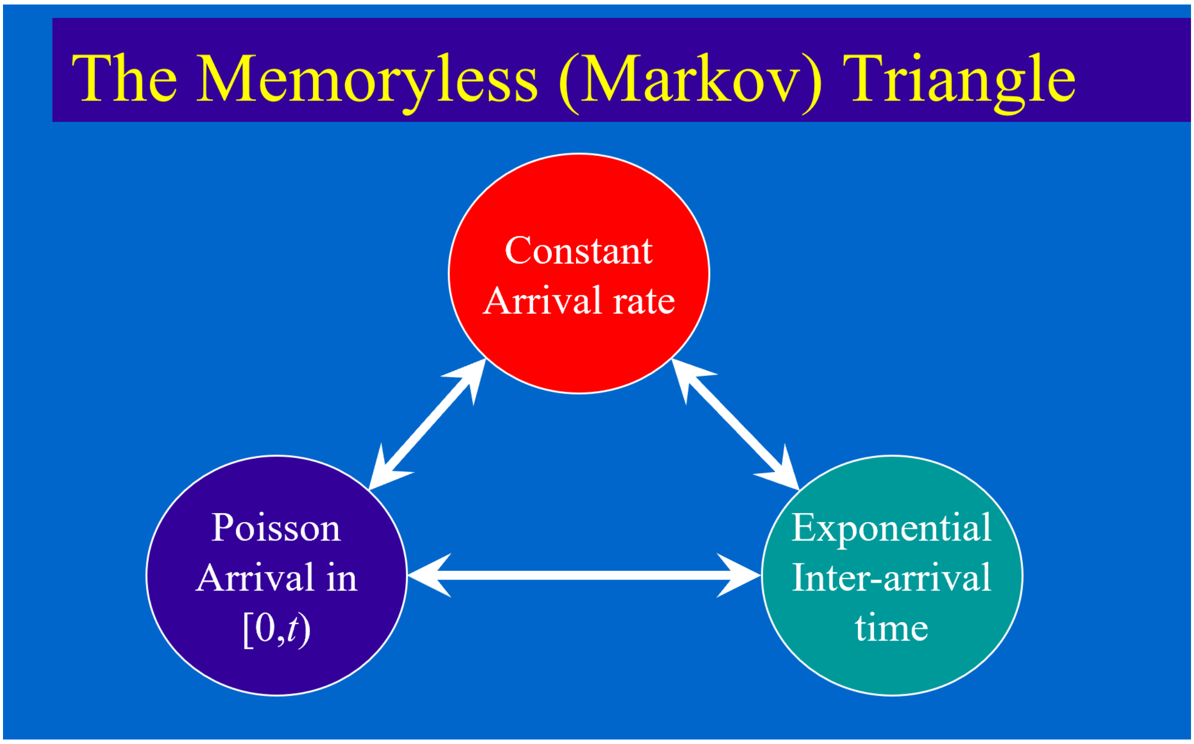
Probability Fundamentals



Memoryless Triangle

> 3 Properties below indicates that arrivals are **independent** and the stochastic process is **memoryless**.

1. Exponential Inter-arrival time:

$$P(T \le t + t_0 | T > t_0) = 1 - e^{-\lambda t} = P(T \le t) \Rightarrow p(t) = \lambda e^{-\lambda t}$$

   Inter-arrival time distribution of the customers is exponential.

2. Constant arrival rate:

$$\begin{aligned} P(\Delta t) &= \lambda \Delta t + o(\Delta t) \\ P(T \ge t) &= \lim_{n \to +\infty} (1 - \lambda \Delta t)^N \\ &= \lim_{n \to +\infty} \left(1 - \lambda \frac{t}{N}\right)^N \\ &= e^{-\lambda t} \end{aligned}$$

3. Poisson arrival in $[0, t)$:

$$P(a = n | T \le t + \Delta t) = P(a = n | T \le t) \cdot P(a = 0 | t \le T \le t + \Delta t) + \\ P(a = n - 1 | T \le t) \cdot P(a = 1 | t \le T \le t + \Delta t)$$

   If there are n arrivals in t, the arrivals obeys the **Poisson distribution**.

> Actually, they're amazing properties of **Poisson process**.