Хохлов Пётр А-07-23
ТР-2
№ (11-16)

№ 11.21) $\int_0^2 \frac{x^3\,dx}{\sqrt{x^4+9}} \;(=)$

из фор-мы $\int x^m(a+bx^n)^p dx \Rightarrow \begin{array}{l} m=3 \\ n=4 \\ p=-\frac{1}{2} \Rightarrow s=2 \end{array} \Bigg| \Rightarrow z^s = a+bx^n$ следует $\Rightarrow$ $x^4+9=z^2$, $4x^3\,dx = 2z\cdot dz$

$\Rightarrow (=) \int_3^5 \frac{x^3\,dz}{|z|\cdot 2x^3}\cdot z = \frac{1}{2}\int_3^5 dz = \frac{z}{2}\Big|_3^5 = \frac{5}{2}-\frac{3}{2} = 1$

№ 12.21) $\int_{1/2}^{1} \operatorname{arctg}\sqrt{5x-1}\,dx = x\cdot\operatorname{arctg}\sqrt{5x-1}\Big|_{1/2}^{1} - \int_{1/2}^{1} x\cdot d(\operatorname{arctg}\sqrt{3x-1}) =$

$= x\cdot\operatorname{arctg}\sqrt{3x-1}\Big|_{1/2}^{1} - \int_{1/2}^{1}\frac{x\,d(\sqrt{5x-1})}{5x} = x\cdot\operatorname{arctg}\sqrt{5x-1}\Big|_{1/2}^{1} - \frac{\sqrt{5x-1}}{5}\Big|_{1/2}^{1} = \operatorname{arctg}2 - \frac{\operatorname{arctg}\sqrt{\frac{3}{2}}}{2} - \left(\frac{2}{5} - \frac{\sqrt{\frac{3}{2}}}{5}\right) =$

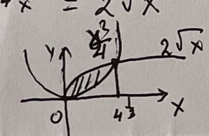
$= \operatorname{arctg}2 - \frac{\operatorname{arctg}\sqrt{\frac{3}{2}}}{2} + \frac{\sqrt{\frac{3}{2}}-2}{5}$

№ 13.21) $\begin{cases} y^2 = 4x \; (*) \\ x^2 = 4y \; (**) \end{cases}$ | S-?

$(*) \to y = \sqrt{4x} = 2\sqrt{x}$

$y = \frac{x^2}{4} \Rightarrow y^2 = \left(\frac{x^2}{4}\right)^2$

$\Rightarrow 4x = \frac{x^4}{4\cdot 4} \Rightarrow \frac{x^4}{x} = 4 \iff \left[\begin{array}{l} x=0 \\ x=4^{\frac{1}{3}} \end{array}\right.$



$S = \int_0^{4^{1/3}} 2\sqrt{x}\,dx - \int_0^{4^{1/3}}\frac{x^2}{4}dx = \frac{2\cdot 2}{3}\cdot x^{\frac{3}{2}}\Big|_0^{4^{1/3}} - \frac{x^3}{12}\Big|_0^{4^{1/3}} = \frac{4}{3}\cdot 4^{\frac{1}{3}\cdot\frac{3}{2}} - 0 - \frac{4^{\frac{1}{3}\cdot 3}}{12} + 0 =$

$= \frac{4\cdot 2}{3} - \frac{1}{3} = \frac{7}{3}$

№ 14.21) $\rho = 4\cdot\cos^2\left(2\varphi - \frac{\pi}{4}\right)$ ⇒ роз - 4 штуки; для нахожд. углов, приравняем длину ρ к нулю ⇒

1) $\cos\left(2\varphi - \frac{\pi}{2}\right) = \frac{\pi}{2} \Rightarrow \varphi = \frac{3\pi}{8}$

2) $\cos\left(2\varphi - \frac{\pi}{2}\right) = \frac{3\pi}{2} \Rightarrow \varphi = \frac{7\pi}{8}$

для 1

$\Rightarrow S = \frac{1}{2}\int_{\varphi_1}^{\varphi_2}\rho^2(\varphi)\,d\varphi = \frac{1}{2}\int_{\frac{3\pi}{8}}^{\frac{3\pi}{8}} 16\cdot\cos^4\left(2\varphi - \frac{\pi}{4}\right)d\varphi = 8\int_{\frac{3\pi}{8}}^{\frac{7\pi}{8}}\cos^4\left(2\varphi - \frac{\pi}{4}\right)d\varphi =$

