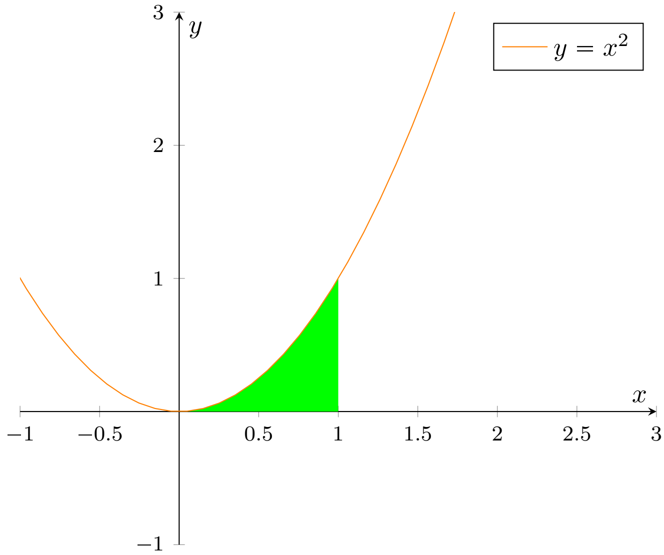
Хатюхин Евгений **А-02-23**

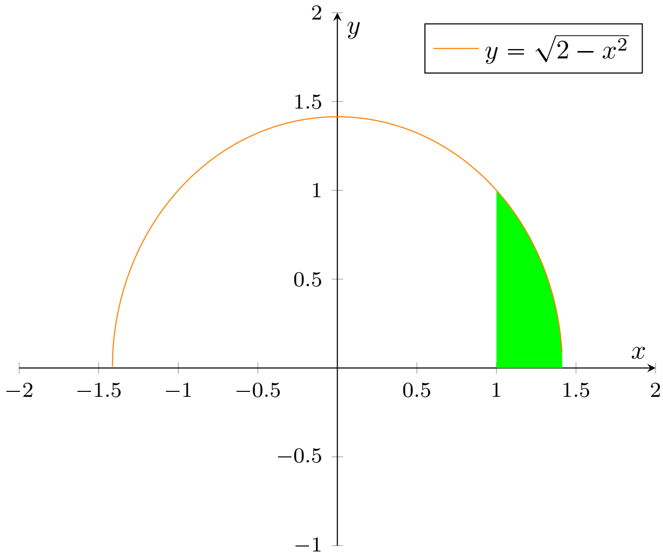
Содержание

2 Книга

1.22

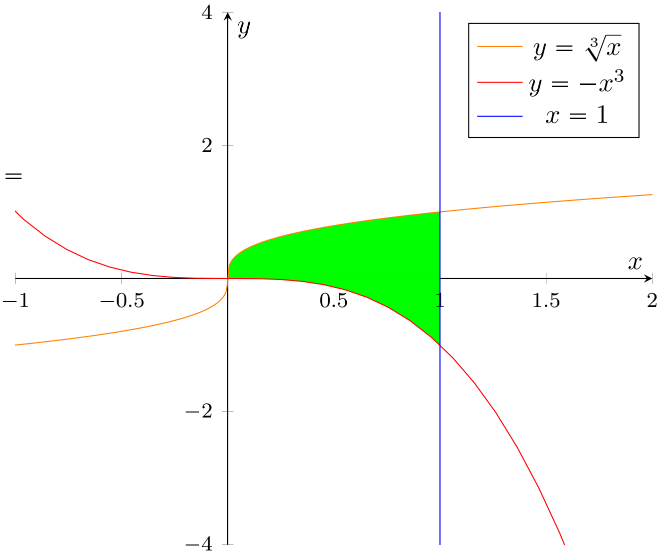
$$\int\limits_0^1 \mathrm{d}x \int\limits_0^{x^2} f \mathrm{d}y + \int\limits_1^{\sqrt{2}} \mathrm{d}x \int\limits_0^{\sqrt{2-x^2}} f \mathrm{d}y = \int\limits_0^1 \mathrm{d}y \int\limits_{\sqrt{y}}^1 f \mathrm{d}x + \int\limits_0^1 \mathrm{d}y \int\limits_1^{\sqrt{2-y^2}} f \mathrm{d}x$$





2.22

$$\iint\limits_D (4xy + 176x^3y^3)\mathrm{d}x\mathrm{d}y;\, D: x = 1, y = \sqrt[3]{x}, y = -x^3;$$



$$\int\limits_0^1 \mathrm{d}x \int\limits_{-x^3}^{\sqrt[3]{x}} (4xy + 176x^3y^3)\mathrm{d}y \ominus$$

$$\int\limits_{-x^3}^{\sqrt[3]{x}} (4xy + 176x^3y^3)\mathrm{d}y = 4x \int\limits_{-x^3}^{\sqrt[3]{x}} y\mathrm{d}y + 176x^3 \int\limits_{-x^3}^{\sqrt[3]{x}} y^3\mathrm{d}y =$$

