Симонов А. ТР к КМ 2.

№6.14

$$\int \frac{x^3+x+2}{(x+2)x^3}\,dx = \int \frac{x^3}{(x+2)\cancel{x^3}}\,dx + \int \frac{\cancel{x+2}}{(\cancel{x+2})x^3}\,dx =$$

$$= \int \frac{dx}{x+2} + \int \frac{dx}{x^3} = \underline{\ln|x+2| - \frac{1}{2x^2} + C}$$

№7.14

$$\int \frac{3x^3+x+46}{(x-1)^2(x^2+9)}\,dx$$

$$\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{(x^2+9)} = \frac{(Ax-A)(x^2+9) + Bx^2+9B + (Cx+D)(x-1)^2}{(x-1)^2(x^2+9)} =$$

$$= \frac{Ax^3+9Ax-Ax^2-9A+Bx^2+9B+Cx^3+Dx^2-2Cx^2-2Dx+Cx+D}{(x-1)^2(x^2+9)} =$$

$$= \frac{(A+C)x^3+(A+B-2C+D)x^2+(-2D+C)x+9B+D}{(x-1)^2(x^2+9)}$$

$$\begin{cases} A+C=3 \\ A+B-2C+D=0 \\ -2D+C=1 \\ 9B+D=46 \end{cases} \qquad \begin{cases} A=3-C \\ 3+B-3C+D=0 \\ -2D+C=1 \\ 9B+D=46 \end{cases}$$

$$\begin{cases} A=3-C \\ B=3C-3-D \\ -2D+C=1 \\ 27C-27-9D+D=46 \end{cases} \qquad \begin{cases} A=3-C \\ B=3C-3-D \\ C=1+2D \\ 27+54D-27-8D=46 \end{cases}$$

$$\begin{cases} A=3-C \\ B=3+2D-3-D \\ C=1+2D \\ D=1 \end{cases} \qquad \begin{cases} A=0 \\ B=5 \\ C=3 \\ D=1 \end{cases}$$

$$\int \frac{3x^3+x+46}{(x-1)^2(x^2+9)}dx = 5\int\frac{dx}{(x-1)^2} + \int\frac{3x+1}{(x^2+9)}dx =$$

$$= -\frac{5}{x-1} + \frac{3}{2}\int\frac{d(x^2+9)}{(x^2+9)} + \int\frac{dx}{x^2+9} = \underline{-\frac{5}{x-1}+\frac{3}{2}\ln(x^2+9)+\frac{1}{3}\operatorname{arctg}\frac{x}{3}+C}$$

№10.14

$$\int_0^{\pi} 2^4\sin^2 x\cos^6 x\,dx = \int_0^{\pi} 2^4\cdot\frac{1}{2^2}\sin^2 2x\,(1+\cos 2x)^2\,dx =$$

$$= \int_0^{\pi}\sin^2 2x\,(1+2\cos 2x+\cos^2 2x)\,dx = \int_0^{\pi}(\sin^2 2x + 2\sin^2 2x\cdot$$

$$\cdot\cos 2x + \sin^2 2x\cos^2 2x)\,dx = \int_0^{\pi}\frac{1}{2}(1-\cos 4x)\,dx + \int_0^{\pi}\sin^2 2x\cos 2x\,dx$$

$$+\int_0^{\pi}\frac{1}{4}\sin^2 4x\,dx = \frac{1}{2}\left(x-\frac{1}{4}\sin 4x\right)\Big|_0^{\pi} + 2\int_0^{\pi}\frac{1}{2}\sin^2 2x\,d(\sin 2x) +$$

$$+\frac{1}{4}\int\frac{1}{2}(1-\cos 8x)\,dx = \frac{\pi}{2}-\frac{1}{2}\cdot 0+\frac{1}{3}\sin^3 2x\Big|_0^{\pi}+\frac{1}{8}\left(x-\frac{1}{8}\sin 8x\right)\Big|_0^{\pi} =$$

