policy, debt covenants, and the magnitude of settlements in debt renegotiations. Section 6 concludes the paper, and an appendix contains the proofs of the propositions.

## 2. Some pre-modeling issues

We assume initially that debt contracts contain no debt covenants, sinking fund provisions, or coupons. This will allow us to focus on the usefulness of these standard debt provisions in mitigating the problems that may arise from management's threats to undertake suboptimal investment policies. We also assume that the firm's creditors cannot neutralize management's threats by forcing the firm into involuntary bankruptcy under a court-appointed trustee. In fact, few firms are ever forced into involuntary bankruptcy by their creditors [White (1989)]. In the first place, as long as the firm is solvent and shareholders can meet their current debt obligations, the courts are unlikely to accede to a bankruptcy petition. Secondly, as long as the management is reasonably subtle in its threats, mismanagement on its part may be quite

difficult to prove. Thirdly, even if creditors could petition for bankruptcy or liquidation of the firm, it may not be in their interest to do so, especially if the firm's product is reputation-dependent [Titman (1984) and Jackson (1986, pp. 203–204)] or if much of the firm's growth potential resides in the firm-specific skills of the management or of major shareholders. For example, Butters et al. (1981, p. 307 – the DTI case) describe a manager-owner extorting concessions from a banker-creditor by threatening to simply walk away from his electronics start-up firm.

Even if creditors are willing and able to force the firm into bankruptcy, it by no means implies that the court will appoint a trustee. The courts have ruled consistently that management shall provide a plan of reorganization and control that process [White (1989, p. 139)]. For the courts to appoint a trustee, creditors must show *cause* which is defined as 'fraud, dishonesty, incompetence, or gross mismanagement' [Section 1104(a) of the Bankruptcy Code, Title 11, United States Codes]. Mere management indiscretion is not sufficient.

In addition, trustees have been generally known to be poor managers, either because they lack the necessary operating expertise or because they are uninterested [Baird (1986) and McCafferty and Holthus (1986)]. Rather than being the salvation of the creditors, the appointment of a trustee can be the effectuation of managements' threats.

Arguably, managers may shun opportunism on behalf of their shareholders in order to maintain a good reputation. But reputation effects only mitigate and do not eliminate post-contractual opportunistic behavior, as pointed out by Klein and Leffler (1981). Furthermore, reputation for business opportunism may not be detrimental to a manager who wants to cultivate an image of shrewdness. If the incumbent management team shuns such a stratagem, a more astute team could be hired or even take over the firm.

### 3. The model

We analyze the life cycle of the firm as a noncooperative game with complete and perfect information (and with uncertainty) played by three players: an entrepreneur, a group of bondholders, and a group of shareholders. (The separation of entrepreneur from shareholders is for ease of exposition only.) We assume that agents are risk-neutral. (Risk-aversion does not alter the chief results.) In order to provide a motive for debt in the capital structure of firms, we assume that corporate earnings are taxed at a rate  $\tau$  and interest expense is tax-deductible. There are no personal taxes.

The game, in its extensive form, is depicted in fig. 1. At time  $t = -5$ , the entrepreneur owns a project which at time  $t = 1$  returns a *random* net operating profit (earnings before interest and taxes) of  $P$  dollars, provided

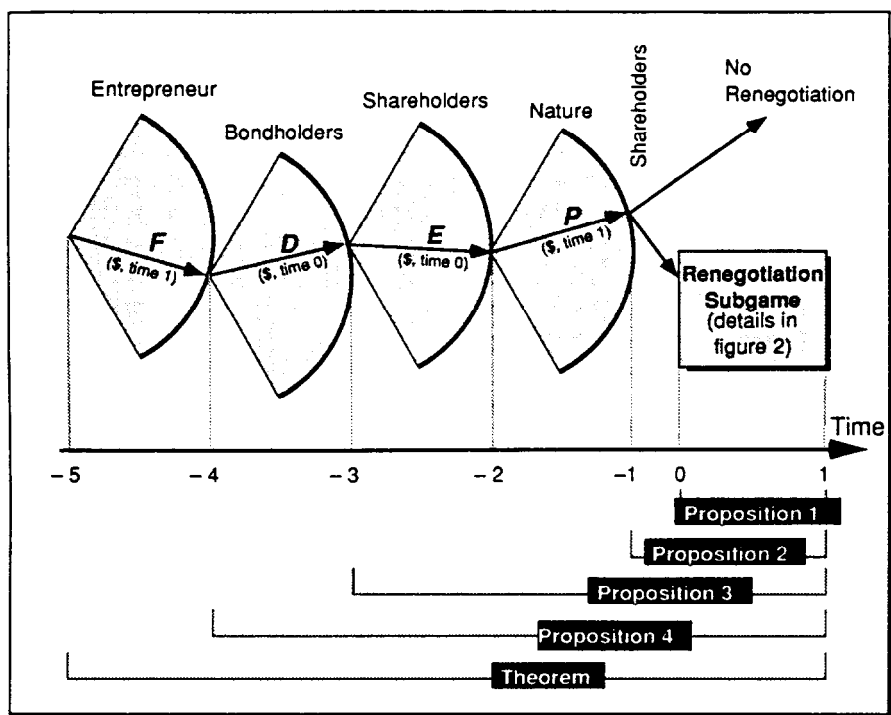


Fig. 1. The global game in extensive form.

An entrepreneur sells claims to his project consisting of bonds with a total face-value  $F$  to bondholders, who collectively bid  $D$ , and the residual claim (equity) to shareholders, who collectively offer  $E$ . Nature subsequently reveals the firm's pre-tax profit  $P$ , and shareholders then decide whether or not to renegotiate the debt contract. The (subgame-perfect) equilibrium in the game is constructed by backwards induction in a retrograding sequence of propositions.

that an optimal investment plan is followed from time zero to time 1. However, if an alternative, suboptimal investment plan is employed until time  $t$  (between 0 and 1) and only from then on is the optimal investment plan adopted and followed, the earning capabilities of the project deteriorate, and the operating profit at time 1 reduces to  $d(t)P$  dollars. We call  $d(\cdot)$  the *deterioration function* and assume that it continuously decreases in  $t$  from  $d(0) = 1$ . The value of the operating profit  $P$  is drawn at time  $t = -2$  according to a probability distribution function  $\Phi(\cdot)$  with density  $\phi(\cdot)$  (both known to the players). It is also assumed that the random profit  $P$  has a finite expectation. Times  $-5$  to  $1$  are so labeled to clarify the sequence of the players' moves, but for simplicity the time interval between  $t = -5$  and  $0$  is assumed to be infinitesimal.

The global game, which begins at time  $t = -5$  and continues until time  $t = 1$ , will be denoted  $G_{-5}$ . At time  $t = -5$ , the entrepreneur incorporates the project and sells claims to the newly formed firm by issuing two types of securities to a competitive capital market: (i) pure discount bonds, whose purchasers are promised collectively  $F$  dollars (the face-value) at time 1, and (ii) stock, whose purchasers control the firm. A strategy for the entrepreneur – acting as a von Stackelberg leader – is to set the level of  $F$ . Given  $F$ , competitive bondholders decide at time  $t = -4$ , when subgame  $G_{-4}(F)$  commences, on the dollar price  $D(F)$  they are willing to pay collectively for the debt issue. Given  $F$  and  $D$ , competitive shareholders decide at time  $t = -3$ , when subgame  $G_{-3}(F, D)$  starts, on the dollar price  $E(F, D)$  they are willing to pay for the equity. (We let the bondholders make their purchase decision before the shareholders, because this greatly simplifies the mechanics of the solution. Reversing the order does not affect the results.)

At time  $t = -1$ , after the debt and the equity are purchased and after the random amount of pre-tax profit  $P$  that the firm can capture – given that it adheres throughout to the optimal investment policy – becomes known, subgame  $G_{-1}(F, D, E, P)$  begins in which the shareholders move first and *decide* whether or not to launch debt renegotiations. If the shareholders decide at time  $t = -1$  not to renegotiate the debt, then the game terminates: the firm earns its profit at time 1, pays its taxes, and distributes the after-tax profit to its claimants according to strict priority rules. If the shareholders decide to renegotiate, then they trigger the debt renegotiation subgame  $G_0(F, D, E, P)$  at time 0 by communicating to the bondholders a threat to adopt the suboptimal investment policy, unless the latter accede to the shareholders' demands to reduce the debt. It is assumed that the firm's deterioration profile  $d(\cdot)$  is known to both negotiating parties. The threat is not revealed to outsiders or to the courts. Although creditors could always divulge shareholders' intent to the courts, the latter would require clear-cut evidence to act. We assume that shareholders are sufficiently sophisticated in their approach that no such evidence is available.

The debt renegotiation subgames  $G_0(F, D, E, P)$ , illustrated in fig. 2, are modeled similarly to Rubinstein's (1982) sequential bargaining game for two players who bargain on how to divide a shrinking asset. There is an essential difference, however. In our model, the size of the contested asset and the bargainers' strategic proposals on how to divide that asset must be determined simultaneously, because it is the *after-tax* profit which is divided between the two contestants. Thus, a proposed distribution to the bondholders determines the amount of tax-deductible interest effectively paid on the debt which, in turn, affects the amount of corporate tax paid, and therefore also the amount of after-tax profit to be divided between the shareholders and the bondholders.

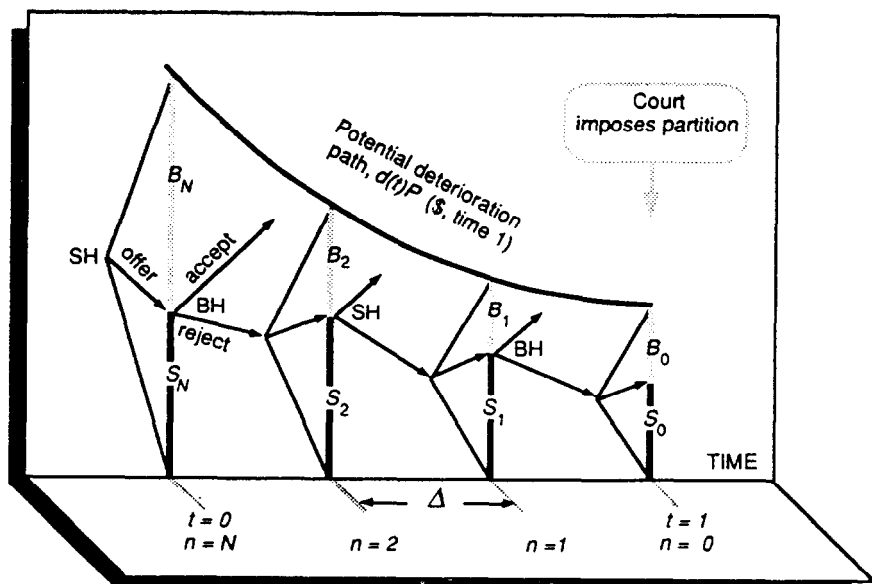


Fig. 2. The renegotiation subgame (see fig. 1).

