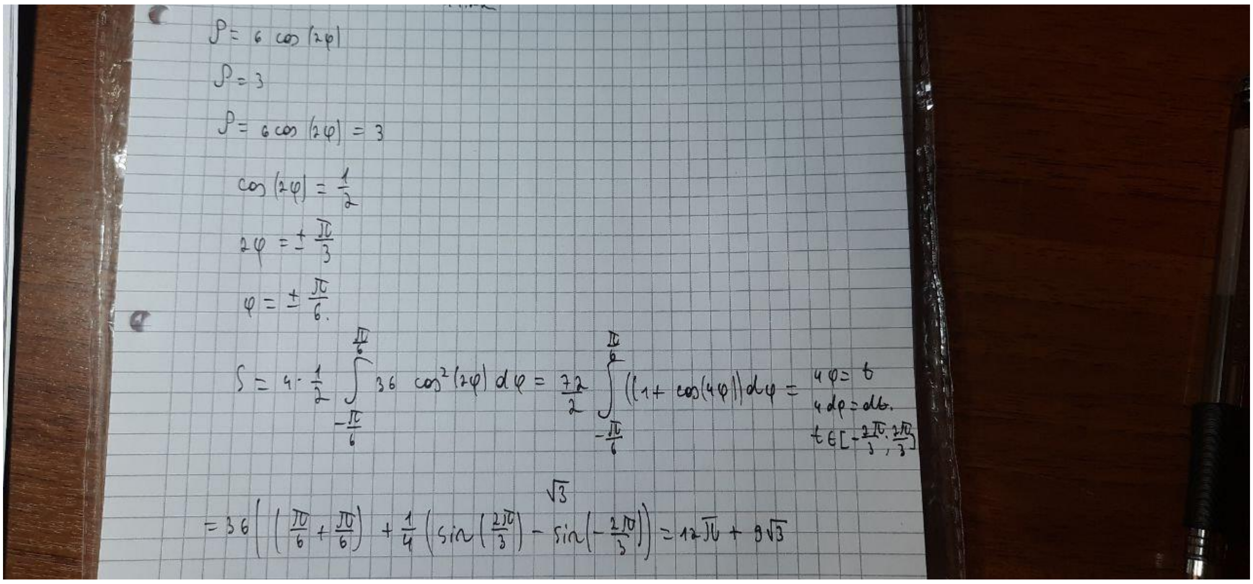
1 книга



$\rho = 6\cos(2\varphi)$

$\rho = 3$

$\rho = 6\cos(2\varphi) = 3$

$\cos(2\varphi) = \frac{1}{2}$

$2\varphi = \pm\frac{\pi}{3}$

$\varphi = \pm\frac{\pi}{6}.$

$$S = 4\cdot\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 36\cos^2(2\varphi)\,d\varphi = \frac{72}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\left((1+\cos(4\varphi)\right)d\varphi =$$