$$= \frac{\cancelto{2}{4}\,x}{\cancel{2}} \left(y^2 \Big|_{-x^3}^{\sqrt[3]{x}} \right) + \frac{\cancelto{44}{176}\,x^3}{\cancel{4}} \left(y^4 \Big|_{-x^3}^{\sqrt[3]{x}} \right) =$$

$$= 2x\left(x^{\frac{2}{3}} - x^6\right) + 44x^3\left(x^{\frac{4}{3}} - x^{12}\right) =$$

$$= 2x^{\frac{5}{3}} - 2x^7 + 44x^{\frac{13}{3}} - 44x^{15}$$

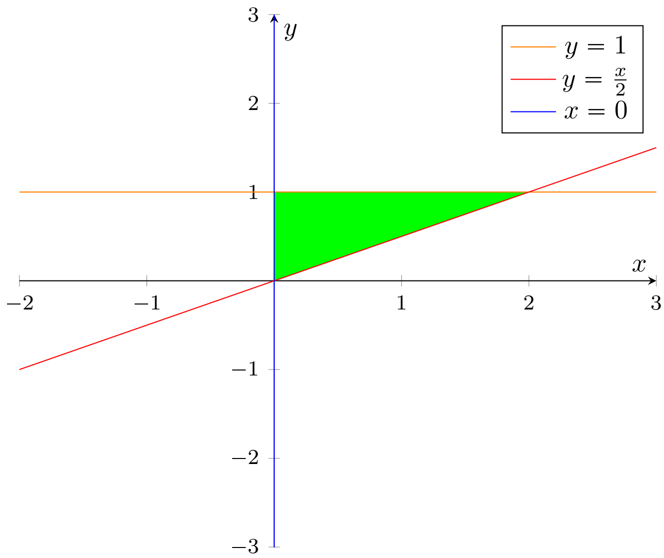
$$\ominus 2\int\limits_0^1 x^{\frac{5}{3}}\mathrm{d}x - 2\int\limits_0^1 x^7\mathrm{d}x + 44\int\limits_0^1 x^{\frac{13}{3}}\mathrm{d}x - 44\int\limits_0^1 x^{15}\mathrm{d}x =$$

$$= \cancel{2} \cdot \frac{3}{\cancelto{4}{8}} \cdot (1-0) - \frac{\cancel{2}}{\cancelto{4}{8}} \cdot (1-0) + \cancelto{11}{44} \cdot \frac{3}{\cancelto{4}{16}} \cdot (1-0) - \frac{\cancelto{11}{44}}{\cancelto{4}{16}} \cdot (1-0) = \frac{3}{4} - \frac{1}{4} + \frac{33}{4} - \frac{11}{4} = \frac{3 - 1 + 33 - 11}{4} =$$

$$= \frac{24}{4} = 6$$

3.22

$$\iint\limits_D y^2 \cdot e^{-\frac{xy}{2}}\mathrm{d}x\mathrm{d}y; \quad D: x = 0;\ y = 1;\ y = \frac{x}{2}$$



$$\int\limits_0^2 \mathrm{d}x \int\limits_{\frac{x}{2}}^{1} y^2 \cdot e^{-\frac{xy}{2}}\mathrm{d}y = \int\limits_0^1 \mathrm{d}y \int\limits_0^{2y} y^2 \cdot e^{-\frac{xy}{2}}\mathrm{d}x \ominus$$

$$\int\limits_0^{2y} y^2 \cdot e^{-\frac{xy}{2}}\mathrm{d}x = y^2 \int\limits_0^{2y} e^{-\frac{xy}{2}}\mathrm{d}x = \left| \begin{array}{l} -\dfrac{xy}{2} = u \\ -\dfrac{y}{2}\mathrm{d}x = \mathrm{d}u \\ u \in [0; -y^2] \end{array} \right| =$$

$$= -y^{\cancel{2}} \cdot \frac{2}{\cancel{y}} \int\limits_0^{-y^2} e^u\mathrm{d}u = -2y \cdot \left(e^u \Big|_0^{-y^2} \right) =$$

$$= -2y \cdot \left(e^{-y^2} - \cancelto{1}{e^0} \right) = -2ye^{-y^2} + 2y$$

$$\ominus \int\limits_0^1 \left(-2ye^{-y^2} + 2y\right)\mathrm{d}y = -2 \cdot \left(\int\limits_0^1 y \cdot e^{-y^2}\mathrm{d}y \right)^{*} + 2 \cdot \int\limits_0^1 y\mathrm{d}y \ominus$$

$$* = \int_0^1 y \cdot e^{-y^2} \mathrm{d}y = \left| \begin{array}{l} -y^2 = v \\ -2y\mathrm{d}y = \mathrm{d}v \\ v \in [0; -1] \end{array} \right| = \frac{1}{-2} \int_0^{-1} e^v \mathrm{d}v$$

$$\ominus \cancelto{1}{\frac{-2}{-2}} \left(e^v \Big|_0^{-1} \right) + \cancel{2} \cdot \frac{1}{\cancel{2}} \left(y^2 \Big|_0^1 \right) = \left(e^{-1} - \cancelto{1}{e^0} \right) + (1 - 0) = \frac{1}{e} - 1 + 1 = \frac{1}{e}$$

