GPS Signal Processing

Pengluo Wang & Yang Mi

Start with Side Dishes

- Use of GPS in the Gulf War
  - Before the Gulf War, localization is mainly achieved by using fixed landmarks as references.
  - Under severe conditions, landmarks will have great uncertainty, making it hard to locate position accurately.
  - GPS allowed the alliance to know their accurate positions, and also enabled precision strike to the enemies.

One More Side

- Xingxin Gao got scolded by Chinese media because of a paper.
  - She derived Beidou's civil PRN code before it has been released by Chinese government four years later.

- Is she really a traitor, as said by some mainstream?
  - No. It actually helped other scientists analyze Beidou signal characteristics on an early stage, which is beneficial for the development of Beidou.

- How did she derive civil code?
  - Can military code be derived in the same manner?

Reference: http://web.stanford.edu/~gracegao/publications//magazine//InsideGNSS%20Compass%2007.pdf, https://en.wikipedia.org/wiki/BeiDou

Outline

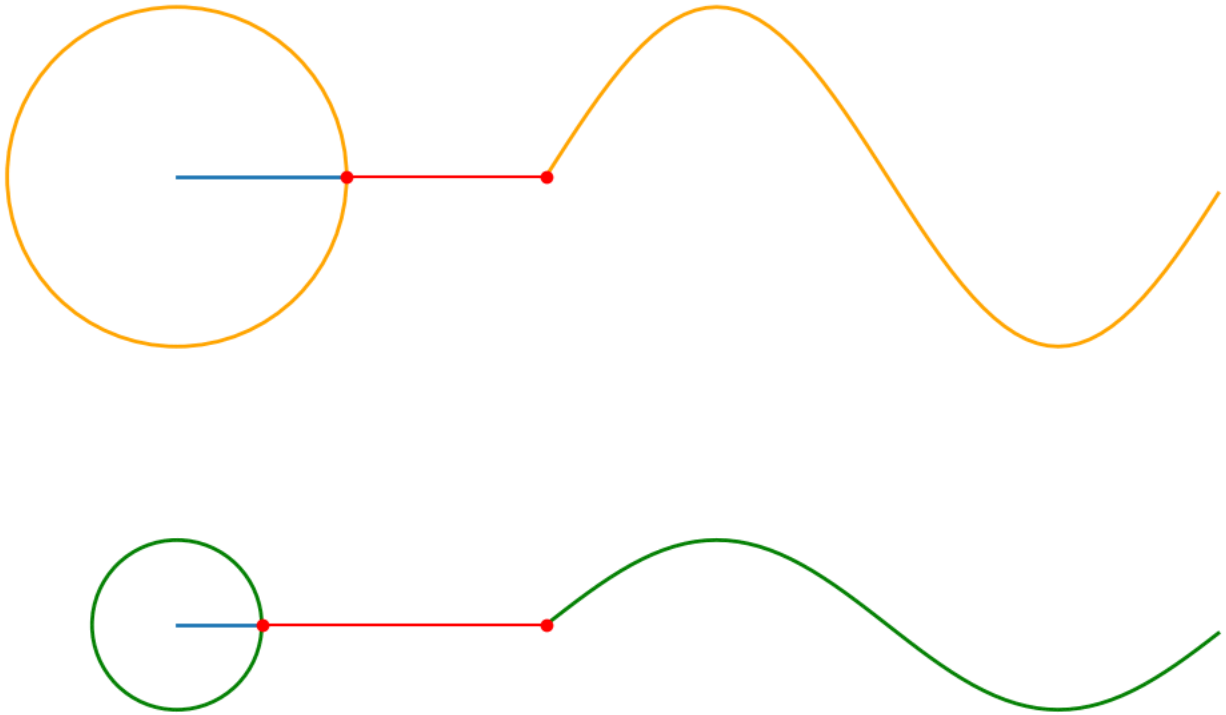
- Digital signal processing (DSP) background
  - Signals, modulation, and correlation
- GPS signal structure
  - Carrier wave, pseudorandom code, and navigation message
- Data demodulation
  - Signal acquisition and tracking

DSP Background

Signals, data modulation, and correlation

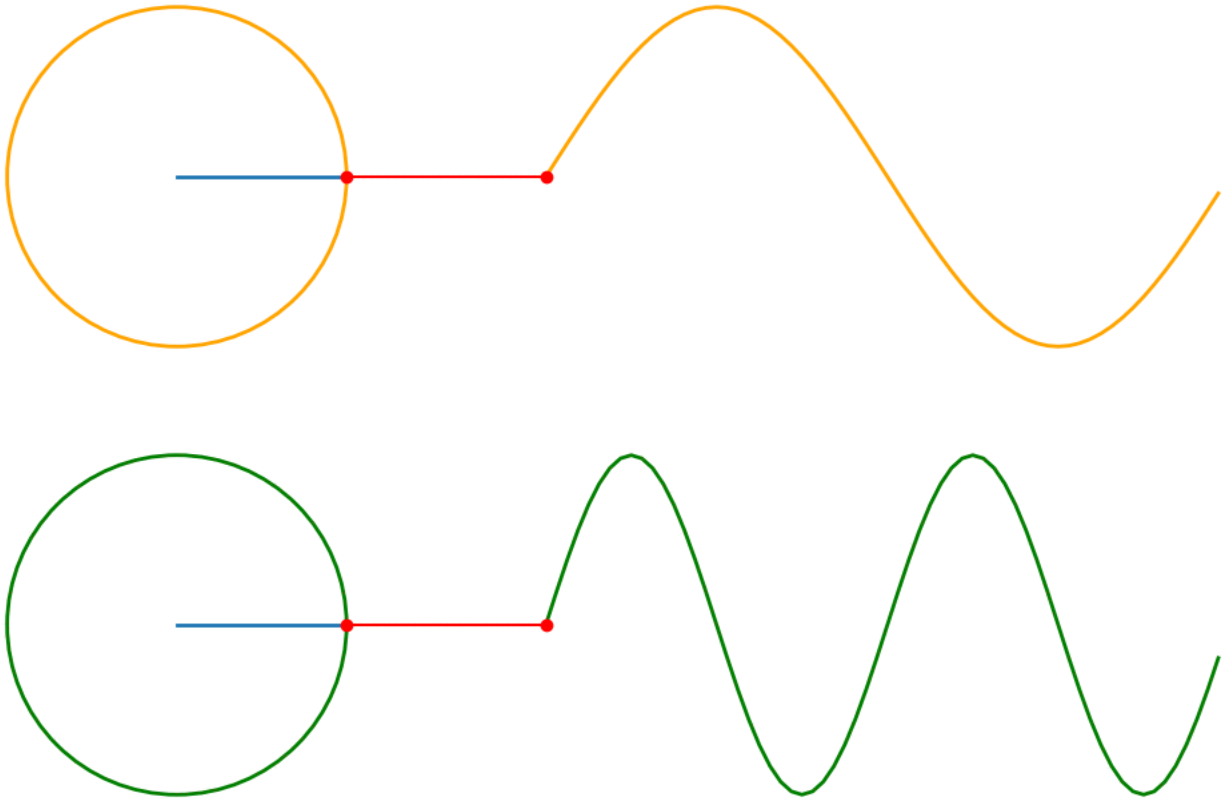
DSP Background – Signals

- Amplitude



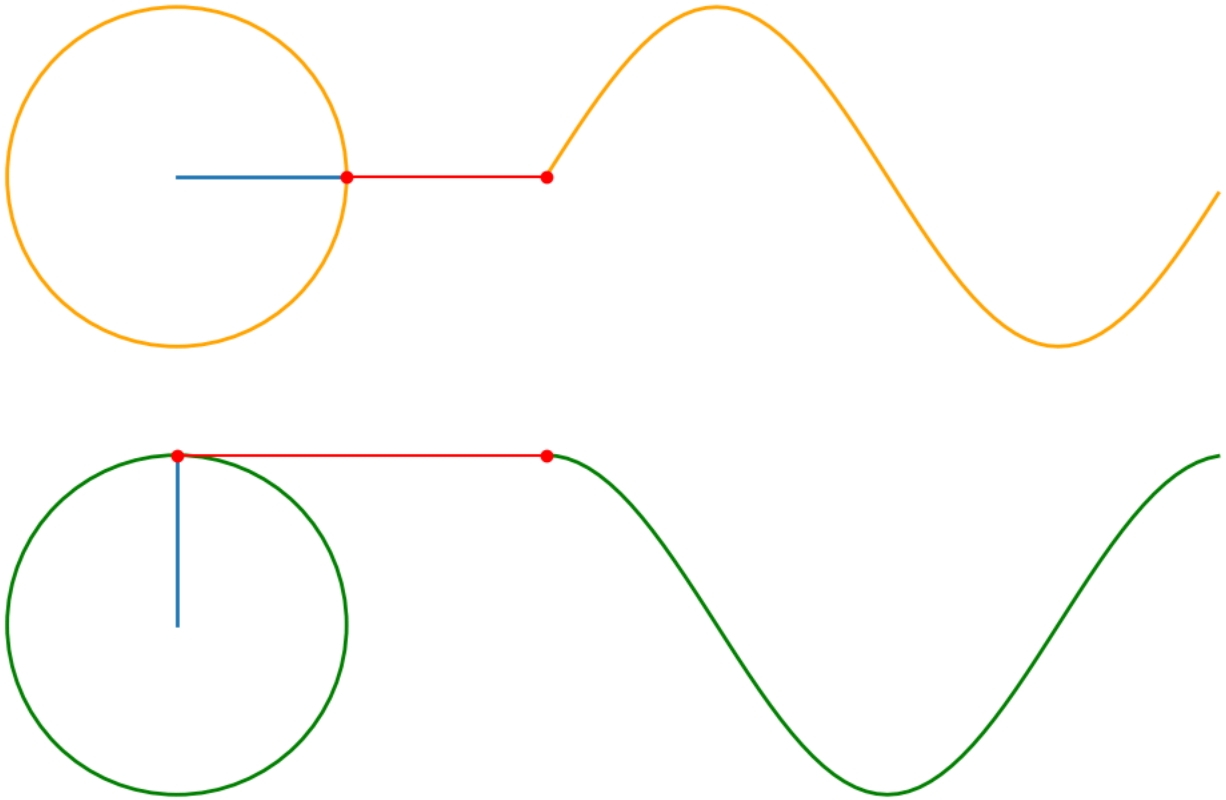
DSP Background – Signals (cont’d)

