Nanomagnetism and Spintronics

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Lecture 2

Magnetization energy of a body, thermodynamics of magnetic materials

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Magnetic work

Problem: Calculate the total work needed in order to create a given distribution of magnetic field **H**(**r**) in a region occupied by a magnetic material

$$\delta L = \delta L' + \delta U_M + \delta L^*$$

note that $\bar{H}(\bar{r}) = \bar{H}_a + \bar{H}_M$

field produced by ext. currents without magnetic materials

demagnetizing field produced by the magnetization itself (also $\bar{H}_d$ in some texts)

$\delta L'$ Work done by the magnet power supply to set a given ***Ha***

δU_M Work corresponding to the energy stored in a given configuration **M**(**r**) and depending on the magnetostatic interaction between local magnetic moments (Magnetostatic energy)

δL^* Additional work to magnetize the body in order to obtain a given **M**(**r**). It depends on the nature of the material (FM, paramagnetic, diamagnetic)

Magnetostatic energy

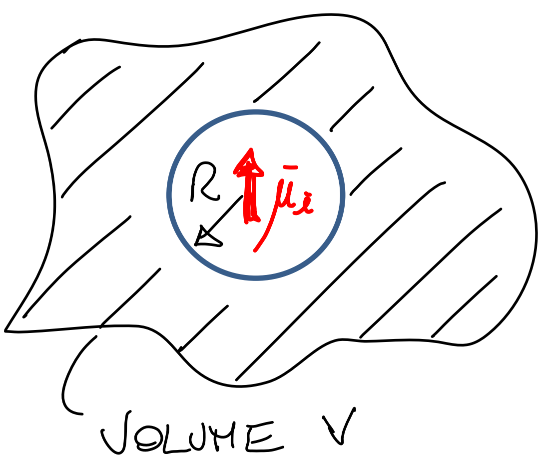
The idea is to calculate the dipole-dipole interaction energy in a solid where a given **M**(**r**) exists. We do not calculate the energy for creating this configuration.

Consider a lattice with mag. moments $\bar{\mu}_i$

$\bar{h}_i$: field intensity at the lattice point i given by all the other dipoles

(a) $U_M = -\frac{\mu_0}{2} \sum_i \bar{\mu}_i \cdot \bar{h}_i$ (without thermal fluctuations)

(ij and ji)



if $R \gg a$ (lattice parameter) the dipoles outside can be treated as a continuum

$$\bar{h}_0 = \bar{H}_M - \left(-\frac{1}{3}\bar{M}\right) + \bar{h}_i'$$

demagnetizing field

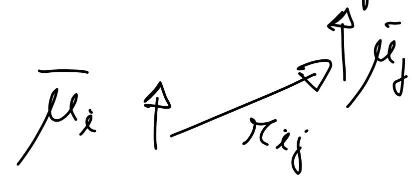
contribution from the small sphere

contribution from dipoles inside

REQUIREMENT $\quad a < R < l_{ex}$ — EXCHANGE LENGTH
Typicale length over which $\bar{M}$ changes

$\bar{M}$ must be constant or slowly variable within the sphere

$$\bar{h}_i' = \sum_{|r_{ij}|<R} \frac{1}{4\pi}\left[-\frac{\bar{\mu}_j}{|\bar{r}_{ij}|^3} + \frac{3(\mu_j \cdot \bar{r}_{ij})\bar{r}_{ij}}{|\bar{r}_{ij}|^5}\right]$$

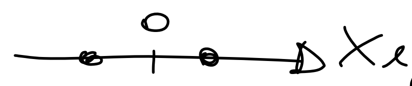


μ_j constant within R

Exercise: demonstrate that h_i is zero for a cubic lattice

For instance

$$h_{ix}' = \frac{1}{4\pi}\sum\left[-\frac{\mu_x}{r_{ij}^3} + \underbrace{\frac{3x_{ij}(\mu_x x_{ij} + \mu_y y_{ij} + \mu_z z_{ij})}{r_{ij}^5}}_{\text{this term is zero for a cubic symmetry}}\right] \qquad (1)$$



Furthermore for a cubic lattice x, y, z are interchangeable:

$$\sum \frac{x_{ij}^2}{r_{ij}^5} = \sum \frac{y_{ij}^2}{r_{ij}^5} = \sum \frac{z_{ij}^2}{r_{ij}^5} = \frac{1}{3}\sum \frac{x_{ij}^2 + y_{ij}^2 + z_{ij}^2}{r_{ij}^5} = \frac{1}{3}\sum \frac{1}{r_{ij}^3}$$

