Nanomagnetism and Spintronics

POLITECNICO
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Lecture 3

Magnetic free energy terms

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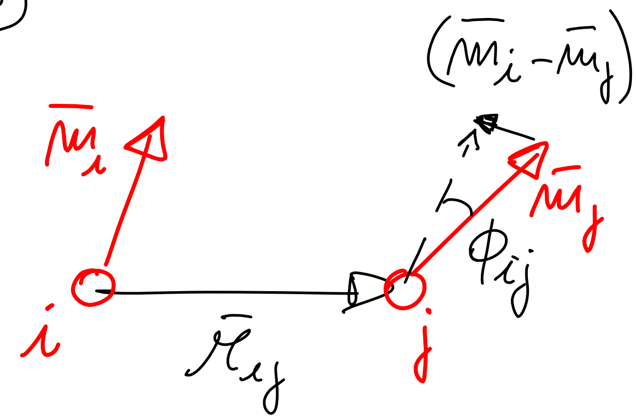
Exchange interaction

$$(E_S - E_t = 2J)$$

$$\hat{\mathcal{H}} = -\sum_{\langle i,j \rangle} J \bar{S}_i \cdot \bar{S}_j \qquad (1)$$



Hp: consider classical spins

$\phi_{ij} \ll 1$

$$E = -JS^2 \sum_{\langle ij \rangle} \cos\phi_{ij} = \text{cost} + \frac{JS^2}{2} \sum_{\langle i,j \rangle} \phi_{ij}^2 \qquad \left\{ \cos\phi_{ij} \simeq 1 - \frac{\phi_{ij}^2}{2} \right\}$$

Define the REDUCED MOMENT $\bar{m} = \bar{M}/M_{sat}$

$$|\phi_{ij}| \simeq |\bar{m}_i - \bar{m}_j| \simeq |(\bar{r}_{ij} \cdot \bar{\nabla})\bar{m}|$$

$\left. \begin{array}{l} m_x \\ m_y \\ m_z \end{array} \right\}$ DIRECTION COSINES

$$E = \frac{JS^2}{2} \sum_{ij} [(\bar{r}_{ij} \cdot \bar{\nabla}) m]^2 =$$

$r_{ij} = a$ LATTICE PARAMETER

$$= \frac{A}{2} \int_V [(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2]\, dv$$

with $A = 2JS^2 \frac{z}{a}$

z: number of sites on the unit cell for cubic lattice

$A = \frac{4\sqrt{2}\, JS^2}{a}$ for hexagonal close packed exercise Find the expression for exchange energy in a simple cubic lattice

$$|\phi_{ij}|^2 = |\bar{m}_i - \bar{m}_j|^2 = |\Delta m_x \bar{i} + \Delta m_y \bar{j} + \Delta m_z \bar{k}|^2 =$$

$$= (\bar{r}_{ij} \cdot \bar{\nabla} m_x)^2 + (\bar{r}_{ij} \cdot \bar{\nabla} m_y)^2 + (\bar{r}_{ij} \cdot \bar{\nabla} m_z)^2 = \left[(\bar{r}_{ij} \cdot \bar{\nabla})\bar{m}\right]^2$$

$$E = \frac{JS^2}{2} \sum_i \sum_j \left\{ (\bar{r}_{ij} \cdot \bar{\nabla} m_x)^2 + (\bar{r}_{ij} \cdot \bar{\nabla} m_y)^2 + (\bar{r}_{ij} \cdot \bar{\nabla} m_z)^2 \right\} =$$