$(=)\ \frac{5\pi}{8}$

N 11.14

$$\int_0^4 e^{\sqrt{\frac{4-x}{4+x}}} \cdot \frac{dx}{(4+x)\sqrt{16-x^2}} = \left| \begin{aligned} & t = \sqrt{\frac{4-x}{4+x}} \\ & dt = -\frac{4}{(4+x^2)\sqrt{\frac{4-x}{4+x}}}\,dx \end{aligned} \right| =$$

$$= -\frac{1}{4}\int_1^0 e^t\,dt = \frac{1}{4}\int_0^1 e^t\,dt = \frac{1}{4}(e-1) = \frac{e-1}{4}$$

12.14

$$\int_0^{\frac{5}{2}} \frac{x^2\,dx}{\sqrt{25-x^2}} = \left| \begin{aligned} & x = 5\sin t \\ & dx = 5\cos t\,dt \\ & \sqrt{5-25\sin^2 t} = 5\sqrt{1-\sin^2 t} = \\ & = 5\cos t \end{aligned} \right| = \int_0^{\pi/6} \frac{25\sin^2 t \cdot 5\cos t\,dt}{5\cos t} =$$

$$= \int_0^{\pi/6} \frac{25}{2}(1-\cos 2t)\,dt = \frac{25}{2}\left(t - \frac{1}{2}\sin 2t\right)\Big|_0^{\pi/6} =$$

$$= \frac{25}{2}\left(\frac{\pi}{6} - \frac{1}{2}\sin\frac{\pi}{3}\right) = \frac{25}{2}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{25}{4}\left(\frac{4\pi - 3\sqrt{3}}{12}\right) =$$

$$= \frac{25(4\pi - 3\sqrt{3})}{48}$$

№ 13.14

$$\int \frac{\sqrt{1+\sqrt[4]{x^3}}}{x^2 \cdot \sqrt[8]{x}}\,dx = \int x^{-\frac{17}{8}}\left(1+x^{3/4}\right)^{\frac{1}{2}} dx \ominus$$

$$\frac{m+1}{n}+p = -1$$

$$x^{-\frac{3}{4}}+1 = z^2$$

$$x = (z^2-1)^{-\frac{4}{3}}$$

$$dx = -\frac{4}{3}(z^2-1)^{-\frac{7}{3}} \cdot 2z\,dz = -\frac{8}{3}z(z^2-1)^{-\frac{7}{3}}dz$$

$$\ominus \int (z^2-1)^{\frac{17}{6}} \cdot \left(1+(z^2-1)^{-1}\right)^{\frac{1}{2}} \left(-\frac{8}{3}z(z^2-1)^{-\frac{7}{3}}\right)dz =$$

$$= -\frac{8}{3}\int (z^2-1)^{\frac{1}{2}}\left(1+(z^2-1)^{-1}\right)^{\frac{1}{2}} z\,dz = -\frac{8}{3}\int \cancel{(z^2-1)} \cdot \frac{z}{\cancel{(z^2-1)}} \cdot z\,dz =$$

$$= -\frac{8}{3}\cdot\frac{z^3}{3}+C = -\frac{8}{9}z^3 + C$$

14.14

$x = \arccos(y)$

$x = 0$

$y = 0$

$y = \cos(x)$

$$S = \int_a^b y(x)\,dx = \int_0^{\frac{\pi}{2}} \cos(x)\,dx = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1$$

15.14

$$\begin{cases} x = 3\cos t \\ y = 8\sin t \end{cases} \qquad y \geq 4$$

$8\sin t = 4$

$\sin t = \frac{1}{2}$

$t = \left\{\frac{\pi}{6} ; \frac{5\pi}{6}\right\}$

$$S = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 3\cos t \cdot 8\cos t\,dt = 24 \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\cos 2t + 1)\,dt =$$

$$= 24\left(\frac{4\pi}{6} + \frac{1}{2}\left(\sin\frac{5\pi}{3} - \sin\frac{\pi}{3}\right)\right) = 12\left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}\right) =$$

$$= 8\pi - 6\sqrt{3}$$