Therefore (1) becomes

$$h'_{ix} = \frac{1}{4\pi} \Sigma \left(-\frac{\mu_x}{r_{ij}^3} + \frac{\mu_x}{r_{ij}^3} \right) = 0 \qquad \text{C.V.D.}$$

In general, for a non cubic symmetry

$$\underset{\text{TENSOR}}{\bar{h}'_i = -\Lambda \cdot \bar{M}}$$ as the summation will give in any case something proportional to $\bar{M}$

The magnetostatic energy is then:

$$U_M = -\frac{\mu_0}{2} \sum_i \bar{\mu}_i \cdot \left(\bar{H}_M + \frac{1}{3} \bar{M} - \Lambda \cdot \bar{M} \right)$$

The summation can be replaced by an integral if $\bar{\mu}_i = \bar{M} d^3r$

$$U_M = -\frac{\mu_0}{2} \int_V \bar{M} \cdot \left(\bar{H}_M + \frac{\bar{M}}{3} - \Lambda \cdot \bar{M} \right) d^3r = \underbrace{-\frac{\mu_0}{2} \int_V \bar{M} \cdot \bar{H}_M \, d^3r}_{(\alpha)} \underbrace{- \frac{\mu_0}{6} \int_V M^2 d^3r}_{(\beta)} + \underbrace{\int_V \bar{M} \cdot \Lambda \cdot \bar{M} \, d^3r}_{(\gamma)}$$

(β) does not depend on the spatial distribution of $\bar{M}(\bar{r})$ – the energy is minimized simply if M is big, but this tendency to FM is much less important than exchange $\Rightarrow$ CAN BE NEGLECTED

(γ) can be included in the magnetic anisotropy term

The MAGNETOSTATIC self ENERGY is then conventionally defined as:

$$E_M = -\frac{\mu_0}{2}\int_V \bar{H}_M \cdot \bar{M}\, d^3r \qquad \bar{B} = \mu_0(\bar{H}_M + \bar{M})$$

But

$$E_M = -\frac{\mu_0}{2}\int_{\text{ALL SPACE}} \bar{H}_M \cdot \bar{M}\, d^3r = -\frac{\mu_0}{2}\int_{A.S.} \bar{H}_M \cdot \left(\frac{\bar{B}}{\mu_0} - \bar{H}_M\right) d^3r =$$

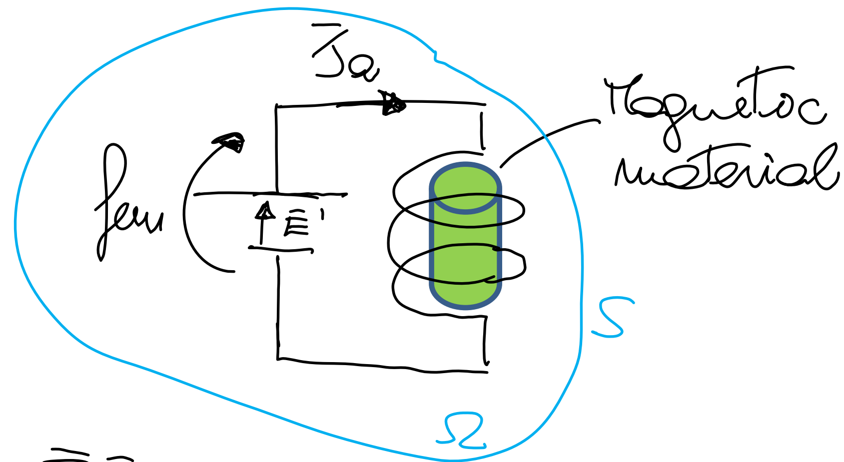
$$= \cancel{-\frac{1}{2}\int_{A.S.} \bar{H}_M \cdot \bar{B}\, d^3r} + \frac{\mu_0}{2}\int_{A.S.} H_M^2\, d^3r$$

PROOF: $\bar{B} = \bar{\nabla} \times \bar{A} \qquad \bar{H} \cdot (\bar{\nabla} \times \bar{A}) = \bar{\nabla} \cdot (\bar{A} \times \bar{H}) + \bar{A} \cdot \cancel{(\bar{\nabla} \times \bar{H})}$ no conduction currents

$$\int_{A.S.} \bar{H}_M \cdot \bar{B}\, d^3r = \int_{A.S.} \bar{\nabla} \cdot (\bar{A} \times \bar{H})\, d^3r = \int_{\substack{\text{surface}\\ \text{at } \infty}} (\underbrace{\bar{A}}_{\sim \frac{1}{r^2}} \times \underbrace{\bar{H}}_{\sim \frac{1}{r^2}}) \cdot \bar{n}\, d^2r \to 0$$

$$E_M = \begin{cases} -\dfrac{\mu_0}{2}\displaystyle\int_V \bar{H}_M \cdot \bar{M}\, d^3r \\[2ex] \dfrac{\mu_0}{2}\displaystyle\int_{A.S.} H_M^2\, d^3r \end{cases}$$