If shareholders decide to launch debt renegotiations, the bargaining subgame commences at time 0. Shareholders simultaneously embark on the suboptimal investment policy, which causes the firm's earning capabilities to deteriorate. Bargaining takes place in  $N$  rounds of duration  $\Delta$ , in which shareholders and bondholders alternate in making offers and counter-offers ( $S_n, B_n$ ) on how to divide the remaining after-tax profits. If there is no agreement, the bankruptcy court steps in and imposes a resolution of the conflict at time 1.

Formally, bargaining takes place in  $N$  rounds of equal length  $\Delta = 1/N$ , spanning the time interval  $[0, 1]$ . Roles alternate from one round to the next; one party proposes how to divide the after-tax profit, and the other either accepts or makes a counteroffer. Because we use backwards induction in solving for the equilibrium, it is convenient to number the rounds backwards so that the first round (at  $t = 0$ ) is numbered  $n = N$  and the last (at  $t = 1$ ) is numbered  $n = 0$ .

Round  $n$  ( $n = N, N - 2, \dots, 1$ ;  $N$  odd) of the debt renegotiation occurs after the suboptimal investment policy has been in effect for the last  $(N - n)$  bargaining rounds, at which time the earning capabilities of the firm have already deteriorated to the point where they can only generate a pre-tax profit of  $\delta_n P \equiv d[(N - n)\Delta]P$  dollars (note this definition of  $\delta_n$ ). The shareholders propose to pay the bondholders  $B_n$  dollars at time  $t = 1$  to discharge the whole debt, which leaves the shareholders  $S_n$  dollars after paying corporate taxes. The assumptions that  $N$  is odd and that the shareholders make

the first proposal are of minor consequence. When the duration of each bargaining round tends to zero, those assumptions become immaterial.

The tax code, assumed here, recognizes as tax-deductible interest that amount which the bondholders *actually* receive over and above what they originally paid for the debt issue, regardless of the amount they were *promised* as repayment. In our model, this tax-deductible interest is:  $\max(B_n - D, 0)$ . Consequently, any proposed partition,  $(S_n, B_n)$ , of the firm's after-tax profits must satisfy what we call the *partition-feasibility condition*:

$$\begin{aligned} \text{After-Tax Profit} &= S_n + B_n \\ &= (1 - \tau) \cdot (\text{Pre-Tax Profit}) + \tau \cdot \max(B_n - D, 0), \end{aligned} \tag{1}$$

where the last term is the tax shield. The feasibility condition (1) demonstrates the distinction between our model and Rubinstein's, namely, that the size of the partitioned asset – after-tax profit – and the partition itself are determined simultaneously.

The bondholders choose to either accept or to reject the shareholders' offer in round  $n$ . If they accept it, the agreement is signed into a new contract – legally termed a *composition* – replacing the original one, and bargaining ends without further loss in the firm's earning capabilities. If, on the other hand, the bondholders reject the offer, then the suboptimal investment policy stays in effect, so that (time 1) profit deteriorates further to  $\delta_{n-1}P$  dollars. In the next round,  $n - 1$ , it is the bondholders' turn to make a counteroffer, that is, an amount  $B_{n-1}$  that they are willing to accept at time  $t = 1$  to discharge the original debt, which also implies an amount  $S_{n-1}$  that the shareholders receive. If the shareholders accept, they switch to the optimal investment policy, deterioration ceases, and bargaining terminates. If they reject the counteroffer, the firm's profit is again allowed to deteriorate until round number  $n - 2$ , when it is again the shareholders' turn to move as in round  $n$ . The bargaining process continues in this fashion, terminating either when an offer is accepted by one of the bargaining parties or at debt maturity time  $t = 1$ .

The model requires that when a player receives a refusal, it cannot retract; the value of the firm must deteriorate for another bargaining round. But realistically there must be some delay between the decision to retract and its implementation. When the duration of a bargaining round is shorter than the delay, the latter becomes an ironclad commitment. Interestingly, this chain of miniature commitments drives the sequential bargaining model.

Although by assumption the bondholders cannot force the firm into bankruptcy prior to the debt maturity time  $t = 1$ , if the shareholders are not



able to pay the face-value of the debt at that time, a trustee is appointed under Chapter 7 proceedings and the firm is liquidated. Alternatively, one can view the negotiations as taking place under Chapter 11 reorganization with  $t = 1$  as the date at which the reorganization attempt is abandoned and a Chapter 7 liquidation occurs. According to this scenario, the bonds have covenants or coupon payments which were violated, leading to a formal bankruptcy reorganization. However, either the courts will not appoint a trustee at the bondholders' behest prior to time 1, because the courts are not convinced of the shareholders' nefarious intent, or the trustee, if appointed, will not save the firm from deterioration. Essentially, although a detailed bargaining process takes place in the background, shareholders are sufficiently adept so that the game remains invisible to the courts.

We assume that the profit at time 1 – whatever the amount – is divided between the bondholders and the shareholders according to *strict priority rules*, because in Chapter 7 liquidations absolute priority rules are bound to hold [White (1989) and Franks and Torous (1989)]. Alternative bankruptcy codes for assigning different priorities in liquidation could be incorporated, but each such rule would imply a different partition of the pie in debt renegotiations.

At time  $t = 1$ , the firm earns the pre-tax operating profit, which is determined by the remaining earning capabilities of the firm according to the outcome of the renegotiation subgame. If debt renegotiations have not taken place, that profit is  $P$ , and if renegotiations occurred and were terminated in round  $n$ , the profit is  $\delta_n P$ . In either case, corporate taxes are paid, the bondholders receive an amount  $B(P; F, D)$  and the shareholders receive  $S(P; F, D)$ .

Recall that the bondholders and the shareholders make their purchase decisions before the uncertainty about the firm's (optimal policy) profit potential is resolved, and that agents are risk-neutral. The payoffs to the three players in the global game  $G_{-5}$ , evaluated as of date 0, are then described as follows: (i) The entrepreneur gets  $D(F) + E(F, D)$ , the combined proceeds from the debt and the equity sales; (ii) the bondholders receive the net present value of their financial transaction,  $\{EB/(1+r) - D(F)\}$ ; and (iii) the shareholders similarly secure  $\{ES/(1+r) + E(F, D)\}$ .  $E$  is the expectation operator taken with respect to the distribution function  $\Phi(\cdot)$  of the random profit  $P$ , and  $r$  is the  $[0, 1]$  holding-period riskless rate of return. Since the debt and the equity are sold in competitive and efficient capital markets, the net present values of the shareholders' and the bondholders' financial transactions *must be zero*. This requirement is then a necessary condition which is imposed on any acceptable equilibrium in the game. (The entrepreneur can earn a positive rent.) Since the bondholders never get more than the debt face-value, i.e.,  $B \leq F$ , it follows from this and

from the bondholders' zero-profit condition that  $D = EB/(1+r) < F$ ; this condition will also be imposed throughout the following discussion.

#### 4. Characterization of the equilibrium in the model

The shareholders' threats must be credible in the sense that the equilibrium of the renegotiation game is subgame-perfect.<sup>2</sup> Intuitively, unless subgame-perfection obtains, it may not be in the shareholders' interest to dissipate firm value. Bondholders, knowing this, would not take their threats seriously. Since information in the global game  $G_{-5}$  is perfect, we can use backwards induction in order to characterize its subgame-perfect equilibria (SPE). Fig. 1 diagrams the way the equilibrium is constructed. We first compute the SPE strategies in the renegotiation subgames  $G_0(F, D, E, P)$  (Proposition 1). These strategies are then extended to those which form SPE in the  $G_{-1}(F, D, E, P)$  subgames (Proposition 2). The latter are then similarly extended to the  $G_{-3}(F, D)$  subgames (Proposition 3), then once more to the  $G_{-4}(F)$  subgames (Proposition 4), and finally, to the whole game  $G_{-5}$  (the Theorem).

In order to characterize the equilibrium in the model, we need the following definitions:

$$\begin{aligned}\sigma_0 &\equiv 0, \\ \sigma_{n+1} &\equiv \sigma_n \equiv (\delta_n - \delta_{n-1}) + (\delta_{n-2} - \delta_{n-3}) + \cdots + (\delta_1 - \delta_0), \\ \sigma &\equiv \sigma_N, \\ \beta_n &\equiv \beta_{n-1} \equiv \delta_n - \sigma_n, \\ \text{for } n &= 1, 3, 5, \dots, N \quad (N \text{ odd}).\end{aligned}\tag{2}$$

For future reference, it should be noted that  $\sigma$  measures the total damage that shareholders could cause the firm in the rounds they make offers.

<sup>2</sup>Subgames are those parts of the game in extensive form that commence at a given node and can be considered as a game on their own. Any strategy for the whole game induces a strategy in every subgame. A *subgame-perfect equilibrium* is an assignment of a strategy to each player, which induces Nash equilibria in every subgame. Intuitively, this amounts to saying that in a subgame-perfect equilibrium no player has reason to change his or her plan when arriving at any node in the game – including those nodes which are not reached when playing the equilibrium strategies. In other words, threats are credible. Selten (1975) defined and developed this refinement of the Nash equilibrium concept.

