GNSS Algorithms and Development

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Overview

- Position, velocity, and time (PVT) and GNSS Observations
- Differential GPS (DGPS) and assisted GPS (A-GPS)
- Beidou overview and development

Previous Takeaways from Part I

- Principle idea behind GPS localization is simple – **trilateration**.
- Due to **clock offset**, a minimum of four satellites is needed to determine user's location.
- **Pseudorange**, calculated based on time delay, is the distance from satellite to user including clock offset and observation errors.
- Some major observations errors include **atmospheric error** and **multipath**.

Previous Takeaways from Part II

- GPS signal consists of carrier wave, **pseudorandom noise (PRN) code**, and navigation message.
- User will generate a local signal matching with signal through signal acquisition and tracking.
- During signal tracking, user will obtain the value of **carrier wave phase** and **doppler drift**, and **code phase** throughout the time.

Pseudorange

$$\begin{aligned} R &= c[T_2 - T_1] \\ &= c[(t_r + dt_r) - (t^s + dt^s)] \\ &= c(t_r - t^s) + c(dt_r - dt^s) \\ &= \rho_r^s + c(dt_r - dt^s) + \varepsilon \end{aligned}$$

where,

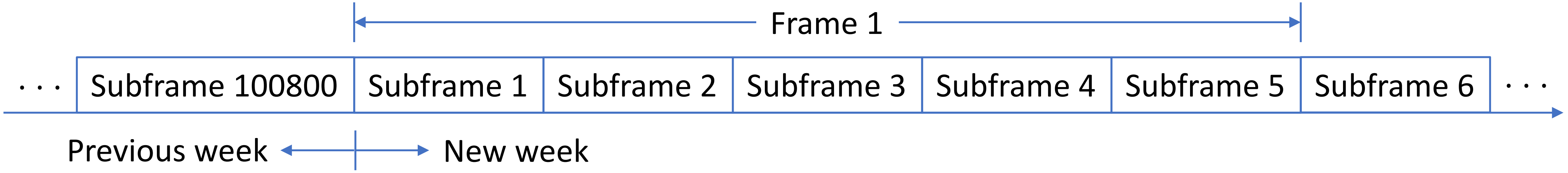
T_2, t_r, dt_r	signal reception time, user's true time and user clock drift
T_1, t^s, dt^s	signal transmission time, satellite's true time and clock drift
ρ_r^s	true distance between satellite and user
$c(dt_r - dt^s)$	user's clock offset w.r.t. satellite (measured in distance)
ε	observation errors (e.g., multipath and atmospheric delay)

Position, Velocity, and Time (PVT) and GNSS Observations

GPS Navigation Message

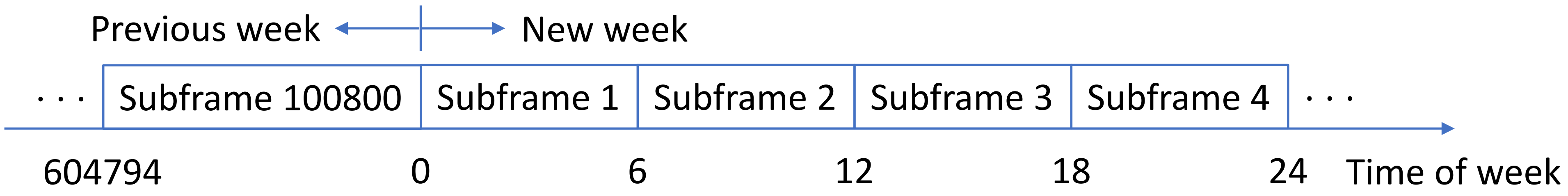
- How to obtain T_2 and T_1 for calculating pseudorange?
  - Signal reception time T_2 can be directly read from user clock.
  - Signal transmission time T_1 can be obtained through GPS navigation message.

- GPS navigation messages are sent by frames.
  - Each frame contains 5 subframes, lasting for 30 seconds.
  - Each subframe contains 300 bits, lasting for 6 seconds.