$= \left\| \begin{array}{l} 2\varphi - \frac{\pi}{4} = u \\ 2\cdot d(\varphi) = du \end{array} \right\| = 8\int_{u_1}^{u_2}\left(\cos^2(u)\right)^2\frac{du}{2} = 4\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\frac{\cos 2u + 1}{2}\right)^2 du = \frac{4}{4}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\cos^2 2u + 2\cdot\cos 2u + 1\right)du = \int_{u_1}^{u_2}\frac{1+\cos 4u}{2}du + \int_{u_1}^{u_2} 2\cos 2u\,du +$

$\frac{3\pi}{2}$ и $\frac{\pi}{2}$

$+ \int_{u_1}^{u_2} du = \frac{u}{2}\Big|_{u_1}^{u_2} + \frac{\sin 4u}{2\cdot 4}\Big|_{u_1}^{u_2} + 2\cdot\frac{\sin 2u}{2}\Big|_{u_1}^{u_2} + u\Big|_{u_1}^{u_2} = \frac{3}{2}u\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \frac{\sin 4u}{8}\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin 2u\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} =$

$= \frac{3}{2}\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + \frac{\underbrace{\sin(6\pi)}_{0} - \underbrace{\sin(2\pi)}_{0}}{8} + \left(\underbrace{\sin 3\pi}_{0} - \underbrace{\sin\pi}_{0}\right) = \frac{3}{2}\cdot\frac{2\pi}{2} = \frac{3\pi}{2}$ ⇒ для 4 лепестков

$S = 4\cdot\frac{3\pi}{2} = 6\pi$

№ 15.21 $L-?$

$$\rho = \cos^4\frac{\varphi}{4} \qquad 0 \le \varphi < \pi$$

$$l = \int_{\varphi_1}^{\varphi_2} \sqrt{\rho^2 + (\rho')^2}\, d\varphi \; ; \; \rho' = \left(\cos^4\frac{\varphi}{4}\right)' = -4\cdot\cos^3\frac{\varphi}{4}\cdot\sin\frac{\varphi}{4}\cdot\frac{1}{4} = -\cos^3\frac{\varphi}{4}\cdot\sin\frac{\varphi}{4}$$

$$l = \int_0^{\pi} \sqrt{\cos^8\frac{\varphi}{4} + \left(\cos^4\frac{\varphi}{4}\right)'^2}\, d\varphi = \int_0^{\pi}\sqrt{\cos^8\frac{\varphi}{4} + \cos^6\frac{\varphi}{4}\cdot\sin^2\frac{\varphi}{4}}\, d\varphi = \int_0^{\pi}\cos^3\frac{\varphi}{4}\sqrt{\cos^2\frac{\varphi}{4}+\sin^2\frac{\varphi}{4}}\, d\varphi = \int_0^{\pi}\left(\cos^3\frac{\varphi}{4}\right)d\varphi =$$

$$= \left\| \begin{matrix} \frac{\varphi}{4} = t \\ \frac{1}{4}\cdot d\varphi = dt \end{matrix} \right\| = 4\cdot\int_0^{\frac{\pi}{4}} \cos^3 t\, dt = 4\int_0^{\frac{\pi}{4}} (1-\sin^2 t)\, d\sin t = 4\cdot\sin t\Big|_0^{\frac{\pi}{4}} - 4\cdot\frac{\sin^3 t}{3}\Big|_0^{\frac{\pi}{4}} =$$

$$= 4\cdot\sin\frac{\pi}{4} - \frac{4}{3}\cdot\sin^3\frac{\pi}{4} = \frac{4\sqrt{2}}{2} - \frac{4}{3}\cdot\left(\frac{\sqrt{2}}{2}\right)^3 = 2\sqrt{2} - \frac{\sqrt{2}}{3} = \sqrt{2}\left(\frac{5}{3}\right)$$

№ 16.21)

$$\int_0^{+\infty} \frac{\operatorname{arctg} 3x}{1+9x^2}\, dx = \frac{1}{3}\int \operatorname{arctg} 3x\cdot d(\operatorname{arctg} 3x) = \frac{\operatorname{arctg} 3x}{3}\Big|_0^{+\infty}$$

$$\lim_{t\to+\infty}\left(\frac{\operatorname{arctg} 3x}{3}\Big|_0^t\right) = \lim_{t\to+\infty}\left(\frac{\operatorname{arctg} 3t}{3} - \frac{\operatorname{arctg} 0}{3}\right) = \lim_{t\to\infty}\left(\frac{\frac{\pi}{2}}{3} - 0\right) = \frac{\pi}{6} \Rightarrow \text{сходится!}$$