4.22

$$\iiint\limits_V y^2 z \cdot ch(xyz) \mathrm{d}x\mathrm{d}y\mathrm{d}z; \quad V = \begin{bmatrix} x = 1; & y = 1; & z = 1; \\ x = 0; & y = 0; & z = 0; \end{bmatrix}$$

$$\int_0^1 \mathrm{d}y \int_0^1 \mathrm{d}z \int_0^1 y^2 z \cdot ch(xyz) \mathrm{d}x \ominus$$

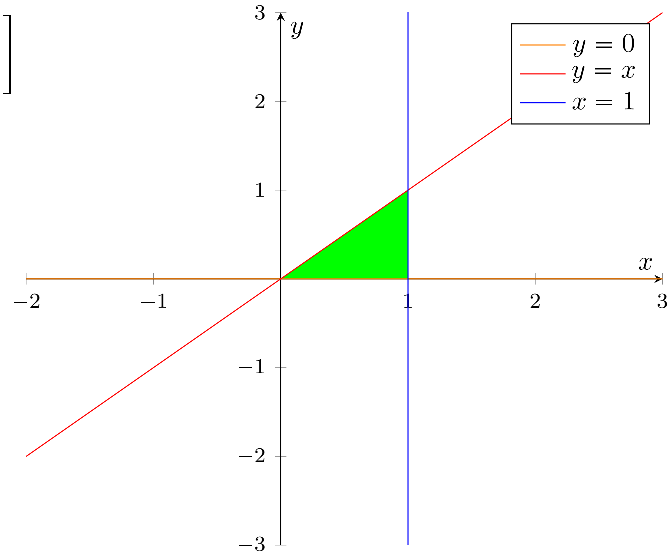
$$\int_0^1 y^2 z \cdot ch(xyz) \mathrm{d}x = \left| \begin{array}{l} xyz = u \\ yz\mathrm{d}x = \mathrm{d}u \\ u \in [0; yz] \end{array} \right| = \frac{y^{\cancel{2}} \cancel{z}}{\cancel{y} \cancel{z}} \int_0^{yz} ch(u) \mathrm{d}u = y \left(sh(u) \Big|_0^{yz} \right) = y(sh(yz) - \cancelto{0}{sh(0)})$$

$$\ominus \int_0^1 \mathrm{d}y \int_0^1 y \cdot sh(yz) \mathrm{d}z \ominus$$

$$\int_0^1 y \cdot sh(yz) \mathrm{d}z = \left| \begin{array}{l} yz = v \\ y\mathrm{d}z = \mathrm{d}v \\ v \in [0; y] \end{array} \right| = \cancelto{1}{\frac{y}{y}} \int_0^y sh(v) \mathrm{d}v = \left(ch(v) \Big|_0^y \right) = (ch(y) - \cancelto{1}{ch(0)})$$

$$\ominus \int_0^1 (ch(y) - 1) \mathrm{d}y = \int_0^1 ch(y) \mathrm{d}y - \int_0^1 \mathrm{d}y = \left(sh(y) \Big|_0^1 \right) - (1 - 0) = sh(1) - \cancelto{0}{sh(0)} - 1 = sh(1) - 1$$

5.22



$$\iiint\limits_V (8y+12z)\mathrm{d}x\mathrm{d}y\mathrm{d}z; \quad V = \begin{bmatrix} y = x;\ y = 0;\ x = 1 \\ z = 0;\ z = 3x^2 + 2y^2 \end{bmatrix}$$

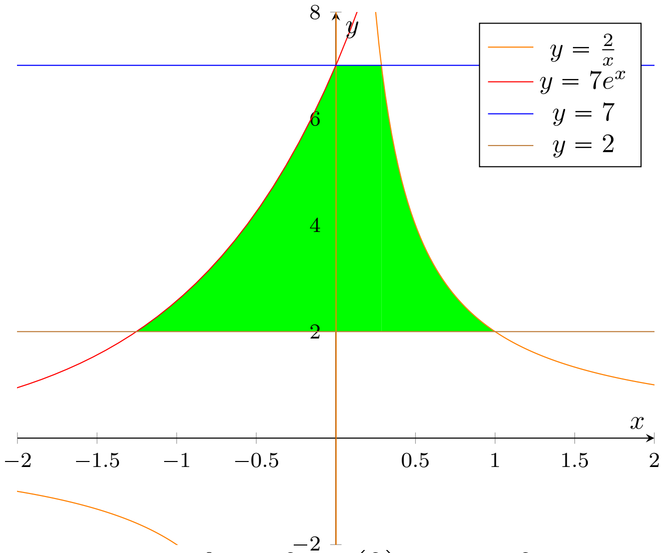
$$\int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \int_0^{3x^2+2y^2} 1\mathrm{d}z = \int_0^1 \mathrm{d}x \int_0^x (3x^2+2y^2)\mathrm{d}y ⊜$$

$$\int_0^x (3x^2+2y^2)\mathrm{d}y = 3x^2 \int_0^x \mathrm{d}y + 2\int_0^x y^2\mathrm{d}y =$$

$$= 3x^2(x-0) + 2(\frac{x^3}{3} - 0) = 3x^3 + \frac{2}{3}\cdot x^3 = \frac{11}{3}\cdot x^3$$

$$⊜\frac{11}{3}\int_0^1 x^3\mathrm{d}x = \frac{11}{3}(\frac{1}{4} - 0) = \frac{11}{12}$$