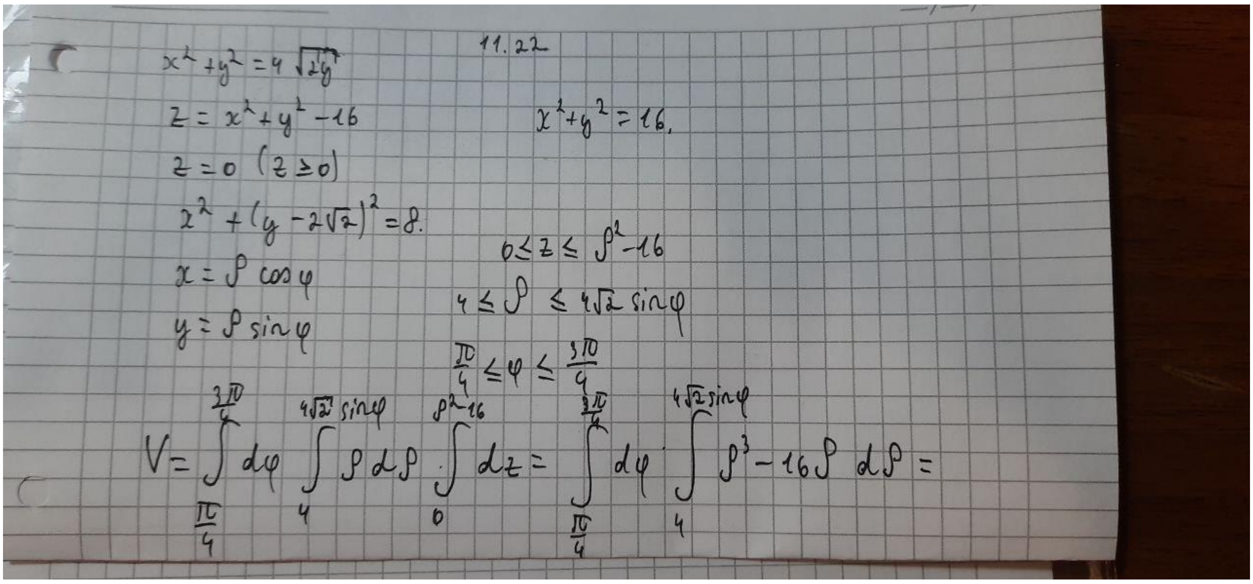
$4\varphi = t$

$4d\varphi = dt.$

$t \in [-\frac{2\pi}{3}; \frac{2\pi}{3}]$

$$= 36\left(\left(\frac{\pi}{6}+\frac{\pi}{6}\right) + \frac{1}{4}\left(\overset{\sqrt{3}}{\sin\left(\frac{2\pi}{3}\right)} - \sin\left(-\frac{2\pi}{3}\right)\right)\right) = 12\pi + 9\sqrt{3}$$