- Frequency



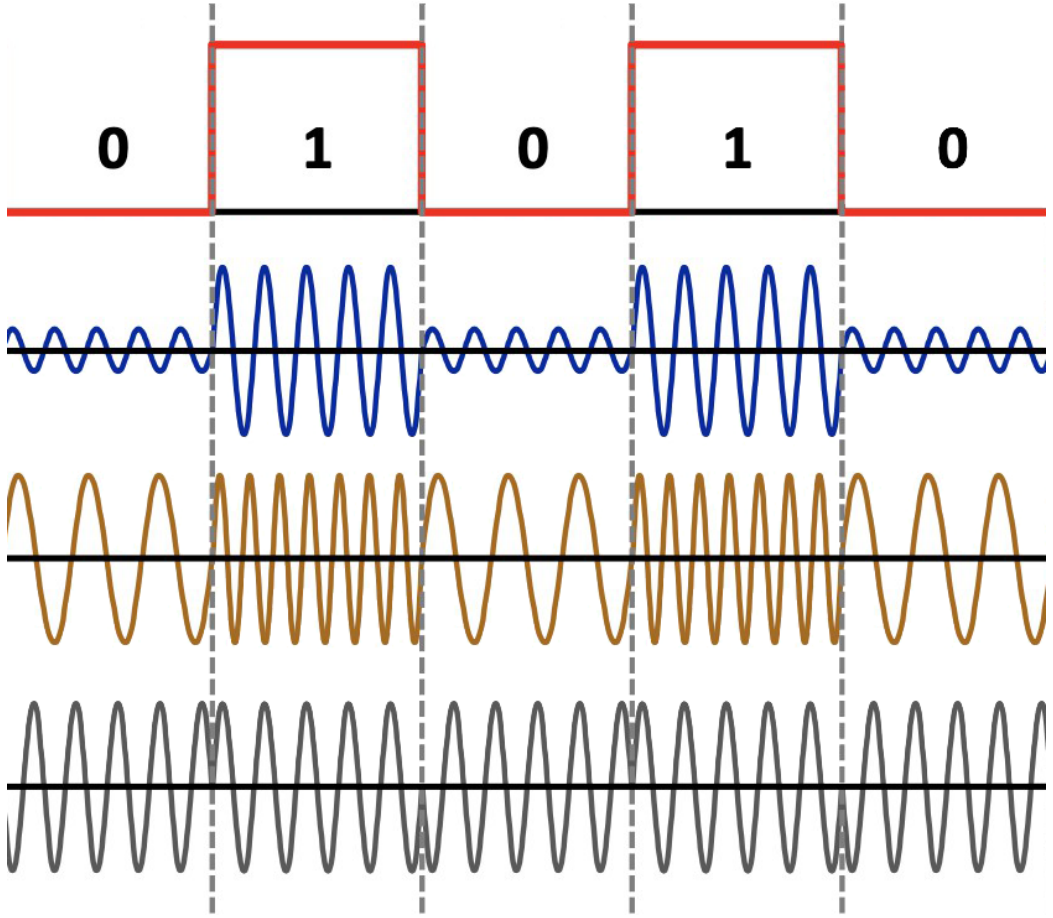
DSP Background – Signals (cont’d)

- Phase



DSP Background – Modulation

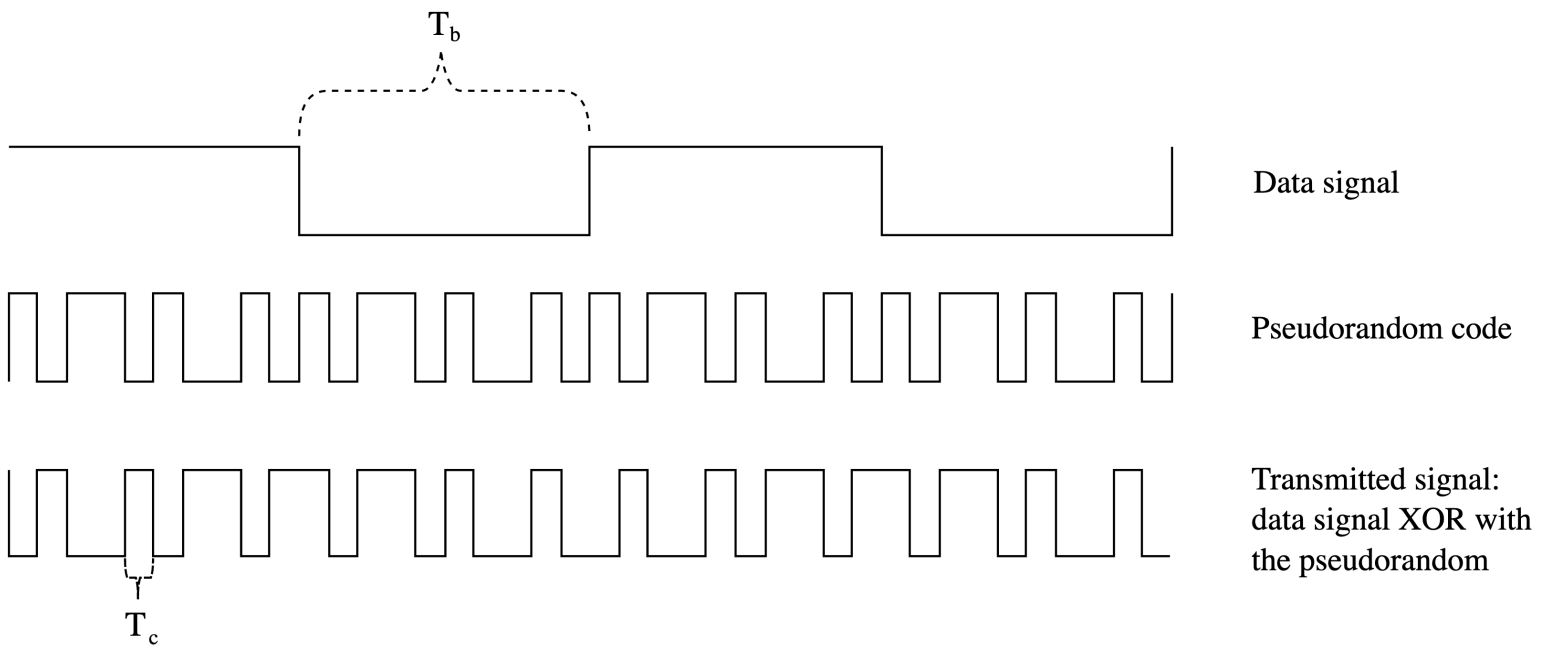
- Amplitude Shift Key (ASK)
  - modulate on amplitude
- Frequency Shift Key (FSK)
  - modulate on frequency
- Phase Shift Key (PSK)
  - modulate on phase
- Only one user is allowed for one frequency band.



Reference: http://www.atlantarf.com/Downloads.php

DSP Background – CDMA

- Code-division multiple access (CDMA) is used for allowing *multiple* users on one frequency band at the same time.
  - Each user has a specific pseudorandom noise (PRN) code for identification.



Reference: https://en.wikipedia.org/wiki/Code-division_multiple_access

DSP Background – Demodulation of CDMA

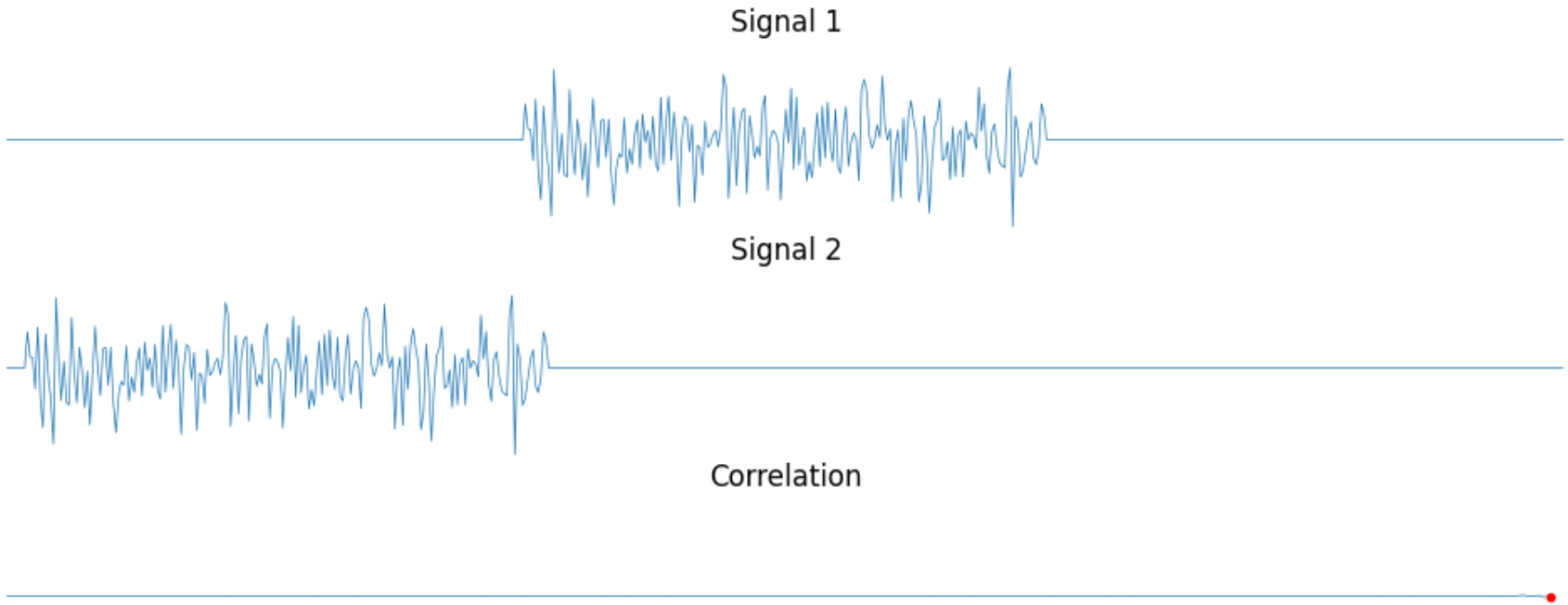
- PRN code functions as a key for communication with received signal and the user.
  - Different users have different keys (PRN codes).
  - One door (data modulated for one user) can be only opened by one key.
  - Inserting the key into the door is done by *correlation*.