GPS Time

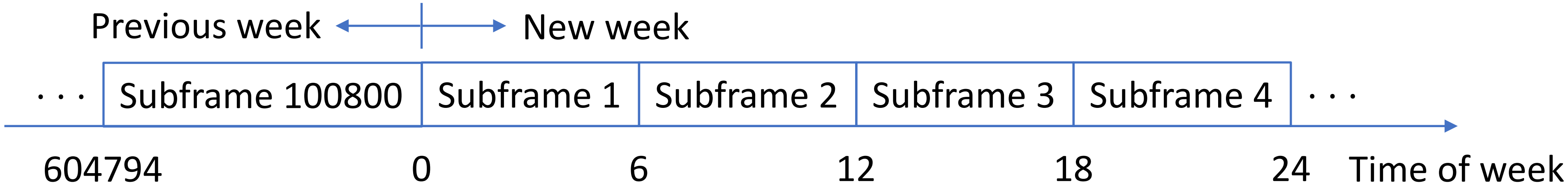
- GPS time consists of two parts
  - GPS week number: number of weeks passed since Jan. 5, 1980.
  - Time of week: how many seconds passed since the beginning of the week.



- GPS week number would overflow if above 1023.
  - Yes, you're right, it's coded as 10 bits in GPS navigation message.

GPS Time (cont'd)

- GPS time is not directed encoded in navigation message
  - Calculate current GPS time based on subframe counter



- GPS time obtained from TOW counter is only accurate to the level of 6 seconds.

Synchronization

- After signal tracking and decoding, user can know when each bit, subframe, and frame start and end in the received signal using prior knowledge, this process is called **synchronization**.

- After synchronization, user would know how many bits have been transmitted in the subframe at current time.
  - One bit lasts for 20 ms, thus using this info user could measure transmission time to an accuracy of milliseconds.
  - Notice 1 ms error leads to an error of 30 km in pseudorange observation.

Ranging Code

- Notice that one bit contains 20 C/A codes, and each C/A code has 1023 chips, thus it’s intuitive to use this information to fine-tune signal transmission time.

- During signal tracking, user knows the code phase of the receiving signal at any time, accurate to decimal place of code chips.
  - Of course, user also knows how many C/A code periods have passed inside the bit.

Ranging Code (cont'd)

- Suppose at signal reception time T_1, the subframe count is n, the bit count is b, and c C/A code period has passed inside the bit, and current code phase is CP, thus signal transmission time is

$$T_2 = 6n + 0.02b + 0.001\times\left(c + \frac{CP}{1023}\right) \quad (sec)$$

- PRN code phase is being used for measuring signal transmission time and derive pseudorange observation, thus both C/A code and P code are also called **ranging code**.

Position, Velocity and Time

- With pseudorange observations, user can solve position and clock offset using trilateration through least square estimation.
  - Thus we know the “P” and “T” in PVT.
- Pseudorange observation contains no information about movement of the user, thus other observations need to be introduced for obtaining user’s velocity
  - Remember the relative speeds of source and receiver determines the Doppler shift, whose value the user already knows during signal tracking.

Doppler Shift

- According to the Doppler effect:

$$s = \lambda \Delta f,$$

Where λ is the wavelength of the transmitting signal (GPS signal frequency), Δf is the Doppler shift, and

$$s = \boldsymbol{e}(\boldsymbol{v}^s - \boldsymbol{v}_r)$$

is the radial speed of the source (GPS satellite $\boldsymbol{v}^s$) and receiver (GPS user $\boldsymbol{v}_r$). $\boldsymbol{e}$ is the unit vector representing the direction of source and receiver.

Doppler Shift (cont'd)

- Similar with pseudorange observation, we have three unknown variable of user's velocity $\boldsymbol{v}_r = (v_x, v_y, v_z)$, and one other extra unknown of user's clock drift (derivative of clock offset).

- A minimum of four Doppler shift observation is needed.
  - Using least square estimation if user has more observations.

GNSS Basic Observations

- There are three basic type of observations in GNSS:
  - Pseudorange,
  - Doppler shift, and
  - Carrier phase.