For a simple cubic there are 6 nearest neighbours

$$\sum_j \{ " \} = 2\left[\left(a\frac{\partial m_x}{\partial x}\right)^2 + \left(a\frac{\partial m_y}{\partial x}\right)^2 + \left(a\frac{\partial m_z}{\partial x}\right)^2\right] + \qquad r_{ij} = \pm a\bar{i}$$

$$+ 2\left[\left(a\frac{\partial m_x}{\partial y}\right)^2 + \left(a\frac{\partial m_y}{\partial y}\right)^2 + \left(a\frac{\partial m_z}{\partial y}\right)^2\right] + \qquad r_{ij} = \pm a\bar{j}$$

$$+ 2\left[\left(a\frac{\partial m_x}{\partial z}\right)^2 + \left(a\frac{\partial m_y}{\partial z}\right)^2 + \left(a\frac{\partial m_z}{\partial z}\right)^2\right] \qquad r_{ij} = \pm a\bar{k}$$

$$\sum_j \{ = \} = 2a^2 \left[(\bar{\nabla} m_x)^2 + (\bar{\nabla} m_y)^2 + (\bar{\nabla} m_z)^2 \right]$$

$$E = \frac{JS^2}{2} \sum_i 2a^2 \left[\quad // \quad \right] = JS^2 \cancel{a^2} \cdot \frac{1}{a^{\cancel{3}}} \int_V \left[= \right] d\bar{r}$$

$$E = \frac{A}{2} \int_V \left[(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2 \right] d\bar{r}$$

with $A = \frac{2JS^2}{a}$ EXCHANGE STIFFNESS

in general for a cubic lattice $A = \frac{2JS^2}{a} z$

$$z = \begin{cases} 1 & \text{simple cubic} \\ 2 & \text{bcc} \\ 4 & \text{fcc} \end{cases}$$

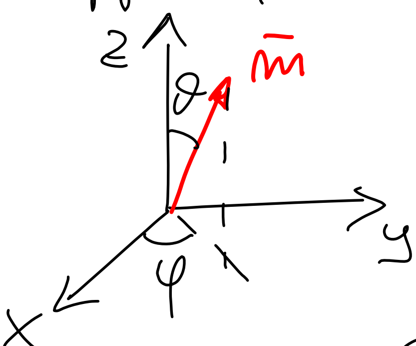
$A_{Co} = 31 \text{ pJ/m}$

$A_{Py} = 10 \text{ pJ/m}$

Anisotropy

We are looking for the dependence of Helmoltz free en. on the orientation of $\bar{M}$

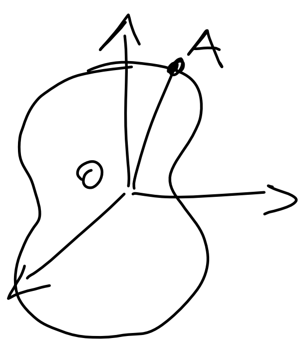
Consider a volume ΔV with $\bar{M}$ uniform inside and suppose $|\bar{M}| = M_{sat}$



$$\bar{m} = \frac{\bar{M}}{M_{sat}} \qquad \begin{cases} m_x = \sin\vartheta\cos\varphi \\ m_y = \sin\vartheta\sin\varphi \\ m_z = \cos\vartheta \end{cases}$$

$$F_{AN}(\bar{m}) = ? \quad \text{or} \quad f_{AN}(\bar{m}) = \frac{F_{AN}}{\Delta V} = ?$$

ANISOTROPY ENERGY SURFACES



$\bar{OA} \sim f_{AN}$ when $\bar{m}$ is directed along $\bar{OA}$

EASY AXES : local minima

HARD AXIS : " maxima

Uniaxial anisotropy

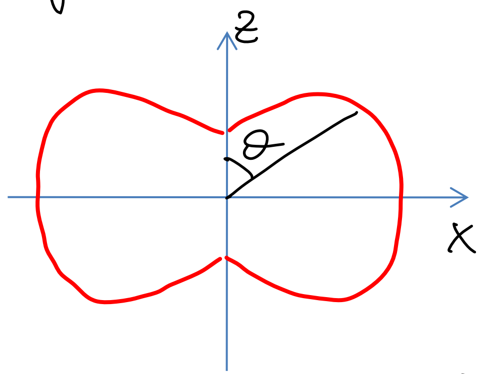
$f_{AN}(\vec{m})$
- must be unchanged upon rotation around the anisotropy axes (z)
- must be an even function of $m_z = \cos\theta$