2книга



11.22

$x^2+y^2 = 4\sqrt{2y}$

$z = x^2+y^2-16$

$z = 0 \quad (z \geq 0)$

$x^2 + (y-2\sqrt{2})^2 = 8.$

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

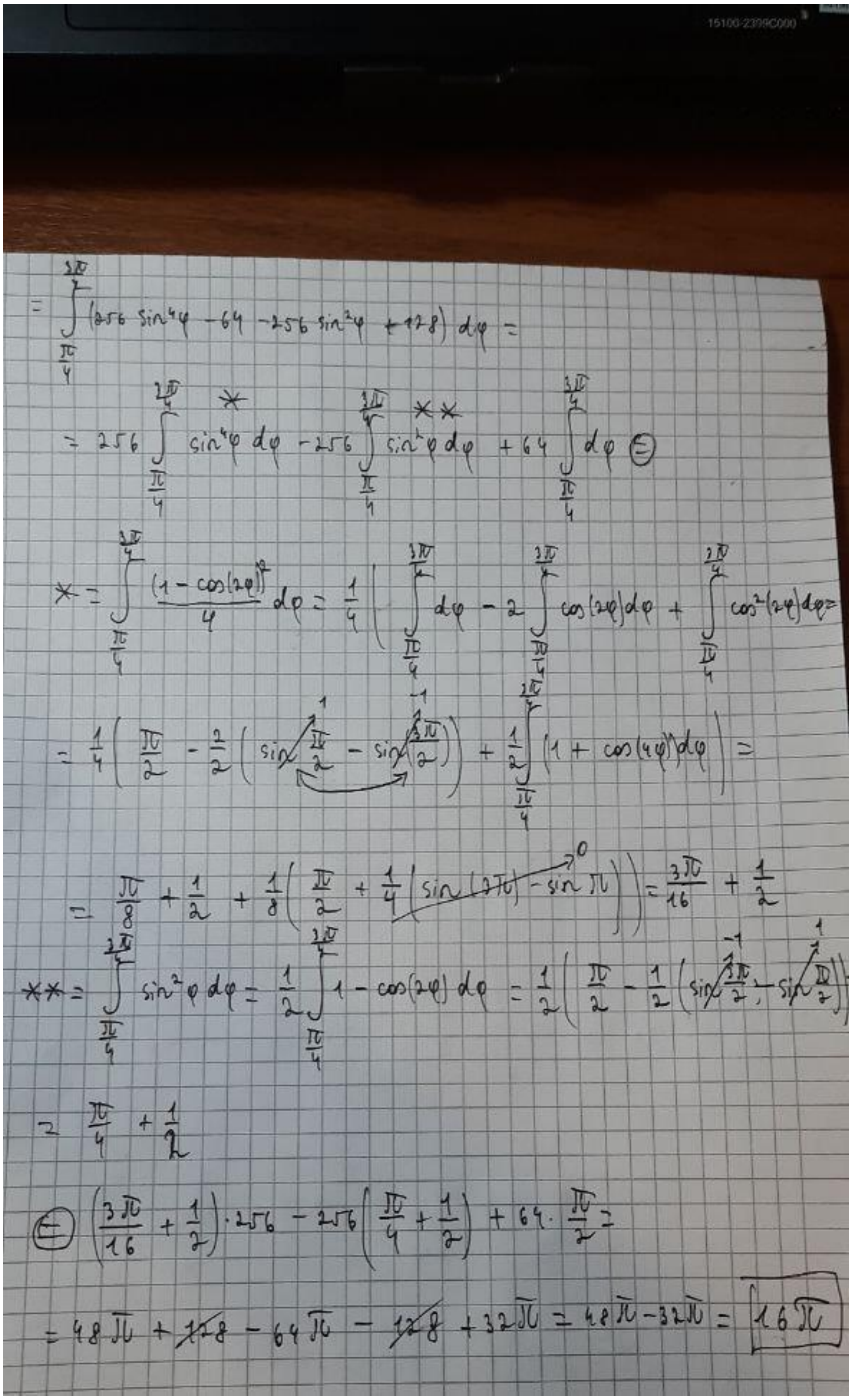
$x^2+y^2 = 16.$

$0 \leq z \leq \rho^2 - 16$

$4 \leq \rho \leq 4\sqrt{2}\sin\varphi$

$\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$

$$V = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_{4}^{4\sqrt{2}\sin\varphi} \rho\,d\rho \int_{0}^{\rho^2-16} dz = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_{4}^{4\sqrt{2}\sin\varphi} \rho^3 - 16\rho\,d\rho =$$



$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (256\sin^4\varphi - 64 - 256\sin^2\varphi + 128)\,d\varphi =$$

$$= 256 \overset{*}{\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^4\varphi\,d\varphi} - 256 \overset{**}{\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\varphi\,d\varphi} + 64\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \;⊜$$

$$* = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(1-\cos(2\varphi))^2}{4}\,d\varphi = \frac{1}{4}\left(\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi - 2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos(2\varphi)\,d\varphi + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos^2(2\varphi)\,d\varphi =\right.$$

$$= \frac{1}{4}\left(\frac{\pi}{2} - \frac{2}{2}\left(\overset{1}{\cancel{\sin\frac{\pi}{2}}} - \overset{-1}{\cancel{\sin\left(\frac{3\pi}{2}\right)}}\right) + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1+\cos(4\varphi))\,d\varphi\right) =$$

$$= \frac{\pi}{8} + \frac{1}{2} + \frac{1}{8}\left(\frac{\pi}{2} + \frac{1}{4}\overset{0}{\cancel{\left(\sin(3\pi) - \sin\pi\right)}}\right) = \frac{3\pi}{16} + \frac{1}{2}$$

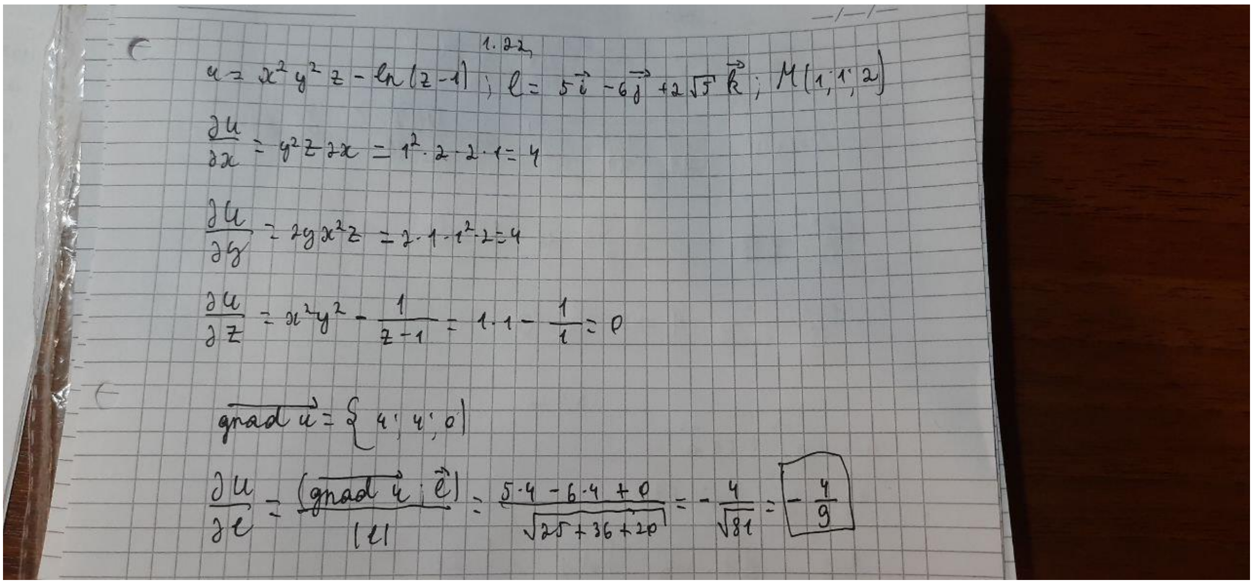
$$** = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\varphi\,d\varphi = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 1 - \cos(2\varphi)\,d\varphi = \frac{1}{2}\left(\frac{\pi}{2} - \frac{1}{2}\left(\overset{-1}{\cancel{\sin\frac{3\pi}{2}}}; - \overset{1}{\cancel{\sin\frac{\pi}{2}}}\right)\right)$$