- Like Pseudorange, which is calculated using the phase of ranging coded, carrier phase is the distance calculated based on the time delay obtained from the phase of carrier wave.
  - Signal tracking loop will keep counting how many periods have passed since the time it locks the carrier wave phase.

Differential GPS (DGPS) and Assisted-GPS (A-GPS)

Selective Availability

- During the 1990s, GPS quality was degraded by the United States government in a program called “Selective Availability” (SA).

- Selective availability will introduce an unknown offset to each satellite’s clock, so the signal transmitting time will be slightly biased compared with true time.
  - Thus all pseudorange observations will be biased and calculated position will be wrong.

Selective Availability (cont'd)

- To express in a mathematical form

$$R = \rho_r^s + c(dt_r - dt^s) + \varepsilon_r^s$$
$$\Downarrow$$
$$R = \rho_r^s + c(dt_r - dt^s) + \varepsilon_r^s + e^s$$

- where e^s is the error introduced by SA for each satellite.

- Thus there will always be more unknowns than the observations.
  - With n satellites, the unknowns will be $n + 4$, consisting of n SA error and 3D position and clock offset.

Differential GPS

- Is there any way to eliminate the SA errors?
  - Notice that satellite is transmitting its signal to all users, so the SA errors are the same for each user in a given area.

- A fixed station with known position can eliminate the SA errors.
  - Use b to stand for the base station:
$$R_b^s = \rho_b^s + c(dt_b - dt^s) + \varepsilon_b^s + e^s$$
  - Compared with user's pseudorange observation:
$$R_r^s = \rho_r^s + c(dt_r - dt^s) + \varepsilon_r^s + e^s$$

Differential GPS (cont'd)

- Make a difference of the two equations:

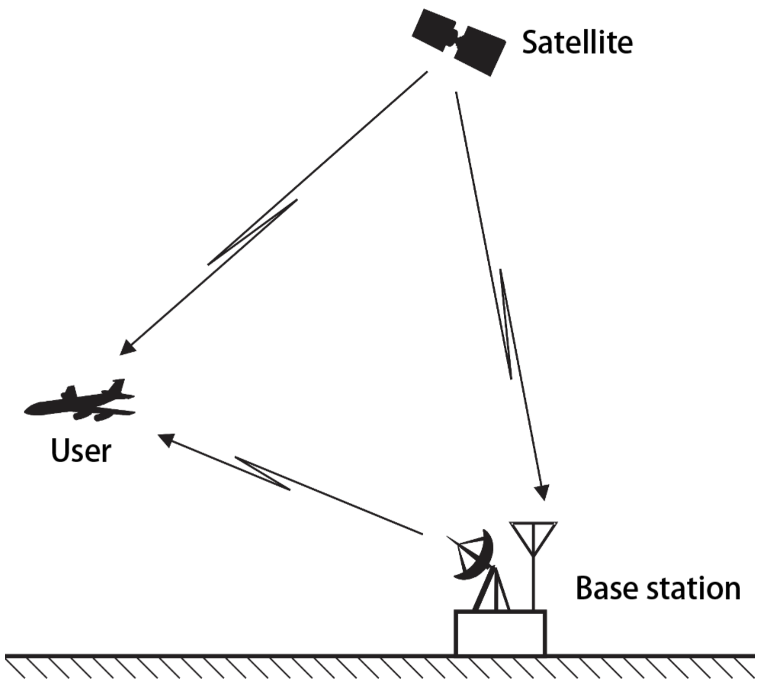
$$R_b^s = \rho_b^s + c(dt_b - dt^s) + \varepsilon_b^s + e^s$$
$$R_r^s = \rho_r^s + c(dt_r - dt^s) + \varepsilon_r^s + e^s$$
$$\Downarrow$$
$$R_b^s - R_r^s = (\rho_b^s - \rho_r^s) + c(dt_b - dt_r) + \varepsilon_b^s - \varepsilon_r^s$$

- SA errors will be eliminated and, same as before, there are four unknowns.
  - $\rho_b^s - \rho_r^s$ contains three unknowns which are user's 3D position.
  - $dt_b - dt_r$ can be treated as one unknown as it's same for all satellites.