6.22



$$y = \frac{2}{x};\ y = 7e^x;\ y = 2;\ y = 7$$

$$\int_2^7 \mathrm{d}y \int_{\ln\frac{y}{7}}^{\frac{2}{y}} \mathrm{d}x = 2\int_2^7 \frac{\mathrm{d}y}{y} - \left(\int_2^7 \ln\frac{y}{7}\mathrm{d}y\right)^{*} ⊜$$

$$* = \int_2^7 \ln\frac{y}{7}\mathrm{d}y = \begin{vmatrix} \frac{y}{7} = t \\ \frac{\mathrm{d}y}{7} = \mathrm{d}t \\ t \in [\frac{2}{7};1] \end{vmatrix} = 7\int_{\frac{2}{7}}^{1} \ln(t)\mathrm{d}u =$$

$$= \begin{vmatrix} u = \ln(t) & \mathrm{d}v = 1 \\ \mathrm{d}u = \frac{\mathrm{d}t}{t} & v = t \end{vmatrix} = 7\cdot\left(t\cdot\ln(t)\Big|_{\frac{2}{7}}^{1} - \int_{\frac{2}{7}}^{1} \cancel{t}\cdot\frac{1}{\cancel{t}}\mathrm{d}t\right)$$

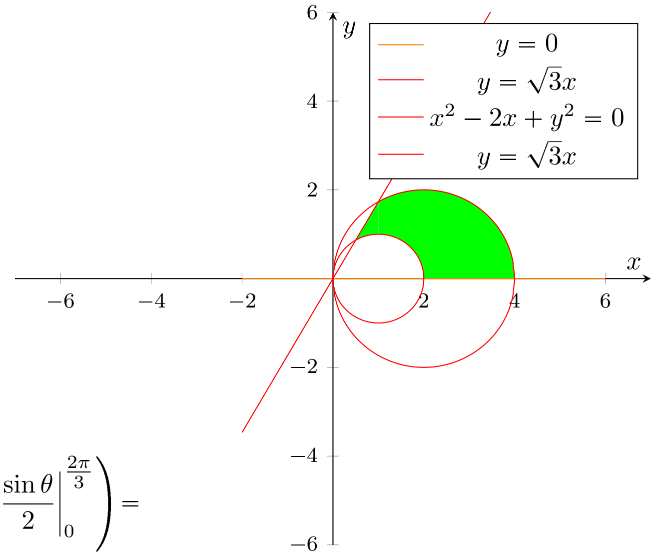
$$⊜2\cdot\left(\ln(y)\Big|_2^7\right) - 7\cdot\left(t\cdot\ln(t)\Big|_{\frac{2}{7}}^{1} - t\Big|_{\frac{2}{7}}^{1}\right) = 2\ln(7) - 2\ln(2) - \cancelto{0}{7\ln(1)} + \cancel{7}\cdot\frac{2}{\cancel{7}}\cdot\ln\left(\frac{2}{7}\right) + 7 - \cancel{7}\cdot\frac{2}{\cancel{7}} =$$

$$= -2\cdot\cancelto{\ln\left(\frac{2}{7}\right)}{(\ln(2) - \ln(7))} + 2\ln\left(\frac{2}{7}\right) + 5 = \cancelto{0}{2\ln\left(\frac{2}{7}\right) - 2\ln\left(\frac{2}{7}\right)} + 5 = 5$$

7.22

$x^2 - 2x + y^2 = 0;\ y = 0$

$x^2 - 4x + y^2 = 0;\ y = \sqrt{3}x$



$x^2 + y^2 = \rho^2;\ x = \rho\cos\phi;\ y = \rho\sin\phi;\ \mathrm{d}S = \rho\mathrm{d}\phi\mathrm{d}rho$

$x^2 + y^2 = 2x \implies \rho^{\cancel{2}} = 2\cancel{\rho}\cos\phi$

$x^2 + y^2 = 4x \implies \rho^{\cancel{2}} = 4\cancel{\rho}\cos\phi$

$$S = \int\limits_0^{\frac{\pi}{3}} \mathrm{d}\phi \int\limits_{2\cos\phi}^{4\cos\phi} \rho\,\mathrm{d}\rho = \frac{1}{2}\int\limits_0^{\frac{\pi}{3}} \left(\rho^2\Big|_{2\cos\phi}^{4\cos\phi}\right)\mathrm{d}\phi =$$