Further define:

$$\underline{P}(D, \beta_n) = \frac{D}{\beta_n(1-\tau)}, \quad \bar{P}(D, F, \beta_n) = \frac{1}{\beta_n} \left[ F + \frac{\tau}{1-\tau} D \right], \quad (3)$$

where  $\underline{P}(D, \beta_n) < \bar{P}(D, F, \beta_n)$  follows from  $D < F$ . Finally, define:

$$\begin{aligned} S_n^*(P; F, D) \\ = \begin{cases} \sigma_n(1-\tau)P & \text{for } 0 \leq P \leq \bar{P}(D, F, \beta_n), \\ (1-\tau)(\delta_n P - F) - \tau D & \text{for } \bar{P}(D, F, \beta_n) < P, \end{cases} \end{aligned} \quad (4)$$

and

$$B_n^*(P; F, D) = \begin{cases} \beta_n(1-\tau)P & \text{for } 0 \leq P < \underline{P}(D, \beta_n), \\ \beta_n P - \left( \frac{\tau}{1-\tau} \right) D & \text{for } \underline{P}(D, \beta_n) \leq P < \bar{P}(D, F, \beta_n), \\ F & \text{for } \bar{P}(D, F, \beta_n) \leq P, \end{cases} \quad (5)$$

#### 4.1. Equilibrium in the renegotiation subgame

Using these definitions, the following proposition characterizes the subgame-perfect equilibrium in the renegotiation subgame  $G_0(F, D, E, P)$ :

*Proposition 1.* Suppose that the entrepreneur chooses  $F$  for the face-value of debt, that the bondholders pay  $D$  dollars for it, and that nature draws  $P$  for the potential profit of the firm. If the shareholders decide to renegotiate the debt, then there exists a unique subgame-perfect equilibrium in the debt renegotiation subgame  $G_0(F, D, E, P)$ , as follows:

In round  $n$  of the negotiations ( $n$  odd) the shareholders propose to take for themselves  $S_n^*(P; F, D)$  dollars of the aftertax profit at time 1; the bondholders accept any offer that gives them  $B_n^*(P; F, D)$  or better, and reject any lesser offer. For  $n$  even, the bondholders propose to take a share equal to  $B_n^*(P; F, D)$ ; the shareholders accept any offer that gives them  $S_n^*(P; F, D)$  or better, and reject anything less.

The outcome of the debt renegotiation subgame  $G_0(F, D, P)$  is that in the opening round the shareholders propose to divide the aftertax profit at time 1 by

taking for themselves  $S_N^*(P; F, D)$  and giving the bondholders  $B_N^*(P; F, D)$ . The bondholders immediately agree.

*Proof.* See appendix.

The unique subgame-perfect equilibrium in the subgame  $G_{-1}(F, D, E, P)$  is described in the next proposition, which considers the shareholders' decision whether to trigger debt renegotiations in the first place.

**Proposition 2.** Define the renegotiation cutoff level,

$$P_{\text{cutoff}}(F, D) \equiv \frac{1}{1-\sigma} \left[ F + \frac{\tau}{1-\tau} D \right]. \quad (6)$$

If the realization of the pre-tax profit,  $P$ , is such that

$$P_{\text{cutoff}}(F, D) \leq P,$$

then the shareholders do not gain from renegotiation of the debt. The firm follows the optimal investment plan throughout its lifetime (the time interval  $[0, 1]$ ). At time 1, it earns the pretax profit  $P$ , repays the bondholders  $F$  dollars (the promised face-value of the debt), and pays  $\tau[(P - (F - D))]$  dollars in corporate taxes. The shareholders receive what is left, namely,  $(1 - \tau)(P - F) - \tau D$  dollars.

If, on the other hand, the realization of the pre-tax profit,  $P$ , is such that

$$P < P_{\text{cutoff}}(F, D),$$

then the shareholders trigger debt renegotiations in which they and the bondholders follow the SPE strategies as in Proposition 1, with the same outcome of an immediate agreement to sign a composition by which the shareholders get  $S_N^*(P; F, D)$  and the bondholders receive  $B_N^*(P; F, D)$ , as follows (subscript  $N$  and asterisks dropped):

$$S(P; F, D) = \begin{cases} \sigma(1 - \tau)P & \text{for } 0 \leq P < P_{\text{cutoff}}(F, D), \\ (1 - \tau)(P - F) - \tau D & \text{for } P_{\text{cutoff}}(F, D) \leq P, \end{cases} \quad (7)$$

$$B(P; F, D) = \begin{cases} (1 - \sigma)(1 - \tau)P & \text{for } 0 \leq P < \underline{P}(D), \\ (1 - \sigma)P - \left( \frac{\tau}{1 - \tau} \right) D & \text{for } \underline{P}(D) \leq P \\ < P_{\text{cutoff}}(D, F), \\ F & \text{for } P_{\text{cutoff}}(D, F) \leq P, \end{cases} \quad (8)$$

where  $\underline{P}(D) \equiv D/[(1 - \sigma)(1 - \tau)] < P_{\text{cutoff}}(F, D)$  and where the last inequality follows from  $D < F$ .

*Proof.* See appendix.

The equilibrium payoffs to the shareholders,  $S(P; F, D)$ , and to the bondholders,  $B(P; F, D)$ , in the renegotiation subgame  $G_{-1}$  are depicted in fig. 3 by the dark lines. For comparison, the respective standard text-book (after-tax) payoffs to the shareholders and the bondholders are depicted by the gray lines. The minimum pre-tax profit for which the firm can still pay bondholders the full promised amount  $F$ , if it so wishes (firm still solvent), is denoted by  $P_0$ .

Propositions 1 and 2 and fig. 3 have the following interpretation: When the realization of the profit leaves the firm insolvent [the region  $[0, P_0)$  in the figure], shareholders have nothing to lose, so they renegotiate the debt to their advantage; when the realization of the profit is larger than  $P_0$ , so that the firm is solvent, both the shareholders and the debtholders stand to lose from carrying out the threat to let the firm deteriorate. But, if the profit is smaller than the cutoff level [in the region  $(P_0, P_{\text{cutoff}}]$ ], the shareholders stand to lose *less* than the debtholders. This lends credibility to the shareholders' threat; they gain from renegotiating the debt, and the bondholders are paid less than the full promised face-value of the debt. On the other hand, if profits are greater than the cutoff level, the shareholders stand to lose *more* than the debtholders; their threat is not credible, and therefore they do not attempt to renegotiate the debt.

In order to appreciate the role which  $\sigma$  plays in the preceding propositions and in those to follow, we will call it the *shareholders' bargaining-power index* [ $\sigma$  is defined<sup>3</sup> in (2)]. The motivation for this name is twofold. First, by definition,  $\sigma$  measures the amount of cumulative damage that the shareholders can inflict on the earning capabilities of the firm in the rounds in which they make offers. Second [by (6)], as  $\sigma$  increases, shareholders become more daring in that they trigger debt renegotiations at an increasingly higher cutoff level of pre-tax profits, and [by (7)] when the shareholders renegotiate the original debt contract, they receive a settlement in proportion to  $\sigma$  (this turns out to be true of the equilibrium outcome in the global game as well).

Because  $\sigma$  measures the amount of damage that the shareholders can cause, it follows that the shareholders' bargaining power index is an operational concept that is measurable in principle. We propose that a good surrogate for the index would be the proportion of intangible assets in the

<sup>3</sup>By the properties of the deterioration function  $d(\cdot)$ ,  $\sigma$  takes values in  $(0, 1)$ . When firm deterioration is impossible, i.e.,  $d(\cdot)$  is constant at 1 on the time interval  $[0, 1]$ , then  $\sigma = 0$ . When a firm deterioration is abrupt, happening all at once at  $t = 0$ , then  $\sigma = 1$ . These two extreme cases are ruled out by the requirement that  $d(\cdot)$  be a continuously decreasing function; nevertheless, we will consider them as limiting cases in Proposition 5 below.

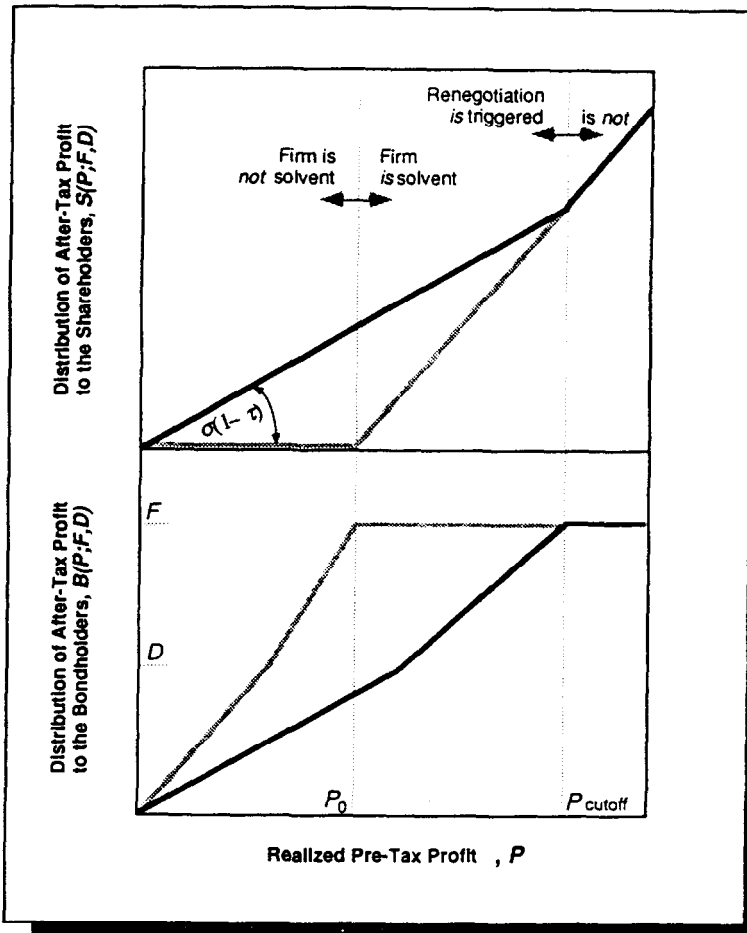


Fig. 3. Distributions of after-tax profit to shareholders and to bondholders as a function of realized pre-tax profit shown in black.