- User will generate PRN code locally to correlate with received signal.
  - If signal matches the local PRN code, the correlation will be high.
  - Data information is indicated by the sign ($\pm$) of correlation result, this process is called data *demodulation*.

Reference: https://en.wikipedia.org/wiki/Code-division_multiple_access

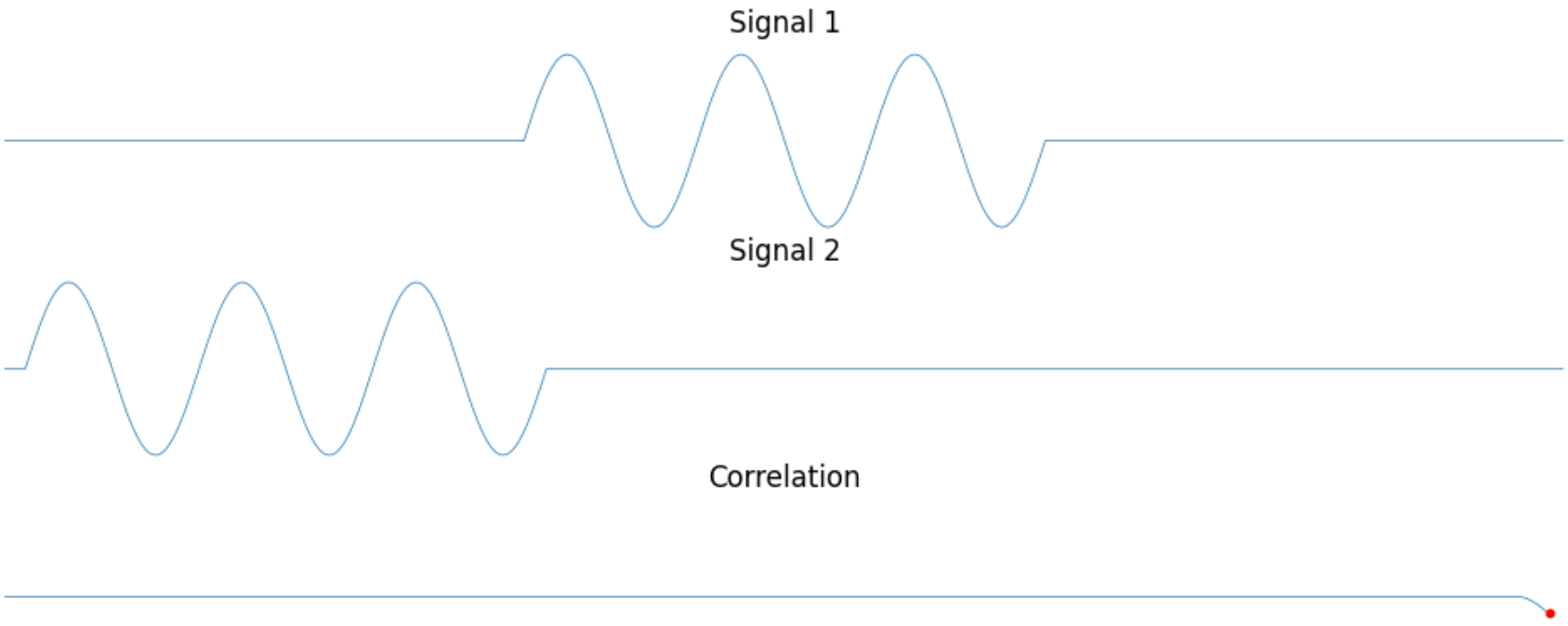
DSP Background – Correlation

- Correlation measures the similarity of two signals.



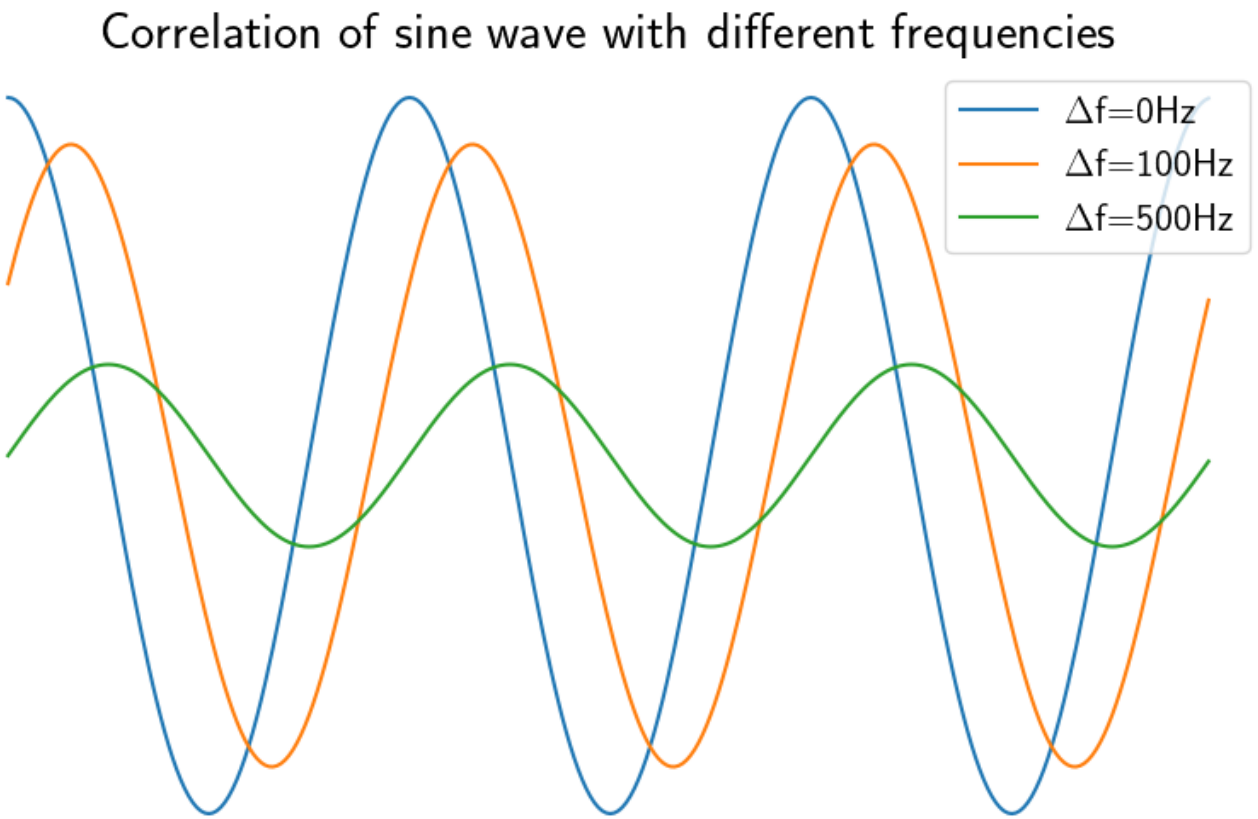
DSP Background – Correlation (cont’d)

- Correlation measures the similarity of two signals.



DSP Background – Correlation (cont'd)

- Correlation measures the similarity of two signals.



DSP Background – Correlation (cont'd)

- Correlation definition

$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]y^*[n-m]$$

- Some key points:
  - Random noise $r[n]$ only correlates with itself

$$R_{rr}[m] = \begin{cases} 1, & m = 0 \\ 0, & otherwise \end{cases}$$

  - The more sinusoid signal frequencies differ, the less their correlation will be.