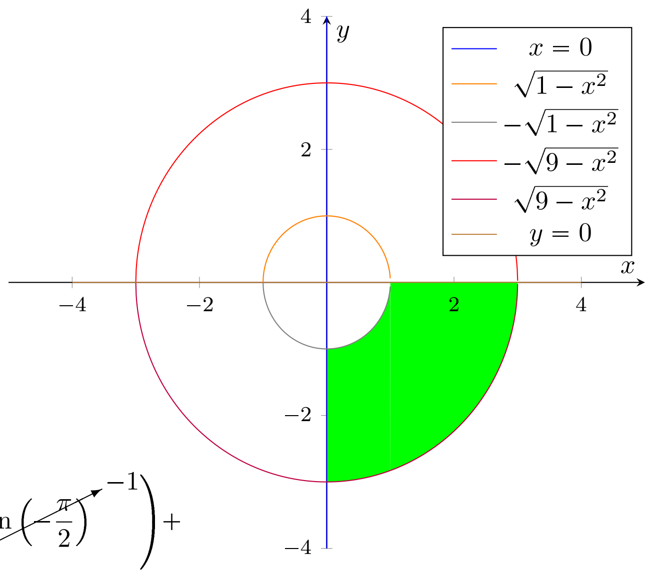
$$= \frac{1}{2}\int\limits_0^{\frac{\pi}{3}} \left(16\cos^2\phi - 4\cos^2\phi\right)\mathrm{d}\phi = 6\int\limits_0^{\frac{\pi}{3}} \cos^2\phi\mathrm{d}\phi =$$

$$= 3\left(\int\limits_0^{\frac{\pi}{3}}\mathrm{d}\phi + \int\limits_0^{\frac{\pi}{3}}\cos 2\phi\mathrm{d}\phi\right) = \left|\begin{matrix} 2\phi = \theta \\ 2\mathrm{d}\phi = \mathrm{d}\theta \\ \theta \in \left[0; \frac{2\pi}{3}\right] \end{matrix}\right| = 3\left(\frac{\pi}{3} + \frac{\sin\theta}{2}\Big|_0^{\frac{2\pi}{3}}\right) =$$

$$= 3\left(\frac{\pi}{3} + \frac{\sin\frac{2\pi}{3}}{2} - \cancelto{0}{\frac{\sin 0}{2}}\right) = \pi + \frac{3\sqrt{3}}{4}$$

8.22

$$D: x^2 + y^2 = 1;\ x^2 + y^2 = 9;\ x = 0;\ y = 0;\ (x \geqslant 0, y \leqslant 0)\ \mu = \frac{2x - y}{x^2 + y^2}$$



$x^2 + y^2 = \rho^2;\ x = \rho\cos\phi;\ y = \rho\sin\phi$

$\phi \in \left[-\frac{\pi}{2}; 0\right]\ \rho^2 = 1 \implies \rho = 1;\ \rho^2 = 9 \implies \rho = 3$

$$\mu = \frac{\cancel{\rho}(2\cos\phi - \sin\phi)}{\rho^{\cancel{2}}}$$

$$M = \int\limits_{-\frac{\pi}{2}}^{0}\mathrm{d}\phi\int\limits_1^3 \frac{\cancel{\rho}(2\cos\phi - \sin\phi)}{\cancel{\rho}}\mathrm{d}\rho =$$

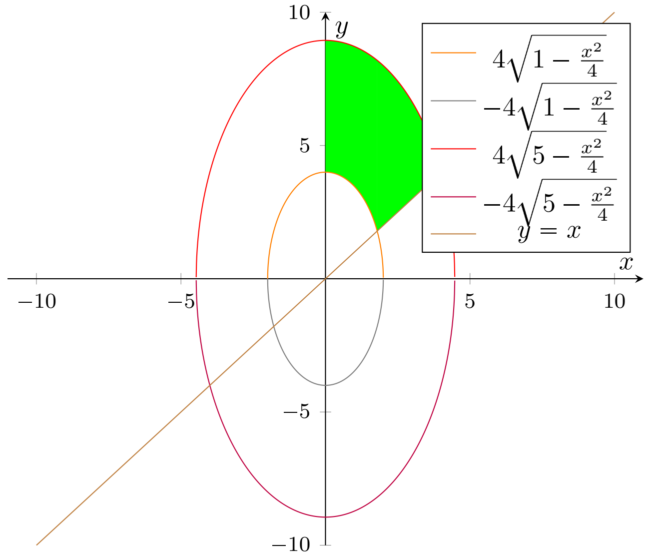
$$= 2\cdot\left(\int\limits_{-\frac{\pi}{2}}^{0} 2\cos\phi\mathrm{d}\phi - \int\limits_{-\frac{\pi}{2}}^{0}\sin\phi\mathrm{d}\phi\right) =$$

$$= 2\cdot\left(2\cdot\sin\phi\Big|_{\frac{\pi}{2}}^{0} + \cos\phi\Big|_{\frac{\pi}{2}}^{0}\right) = 2\cdot\left(2\cdot\left(\cancelto{0}{\sin 0} - \cancelto{-1}{\sin\left(-\frac{\pi}{2}\right)}\right) + \right.$$

$$\left. + \left(\cancelto{1}{\cos 0} - \cancelto{0}{\cos\left(-\frac{\pi}{2}\right)}\right)\right) = 2\cdot(2+1) = 6$$