$$= \frac{\pi}{4} + \frac{1}{2}$$

$$⊜ \left(\frac{3\pi}{16} + \frac{1}{2}\right)\cdot 256 - 256\left(\frac{\pi}{4} + \frac{1}{2}\right) + 64\cdot\frac{\pi}{2} =$$

$$= 48\pi + \cancel{128} - 64\pi - \cancel{128} + 32\pi = 48\pi - 32\pi = \boxed{16\pi}$$

3 книга



1.22

$u = x^2y^2z - \ln(z-1)$; $l = 5\vec{i} - 6\vec{j} + 2\sqrt{5}\vec{k}$; $M(1;1;2)$

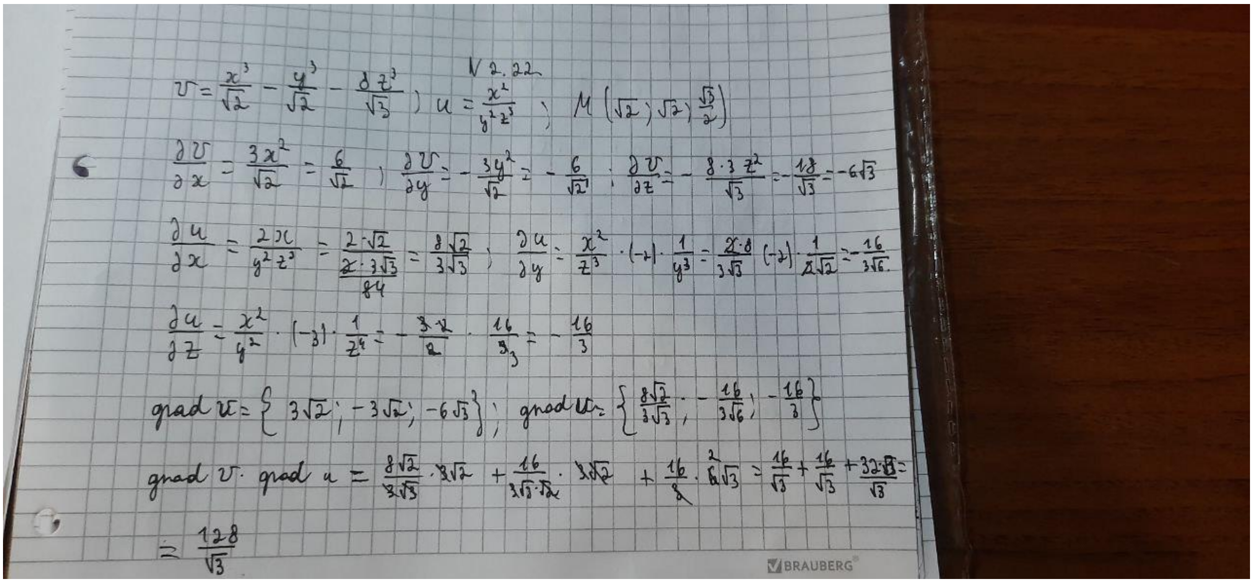
$\frac{\partial u}{\partial x} = y^2z2x = 1^2\cdot2\cdot2\cdot1 = 4$

$\frac{\partial u}{\partial y} = 2yx^2z = 2\cdot1\cdot1^2\cdot2 = 4$

$\frac{\partial u}{\partial z} = x^2y^2 - \frac{1}{z-1} = 1\cdot1 - \frac{1}{1} = 0$

$\overrightarrow{grad\,u} = \{4;4;0\}$

$\frac{\partial u}{\partial l} = \frac{(\overrightarrow{grad\,u};\vec{l})}{|l|} = \frac{5\cdot4-6\cdot4+0}{\sqrt{25+36+20}} = -\frac{4}{\sqrt{81}} = \boxed{-\frac{4}{9}}$