GPS Signal Structure

Carrier wave, pseudorandom code, and navigation message

GPS Signal Structure

Carrier wave

×

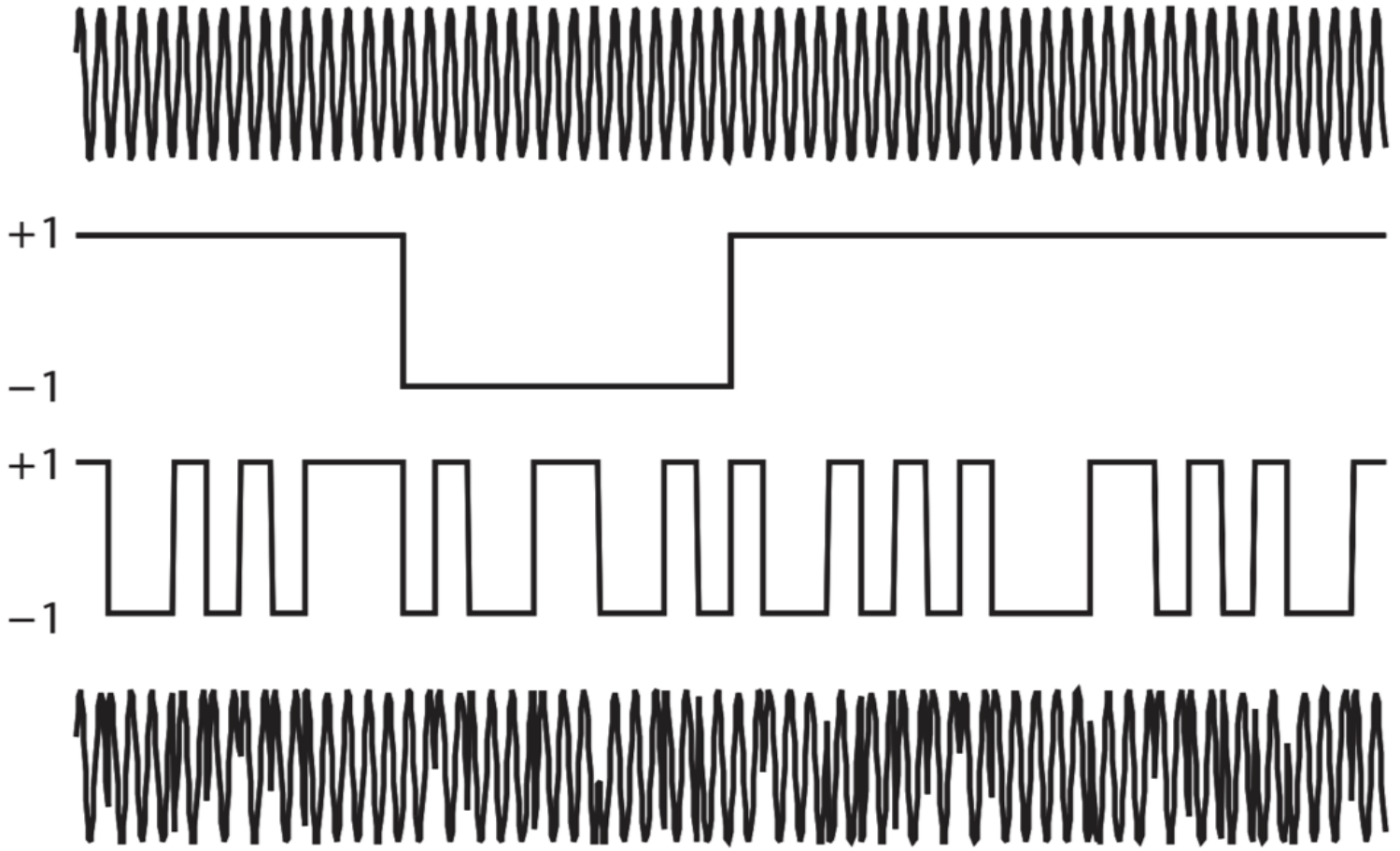
Navigation message

×

Pseudorandom code

⇓

GPS signal (w/o noise)



Carrier Wave

- Sine wave with frequency of 1575.42 MHz.
  - High frequency ensures users to receive accurate data under all weather conditions.
  - High frequency will also lead to strong degradation, thus received signals are rather weak, like the light from a 25W lightbulb shining 20,00 km away.

- Due to Doppler effect, received carrier frequency may be different.
  - The faster the user moves, the bigger the frequency difference will be.
  - Typically Doppler shift (frequency difference) is within $\pm$ 10 kHz.

Reference: https://www.cbo.gov/sites/default/files/112th-congress-2011-2012/reports/10-28-GPS.pdf

C/A Code and P code

- Two kinds of *pseudorandom noise* (PRN) codes are modulated.

- Coarse/acquisition (C/A) code with 1023 chips, chip rate 1.023 MHz.
  - Code sequence is open for civil use.
  - Each satellite will have a specific code with a period of 1 ms.

- Precise (P) code with 2.35×10^{14} chips, chip rate 10.23 MHz.
  - The original period of P code is more than half a year, it's been cut to a period of 1 week when broadcast by each satellite.

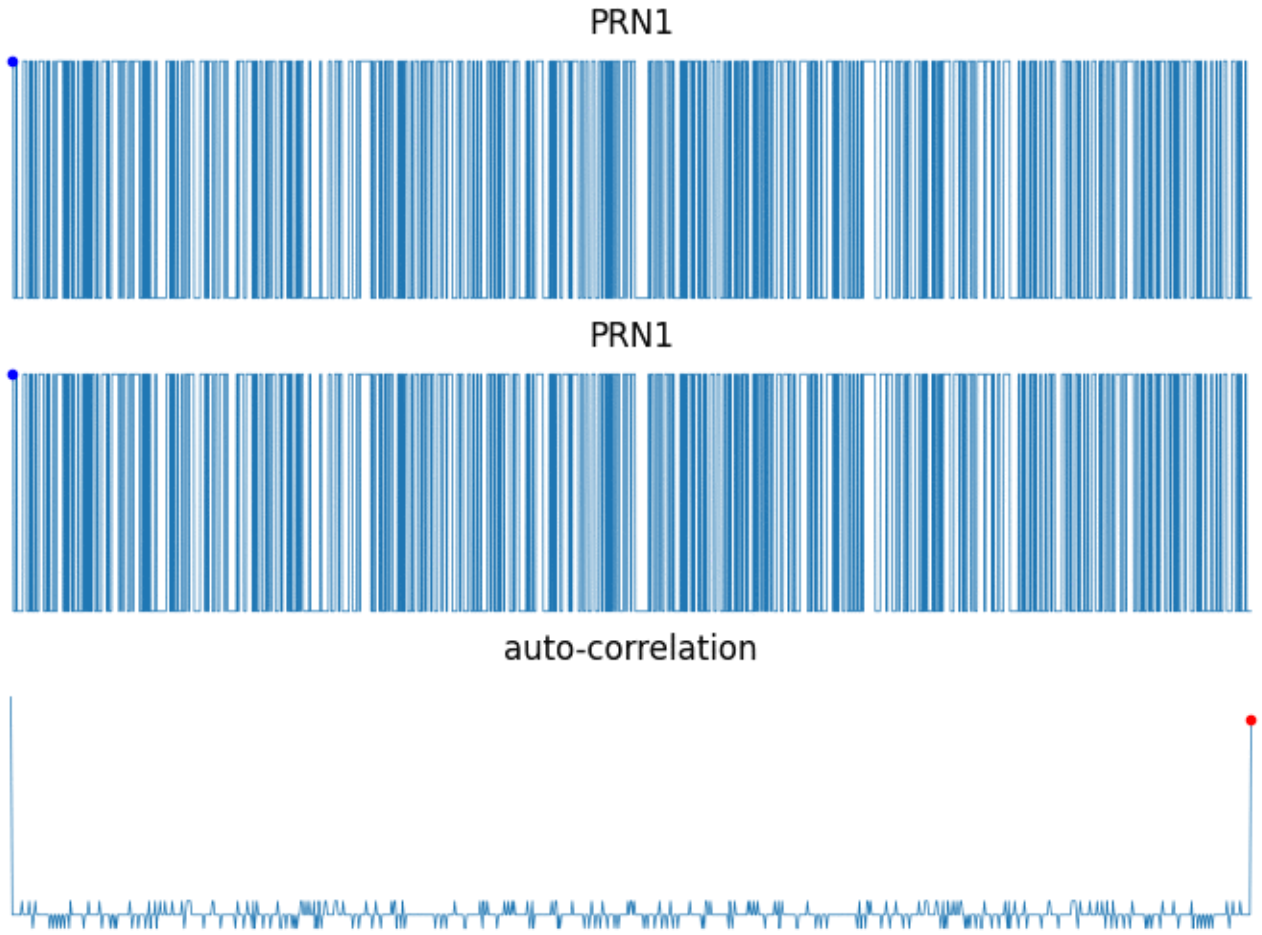
More on C/A Code and P Code (cont'd)

- Recall Xingxin Gao's work to derive BeiDou C/A code.

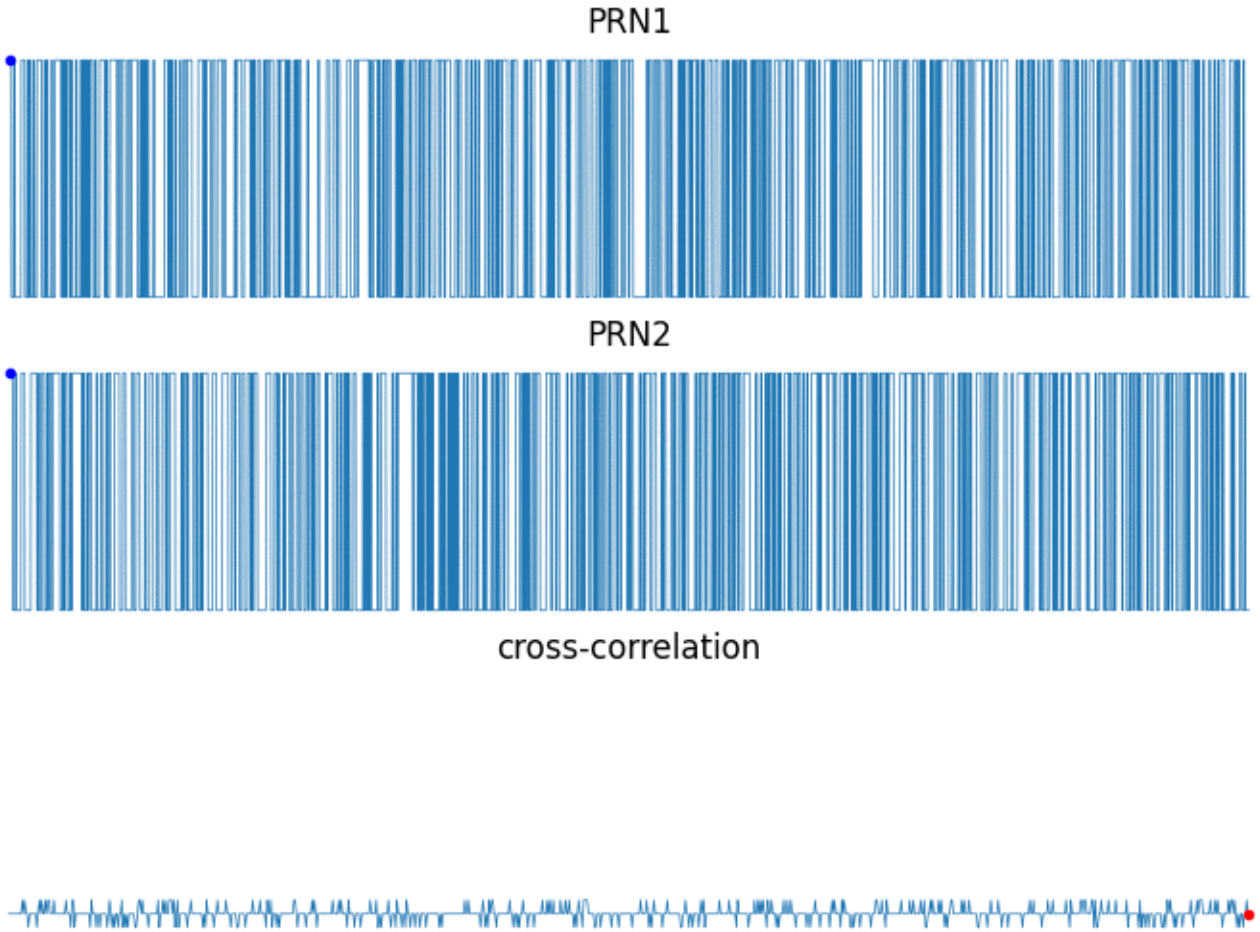
- Deriving C/A code sequence is not hard, but
  - it requires a received signal lasting for 50 C/A code period (50 ms).