9.22

$$D: 1 \leqslant \frac{x^2}{4} + \frac{y^2}{16} \leqslant 5;\ x \geqslant 0;\ y \geqslant 2x;\ \mu = \frac{x}{y}$$



$$x = 2\rho\cos\phi;\ y = 4\rho\sin\phi$$

$$1 \leqslant \frac{\cancel{4}\rho^2\cos^2\phi}{\cancel{4}} + \frac{\cancel{16}\rho^2\sin^2\phi}{\cancel{16}} \leqslant 5$$

$$1 \leqslant \rho^2 \cancelto{1}{\left(\cos^2\phi + \sin^2\phi\right)} \leqslant 5$$

$$1 \leqslant \rho \leqslant \sqrt{5}$$

$$y = 2x \implies \cancel{4}\rho\cos\phi = \cancel{4}\rho\sin\phi$$

$$\cos\phi = \sin\phi \implies \phi = \frac{\pi}{4}$$

$$\phi \in \left[\frac{\pi}{4}; \frac{\pi}{2}\right]\ \rho \in [1; \sqrt{5}]\mu = \frac{\cancel{2\rho}\cos\phi}{\cancelto{2}{\rho 4}\ \sin\phi} = \frac{\cos\phi}{2\sin\phi}$$

$$M = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \mathrm{d}\phi \int\limits_{1}^{\sqrt{5}} \frac{\operatorname{ctg}\phi}{2} \cdot 8\rho\mathrm{d}\rho = 2(5-1)\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{ctg}\phi\mathrm{d}\phi \ominus$$

$$= \int \frac{\operatorname{ctg}\phi \cdot \sin^2\phi}{\sin^2\phi}\mathrm{d}\phi = \int \frac{\operatorname{ctg}\phi}{\sin^2\phi(1+\operatorname{ctg}^2\phi)} \left| \begin{matrix} \operatorname{ctg}\phi = t \\ \dfrac{\mathrm{d}\phi}{\sin^2\phi} = \mathrm{d}t \end{matrix} \right| = -\int \frac{t\mathrm{d}t}{1+t^2} = \left| \begin{matrix} t^2 + 1 = u \\ 2t\mathrm{d}t = \mathrm{d}u \end{matrix} \right| = -\frac{1}{2}\int \frac{\mathrm{d}u}{u} =$$

$$= -\ln\sqrt{t^2+1} = -\ln\sqrt{\operatorname{ctg}^2\phi + 1} = \ln\sqrt{\frac{1}{\sin^2\phi}} = -\ln 1 + \ln|\sin\phi| = \ln|\sin\phi|$$

$$\ominus 8 \cdot \left(\cancelto{0}{\ln\left|\sin\frac{\pi}{2}\right|} - \ln\left|\sin\frac{\pi}{4}\right| \right) = -8 \cdot \ln\frac{\sqrt{2}}{2} = -8\left(\frac{1}{2}\ln 2 - \ln 2\right) = 4\ln 2$$

10.22

$$x = \frac{5\sqrt{y}}{3};\ x = \frac{5y}{9};\ z = 0;\ z = \frac{5(3+\sqrt{y})}{9}$$

$$x^2 = \frac{25y}{9} \implies y = \frac{9x^2}{25};\ y = \frac{9x}{5}$$

$$V = \int\limits_0^9 \mathrm{d}y \int\limits_{\frac{5y}{9}}^{\frac{5\sqrt{y}}{3}} \mathrm{d}x \int\limits_0^{\frac{5(3+\sqrt{y})}{9}} \mathrm{d}z = \frac{5}{9}\int\limits_0^9 (3+\sqrt{y})\mathrm{d}y \int\limits_{\frac{5y}{9}}^{\frac{5\sqrt{y}}{3}} \mathrm{d}x =$$

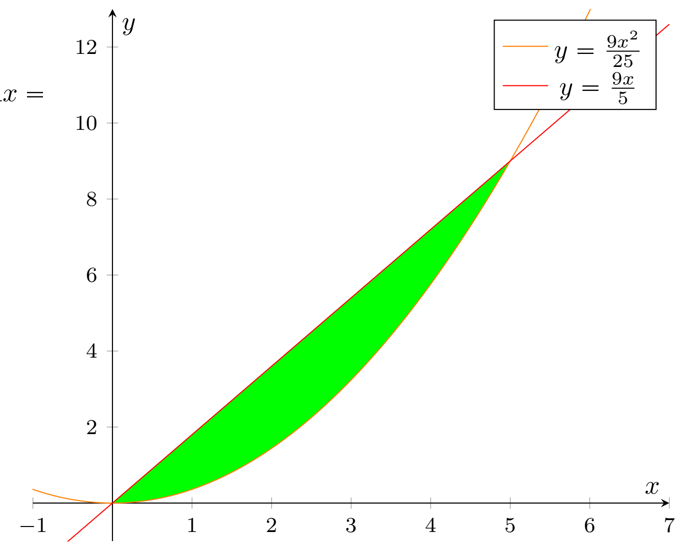
$$= \frac{5}{9}\int\limits_0^9 (3+\sqrt{y})\left(\frac{5\sqrt{y}}{3} - \frac{5y}{9}\right)\mathrm{d}y =$$

$$= \frac{25}{81}\int\limits_0^9 (9\sqrt{y} \cancel{-3y} \cancel{+3y} - y^{\frac{3}{2}})\mathrm{d}y =$$