$F$  is the debt face-value,  $D$  is the price paid for the debt, and  $P_0$  is the profit level that separates between solvency and insolvency of the firm. If pre-tax profit is greater than the cutoff level,  $P_{\text{cutoff}}$ , the debt is not renegotiated. If it is less, the debt is renegotiated to the shareholders' advantage. The slope of the shareholders' renegotiated payoff is proportional to their bargaining-power index,  $\alpha$ ; the larger it is, the more they get. The respective text-book after-tax payoffs to the shareholders and the bondholders are drawn in gray for comparison.

total value of the firm. Since by (7) the settlement that the shareholders receive in debt renegotiations is proportional to their bargaining power index, this settlement should be (cross-sectionally) positively correlated with the proportion of intangible assets of the firm.<sup>4</sup>

Despite the rather detailed bargaining protocol underlying Proposition 2, bargaining will conclude with an agreement in the first round. Firm value is not dissipated, and the outcome is efficient. Driving this result is the assumption of complete and perfect information in the renegotiation game (and the absence of indeterminacies in the bargainers' payoff structure). It is important to emphasize, however, that when information is *incomplete*, the resolution of the bargaining process is not immediate [e.g., Fudenberg and Tirole (1983) and Grossman and Perry (1986)]. In that case, as Rubinstein (1987) notes, the series of offers and responses becomes a communication channel between the players. Each player tries to extract from the other's move information about the opponent's nature (i.e., preferences, beliefs, etc.), and each may try to mislead the other regarding the strength of their bargaining position. In that case, the bargaining process may last many rounds with an inefficient outcome the inevitable result, as value is dissipated until agreement is reached, if ever.

Haugen and Senbet (1978, 1988) have advanced a strong case that (indirect) bankruptcy costs are immaterial to the theory of optimal capital structure. They argue that financial distress should, at most, only slightly impair the firm's ability to conduct business. They maintain that customers, employees, etc. are mainly concerned with the tenure of the firm, which depends on its asset characteristics and not on the way those assets are financed. Our theory sheds new light on this issue. In order for Haugen and Senbet's argument to hold, it is necessary that some ideal conditions be satisfied: either shareholders have very little bargaining power (in the sense of our model) to renegotiate the debt during financial distress, or else debt renegotiations are conducted in an environment of near-*complete* information, so that an agreement between the parties is quickly reached without significant deterioration of the firm. If these two conditions are not satisfied, then, as indicated earlier, the value of the firm would be deliberately dissipated until agreement is reached, if ever; the goodwill that a firm has with its suppliers, customers, etc. would be among the first intangible assets to evaporate, and the costs subsequently induced by financial distress might well comprise a substantial portion of the firm's value. We further suggest that Myers's (1977) model can be reinterpreted as being embedded in a similar environment, in which his underinvestment problem cannot be miti-

<sup>4</sup>In a recent Wall Street Journal article (October 19, 1990 – after this paper was written), a creditor-banker commenting on his institute's position on accommodating Rupert Murdoch's financially distressed News Corp. was quoted: 'We're backing the man. He is one of the intangible assets.'

gated by debt renegotiations, because it is precisely those renegotiations which may cause inefficiencies in financial distress. [Giammarino (1989) also points out, in a different framework, the relationship between incomplete information and inefficiencies in a firm in financial distress.]

#### 4.2. Equilibrium in the global game

The next proposition determines the shareholders' optimal response to any (including out-of-equilibrium) moves by the entrepreneur and the bondholders, i.e., the shareholders' bid for the equity as a function of (i) the face-value of debt set by the entrepreneur and (ii) the bondholders' bid for the debt.

*Proposition 3.* At time  $t = -3$ , after the entrepreneur has set the face-value of the debt issue at  $F$  and the bondholders have purchased it for  $D$ , risk-neutral shareholders collectively purchase the equity in a competitive and efficient asset market for its discounted expected value:

$$E(F, D) = \frac{1}{1+r} \int_0^\infty S(P; F, D) d\Phi(P).$$

*Proof.* Competition among potential shareholders drives the net present value of the equity transaction to zero.

Recall that nature draws  $P$  according to the probability distribution function  $\Phi(\cdot)$ . Its support<sup>5</sup> is, obviously, either bounded or unbounded. An example of a bounded support is the uniform distribution; a lognormal distribution has an unbounded support. Since the two cases – both plausible – require somewhat different treatments, we will distinguish between them. Therefore we define

$$P^{(\text{sup})} = \begin{cases} \sup \{ \text{support } \Phi(\cdot) \} & \text{for a bounded support,} \\ \infty & \text{for an unbounded support.} \end{cases}$$

For simplicity, we assume that the infimum of the support of  $\Phi(\cdot)$  is zero.

The following proposition determines the bondholders' optimal response to any (including out-of-equilibrium) move by the entrepreneur, i.e., the bondholders' bid for the debt as a function of the debt face-value set by the entrepreneur. It again imposes the competitive condition that the net present value of the debt transaction is zero.

<sup>5</sup>The *support* of a distribution function  $\Phi(\cdot)$  is a collection of real numbers. A number  $x$  belongs to the support iff for every  $\varepsilon > 0$ ,  $\Phi(x + \varepsilon) - \Phi(x - \varepsilon) > 0$ . Informally, the support is the collection of probable values of the corresponding random variable.



*Proposition 4.* At time  $t = -2$ , after the entrepreneur has set the face-value of the debt issue at  $F$ , risk-neutral bondholders collectively purchase it in a competitive and efficient asset market for its discounted expected value  $D^*(F)$ , which is the unique solution to the implicit equation

$$D = \frac{1}{1+r} \int_0^\infty B(P; F, D) d\Phi(P). \quad (9)$$

Furthermore,  $D^*(F)$  is continuous on  $[0, \infty)$ ,  $D^*(0) = 0$ , and  $0 < D^*(F) < F$  on  $(0, \infty)$ . If  $P^{(\sup)} = \infty$ , then  $D^*(F)$  is increasing on  $[0, \infty)$  tending to a finite limit  $D^*(\infty)$ . If  $P^{(\sup)} < \infty$ , then there exists a unique and positive solution  $F^*$  to the equation

$$D^*(F) = \frac{1-\tau}{\tau} [(1-\sigma)P^{(\sup)} - F].$$

In this case,  $D^*(F)$  is increasing on  $[0, F^*)$ , and then stays constant on  $[F^*, \infty)$  at the value

$$D^*(F^*) = \frac{1-\tau}{\tau} [(1-\sigma)P^{(\sup)} - F^*] > 0.$$

The shareholders pay for the equity  $E^*(F) = E(F, D^*(F))$ , which is positive and continuous on  $[0, \infty)$ , with  $E^*(0) = (1+r)^{-1}(1-\tau)EP$ , where  $EP$  is the finite expectation of the random profit. If  $P^{(\sup)} = \infty$ , then  $E^*(F)$  is decreasing on  $[0, \infty)$ , and  $E^*(F) \rightarrow \sigma E^*(0)$  as  $F \rightarrow \infty$ . If  $P^{(\sup)} < \infty$ , then  $E^*(F)$  is decreasing on  $[0, F^*)$  and then stays constant on  $[F^*, \infty)$  at the value  $E^*(F^*) = \sigma E^*(0) > 0$ .

The value of the firm  $V^*(F) \equiv D^*(F) + E^*(F)$  is continuous on  $[0, \infty)$ . If  $P^{(\sup)} = \infty$ , then  $V^*(F)$  is increasing on  $[0, \infty)$ , and  $V^*(F) \rightarrow D^*(\infty) + \sigma E^*(0)$  as  $F \rightarrow \infty$ . If  $P^{(\sup)} < \infty$ , then  $V^*(F)$  is increasing on  $[0, F^*)$ , and then stays constant on  $[F^*, \infty)$  at the value

$$\begin{aligned} V^*(F^*) &= D^*(F^*) + E^*(F^*) \\ &= \frac{1-\tau}{\tau} [(1-\sigma)P^{(\sup)} - F^*] + \frac{\sigma(1-\tau)}{1+r} EP. \end{aligned}$$

*Proof.* See appendix.

In the spirit of Myers (1977), we give the name *debt capacity* to  $D^*(F^*)$  or  $D^*(\infty)$ , the maximum debt available when  $P^{(\sup)} < \infty$  or  $P^{(\sup)} = \infty$ , respectively.

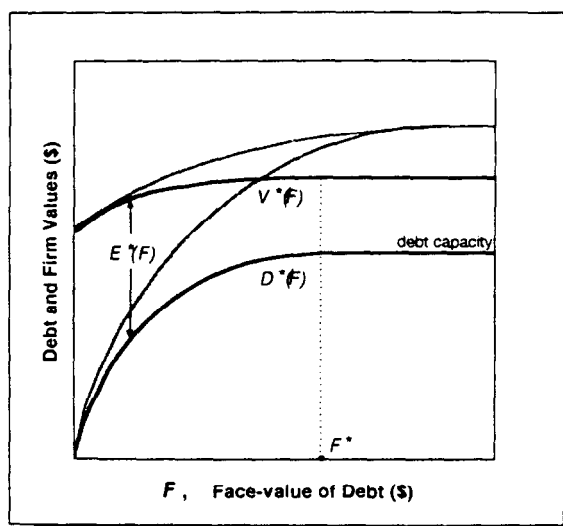


Fig. 4. The optimal bids by bondholders,  $D^*(F)$ , and by shareholders,  $E^*(F)$ , in response to the entrepreneur's designation of a debt face-value,  $F$ , and the value of the firm,  $V^*(F)$  – the sum of the two bids (shown in black).

Here the profit distribution has finite support, hence (by Proposition 4) at  $F^*$  the values of debt,  $D^*(\cdot)$ , and of the firm,  $V^*(\cdot)$ , level off at their maxima. Consequently (by the Theorem), debt face-values of  $F^*$  or greater generate equilibria with the common property that  $V^*(F^*)$  is the optimal firm value, and  $(D^*(F^*), E^*(F^*))$  is the interior optimal capital structure. The firm is then purely debt financed at the optimum – the Modigliani–Miller result. Debt and firm values in the case of zero shareholders' bargaining power ( $\sigma = 0$ ) are drawn in gray for comparison.