№2.22

$v = \frac{x^3}{\sqrt2} - \frac{y^3}{\sqrt2} - \frac{8z^3}{\sqrt3}$; $u = \frac{x^2}{y^2z^3}$; $M(\sqrt2;\sqrt2;\frac{\sqrt3}{2})$

$\frac{\partial v}{\partial x} = \frac{3x^2}{\sqrt2} = \frac{6}{\sqrt2}$; $\frac{\partial v}{\partial y} = -\frac{3y^2}{\sqrt2} = -\frac{6}{\sqrt2}$; $\frac{\partial v}{\partial z} = -\frac{8\cdot3z^2}{\sqrt3} = -\frac{18}{\sqrt3} = -6\sqrt3$

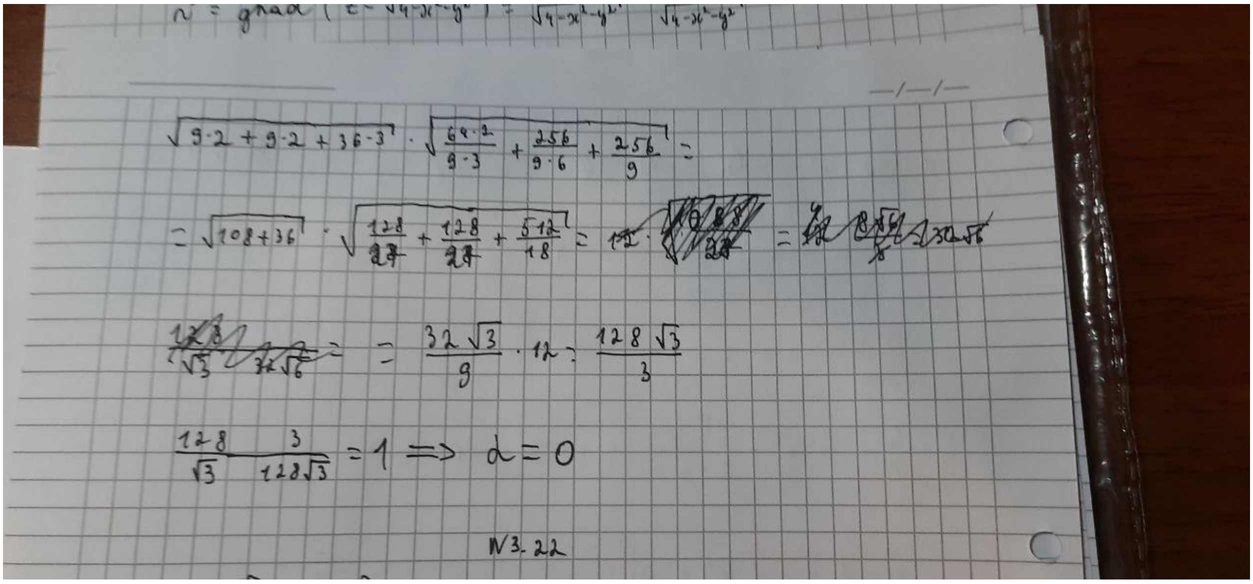
$\frac{\partial u}{\partial x} = \frac{2x}{y^2z^3} = \frac{2\cdot\sqrt2}{\frac{\cancel{2}\cdot3\sqrt3}{84}} = \frac{8\sqrt2}{3\sqrt3}$; $\frac{\partial u}{\partial y} = \frac{x^2}{z^3}\cdot(-2)\cdot\frac{1}{y^3} = \frac{\cancel{2}\cdot8}{3\sqrt3}(-2)\cdot\frac{1}{\cancel{2}\sqrt2} = -\frac{16}{3\sqrt6}$

$\frac{\partial u}{\partial z} = \frac{x^2}{y^2}\cdot(-3)\cdot\frac{1}{z^4} = -\frac{\cancel{3}\cdot\cancel{2}}{\cancel{2}}\cdot\frac{16}{\cancel{9}_3} = -\frac{16}{3}$

$grad\,v = \{3\sqrt2; -3\sqrt2; -6\sqrt3\}$; $grad\,u = \{\frac{8\sqrt2}{3\sqrt3}; -\frac{16}{3\sqrt6}; -\frac{16}{3}\}$