Magnetic work calculation



Without magnetic material:

$$\bar{B}_a = \mu_0 \bar{H}_a \qquad \begin{cases} \bar{\nabla}\cdot\bar{H}_a = 0 \\ \bar{\nabla}\times\bar{H}_a = \bar{J}_a \end{cases}$$

With magnetic material $(\bar{M}(\bar{r}))$

$$\begin{cases} \bar{H} = \bar{H}_a + \bar{H}_M \\ \bar{B} = \mu_0(\bar{H}_a + \bar{H}_M + \bar{M}) \end{cases} \qquad \begin{cases} \bar{\nabla}\cdot\bar{H}_M = -\bar{\nabla}\cdot\bar{M} \\ \bar{\nabla}\times\bar{H}_M = 0 \end{cases}$$

$$U_H = \int_\Omega H\cdot B\, d\tau$$

POYNTING THEOREM

$$\int_\Omega \bar{H}\cdot\frac{\partial\bar{B}}{\partial t}\, d^3r + \left[\int_\Omega \frac{J^2}{\sigma}\, d^3r\right] = -\overbrace{\int_S (\bar{E}\times\bar{H})\cdot d\bar{S}}^{(*)} + \overbrace{\int_\Omega \bar{J}_a\cdot\bar{E}'\, d^3r}^{\delta L/\delta t}$$

if $\Omega \to$ ALL SPACE $(*) = 0$

if we neglect energy dissipation due to eddy currents $[\ ] = 0$

$$\delta L = \delta t \cdot \int_\Omega \bar{J}_a\cdot\bar{E}'\, d^3r = \delta t \cdot \int_\Omega \bar{H}\cdot\frac{\partial\bar{B}}{\partial t}\, d^3r = \int_\Omega \bar{H}\cdot\delta\bar{B}\, d^3r$$

A portion of this work corresponds to which is needed for setting $\bar{H}_a$ without magnetic bodies:

$$\delta L' = \int_\Omega \bar{H}_a\cdot\delta\bar{B}_a\, d^3r = \int_\Omega \mu_0 \bar{H}_a\cdot\delta\bar{H}_a\, d^3r$$

The extra work for the presence of magnetic bodies is:

$$\delta L_M = \delta L - \delta L' = \int_{A.S.} (\bar{H}\cdot\delta\bar{B} - \mu_0 \bar{H}_e \cdot \delta\bar{H}_e)\, d^3r =$$

$$= \int_{AS} \left[(\bar{H}_e + \bar{H}_M)\cdot \mu_0\, \delta(\bar{H}_e + \bar{H}_M + \bar{M}) - \mu_0 \bar{H}_e \cdot \delta \bar{H}_e \right] d^3r =$$

$$= \mu_0 \cancel{\int_{AS} \delta(\bar{H}_e \cdot \bar{H}_M)\, d^3r} + \underbrace{\mu_0 \int_{AS} \bar{H}_M \cdot \delta\bar{H}_M\, d^3r}_{\delta E_M = \delta \frac{\mu_0}{2} \int_{AS} H_M^2\, d^3r} + \mu_0 \int_{AS} \bar{H}\cdot\delta\bar{M}\, d^3r$$

$$\begin{cases} \bar{\nabla}\cdot\bar{H}_e = 0 \\ \bar{\nabla}\times\bar{H}_M = 0 \end{cases}$$

$$\boxed{\delta L_M = \delta E_M + \mu_0 \int_{\Omega} \bar{H}\cdot\delta\bar{M}\, d^3r} \qquad (M = 0 \text{ out of } \Omega)$$

δL_M includes magnetostatic energy!