To help understand the implications of Proposition 4, fig. 4 depicts the values of the debt, the equity, and the firm as a function of the face-value announced by the entrepreneur (for a finite support). For comparison, we show the same schedules for the case where the shareholders cannot threaten to adopt a suboptimal investment policy, i.e., when their bargaining power index is zero. The latter schedules result from substituting  $\sigma = 0$  in the formulas above.

The following theorem characterizes the equilibrium of the global game, which implies an interior optimal capital structure with taxes as the bondholders' foresighted response to the shareholders' opportunistic behavior in debt renegotiations.

**Theorem.** *The subgame-perfect equilibria of the global game,  $G_{-5}$ , are characterized by the entrepreneur designing the debt-issue attributes to maximize the combined debt and equity proceeds, i.e., the value of the firm.*

*Case 1. When  $P^{(\text{sup})} < \infty$ , there are multiple subgame-perfect equilibria which are operationally equivalent. The entrepreneur sets the face-value of the debt at any  $F^{(\text{eq})}$  that is greater than or equal to  $F^*$ , which is equivalent to setting the equilibrium cutoff level,  $P_{\text{cutoff}}(F^*, D^*(F^*))$ , at the supremum of the distribution support,  $P^{(\text{sup})}$ . The bondholders collectively purchase the debt issue for  $D^*(F^*) > 0$  (Proposition 4), and the shareholders buy the equity for  $E^*(F^*) > 0$ .*

*Case 2. When  $P^{(\text{sup})} = \infty$ , there is a unique subgame-perfect equilibrium. The entrepreneur designs the debt so that the bondholders are promised the entire after-tax profit – in effect setting the face-value of the debt at infinity. The bondholders collectively purchase the debt issue for  $D^*(\infty) > 0$ , and the shareholders buy the equity for  $\sigma E^*(0) > 0$ .*

*In either case, nature next draws the potential pre-tax profit; then the shareholders and the bondholders always renegotiate the debt, and immediately agree to rewrite the debt contract according to Proposition 1.*

*Evidently, the equilibrium values of the debt and the equity are both positive, which implies an interior optimal capital structure of the firm.*

*Proof.* See appendix.

According to the Theorem, the entrepreneur appears to be *trying* to finance the firm purely by debt in order to take full advantage of the debt-favoring tax code, knowing perfectly well that in ‘bad’ states of nature (profit below cutoff) the shareholders and the bondholders will undo his original debt contract and sign a renegotiated one. The entrepreneur cannot, therefore, finance the firm purely by debt: no matter how large he sets the face-value of the debt, he cannot sell it for more than debt capacity, which is determined by the nature of the firm’s assets.

The model yields an interior optimal capital structure even in the *absence of bankruptcy costs*. Moreover, as we will see, depending on the value of the shareholders’ bargaining-power index, any value of debt-to-equity ratio can be obtained. This has important empirical implications. If the interior optimum is generated by a tradeoff between bankruptcy costs and tax deductibility of interest expense [e.g., Kraus and Litzenberger (1973), Scott (1976), Kim (1978)], then it is crucial that realized bankruptcy costs be significant. Given that an interior capital structure obtains even in the absence of bankruptcy costs, the size of those costs becomes less important.

#### 4.3. Example: An analytic solution when the profit distribution is uniform

When pre-tax  $P$  is distributed uniformly on  $[0, P^{(\text{sup})}]$ , eq. (9) has the following analytic solution, which is the bondholders’ optimal collective bid for the debt issue in response to the entrepreneur’s choice of debt face-

value  $F$ :

$$D^*(F) = \begin{cases} F + K - \sqrt{\frac{1}{\tau}F^2 + \frac{2Kr}{1+r}F + K^2} & \text{for } 0 \leq F < F^*, \\ \frac{1-\tau}{\tau}[(1-\sigma)P^{(\text{sup})} - F^*] & \text{for } F^* \leq F, \end{cases}$$

where

$$K = \frac{1-\tau}{\tau}(1+r)(1-\sigma)P^{(\text{sup})}$$

and

$$F^* = (1-\sigma)P^{(\text{sup})} \left[ \sqrt{[1+r(1-\tau)]^2 - \tau} - r(1-\tau) \right].$$

The price that the shareholders are willing to pay for the equity issue is then given by

$$E^*(F) = \begin{cases} \frac{(1-\tau)}{2(1+r)} \left[ P^{(\text{sup})} - 2(1-\sigma)\bar{p}(F) + \frac{(1-\sigma)\bar{p}^2(F)}{P^{(\text{sup})}} \right] & \text{for } 0 \leq F < F^*, \\ \frac{(1-\tau)\sigma P^{(\text{sup})}}{2(1+r)} & \text{for } F^* \leq F, \end{cases}$$

where

$$\bar{p}(F) = \bar{P}(F, D^*(F)) = \frac{1}{1-\sigma} \left[ F + \frac{\tau}{1-\tau} D^*(F) \right].$$

Note the way both functions,  $D^*(\cdot)$  and  $E^*(\cdot)$ , conform to the results of Proposition 4.

#### 4.4. *Equilibrium with renegotiation costs that are realized with positive probability*

The theorem implies that the optimal capital structure of the firm leads to debt renegotiation with probability one. Of course, in our model there are no costs of debt renegotiations, and the tax benefit to debt provides an incentive to increase the amount of debt in the firm's capital structure. In the absence of renegotiation costs the entrepreneur sells the maximum amount of debt

available, i.e., he issues at debt capacity, without being concerned about the resulting inevitability of debt renegotiations. However, if debt renegotiations involve realized costs, increasing the amount of debt also increases the probability for debt renegotiation, and with it the present value of those renegotiation costs that the entrepreneur must bear. Such costs act then as a disincentive to increasing the amount of debt, and will produce a probability of debt renegotiation that is less than one.

## 5. Further empirical implications of the model

### 5.1. Capital structure policy and shareholders' bargaining power

*Proposition 5. Other things equal, smaller values of the shareholders' bargaining-power index  $\sigma$  are associated with greater equilibrium values of the firm  $V$ , of the debt  $D$ , and of the ratio of debt to firm value  $D/V$ . In particular, for the maximum value of the shareholders' bargaining-power index,  $\sigma = 1$ , the firm is purely equity financed in equilibrium; for the minimum value,  $\sigma = 0$ , it is purely debt financed.*

*Proof.* See appendix.

Fig. 5 illustrates Proposition 5 for the following parametric values:  $P^{(\text{sup})} = \$1$ ,  $\tau = \frac{1}{3}$ , and  $r = 0.5$  (per five-year holding period, say).

The intuition behind Proposition 5 is fairly clear. The greater is the shareholders' bargaining-power index, the less debt the firm can issue in equilibrium, hence the less tax benefit it can capture, and consequently the smaller is the value of the firm.

This proposition also has implications for industry capital structure. It is well known [Brealey and Myers (1988), p. 407] that capital-intensive industries like airlines, utilities, steel, chemicals, etc. rely heavily on debt, while firms like high-tech companies and advertising agencies, whose assets are mostly composed of intangible assets, are predominantly equity financed. Our theory suggests an explanation for this phenomenon. Managements of firms with a larger proportion of intangibles to total assets are in a better position to successfully renegotiate away value from their debtholders. Consequently, such firms can borrow less in equilibrium. A larger proportion of intangibles entails a larger shareholders' bargaining-power index, which in turn implies, by Proposition 5, a smaller optimal debt-equity ratio.

### 5.2. Debt covenants

It is clear from Proposition 5 and fig. 5 that given the choice, the firm would select the smallest possible shareholders' bargaining-power index,

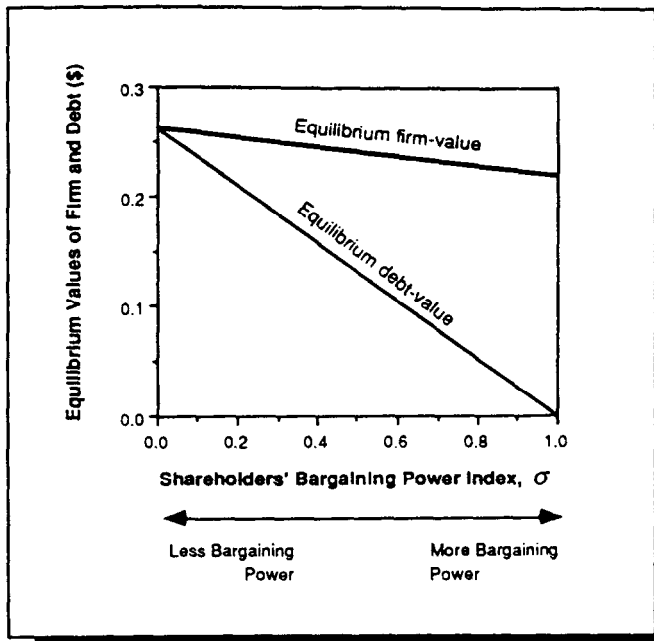


Fig. 5. Equilibrium values of the firm and of the debt are decreasing functions of the shareholders' bargaining-power index,  $\sigma$ .

When shareholders have zero bargaining power ( $\sigma = 0$ ), the firm is purely debt financed (MM); when they have maximum bargaining power ( $\sigma = 1$ ), the firm is purely equity financed. The graph is drawn based on  $P^{(\text{sup})} = \$1$ ,  $\tau = \frac{1}{3}$ , and  $r = 0.5$ .

because then the firm could increase the proportion it borrows in equilibrium, thus capturing more of the tax advantage of debt. We suggest that firms achieve the goal of curbing *ex ante* the shareholders' ability to renegotiate the debt by incorporating certain debt covenants which restrain management from adopting suboptimal investment policies; breaching such a covenant hastens formal default, and therefore shortens the amount of time for opportunistic maneuvers by management.

Consider, for example, the debt covenant that requires the firm to maintain its assets. Absent the rationale suggested above, this covenant seems redundant at best. If slackening maintenance of the firm's assets is an optimal policy for the firm – as is sometimes the case just before the firm scraps old machinery and retools – then a covenant that bans such optimal policy *hurts* the creditors (as well as the owners), and therefore should not be incorporated into the debt contract. If, on the other hand, meticulous maintenance of the firm's assets is optimal, then failing to maintain hurts the

owners (as well as the creditors), and inclusion of that covenant, whose effect is then to prevent the owners from hurting *themselves*, is redundant. Our model suggests that a covenant that requires the firm to maintain its assets serves the purpose of limiting management's ability to use the purposeful dereliction of the firm's assets as a bargaining device. A similar rationale helps explain debt covenants that restrict disposition of the firm's assets.