$m_x^2 + m_y^2 = 1 - m_z^2 = 1 - \cos^2\theta = \sin^2\theta$ is the good variable for the expansion of f_{AN}

$$f_{AN} = K_0 + K_1 \sin^2\theta + K_2 \sin^4\theta + K_3 \sin^6\theta + \dots \qquad K : \frac{\text{energy}}{\text{volume}}$$

Usually $f_{AN} = K_0 + K_1 \sin^2\theta$

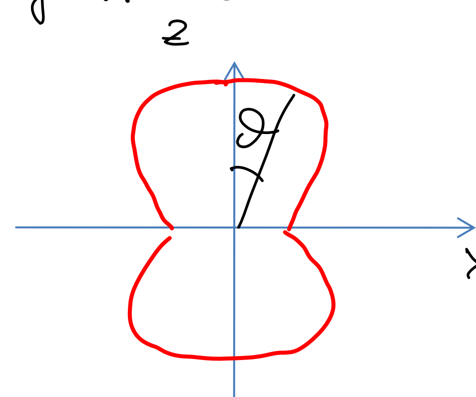
a) if $K_1 > 0$



EASY AXIS ANISOTROPY

(MINIMA FOR $\theta = 0, \pi$)

b) if $K_1 < 0$



EASY PLANE ANISOTROPY

(MINIMA FOR $\theta = \frac{\pi}{2}$)

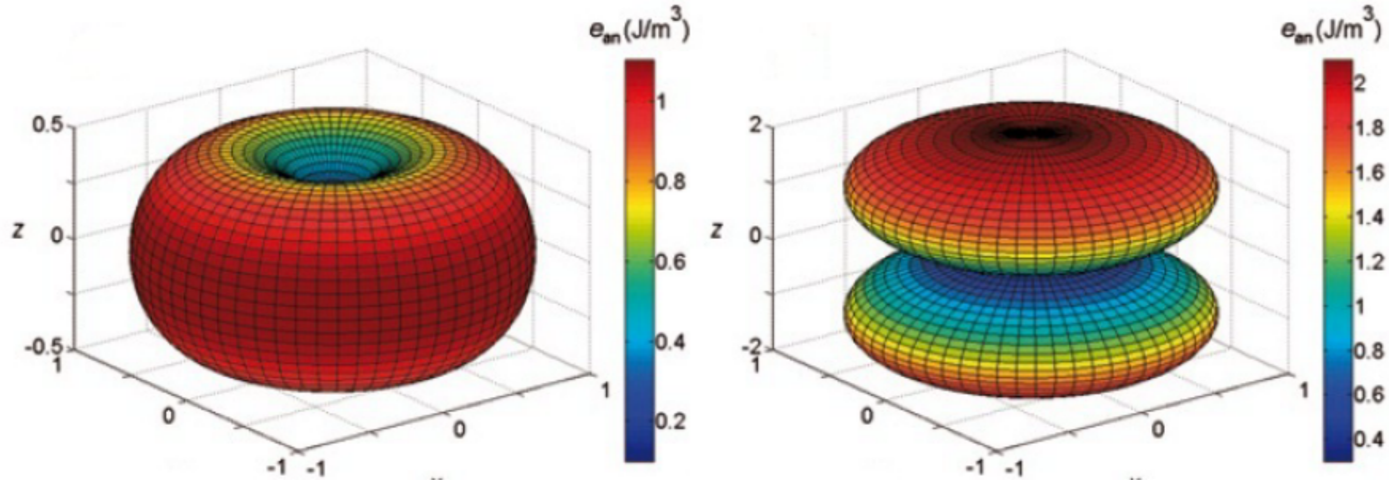
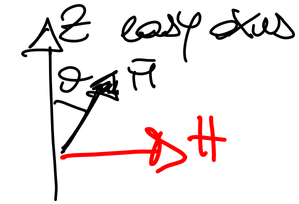


Figure 2.1: Uniaxial anisotropies represented by energy surfaces. The length of the plotted radial coordinate is proportional to the energy density for that direction. The anisotropy constants are chosen to illustrate different cases at similar energy scales. (left) Easy perpendicular direction (right) Easy plane

ANISOTROPY FIELD ($\bar{H}_a$) (FOR EASY AXIS ANISOTROPY)

H_a is defined as the field needed to saturate $\bar{M}$ along the hard axis.