δE_M can also be written as $\delta \left\{ -\frac{\mu_0}{2} \int_{\Omega} \bar{H}_M \cdot \bar{M}\, d^3r \right\}$

$$\Rightarrow \delta E_M = -\frac{\mu_0}{2} \int_{\Omega} \bar{H}_M \cdot \delta\bar{M}\, d^3r - \frac{\mu_0}{2} \int_{\Omega} \bar{M}\cdot\delta\bar{H}_M\, d^3r$$

$\bar{M}$ gives $\bar{H}_M$
$\delta\bar{M}$ " $\delta\bar{H}_M$ } a reciprocity theorem states that $\int \bar{H}_M \cdot \delta\bar{M}\, d^3r = \int \bar{M}\cdot\delta\bar{H}_M\, d^3r$

$$\delta E_M = -2\frac{\mu_0}{2} \int \bar{H}_M \cdot \delta\bar{M}\, d^3r$$

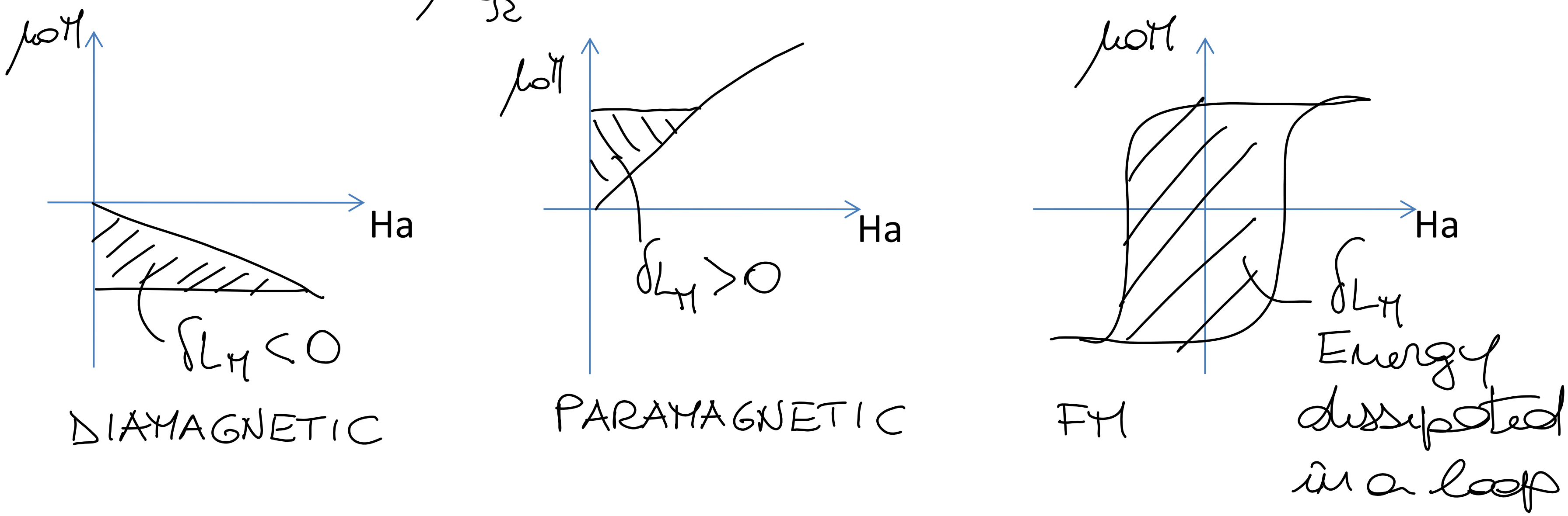
$$\delta L_M = \delta E_M + \mu_0 \int_\Omega \bar{H}\cdot\delta\bar{M}\,d^3r = -\mu_0\int_\Omega \bar{H}_M\cdot\delta\bar{M}\,d^3r + \mu_0\int_\Omega \bar{H}\cdot\delta\bar{M}\,d^3r =$$

$$\boxed{\delta L_M = \mu_0\int_\Omega \bar{H}_a\cdot\delta\bar{M}\,d^3r} = \delta U_M + \delta L^*$$

THIS IS IMPORTANT BECAUSE $\bar{H}_a$ is the external variable!

If we disentangle the contribution of magnetostatic energy

$$\delta L^* = \delta L_M - \delta E_M = \mu_0\int_\Omega \bar{H}\cdot\delta\bar{M}\,d^3r$$



Experimental: for a long cylinder $H = H_a$!

$$\delta L^* = \delta L_M$$

Thermodynamics of magnetic materials

The first law is written in terms of pairs of conjugate variables

H_x : external action on the system $(-p)$

X : state variable (V)

$$dU = \underbrace{H_x dX}_{\delta L} + \delta Q \qquad \left(\text{e.g. } dU = -p dV + dQ\right)$$

$$\delta Q \leq T dS$$

Four thermodynamic potentials are defined if we fix two variables leaving the other free

1. INTERNAL ENERGY : $U(X, S)$
2. ENTHALPY : $E(H_x, S)$
3. HELMHOTZ FREE ENERGY : $F(X, T)$
4. GIBBS FREE ENERGY : $G(H_x, T)$

For T fixed the more relevant are:

$$F = U - TS \qquad dF \leq H_x dX - S dT$$

$$G = F - H_x X \qquad dG \leq -X dH_x - S dT$$

The magnetic work done on a magnetic body, including that of power supplies is

$$\delta L_M = \mu_0 \int_\Omega \bar{H}_e \cdot \delta \bar{M} \, d\hat{}$$ (no dissipations!)