Our model also explains covenants that restrict the firm's ability to acquire financial assets. As noted by Smith and Warner (1979, fn. 16), 'financial assets are negative value projects whose acquisition reduces the value of the firm'. To see why (the following is our interpretation), consider, for example, firm A that faces a decision between two alternatives: (1) investing directly in a given project and (2) passively investing the same amount in the stock of firm B within the same risk-class (in the sense of Miller–Modigliani) as the project. Under the first alternative, firm A pays corporate taxes on the earnings from the project *only once*. Under the second alternative, firm B pays corporate taxes on the earnings, and then pays out the after-tax earnings as dividend to firm A (in proportion to its holdings). According to current US tax laws, firm A must *again* pay corporate taxes on 20% of the dividend, which leaves firm A with a smaller after-tax cash flow than under the first alternative. This implies that an efficient capital market would devalue the stock of firm A by the present value of the avoidable tax on the dividend as soon as firm A purchases the stock of firm B.

Smith and Warner's explanation of the use of covenants that restrict the firm's ability to acquire financial assets focuses on shareholders' incentive to purchase financial assets which *increase* the variability of firm cash flows, thereby inducing a shift of value from debt to equity. However, they don't explain restrictions on holding low-risk financial assets that tend to *decrease* the variability of the firm's cash flows. Our model suggests that a debt covenant which restricts the firm's ability to acquire financial assets makes it difficult for management to use that investment policy in order to run down the value of the firm as a bargaining gambit.

It is important to emphasize, however, that although the covenants described above can help to mitigate management's opportunistic behavior, they cannot eliminate it completely. Breach of a covenant places the firm in default, but this is of no great hindrance to a management that threatens to default anyway. Attesting to the insufficiency of covenants in this respect are the recent proposals for insolvency law reforms by the U.K. government-appointed Cork Committee [see London (1984)]. Among other recommendations by the committee:

[T]he Government has accepted that directors should in future be personally liable for debts incurred by a company through 'wrongful trading'. Personal liability will apply when it appears that: 1. a company

was allowed to continue trading [do business] with the result that the position of existing creditors is worsened and/or additional liabilities are incurred which are not paid; and 2. the directors knew or ought to have known that there was no reasonable prospect of avoiding that situation.

### 5.3. *Private placement versus public offering of debt*

We argued above that firms would employ self-binding devices designed to allay debtholders' misgivings concerning management's opportunistic behavior, in order to increase debt capacity and with it the equilibrium value of the firm. One such device can be inferred from Smith and Warner's (1979) observation: '[T]he consent of 100% of the debtholders is required in order to change the maturity date or the principal amount of the bonds. In private placements involving few lenders, renegotiation is typically easier'. The device is the public offering of debt, which is more difficult to renegotiate. Public offerings have a disadvantage, though; they are costlier to float than private placements [Blackwell and Kidwell (1988)]. Nonetheless, a firm with a high proportion of *intangible* assets, whose need to employ self-binding devices is more pronounced, would tend to float debt publicly despite the extra flotation costs involved. A firm with a high proportion of *tangible* assets, on the other hand, can rely on those assets as a commitment device, and would tend to use private placement of debt to take advantage of the lower associated costs.

The above is only one possible determinant of the choice of debt-flotation method. In fairness, one could argue in the opposite direction as well: A firm with a large proportion of intangible assets tends to possess information which it tries to conceal from competitors and the public, and so prefers to place debt privately to better guard its business secrets. Which argument dominates is an empirical issue. The study by Blackwell and Kidwell does not resolve the question, since their sample involves only public utilities, which are capital-intensive.

### 5.4. *Capital structure and the corporate tax rate*

*Proposition 6. Other things equal, when the tax rate increases, so does the ratio of equilibrium debt to firm values.*

*Proof.* In the appendix.

Proposition 6 is illustrated in fig. 6, which is drawn for the same parameter values as in fig. 5. This result is similar to the one obtained from the bankruptcy-costs theory of optimal capital structure, but it is emphasized



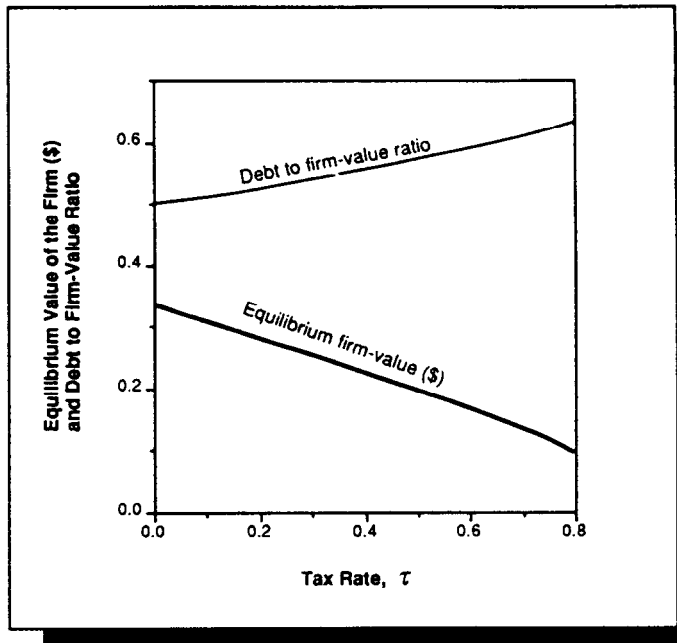


Fig. 6. Equilibrium firm value decreases with the tax rate, while the debt to firm value ratio increases.

again that here the result follows without recourse to any realized bankruptcy costs.

## 6. Summary and conclusions

The model presented in this paper has important theoretical and empirical implications for many aspects of corporate finance, including the capital structure of the firm, the structure of debt contracts, and debt renegotiations. The main findings are:

- (1) The debt capacity of the firm – the maximum amount of debt it can issue in equilibrium – is inversely related to the shareholders' bargaining power in debt renegotiations as measured by a well-defined index  $\sigma$ . This theoretical index is made operational by correlating it with the ratio of intangible assets to the total value of the firm.
- (2) In the absence of incentives to issue debt (like debt-favoring taxes), firm value is independent of the amount of debt issued up to debt capacity; debt levels beyond that are unattainable.

- (3) In the presence of debt-favoring corporate taxes, and even in the *absence* of any costs of debt renegotiations or of bankruptcy, an *internal* optimal capital structure obtains (positive amounts of debt and equity). Debt is then optimally issued at capacity, and consequently, it is always renegotiated.
- (4) There will be a *positive* (cross-sectional) correlation between the renegotiation settlement (or the formal bankruptcy reorganization settlement) accruing to shareholders and the ratio of *intangible assets* to the total value of the firm.
- (5) The greater the proportion of intangibles in the firm's asset structure (the greater is  $\sigma$ ), the smaller the firm's debt-equity ratio. In particular, capital-intensive industries will be more heavily debt financed by comparison to high-tech and service industries.
- (6) Debt covenants that appear to be redundant, because they are designed to prevent management from hurting the firm, are shown to be consequential.
- (7) Firms with a large proportion of intangible assets will tend to offer debt publicly by comparison to firms with a small proportion of intangibles, which will tend to place debt privately.
- (8) An increase in the corporate tax rate will increase the firm's debt-equity ratio.

This paper takes the contractual forms of equity and debt as given. However, a comprehensive analysis that tries to derive the optimal financial contracts endogenously and, in particular, to ascertain which claimholders class should have control of the investment decisions of the firm [see, for example, Harris and Raviv (1989)], would inevitably have to account for post-contractual opportunism along the lines posed here. Thus, this paper can also be viewed as a contribution towards this goal.

## Appendix

### *Proof of Proposition 1*

To construct the subgame-perfect strategies, we proceed from the last negotiation round ( $n = 0$ ) backwards to the first ( $n = N$ ), using backwards induction. The underlying idea is that each bargainer offers, in his or her turn, what the other can force in the next round. In round  $n = 0$ , the firm is capable of generating a *pre-tax* profit of only  $\delta_0 P = \beta_0 P$  dollars. By assumption, the bankruptcy court steps in then and partitions the firm's *after-tax* profits allocating  $S_0^*$  to the shareholders and  $B_0^*$  to the bondholders. This partition must satisfy both the feasibility condition (1),

$$S_0^* + B_0^* = (1 - \tau)\beta_0 P + \tau \cdot \max(B_0^* - D, 0), \quad (\text{A.1})$$

and the strict priority rule which is formalized by

$$\text{if } B_0^* < F, \text{ then } S_0^* = 0$$

and (A.2)

$$\text{if } B_0^* = F, \text{ then } S_0^* \geq 0.$$

It will be convenient to redenote

$$\underline{P}_n \equiv \underline{P}(D, \beta_n), \quad \bar{P}_n \equiv \bar{P}(D, F, \beta_n).$$

Distinguish between the following three regions of the pre-tax profit  $P$  [paralleling those in the definition of  $B_n^*$  in (5)]:

- (i) When  $0 \leq P < \underline{P}_0$ , then  $\beta_0(1-\tau)P < D$ . Hence the only partition that satisfies both (A.1) and (A.2) is  $B_0^* = (1-\tau)\beta_0 D$  and  $S_0^* = 0$ .
- (ii) When  $\underline{P}_0 \leq P < \bar{P}_0$ , then  $D \leq \beta_0 P - (\tau/(1-\tau))D < F$ , whence the only partition that satisfies both (A.1) and (A.2) is  $S_0^* = 0$  and  $B_0^* = \beta_0 P - (\tau/(1-\tau))D$ .
- (iii) When  $\bar{P}_0 \leq P$ , then  $B_0^* = F$  and  $S_0^* = (1-\tau)\beta_0(P - \bar{P}_0)$ .