If the uniaxial anisotropy is the only energy term to be considered in F



$$g_L = K_1 \sin^2\theta - \mu_0 M_s H \cos\left(\frac{\pi}{2} - \theta\right)$$

$$= K_1 \sin^2\theta - \mu_0 M_s H \sin\theta$$

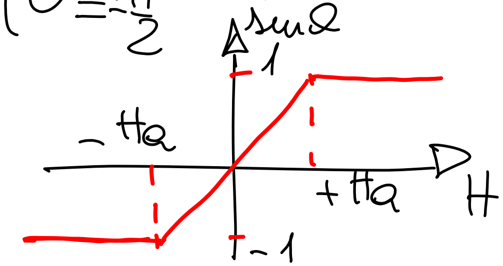
$$\frac{\partial g_L}{\partial \theta} = \left(2K_1 \sin\theta - \mu_0 M_s H\right)\cos\theta$$

$$\begin{cases} \sin\theta = \mu_0 \dfrac{M_s H}{2K_1} \\ \theta = \pm\dfrac{\pi}{2} \end{cases}$$

$$\frac{\partial^2 g_L}{\partial \theta^2} = +2K_1 \cos 2\theta + \mu_0 M_s H \sin\theta$$

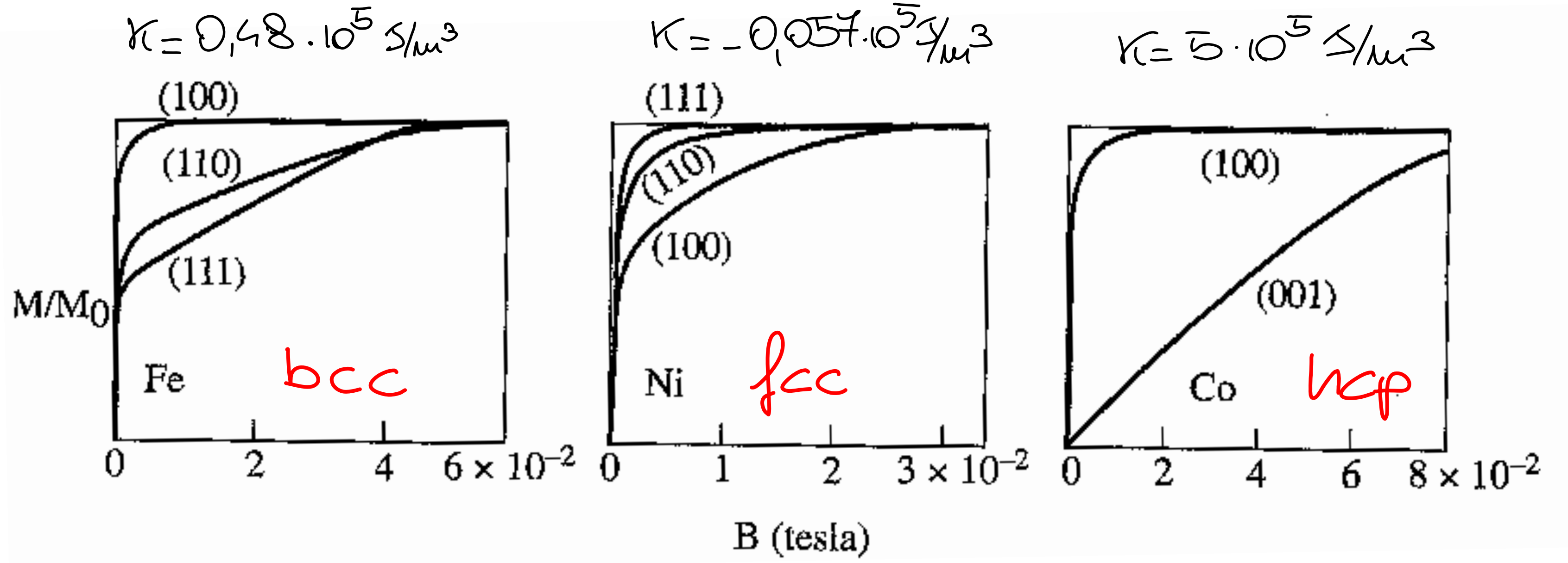
$$\frac{\partial^2 g_L}{\partial \theta^2}\left(\theta = -\frac{\pi}{2}\right) = -2K_1 - \mu_0 M_s H > 0$$