$$= \frac{25}{81}\left(\left(\frac{\cancelto{6}{18}}{\cancel{3}}\cdot(9\sqrt{9} - 0)\right) - \frac{2}{5}\left(9^2\cdot\sqrt{9}\right)\right) =$$

$$= \frac{25}{81}\cdot\left(162 - \frac{486}{5}\right) = \frac{25 * \cancelto{2}{162}}{\cancel{81}} -$$

$$- \frac{\cancel{81} * 6 * \cancelto{5}{25}}{\cancel{5}} = 50 - 30 = 20$$



12.22

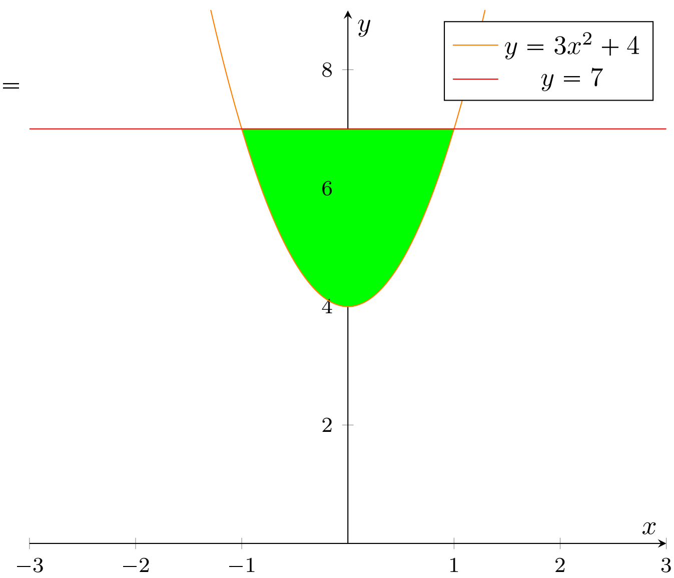
$$y = 3x^2 + 4;\ y = 7;\ z = 5 - \sqrt{2x^2 + 3y^2};\ z = 1 - \sqrt{2x^2 + 3y^2}$$

$$V = \int\limits_{-1}^1 \mathrm{d}x \int\limits_{3x^2+4}^{7} \mathrm{d}y \int\limits_{1-\sqrt{2x^2+3y^2}}^{5-\sqrt{2x^2+3y^2}} \mathrm{d}z = 4\int\limits_{-1}^1 \mathrm{d}x \int\limits_{3x^2+4}^{7} \mathrm{d}y =$$

$$= 4\int\limits_{-1}^1 (7 - 4 - 3x^2)\mathrm{d}x = 12\left(-\int\limits_{-1}^1 x^2\mathrm{d}x + \int\limits_{-1}^1 \mathrm{d}x\right) =$$

$$= 12\cdot\left(\left(\frac{1}{3} + \frac{1}{3}\right) + (1+1)\right) = 12\cdot\left(-\frac{2}{3} + 2\right) =$$

$$= \frac{\cancelto{4}{12}\cdot 4}{\cancel{3}} = 16$$



13.22

$z = 9\sqrt{x^2 + y^2}$; $z = 22 - x^2 - y^2$;

$x^2 + y^2 = \rho^2$; $x = \rho\cos\phi$; $y = \rho\sin\phi$

$z = 9\rho$; $z = 22 - \rho^2$;

$9\rho = 22 - \rho^2 \implies \rho^2 + 9\rho - 22 = 0\,(\rho = 2)$

$$V = \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_0^2 \rho \mathrm{d}\rho \int\limits_{9\rho}^{22-\rho^2} \mathrm{d}z = \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_0^2 \rho(22 - \rho^2 - 9\rho)\mathrm{d}\rho = \int\limits_0^{2\pi} \left(22\int\limits_0^2 \rho\mathrm{d}\rho - \int\limits_0^2 \rho^3\mathrm{d}\rho - 9\int\limits_0^2 \rho^2\mathrm{d}\rho\right)\mathrm{d}\phi =$$

$$\int\limits_0^{2\pi} \left(22\cdot\left(\frac{4}{2} - 0\right) - \left(\frac{16}{4} - 0\right) - 9\cdot\left(\frac{8}{3} - 0\right)\right)\mathrm{d}\phi = \int\limits_0^{2\pi} (44 - 4 - 24)\,\mathrm{d}\phi = 16\int\limits_0^{2\pi} \mathrm{d}\phi = 32\pi$$

14.22

$z = 24((x + 1)^2 + y^2) + 1$; $z = 48x + 49$;