Now assume the induction assumption that (4) and (5) are the SPE strategies up to and including round  $n-1$ . The remaining pre-tax profit capability in round  $n$  is  $\delta_n P$ , and consider that  $n$  is odd first. Since by the induction assumption, the bondholders can force  $B_{n-1}^*$  for themselves in the next round, the shareholders, whose turn it is to make an offer in round  $n$  odd, offer exactly that, i.e.,  $B_n^* = B_{n-1}^*$ . Again distinguish between three regions.

In region (i),  $0 \leq P < \bar{P}_n \equiv D/[(1-\tau)\beta_n]$ . Then  $B_n^* = B_{n-1}^* = (1-\tau)\beta_{n-1}P = (1-\tau)\beta_n P$ . This is so because by definition  $\beta_n = \beta_{n-1}$ , hence  $\underline{P}_n = \underline{P}_{n-1}$ , and  $P$  is within region (i) in round  $n-1$  as well, from which the equality above  $B_{n-1}^* = (1-\tau)\beta_{n-1}P$ , follows by the induction assumption. Now, substituting  $B_n^* = (1-\tau)\beta_n P$  in the partition-feasibility condition (1) and noting that in region (i)  $B_n^* < D$ , results in  $S_n^* = (1-\tau)(\delta_n - \beta_n)P = (1-\tau)\sigma_n P$ . By the induction assumption, the shareholders will be offered  $S_{n-1}^* = (1-\tau)\sigma_{n-1}P$  in the next round ( $n-1$ ). It must then be that  $S_n^* \geq S_{n-1}^*$ , otherwise the shareholders should not be offering  $S_n^*$  in round  $n$ . But the last inequality is equivalent to  $\delta_n \geq \delta_{n-1}$  ( $n$  odd), which is true because the deterioration function is decreasing. This completes the induction proof for  $n$  odd, when  $P$  is in region (i). The proofs for region (ii) and (iii) are similar, and therefore omitted.

Now consider that  $n$  is even. Since by the induction assumption the shareholders can force  $S_{n-1}^* = (1-\tau)\sigma_{n-1}P$  for themselves in the next round

$(n - 1)$ , the bondholders, whose turn is to make an offer in round  $n$  even, offer exactly that, i.e.,  $S_n^* = S_{n-1}^*$ . Since by definition  $\sigma_n = \sigma_{n-1}$ , then  $S_n^* = (1 - \tau)\sigma_n P$ , which was to be proved. Substituting  $S_n^*$  in the partition-feasibility condition, noting that the pre-tax profit is  $\delta_n P$  and that  $\beta_n \equiv \delta_n - \sigma_n$ , yields

$$B_n^* = (1 - \tau)\beta_n P + \tau \cdot \max(B_n^* - D, 0). \quad (\text{A.3})$$

Again distinguish between the three regions. In region (i)  $0 \leq P < \bar{P}_n$ , then  $(1 - \tau)\beta_n P < D$  and the unique solution to (A.3) is  $B_n^* = (1 - \tau)\beta_n P$ . Of course, it must be that  $B_n^* \geq B_{n-1}^*$ , which is equivalent to  $\delta_n \geq \delta_{n-1}$  ( $n$  even). Otherwise the bondholders should not be offering  $B_n^*$  in round  $n$ . But the last inequality is valid because the deterioration function is decreasing. The proofs for the other two regions are similar, and therefore omitted. ■

### *Proof of Proposition 2*

By Proposition 1 and by (4) and (5), if the shareholders decide to renegotiate the debt, then the outcome allocates to them (recall that  $\delta_N = 1$ ,  $\sigma_N = \sigma$ , and  $\beta_N = 1 - \sigma$ )

$$\begin{aligned} S_N^*(P; F, D) \\ = \begin{cases} \sigma(1 - \tau)P & \text{for } 0 \leq P < \bar{P}(F, D, \beta_N), \\ (1 - \tau)(P - F) - \tau D & \text{for } \bar{P}(F, D, \beta_N) \leq P. \end{cases} \end{aligned}$$

On the other hand, if the shareholders decide *not* to renegotiate, then their payoff is the usual one in the presence of taxes, namely,

$$\begin{aligned} S_{\text{no renegotiation}}(P; F, D) \\ = \begin{cases} 0 & \text{for } 0 \leq P < \bar{P}(F, D, \beta_N), \\ (1 - \tau)(P - F) - \tau D & \text{for } \bar{P}(F, D, \beta_N) \leq P. \end{cases} \end{aligned}$$

Obviously,  $S_N^*(P; F, D)$  is larger than  $S_{\text{no renegotiation}}(P; F, D)$  for  $P < \bar{P}(F, D, \beta_N)$ , hence the shareholders find it optimal to renegotiate the debt at those values of  $P$ . On the other hand, for  $\bar{P}(F, D, \beta_N) \leq P$ , the shareholders get the same payoff whether or not they renegotiate the debt, so *not* renegotiating is a best response (as is renegotiating). It then follows that

$$P_{\text{cutoff}}(F, D) \equiv \bar{P}(F, D, \beta_N) = \frac{1}{1 - \sigma} \left[ F + \frac{\tau}{1 - \tau} D \right]$$

is the profit cutoff level that separates low profit realizations, at which it is optimal for the shareholders to renegotiate the debt, from high realizations, at which it is optimal not to renegotiate. Eq. (8) is similarly obtained from (5) by setting  $n = N$ . ■

*Proof of Proposition 4*

Rewrite (9) in the form

$$h(F, D) = 0 \quad \text{on the half-plane } D < F, \quad (\text{A.4})$$

where, by (8),

$$\begin{aligned} h(F, D) = & \int_0^{\underline{P}(D)} (1 - \tau)(1 - \sigma) P \phi(P) dP \\ & + \int_{\underline{P}(D)}^{P_{\text{cutoff}}(F, D)} \left[ (1 - \sigma) P - \frac{\tau}{(1 - \tau)} D \right] \phi(P) dP \\ & + \int_{P_{\text{cutoff}}(F, D)}^{\infty} F \phi(P) dP - (1 + r) D. \end{aligned}$$

It is easy to see that all the integrands above are continuous in  $F$  and in  $D$ , and their partial derivatives with respect to  $F$  and  $D$  are continuous as well. Also,  $\underline{P}(D)$  and  $P_{\text{cutoff}}(F, D)$  have partial derivatives with respect to  $F$  and  $D$ . Then, by Leibniz's rule, the partials of  $h(\cdot, \cdot)$  exist and are given by

$$\begin{aligned} h'_F(F, D) &= 1 - \Phi(P_{\text{cutoff}}(F, D)), \\ h'_D(F, D) &= -\frac{\tau}{1 - \tau} [\Phi(P_{\text{cutoff}}(F, D)) - \Phi(\underline{P}(D))] - (1 + r) < 0, \end{aligned} \quad (\text{A.5})$$

where, recall,  $\Phi(\cdot)$  is the pre-tax profit distribution function. Since  $h(F, D)$ ,  $h'_F(F, D)$ , and  $h'_D(F, D)$  are continuous, and  $h'_D(F, D)$  is always negative, there exists – by the implicit function theorem – a unique explicit solution to (A.4):

$$D = D^*(F).$$

Moreover,  $D^*(F)$  is continuous and has a continuous first derivative

$$\frac{d}{dF} D^*(F) = -\frac{h'_F(F, D^*(F))}{h'_D(F, D^*(F))}. \quad (\text{A.6})$$

It is easy to see that  $h(0, 0) = 0$ , which implies that  $D^*(0) = 0$ . It is also straightforward to verify that  $0 < D^*(F) < F, \forall F > 0$ , hence also  $\underline{P}(D^*(F)) <$

$P_{\text{cutoff}}(F, D^*(F))$ , from which it follows by (A.5) and by (A.6) that

$$\begin{aligned} & \frac{d}{dF} D^*(F) \\ &= \frac{1 - \Phi(P_{\text{cutoff}}(F, D^*(F)))}{\frac{\tau}{1 - \tau} [\Phi(P_{\text{cutoff}}(F, D^*(F))) - \Phi(\underline{P}(D^*(F)))] + (1 + r)} \\ &\geq 0. \end{aligned} \quad (\text{A.7})$$

*Case 1.*  $P^{(\text{sup})} = \infty$ . Then  $1 - \Phi(P_{\text{cutoff}}(F, D^*(F))) > 0$ , for any positive  $F$ , and therefore  $(d/dF)D^*(F) > 0$ ,  $\forall F > 0$ , i.e.,  $D^*(F)$  increases on  $[0, \infty)$ . Since  $D^*(F)$  is also bounded by  $(1 + r)^{-1}EP$  on  $[0, \infty)$ , it has a finite limit  $D^*(\infty)$ .

*Case 2.*  $P^{(\text{sup})} < \infty$ . Define on  $[0, \infty)$  the following two functions:

$$L(F) \equiv \frac{1 - \tau}{\tau} [(1 - \sigma)P^{(\text{sup})} - F],$$

$$g(F) \equiv L(F) - D^*(F).$$

Note, first, that  $g(\cdot)$  is continuous and decreasing on  $[0, \infty)$  since  $g'(F) = -(1 - \tau)/\tau - (d/dF)D^*(F) < 0$  by (A.7). Second,  $g(0) = L(0) > 0$  and  $g((1 - \sigma)P^{(\text{sup})}) = -D^*((1 - \sigma)P^{(\text{sup})}) < 0$ . Hence,  $g(\cdot)$  attains the value zero only once at a point  $F^*$  in the interval  $(0, (1 - \sigma)P^{(\text{sup})})$ . To the left of  $F^*$ ,  $g(F)$  is positive, and to the right, it is negative. It follows that

$$\begin{aligned} D^*(F) \leq D^*(F^*) = L(F^*) < L(F) &= \frac{1 - \tau}{\tau} [(1 - \sigma)P^{(\text{sup})} - F], \\ &\forall F \in [0, F^*). \end{aligned}$$

Hence  $P_{\text{cutoff}}(F, D^*(F)) < P^{(\text{sup})}$  for any  $F$  in  $[0, F^*)$ . From the last inequality it follows that  $1 - \Phi(P_{\text{cutoff}}(F, D^*(F))) > 0$ , whence, by (A.7),  $(d/dF)D^*(F) > 0$  for any  $F$  in  $[0, F^*)$ , i.e.,  $D^*(F)$  increases there.