If we consider now a small body where $\bar{M}$ is uniform

$$\delta L_M = \mu_0 \bar{H}_e \cdot \delta \int_\Omega \bar{M} \, d\hat{} = \mu_0 \bar{H}_e \cdot \delta \bar{m}$$

$$\delta L_M = \mu_0 \bar{H}_e \cdot \delta \bar{m} = H_x \, dX$$

$$dU \leq \mu_0 \bar{H}_e \cdot \delta \bar{m} + T dS$$

$$dF \leq \mu_0 \bar{H}_e \cdot \delta \bar{m} - S dT$$ but $\bar{M}$ is not the ext. variable!

$$G = F - H_x X \Rightarrow dG \leq \left(\cancel{\mu_0 \bar{H}_e \cdot \delta \bar{m}} - S dT\right) - \cancel{\mu_0 \bar{H}_e \cdot \delta \bar{m}} - \mu_0 \bar{m} \cdot \delta \bar{H}_e$$

$$dG \leq -S dT - \mu_0 \bar{m} \cdot \delta \bar{H}_e$$

For fixed $\bar{H}_e$ and T

$dG \leq 0$ The equilibrium corresponds to a minimum for G

For $(\bar{H}_a, T)$ fixed we should have only one $\bar{m}$ - But this is not exactly the case (hysteresis!).

$\bar{m}$ is an INTERNAL DEGREE OF FREEDOM

LANDAU FREE ENERGY $G_L(\bar{H}_a, T, \bar{m})$

This is the functional to be minimized for finding the equilibrium

NOTE: They differ for the energy of $\bar{m}$ in $\bar{H}_a$

$$G_L(\bar{H}_a, T, \bar{m}) = F(\bar{m}, T) - \mu_0 \bar{H}_a \cdot \bar{m}$$

we are assuming that out of eq. the equation of state $\bar{m} = f(\bar{H}_a, T)$ is not valid

EQUILIBRIUM : $\left.\dfrac{\partial G_L}{\partial m}\right|_{H_a, T} = 0 \qquad \left.\dfrac{\partial^2 G_L}{\partial m^2}\right|_{H_a, T} > 0$

$$\left.\frac{\partial F}{\partial m}\right|_{H_a, T} = \mu_0 H_a$$

For an extended body we will consider

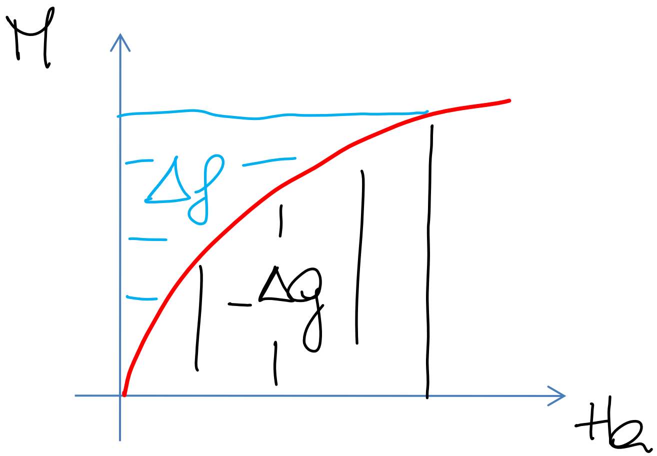
$$G_L = F - \int_\Omega \mu_0 \bar{H}_a \cdot \bar{M} \, d\tau$$

For reversible transformations

@ T = const

$$df = \frac{dF}{d\tilde{v}} = \mu_0 H_a \cdot d\bar{M}$$

$$dg = \frac{dG}{d\tilde{v}} = -\mu_0 \bar{M} \cdot dH_a$$



Next step

Write a reasonable form for $F = U - TS$

The contributions to F are:

1. Exchange energy
2. Anisotropy: Magnetocrystalline; Shape
3. Other terms arising from "interfacial" interactions