- Well, using same method for hacking military code, you have to
  - collect signal for almost a year (50 weeks), and
  - gamble that the military encrypted code won't change during that time.

PRN Code – Auto-correlation



PRN Code – Cross-correlation

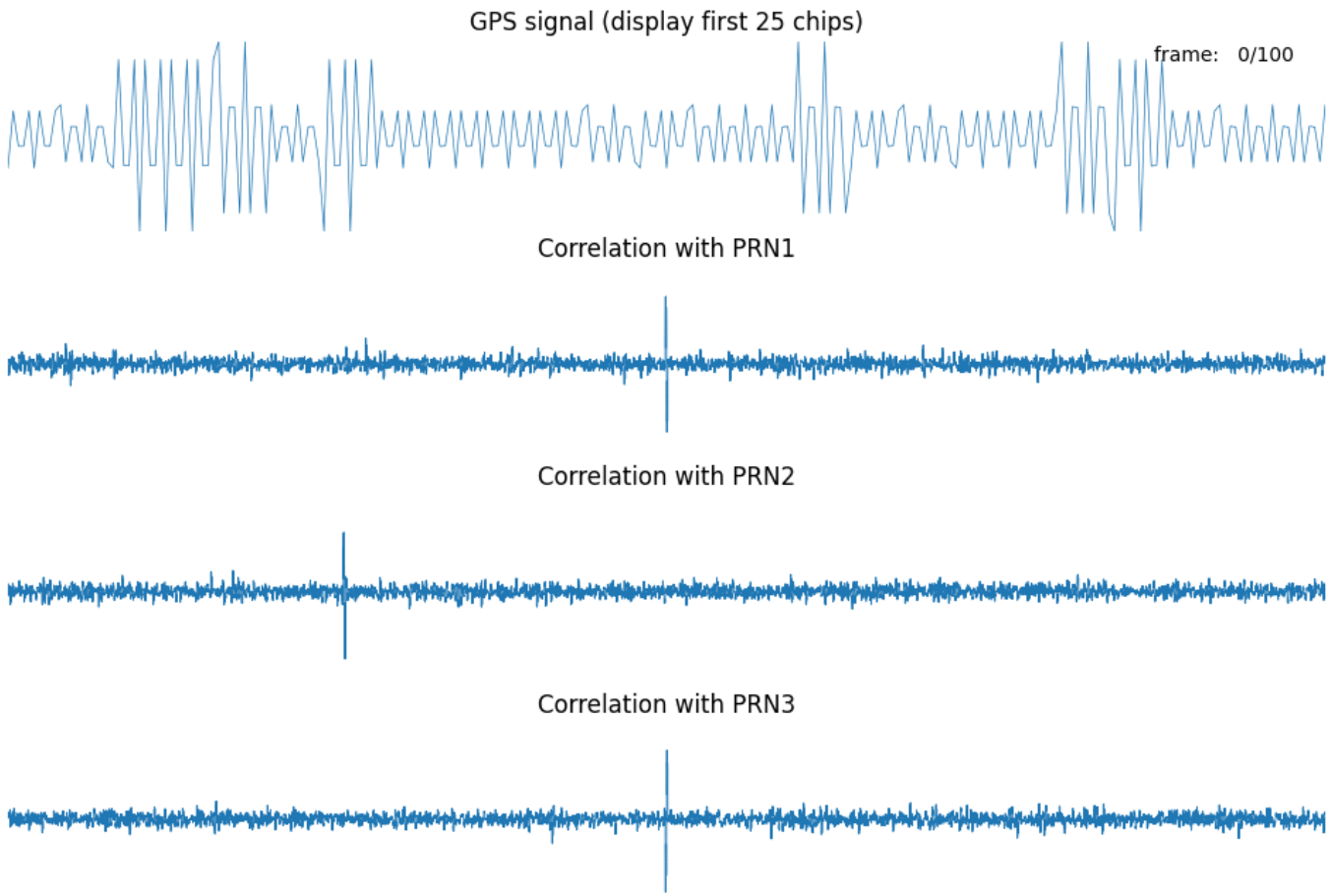


PRN Code – GPS Signal Correlation

- Signal received by the user is a combination of
  - signals transmitted from different satellites,
  - plus noise caused by receiver, multipath, etc.

- What would be the results if we correlate received signal with different PRN code?
  - Received signal will have signals transmitted by three satellites.
  - PRN1 will have no phase shift, no Doppler (carrier frequency difference).
  - PRN2 will have phase shift, no Doppler.
  - PRN3 will have no phase shift, but Doppler is changing across time.

PRN Code – GPS Signal Correlation (cont'd)



PRN Code – GPS Signal Correlation (cont'd)

- Location of correlation peak indicates code phase shift value.
  - Peak should locate at the center to demodulate data successfully.
  - Need to know initial code phase of received signal to "move" the peak at the center, by generating a local PRN signal with same phase.

- Doppler frequency difference will impact correlation peak amplitude.
  - PRN code is modulated on carrier wave, and a large difference of sine wave frequency or phase will lead to small correlation peak
  - Need to know initial Doppler frequency and phase to ensure a correlation peak with carrier wave.

Navigation Message

- Broadcast at 50 bps.
  - A total of 25 frames, each frame takes 30 seconds to be transmitted.
  - Each satellite transmits its own navigation messages all the time.

Satellite parameters	Contains signal transmission time, SV health status, etc.	Data are updated per 2 hours.
Ephemeris	Contains detailed orbital information for each satellite	Data are updated per 2 hours.
Almanac	Contains inaccurate position over time for *all* satellites	Data are broadcast across 25 frame and updated every week approximately.

Data Demodulation

Signal acquisition and tracking

GPS Signal Demodulation

- Data modulation can be implemented by correlation.
  - After receiving GPS signal, the user will also generate a local signal with carrier wave and PRN code signal.
  - Sign of the correlation results indicates if the data modulated is 1 or -1.

- For generating suitable local signals, the user needs to know
  - satellite ID (if satellite is visible),
  - carrier frequency (with Doppler shift),
  - initial carrier phase, and
  - initial PRN code phase.

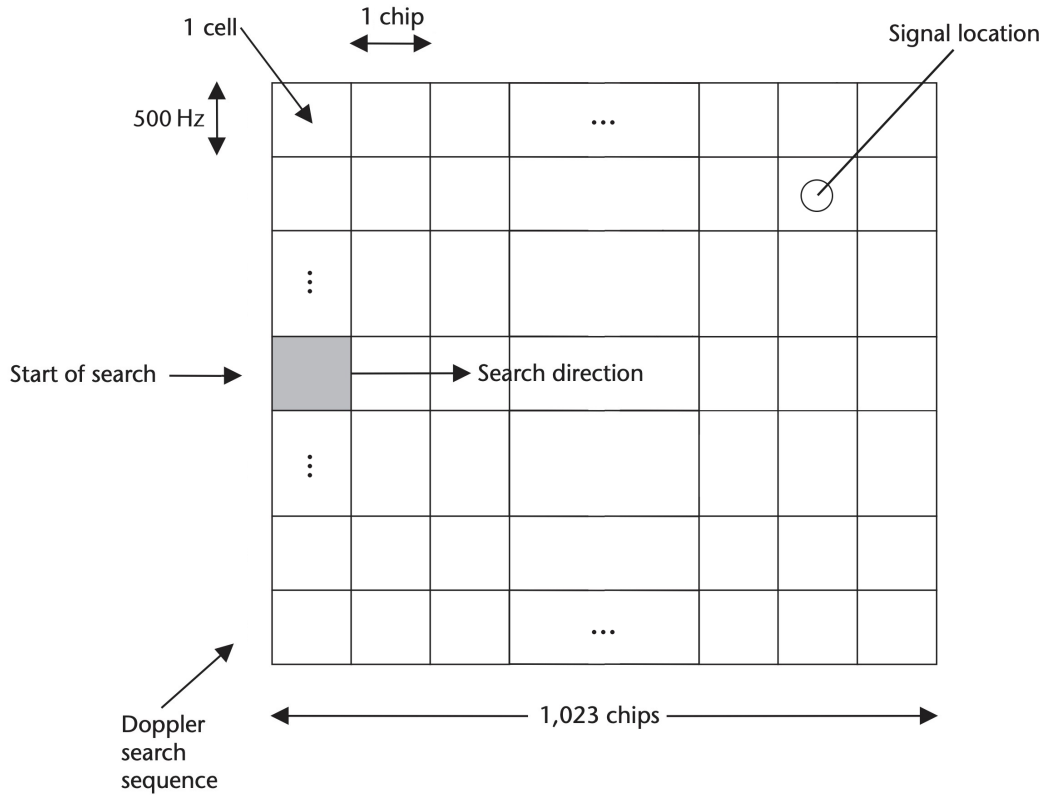
Signal Acquisition

- Satellites have different distance and relative speed w.r.t. user,
  - thus received satellite signals have different carrier/code phase (decided by distance), and difference Doppler shift (decided by relative speed).