Differential GPS (cont'd)

- Differential GPS also helps to eliminate atmospheric error, which could be a major contribution to pseudorange error.

- When base station is near to the user (less than 200 km), the atmospheric error can be regarded as the same.
  - $\varepsilon_b^s - \varepsilon_r^s$ will not have atmospheric error.



Differential GPS (cont'd)

- The accuracy of standard GPS solution is about 10 to 15 meters, and with differential GPS the accuracy is around 1 meter.
  - The distance between base station and the user matters.

- If the base station is near enough (less than 20 km with the user), using carrier phase observations can provide up to centimeter-level accuracy.
  - This procedure is called real-time-kinematic (RTK).

- The DGPS network is also called ground-based augmentation system.

GPS Startup Time

- Almanac contains inaccurate position over time for *all* satellites.
  - It is transmitted repeatedly over 12.5 minutes.
  - All satellites will broadcast same almanac data.

- Ephemeris contains satellite's detailed orbital information.
  - Each satellite broadcasts its ephemeris data every 30 seconds.

- If the user doesn’t have almanac or ephemeris data, typically it will wait to collect enough data before giving any position solution.
  - The waiting time is called startup time.

Reference: https://en.wikipedia.org/wiki/Time_to_first_fix

Assisted GPS

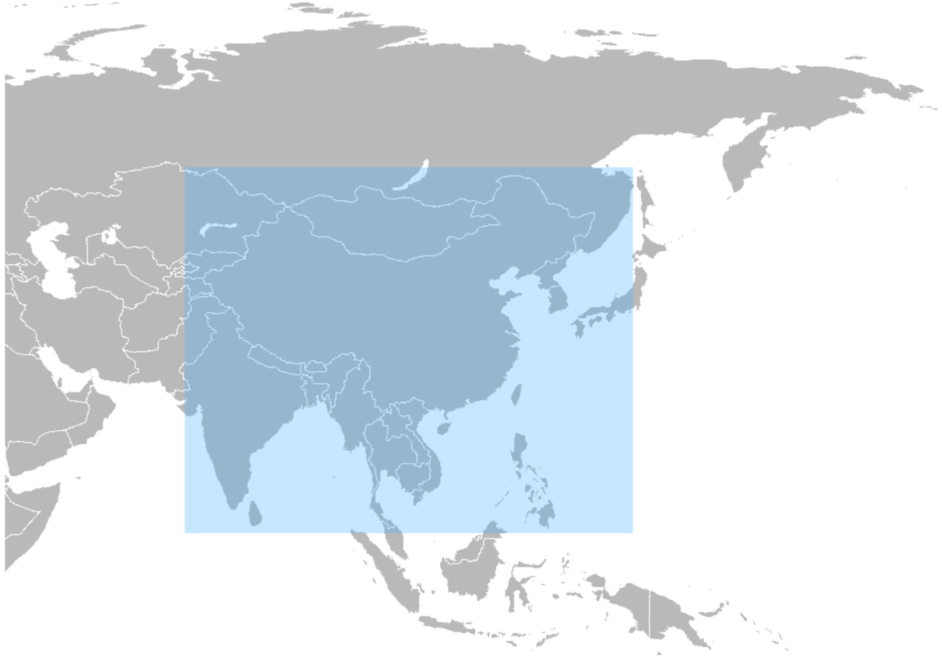
- Typical startup performance for each mode:
  - Cold (no almanac data): 15 minutes
  - Warm (with almanac, no ephemeris): 45 seconds
  - Hot (almanac and ephemeris): < 10 seconds

- Assisted GPS improves the startup performance by
  - sending almanac and ephemeris data to the user through cellular data, so the user would always have a hot start.
  - That’s why one will get an accurate position through the mobile right away when opening navigation apps.

Beidou Overview and Development

Beidou-I

- Beidou-1 consisted of only four satellites and ceases to work after 2012.
- It’s an experimental regional navigation system which only covered limited area around China.
- Beidou-I used active positioning and only allowed limited users. It had no ability for measuring velocity either.