$$H < -\frac{2K_1}{\mu_0 M_s} = -H_a$$

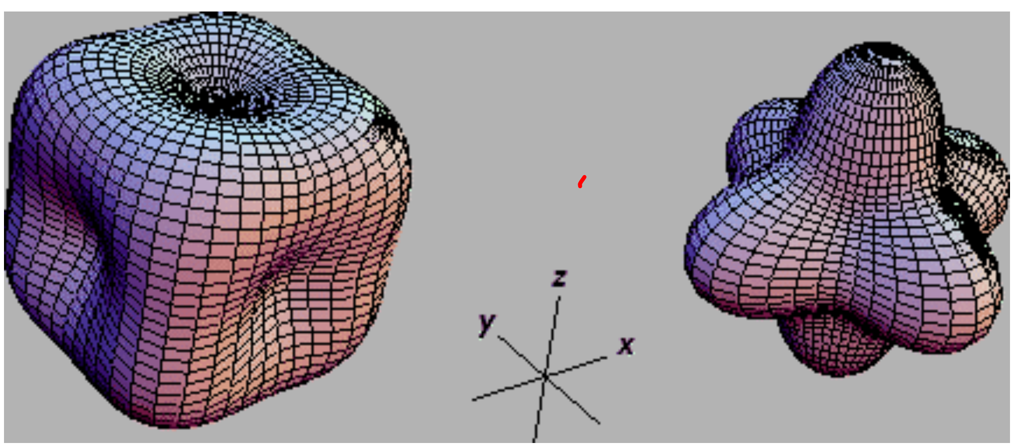


$H_a = \dfrac{2K_1}{\mu_0 M_s}$ indicates the strength of the anisotropy

Magnetocrystalline anisotropy



Magnetocrystalline anisotropy energy surfaces



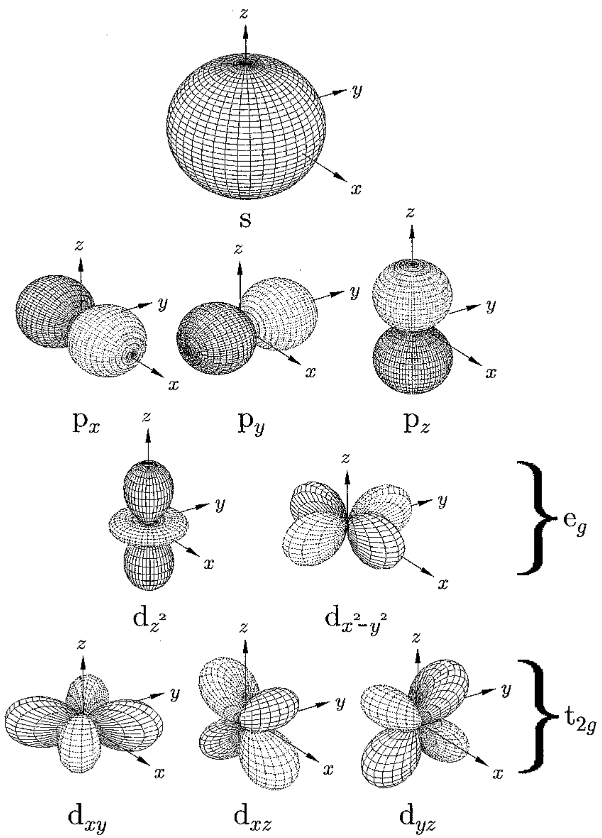
Fe (bcc)