- Signal acquisition is a 3D search in the following dimensions:
  - satellite ID (PRN code sequence) ⇒ satellite dimension
  - carrier frequency (with Doppler shift) ⇒ frequency dimension
  - initial PRN code phase ⇒ code dimension

Search Space

- Considering search only in 2D, search in carrier frequency and code.



Reference: Elliott D. Kaplan. Understanding GPS : Principles and Applications. Boston :Artech House, 1996.

Search Space (cont'd)

- Signal acquisition search space:
  - satellite dimension = 31 (total number of GPS satellites)
  - frequency dimension = 41 (if step size = 500 Hz within $\pm$ 10 kHz range)
  - code dimension = 1023 (if step size = 1 chip for C/A code)
  - thus total search space = $31 \times 41 \times 1023$. (It's HUGE!)

- It would be very slow if searching each bin one by one.
  - Doppler and code phase search can be speeded up using FFT.
  - Some prior can be used when searching.

Priors for Signal Acquisition

- With almanac data, the user will know beforehand the visible satellites, thus reducing one dimension for satellite search.
- With ephemeris data, the user could estimate Doppler shift and initial code phase, greatly reducing the remaining 2D search space.
- After signal acquisition, almanac is no longer useful.

More on C/A Code and P Code

- For P code the search space at code dimension is 6.18×10^{12}!
  - It's impossible to acquire code phase in real time without any help of prior.
  - The previous search method requires receiving signal for at least one PRN code period. Recall that P code period is one week!

- After tracking using C/A code to obtain current time first, the user can estimate P code phase, reducing the 2D search space.

- That's why C/A code is called **coarse** acquisition code.

Signal Tracking

- After signal acquisition, the user has a rough estimate for Doppler shift and code phase.
- Signal tracking can fine-tune the estimation to an accurate level, and also obtain carrier phase accurately.
- User will track the status of received signal.
  - Adjust the estimation of carrier frequency, carrier phase, and code phase, which may slowly across time.

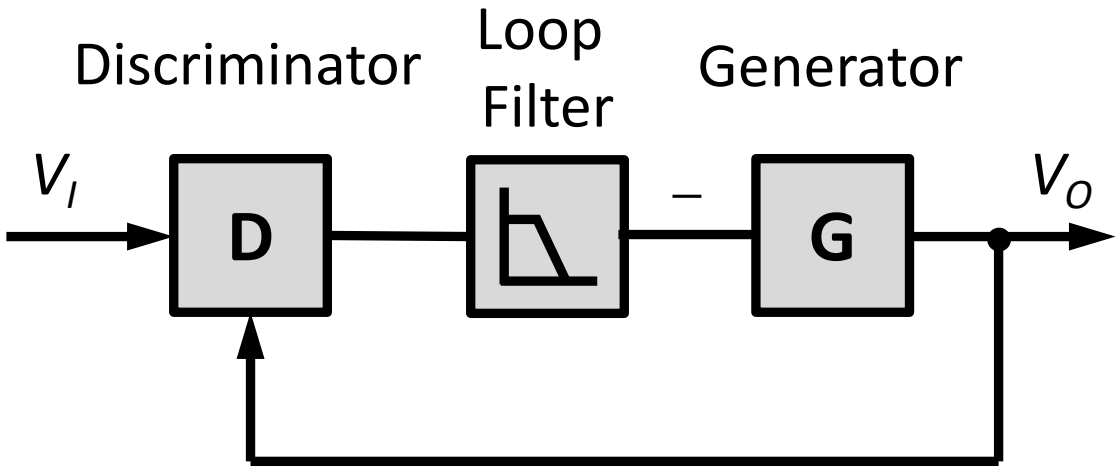
Tracking Modules

- Frequency-locked loop (FLL) ⇒ tracking carrier frequency
- Phase-locked loop (PLL) ⇒ tracking carrier phase
- Delay lock loop (DLL) ⇒ tracking code phase

- All these lock loops are electronic control systems based on *negative feedback loop*.

Negative Feedback Loop

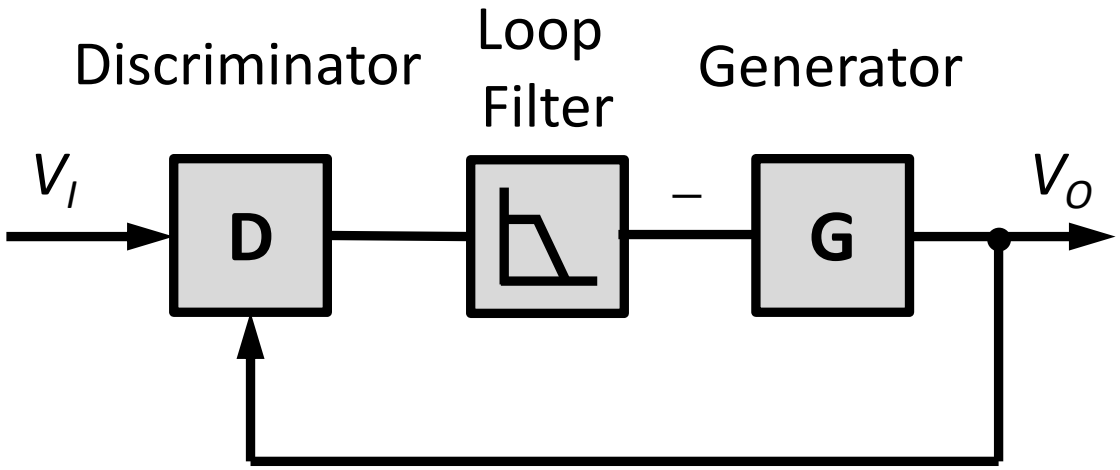
- Negative feedback loop structure



- Discriminator will calculate difference between input and output.
  - Either frequency (FLL), carrier phase (PLL), or code phase (DLL).

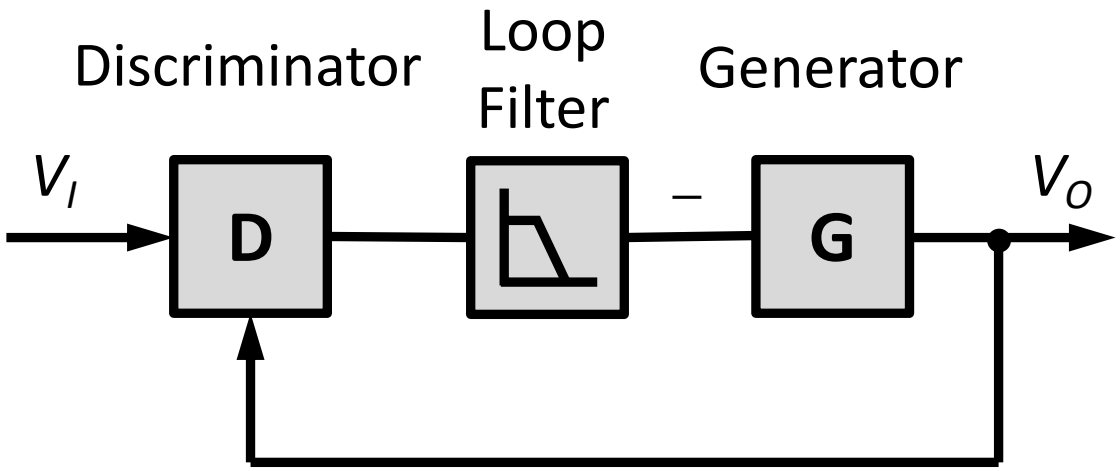
Negative Feedback Loop (cont'd)

- Negative feedback loop structure



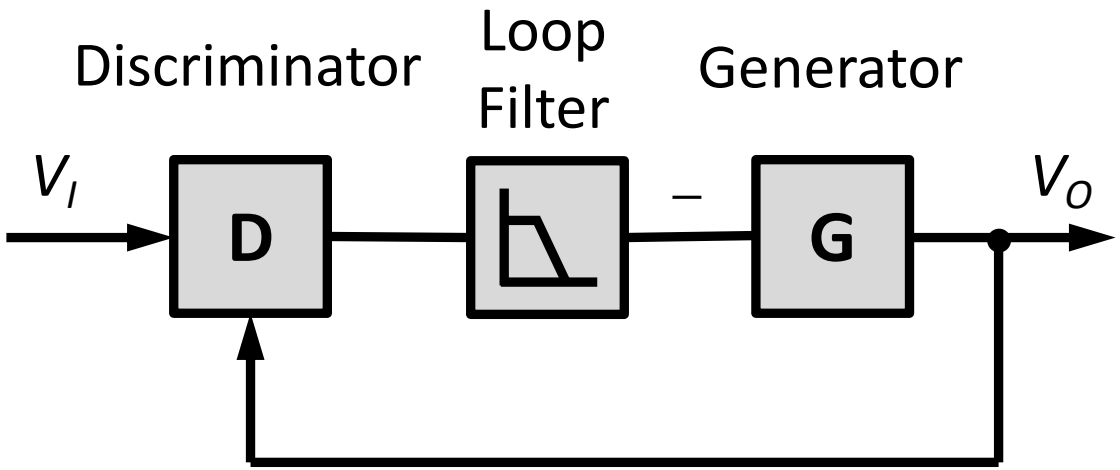
- Loop filter will output a value to control generator.
  - The value is calculated based on previous history.
  - Filter output is the negative of generator's input, hence "negative" feedback.