On the other hand,  $\forall F \in [F^*, \infty)$ ,  $g(F) \leq 0$ , hence,

$$D^*(F) \geq D^*(F^*) = L(F^*) \geq L(F) = \frac{1 - \tau}{\tau} [(1 - \sigma)P^{(\text{sup})} - F],$$

whence

$$P_{\text{cutoff}}(F, D^*(F)) \geq P^{(\text{sup})}, \quad \forall F \in [F^*, \infty). \quad (\text{A.8})$$

It follows from (A.8) that  $1 - \Phi(P_{\text{cutoff}}(F, D^*(F))) = 0$ ; hence by (A.7),  $(d/dF)D^*(F) = 0$  for any  $F$  in  $[F^*, \infty)$ , i.e.,  $D^*(F)$  is constant there, or

$$D^*(F) = D^*(F^*) = L(F^*) = \frac{1 - \tau}{\tau} [(1 - \sigma)P^{(\text{sup})} - F^*], \quad (\text{A.9})$$

$$\forall F \geq F^*.$$

This completes the proof of the properties of  $D^*(F)$ , when the support of the profit distribution is finite.

The competitive shareholders pay for the equity its discounted expected value, given the bondholders' optimal response to the entrepreneur's move  $F$ . Hence, by (A.7),

$$\begin{aligned} E^*(F) &= E(F, D^*(F)) = \frac{1}{1+r} \int_0^\infty S(P; F, D^*(F)) d\Phi(P) \\ &= \frac{1}{1+r} \int_0^{P_{\text{cutoff}}(F, D^*(F))} (1-\tau)\sigma P \phi(P) dP \\ &\quad + \frac{1}{1+r} \int_{P_{\text{cutoff}}(F, D^*(F))}^\infty [(1-\tau)(P-F) - \tau D^*(F)] \phi(P) dP. \end{aligned}$$

Substituting  $F=0$  in the last equation and noting that we have shown that  $D^*(0) = 0$  results in  $E^*(0) = (1+r)^{-1}(1-\tau)EP$ . Noting that  $\lim_{F \rightarrow \infty} P_{\text{cutoff}}(F, D^*(F)) = \infty$  and taking the limit of the integral above as  $F \rightarrow \infty$  results in  $E^*(\infty) = \sigma E^*(0)$ . Using Leibniz's rule

$$\begin{aligned} \frac{d}{dF} E^*(F) &= \frac{-1}{(1+r)} \left[ (1-\tau) + \tau \frac{d}{dF} D^*(F) \right] \\ &\quad \times [1 - \Phi(P_{\text{cutoff}}(F, D^*(F)))] \leq 0. \end{aligned}$$

Adding  $(d/dF)D^*(F)$  to both sides of the last equation and using (A.7) results in

$$\begin{aligned} \frac{d}{dF} V^*(F) &= \frac{d}{dF} E^*(F) + \frac{d}{dF} D^*(F) \\ &= \frac{[1 - \Phi(P_{\text{cutoff}}(F, D^*(F)))] [\tau + \Phi(P_{\text{cutoff}}(F, D^*(F)))]}{(1+r) \left\{ \frac{\tau}{1-\tau} [\Phi(P_{\text{cutoff}}(F, D^*(F))) - \Phi(P(D^*(F)))] + (1+r) \right\}} \geq 0. \end{aligned}$$

Applying to the last two equations a similar analysis to the one used on  $(d/dF)D^*(F)$  above yields the properties of  $E^*(F)$  and of  $V^*(F)$  as stated in the proposition. ■

### Proof of the Theorem

*Case 1.*  $P^{(\sup)} = \infty$ . By Proposition 4,  $V^*(F)$  increases on  $[0, \infty)$  (with a finite limit at  $\infty$ ). Hence, the entrepreneur's best strategy is to design the debt contract so that the bondholders are *promised* all the realized profit (as if setting  $F = \infty$ ). This, obviously, results in debt renegotiation with probability 1. Consequently, the entrepreneur is able to sell the debt for  $D^*(\infty)$  and the equity – for  $E^*(\infty) = \sigma E^*(0)$ .

*Case 2.*  $P^{(\sup)} < \infty$ . According to Proposition 4,  $V^*(F)$  attains a global maximum throughout  $[F^*, \infty)$  [it is constant there]. Hence, the entrepreneur's best strategy is to set the debt face-value at  $F^*$  or above. By (A.8), the probability for debt renegotiation is then 1. ■

### Proof of Proposition 5

Following is the proof for the finite-support case; the proof for the infinite support is similar. For ease of notation, only the dependence of the different functions on  $\sigma$  will be shown explicitly. For instance, instead of fully spelling out the equilibrium debt value as  $D^*(F^*(\sigma, \tau, r, P^{(\sup)}); \sigma, \tau, r, P^{(\sup)})$ , I will simply write  $D^*(F^*(\sigma); \sigma)$ .

By the 'envelope' theorem,

$$\frac{d}{d\sigma} D^*(F^*(\sigma); \sigma) = \frac{\partial}{\partial \sigma} D^*(F^*(\sigma); \sigma).$$

Hence, by (A.5) and (A.8),

$$\begin{aligned} \frac{\partial}{\partial \sigma} D^*(F^*(\sigma); \sigma) &= - \left( \frac{\partial h}{\partial \sigma} / \frac{\partial h}{\partial D} \right)_{(F^*(\sigma), D^*(F^*(\sigma); \sigma))} \\ &= - \frac{(1 - \tau)EP + \tau \int_{P(D^*(F^*(\sigma); \sigma))}^{P^{(\sup)}} P \phi(P) dP}{\frac{\tau}{1 - \tau} [1 - \Phi(P(D^*(F^*(\sigma); \sigma)))] + (1 + r)} \\ &< 0. \end{aligned}$$

Hence,  $D^*(F^*(\sigma); \sigma)$  is decreasing in  $\sigma$ .



Further, add

$$\frac{d}{d\sigma} E^*(F^*(\sigma); \sigma) = \frac{1-\tau}{1-r} EP$$

to the previous equation to get

$$\begin{aligned} & \frac{d}{d\sigma} V^*(F^*(\sigma); \sigma) \\ &= \frac{\frac{\tau}{1-r} [1 - \Phi(P(\underline{D}^*(F^*(\sigma); \sigma)))] EP - \tau \int_{P(\underline{D}^*(F^*(\sigma); \sigma))}^{P^{(\sup)}} P \phi(P) dP}{\frac{\tau}{1-\tau} [1 - \Phi(P(\underline{D}^*(F^*(\sigma); \sigma)))] + (1+r)} \\ &< 0. \end{aligned}$$

The last inequality follows from  $r > 0$  and from the Lemma that succeeds the present proof. It follows then that  $V^*(F^*(\sigma); \sigma)$  is decreasing in  $\sigma$ .

Using the properties of  $D^*(F^*(\sigma); \sigma)$ , of  $E^*(F^*(\sigma); \sigma)$  and of their derivatives with respect to  $\sigma$  yields

$$\frac{d}{d\sigma} \left( \frac{D^*}{E^*} \right)_{(F^*(\sigma); \sigma)} = \left( \frac{E^* \frac{dD^*}{d\sigma} - D^* \frac{dE^*}{d\sigma}}{(E^*)^2} \right)_{(F^*(\sigma); \sigma)} < 0.$$

Hence  $D^*(F^*(\sigma); \sigma)/E^*(F^*(\sigma); \sigma)$  is decreasing in  $\sigma$ ; therefore  $E^*/D^*$  is increasing in  $\sigma$ ; and so is  $E^*/D^* + 1 = V^*/D^*$ . Hence  $D^*/V^*$  is decreasing in  $\sigma$ .

We now proceed to show that when  $\sigma = 1$ , the equilibrium debt-value is zero; namely, we want to show that

$$D^*(F^*(\sigma); \sigma)_{\sigma=1} = 0. \quad (\text{A.10})$$

By (A.9),

$$D^*(F^*(\sigma); \sigma) = \frac{1-\tau}{\tau} [(1-\sigma) P^{(\sup)} - F^*(\sigma)],$$

hence

$$D^*(F^*(1); 1) = -\frac{1-\tau}{\tau} F^*(1). \quad (\text{A.11})$$

But it was shown in Proposition 4 that  $D^*(0; 1) = 0$  and that  $0 < D^*(F; 1) < F$ ,  $\forall F > 0$ . Hence,  $F^*(1) = 0$  is the unique solution to (A.11), which proves (A.10). Furthermore, by Proposition 4,  $E^*(F^*(\sigma); \sigma) = \sigma E^*(0; \sigma)$ . Hence  $E^*(F^*(\sigma); \sigma)_{\sigma=0} = 0$ , and therefore

$$D^*(F^*(\sigma); \sigma)_{\sigma=0} = V^*(F^*(\sigma); \sigma)_{\sigma=0}. \quad \blacksquare$$

*Lemma* (in support of the proof of Proposition 5). *Let a random variable  $X$  with a probability distribution function  $\Phi(\cdot)$  and with a density  $\phi(\cdot)$  have a finite support, then for any  $\alpha$  in that support:*

$$\int_{\alpha}^{\sup(\text{support } \phi(\cdot))} x \phi(x) dx \geq [1 - \Phi(\alpha)] EX.$$

*Proof.* Denote  $u \equiv \sup \{\text{support } \phi(\cdot)\}$ ,  $v \equiv \inf \{\text{support } \phi(\cdot)\}$ , and define

$$g(\alpha) \equiv \frac{\int_{\alpha}^u x \phi(x) dx}{1 - \Phi(\alpha)}.$$

Then

$$g'(\alpha) = \frac{\phi(\alpha) \int_{\alpha}^u (x - \alpha) \phi(x) dx}{[1 - \Phi(\alpha)]^2} \geq 0.$$

Hence  $g(\alpha) \geq g(v) = EX$ ,  $\forall \alpha \geq v$ , which completes the proof of the lemma.  $\blacksquare$

The proof of Proposition 6 is similar in spirit and methodology to that of Proposition 5, and is therefore omitted.

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