Reference: https://en.wikipedia.org/wiki/BeiDou, https://blog.csdn.net/Times_poem/article/details/51992699

Beidou-II & III

- Beidou-II provides regional navigation service for Asia-Pacific users and consists of 16 satellites.

- Beidou-III provides global services and consists of 35 satellites and has better accuracy.

- Beidou II & III provide PVT service globally with an accuracy of 10 m for position, 0.2 m/s for velocity and 20 ns for time.
  - At Asia-Pacific area the accuracy improves by 50 %.

Reference: http://www.beidou.gov.cn/xt/gfxz/201912/P020191227337020425733.pdf

Beidou & GPS Comparison

- Some unique features compared with GPS:
  - Beidou has SMS capability, which could help in emergency situations where cellular data is unavailable.
  - Beidou is currently the only GNSS with inter-satellite link ability, which helps improve accuracy and eliminate the need of global ground control segment.

- Some other advantages of Beidou:
  - Beidou has more satellites in total and will have more visible satellites especially in Asia-Pacific area.

Accuracy of Beidou & GPS

	GPS (global)	Beidou (global)	Beidou (Asia-Pacific)
Position (m)	8	10	5
Velocity (m/s)	0.2	0.2	0.1
Time (ns)	30	20	10

These data come from the official report and are quite conservative (95 percentile), on an average basis the accuracy would be much better. For example, the actual measured average global positioning accuracy of BDS is 2.34 m. And GPS is expected to have similar performance if not better.

Reference: https://www.gps.gov/technical/ps/2020-SPS-performance-standard.pdf, http://en.beidou.gov.cn/WHATSNEWS/202008/t20200803_21013.html

Atomic Clock of GPS & Beidou

- The accuracy and stability of the atomic clock is essential to the accuracy of pseudorange observation, and thus is a key contribution of positioning accuracy.

	300 s	**1000 s**	**1 d**
GPS	8.91×10^{-14}	4.14×10^{-14}	4.52×10^{-15}
Beidou-II	2.02×10^{-13}	4.52×10^{-14}	2.08×10^{-14}
Beidou-III	9.43×10^{-14}	2.49×10^{-14}	8.64×10^{-15}

Reference: https://www.mdpi.com/2072-4292/11/24/2895

Beidou Status and Future

- More than 70 % of smartphones in China already support the BeiDou navigation service.
  - Huawei P40, Mate 30
  - One plus 8
  - Xiaomi Mi 10, Redmi Note 9
  - Oppo F17, Reno 4
  - Google Pixel 5
  - Samsung Galaxy S20, M51 ...

Reference: http://www.xinhuanet.com/2020-05/21/c_1126015789.htm

Beidou Status and Future (cont’d)

- Currently in civil GNSS receiver, GPS and Beidou are not competitors, but cooperators.
  - Most chips support many GNSS such as GPS, Beidou, GLONASS, Galileo, and even QZSS, in order to have more observations and achieve better accuracy.
- The price of Beidou chips has been greatly reduced since 2012, when Beidou-II began providing positioning service.

Beidou Status and Future (cont'd)

- Continuous development of ground-based augmentation systems such as QianXun has substantially enhanced Beidou service accuracy and reliability.

- Next generation of Beidou is on the way!
  - Based on Beidou-III, Beidou-IV aims to further improve PVT, SBAS, SMS, international rescue, PPP, and ISL before 2035.

Reference: https://www.chainnews.com/articles/146822831403.htm, http://www.chinanews.com/gn/2018/05-27/8523949.shtml

Takeaways

- GNSS has PVT ability, solving users' position, velocity and time.
- Basic GNSS observations include pseudorange, Doppler and carrier phase.
- Ground-based augmentation system uses differential GPS technique to enhance positioning performance and may lead to cm accuracy.
- Assisted GPS is commonly used in mobiles to reduce GNSS startup time.

Advertisements ▪ᴗ▪

Github for GPS simulation:
@PenroseWang/SimGPS

Wechat Official Account:
luoxiaoheidebiji

Medium: @penrosewang