Negative Feedback Loop (cont'd)



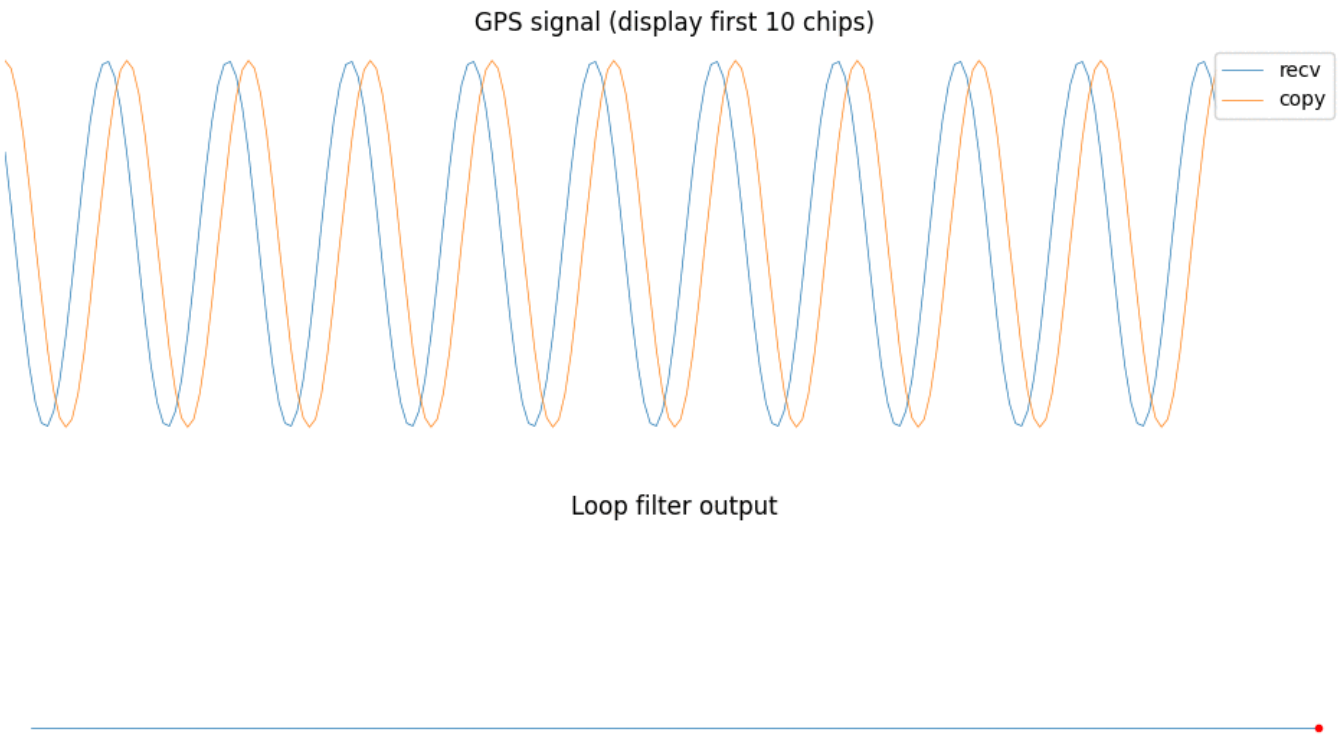
- Generator will generate new output signal based on its input.
  - Taking FLL as an example.
  - If generator input is f_i, and previously the generated signal frequency is f_o.
  - Next time generator will generate new output signal with frequency $f_i + f_o$.

Negative Feedback Loop (cont'd)



- If input and output signal are the same:
  - Discriminator's output will be 0 ($f_I = f_O$).
  - Loop filter's output will be 0 after a short time (considering the filter history).
  - Generator will have zero input ($f_i = 0$), thus keep generating previous output with frequency $f_o = f_i + f_O = f_O$.