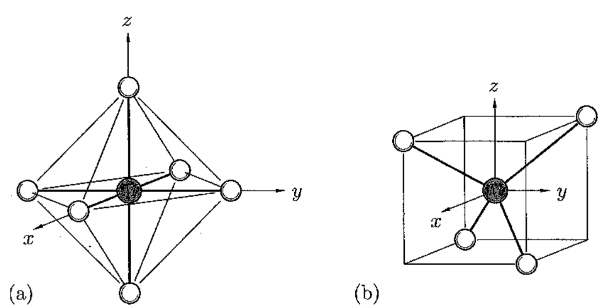
Ni (fcc)

ORIGIN → SINGLE ION ANISOTROPY

ORIGIN → TWO-IONS ANISOTROPY

Single-ion anisotropy





Due to electrostatic interaction of the orbitals containing unpaired electrons with the crystal field.

Crystal field tends to stabilize a particular orbital (e.g. t_{2g} in octahedral symmetry): L is fixed!

Don't forget spin orbit interaction

$$\mathcal{H}_{SO} = \alpha \bar{L} \cdot \bar{S}$$

$\bar{M}$ is given by $\bar{S}$ and therefore the easy axis is that along which spin-orbit energy is minimized

NOTE: This is the case of weak spin orbit coupling

For strong spin orbit, it induces a def. of the orbitals if $\bar{S}$ follows $\bar{M}$ $\Rightarrow$ ENERGY COST!

Two-ions anisotropy

This term reflects the anisotropy of dipolar interaction between two magnetic moments

Parallel
P

Head-To-Tail
HT

For two interacting dipoles

$$E_P - E_{HT} = \frac{3\mu_0 m^2}{4\pi r^3}$$

However the dipole-dipole interaction over the entire lattice must be considered.

For a cubic lattice we have seen that there is no net dipolar contribution to the local field $\Rightarrow$ NO TWO-IONS ANISOTROPY

In non-cubic lattices dipolar interaction can give appreciable magnetic anisotropy.

Cubic anisotropy

$f_{AN}(\bar{m})$ must respect the cubic symmetry $\Rightarrow$ expansion in terms containing:

$$\left.\begin{array}{l}\left[m_x^2 m_y^2 + m_y^2 m_z^2 + m_z^2 m_x^2\right] \\ \left\{m_x^2 m_y^2 m_z^2\right\}\end{array}\right\} \text{unchanged upon exchange of } x, y, z$$

NOTE: The other possible combination $(m_x^4 + m_y^4 + m_z^4)$ depends on the other two:

$$\left\{m_x^4 + m_y^4 + m_z^4\right\} + 2\left[m_x^2 m_y^2 + m_y^2 m_z^2 + m_z^2 m_x^2\right] = \left(m_x^2 + m_y^2 + m_z^2\right)^2 = 1$$

$$f_{AN}(\bar{m}) = K_0 + K_1 [\,//\,] + K_2 \{\quad\} + \ldots$$

in spherical coordinates:

$$f_{AN}(\bar{m}) = K_0 + K_1 \left(\frac{\sin^2\theta \sin^2 2\phi}{4} + \cos^2\theta\right) \sin^2\theta + K_2 \frac{\sin^2 2\phi \sin^2 2\theta}{16} + \ldots$$

if $\theta = \pi/2$ (m stay in the film plane, for instance)

$$f_{AN} = K_0 + K_1 \frac{\sin^2 2\phi}{4}$$

Domain formation

In any case it costs energy to make a DW; why domains form?