$x^2 + y^2 = \rho^2$; $x = \rho\cos\phi$; $y = \rho\sin\phi$

$24(x^2 + 2x + 1 + y^2) = 48x + 48$

$x^2 + y^2 = 1 \implies \rho \in [0; 1]$

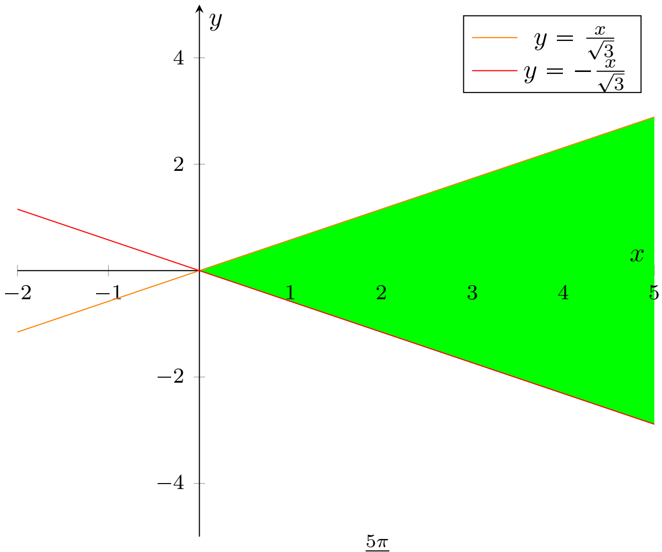
$24x^2 + 48x + y^2 + 25 = 24\rho^2\cos^2\phi + 48\rho\cos\phi + 24\rho^2\sin^2\phi + 25 = 24\rho^2 + 48\rho\cos\phi + 25$

$$V = \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_0^1 \rho\mathrm{d}\rho \int\limits_{24\rho^2+48\rho\cos\phi+25}^{48\rho\cos\phi+49} \mathrm{d}z = \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_0^1 \rho(48\rho\cos\phi + 49 - 25\rho^2 - 48\rho\cos\phi + 25)\mathrm{d}\rho =$$

$$= \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_0^1 (24\rho - 24\rho^3\mathrm{d}\rho = 24\int\limits_0^{2\pi} \left(-\frac{1}{4} + 0\right) + \left(\frac{1}{2} - 0\right)\mathrm{d}\phi = 24\cdot\frac{1}{4}\cdot 2\pi = 12\pi$$

15.22

$$49 \leqslant x^2+y^2+z^2 \leqslant 144;\ z \leqslant -\sqrt{\frac{x^2+y^2}{99}};\ y \geqslant \frac{x}{\sqrt{3}};\ y \geqslant -\frac{x}{\sqrt{3}}$$



$$x = \rho\cos\phi\cos\theta$$

$$y = \rho\sin\phi\sin\theta$$

$$z = \rho\cos\phi 7 \leqslant \rho \leqslant 12;$$

$$y = \frac{x}{\sqrt{3}} \implies \tan\theta = \frac{\sqrt{3}}{3} \implies \theta = \frac{\pi}{6};$$

$$y = -\frac{x}{\sqrt{3}} \implies \tan\theta = -\frac{\sqrt{3}}{3} \implies \theta = \frac{5\pi}{6};$$

$$\tan\beta = \frac{\rho}{|z|} = \sqrt{99} \implies \pi - \arctan\sqrt{99} \leqslant \phi \leqslant \pi$$

$$V = \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \mathrm{d}\theta \int\limits_{\pi-\arctan\sqrt{99}}^{\pi} \sin\phi\,\mathrm{d}\phi \int\limits_{7}^{12} \rho^2\mathrm{d}\rho =$$

$$= \left(\frac{12^3}{3} - \frac{7^3}{3}\right) \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \mathrm{d}\theta \int\limits_{\pi-\arctan\sqrt{99}}^{\pi} \sin\phi\,\mathrm{d}\phi = -\frac{1385}{3}\cdot\left(\cos\pi - \cos\left(\pi - \arctan\sqrt{99}\right)\right) \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \mathrm{d}\theta =$$

$$= \frac{1385}{3}\cdot\frac{2\pi}{3}\cdot\left(-1+\cos\left(\arctan\sqrt{99}\right)\right) = \frac{1385\cdot 2\pi}{9}\left(-1+\frac{1}{\sqrt{1+\tan^2\left(\arctan\sqrt{99}\right)}}\right) = -\frac{1385\cdot 2\pi}{9}\cdot\left(-1+\frac{1}{10}\right) =$$

$$= \frac{\cancel{1385}^{277}\cdot\cancel{2}\cdot\pi}{\cancel{9}}\cdot\frac{\cancel{9}}{\cancel{10}} = 277\pi$$

16.22

$$x^2+y^2+z^2 = 16;\ x^2+y^2 = 4(x^2+y^2 \leqslant 4);\ \mu = |z|$$

$$x^2+y^2 = \rho;\ x = \rho\cos\phi;\ y = \rho\sin\phi;$$

$$z = \sqrt{16-\rho^2} \implies 0 \leqslant z \leqslant \sqrt{16-\rho^2};$$

$$0 \leqslant \phi \leqslant 2\pi;\ 0 \leqslant \rho \leqslant 2$$

$$V = 2\cdot\int\limits_0^{2\pi}\mathrm{d}\phi\int\limits_0^2\rho\mathrm{d}\rho\int\limits_0^{\sqrt{16-\rho^2}} z\mathrm{d}z = \frac{2}{2}\int\limits_0^{2\pi}\mathrm{d}\phi\int\limits_0^2\left(16\rho-\rho^3\right)\mathrm{d}\rho =$$

$$= \int\limits_0^{2\pi}\left(16\cdot\frac{4}{2} - \frac{16}{4}\right)\mathrm{d}\phi = 28\cdot 2\pi = 56\pi$$