FLL

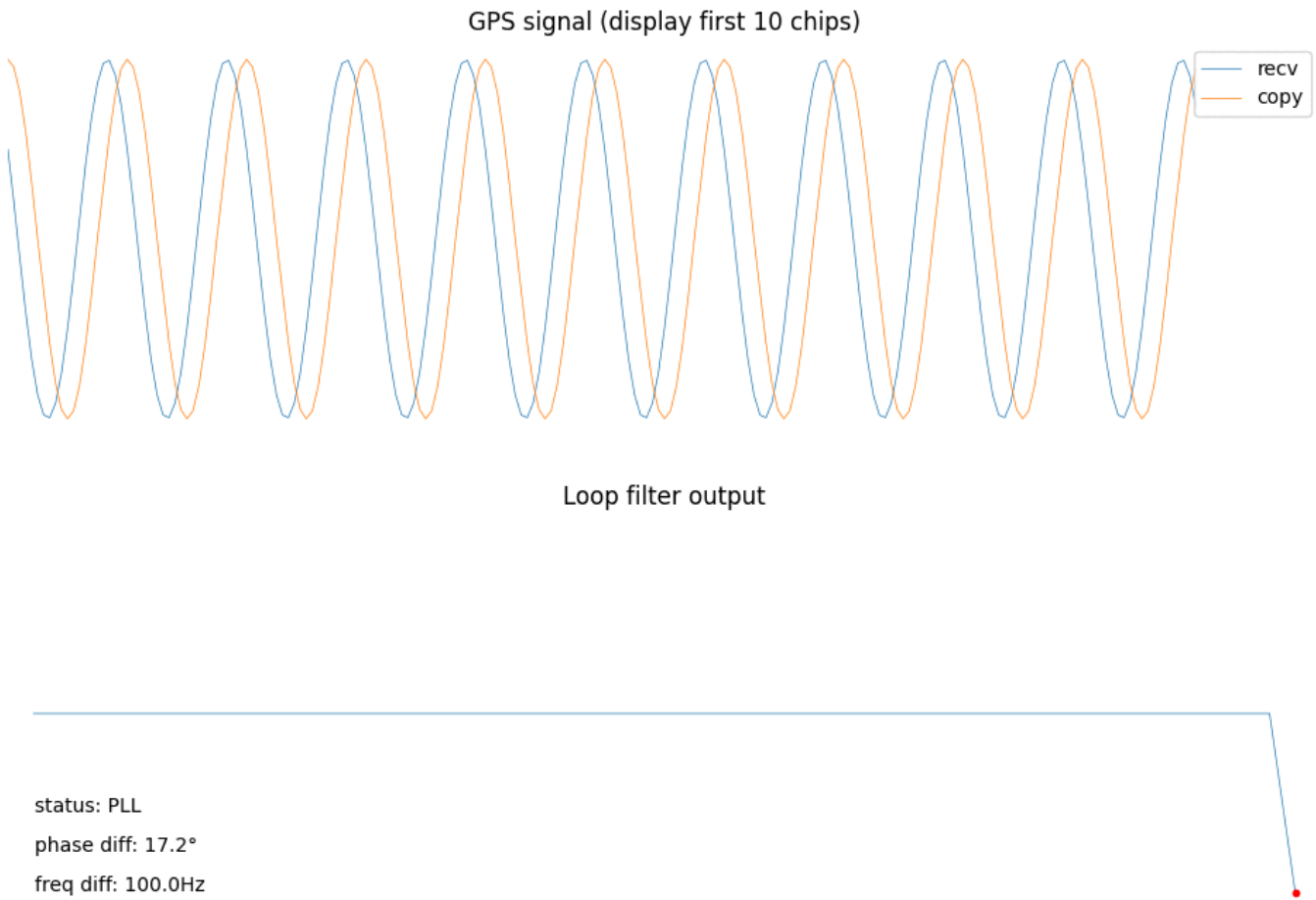


status: FLL

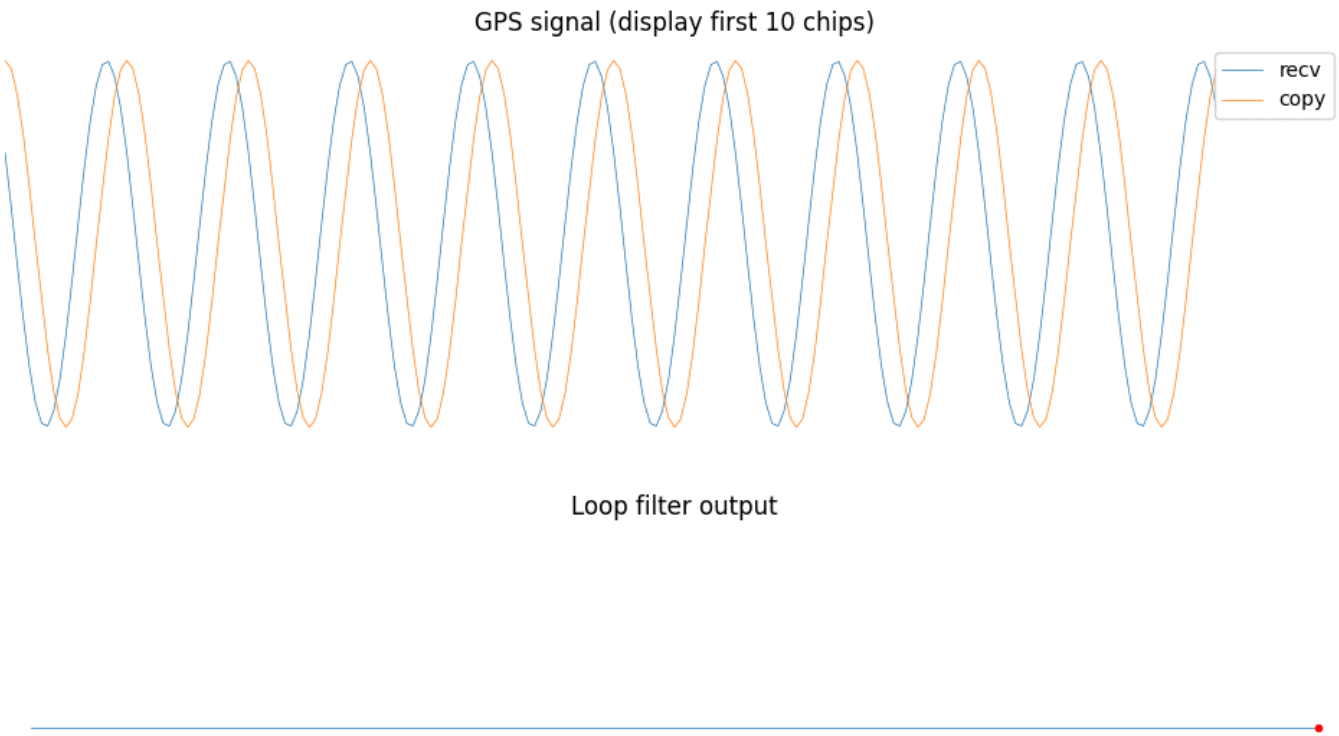
phase diff: 17.2°

freq diff: 100.0Hz

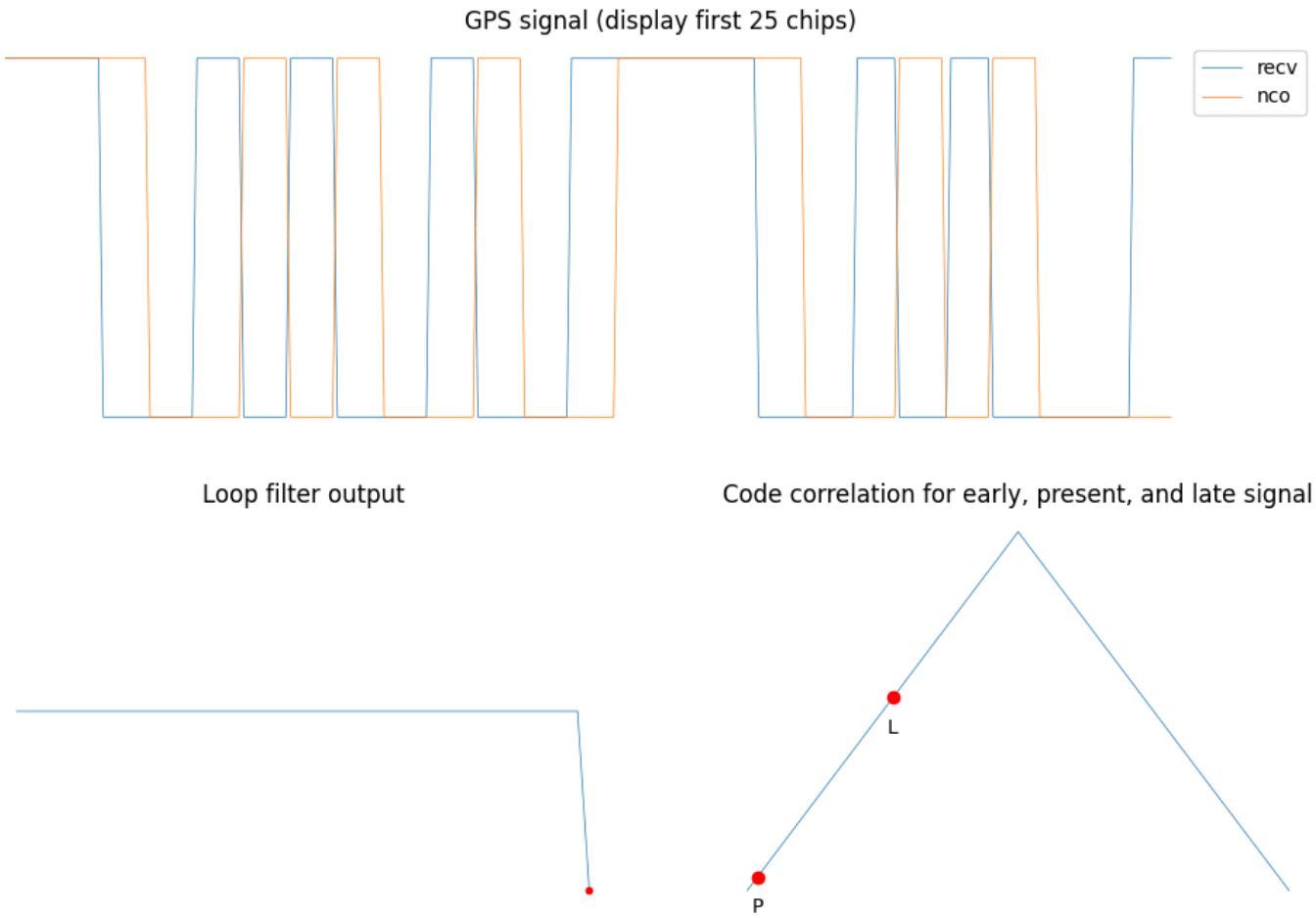
PLL


GPS signal (display first 10 chips)
recv
copy
Loop filter output
status: PLL
phase diff: 17.2°
freq diff: 100.0Hz

FLL + PLL


GPS signal (display first 10 chips)
recv
copy
Loop filter output
status: FLL
phase diff: 17.2°
freq diff: 100.0Hz

DLL

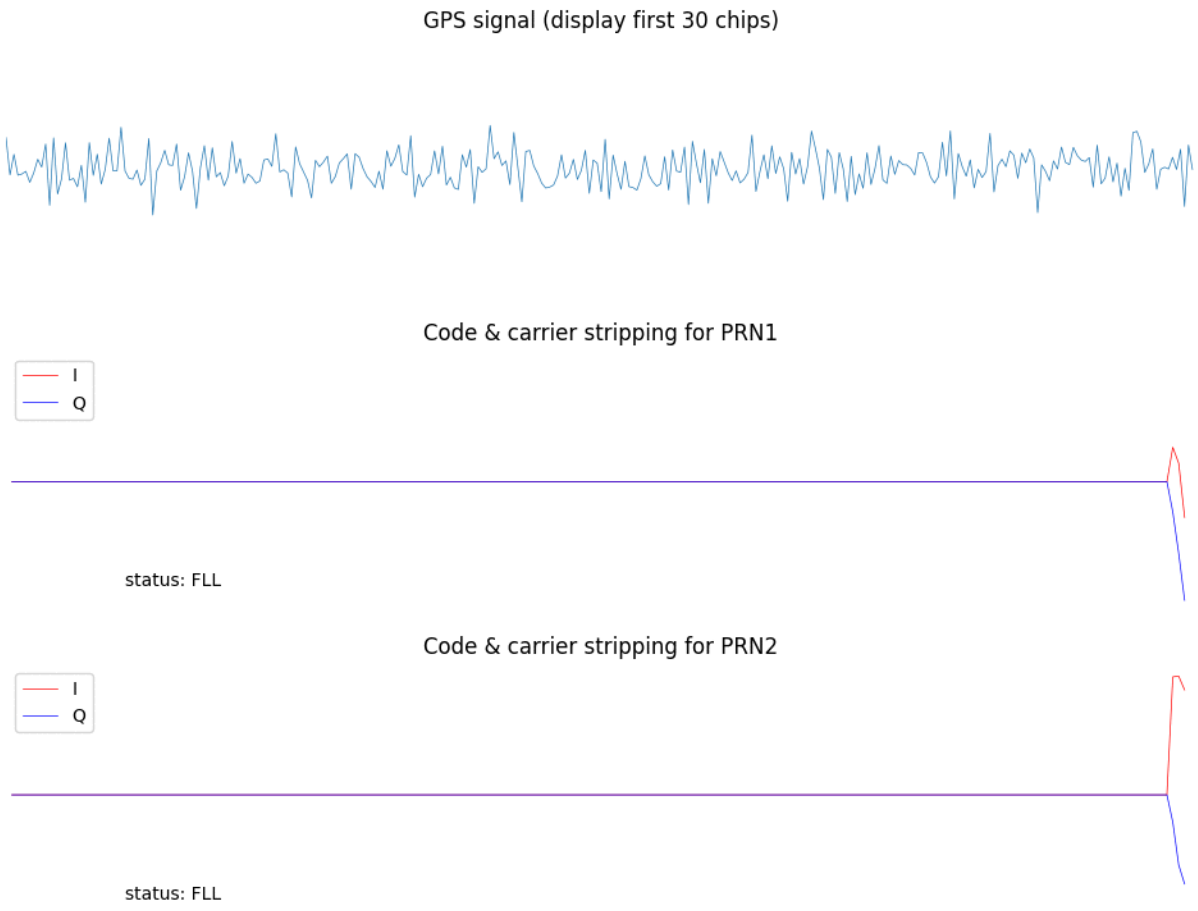

GPS signal (display first 25 chips)
recv
nco
Loop filter output
Code correlation for early, present, and late signal
L
P

Demodulation

- PLL and DLL make sure the copied signal has the maximum correlation results with the received signal.

- Data demodulation is then indicated by correlation result.
  - If correlation is above 0, then the data modulated on the signal is +1, otherwise it’s -1.

Demodulation



Some Takeaways

- Pseudorandom noise (PRN) code is used for GPS signal modulation.
- C/A code is released for public use and can be easily hacked, but military code is impossible to hack.
- User will generate a local signal matching with signal through signal acquisition and tracking.
- Data demodulation is achieved by correlation.

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