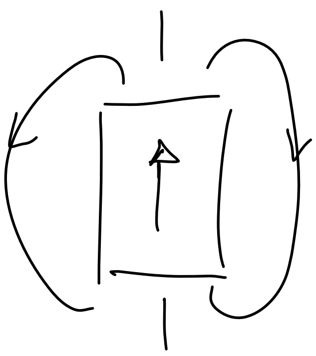
The reason is the minimization of

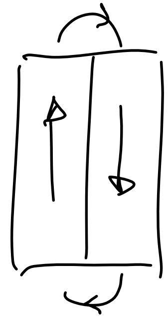
demagnetizing energy =

= magnetostatic energy =

= dipolar energy

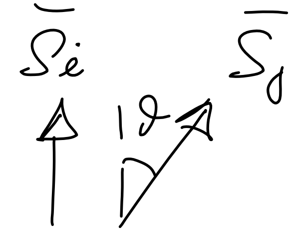
$$E_D = -\frac{\mu_0}{2}\int_V \bar{M}\cdot\bar{H}_d\, d\hat{r} = \frac{\mu_0}{2}\int_{A.S.} H_d^2\, d\hat{r}$$





CLOSURE DOMAIN STRUCTURE

Domains and domain walls



$$E = -2J\bar{S}_1\cdot\bar{S}_2 =$$
$$= -2JS^2\cos\theta$$

The energy cost for $\theta \neq 0$ is:

$$\Delta E = JS^2\theta^2 \quad \text{if } \theta \ll 1 \quad \left\{\cos\theta \simeq 1 - \frac{\theta^2}{2}\right\}$$

FOR A BLOCH DW the spins rotate by π over N sites

$$\Delta E = N \cdot \left(JS^2\left(\frac{\pi}{N}\right)^2\right) = JS^2\frac{\pi^2}{N}$$

The energy per unitarea is

$$\sigma_{BW} = \frac{\Delta E}{a^2} = \frac{JS^2\pi^2}{Na^2}$$

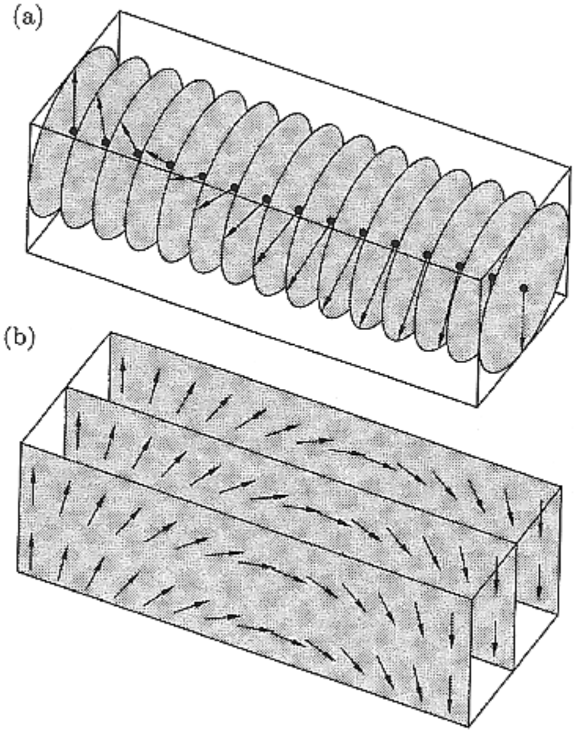


Fig. 6.20 (a) A Bloch wall. (b) A Néel wall.

$$\lim_{N\to\infty}\sigma_{BW} = 0 \Rightarrow \text{SINGLE DOMAIN CONFIG. SHOULD BE FAVORED !!!}$$

But we must consider also anisotropy!

Assume $E_{an} = K \sin^2\theta$ $(K>0)$

The anisotropy energy contribution associated to a BW:

$$\sum_{i=1}^{N} K \sin^2\vartheta_i \simeq \frac{N}{\pi}\int_0^{\pi} K \sin^2\vartheta \, d\theta = \frac{NK}{2}$$

in terms of energy per unit area: $\frac{NK}{2} \cdot a$

The total energy cost of a BW is:

$$\sigma_{BW} = \frac{JS^2\pi^2}{Na^2} + \frac{NKa}{2}$$

$\frac{JS^2\pi^2}{Na^2} \to$ BIG DOMAIN

$\frac{NKa}{2} \to$ SMALL DOMAINS

Equilibrium configuration: $\frac{dE_{BW}}{dN} = 0$

$$N = \pi S \sqrt{\frac{2J}{Ka^3}}$$

the width of the DW is:

$$\delta = Na = \pi S \sqrt{\frac{2J}{Ka}}$$

large K $\Rightarrow$ thin
large J $\Rightarrow$ thick

$$\sigma_{BW_{eq}} = \pi S \sqrt{\frac{2JK}{a}}$$

for the exchange energy we used $A = 2JS^2 \frac{z}{a}$

$$\begin{cases} \delta = \pi \sqrt{\frac{A}{K}} \\ \sigma_{BW_{eq}} = \pi \sqrt{AK} \end{cases}$$

Width and energy per unit surface of a Bloch DW

	Ms (MA m^{-1})	A (pJ m^{-1})	K1 (kJ m^{-3})	δ (nm)
$Ni_{80}Fe_{20}$	0.84	10	0.15	800
Fe	1.71	21	48	64
Co	1.44	31	410	24

For Fe

$$\delta = \pi \sqrt{\frac{21 \cdot 10^{-12}}{48 \cdot 10^{3}}} \simeq 6{,}4 \cdot 10^{-8}\, m$$

$$\sigma = \pi \sqrt{21 \cdot 10^{-12} \cdot 48 \cdot 10^{3}} = 31{,}4 \cdot 10^{-4}\, J/m^2$$

Exercise

1. Si consideri un film sottile di Co nella configurazione micro magnetica a chiusura di flusso rappresentata in figura 1, con domini prevalenti aventi bordi paralleli all'asse di anisotropia uniassiale ($K = 5 \cdot 10^5\ Jm^{-3}$) e domini triangolari, agli estremi, con magnetizzazione perpendicolare all'asse di anisotropia. L'energia per unità di superficie associata alle pareti di dominio a 180° vale $\sigma_W = 4 \times 10^{-3}\ Jm^{-2}$, mentre il contributo all'energia totale delle pareti a 90° ai bordi può essere trascurato nell'ipotesi che $L \gg D$.
   a) Dimostrare che in queste condizioni la larghezza dei domini D si può esprimere come:
   $$D = \sqrt{\frac{2\sigma_W L}{K}}$$
   b) Nell'ipotesi che L sia pari a 5 mm calcolare la larghezza dei domini

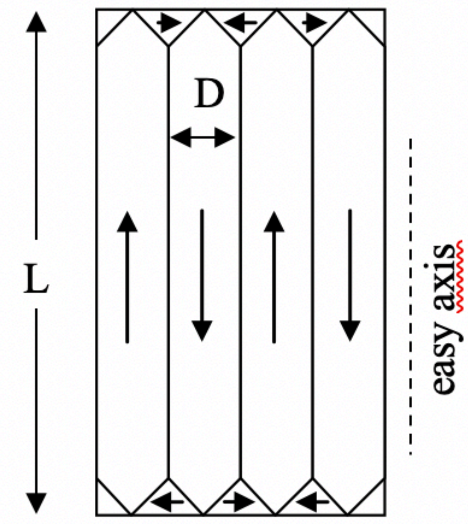
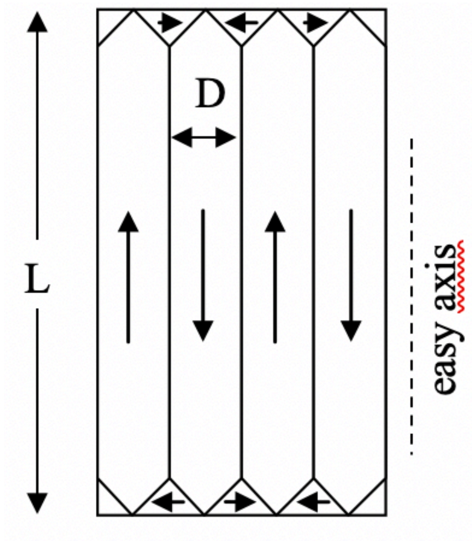


Fig. 1: Vista dall'alto della configurazione magnetica del film di Co



D
L
easy axis