$grad\,v\cdot grad\,u = \frac{8\sqrt2}{\cancel{3}\sqrt3}\cdot\cancel{3}\sqrt2 + \frac{16}{3\sqrt3\cdot\sqrt2}\cdot3\sqrt2 + \frac{16}{\cancel{3}}\cdot\overset{2}{\cancel{6}}\sqrt3 = \frac{16}{\sqrt3} + \frac{16}{\sqrt3} + \frac{32\cdot3}{\sqrt3} =$

$= \frac{128}{\sqrt3}$



$\sqrt{9\cdot2+9\cdot2+36\cdot3}\cdot\sqrt{\frac{64\cdot2}{9\cdot3}+\frac{256}{9\cdot6}+\frac{256}{9}} =$

$= \sqrt{108+36}\cdot\sqrt{\frac{128}{27}+\frac{128}{27}+\frac{512}{18}} = \cancel{12}\cdot$ [illegible] $=$ [illegible] $\cancel{32\sqrt6}$

$\cancel{\frac{128}{\sqrt3}\cdot\frac{3}{32\sqrt6}} = \ = \frac{32\sqrt3}{9}\cdot12 = \frac{128\sqrt3}{3}$

$\frac{128}{\sqrt3}\frac{3}{128\sqrt3} = 1 \Rightarrow \alpha = 0$

№3.22

$$\frac{128}{\sqrt{3}} \frac{3}{128\sqrt{3}} = 1 \Rightarrow \alpha = 0$$

№3.22

$$a = 2y\vec{j} + 6z\vec{k}$$

$$a_y = 2y \,;\; a_z = 6z$$

$$\frac{dy}{a_y} = \frac{dz}{a_z} \Rightarrow \int \frac{dy}{2y} = \int \frac{dz}{6z}$$

$$\frac{1}{2} \ln y = \frac{1}{6} \ln z + C$$

$$3 \ln y - \ln z = 6C$$

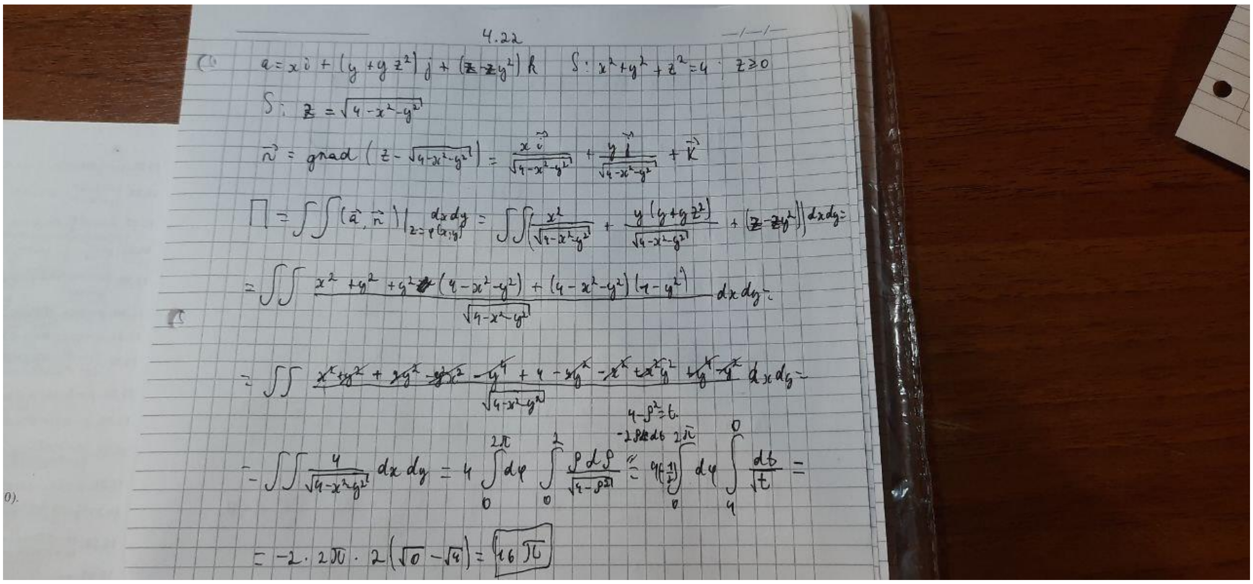
$$\ln \left(\frac{y^3}{z}\right) = C_1$$

$$\frac{y^3}{z} = C_1$$

$$z = \frac{y^3}{C_1}$$

Ответ: $z = \frac{y^3}{C_1}$

$$\int^{0} \left(2 \cdot \left(\frac{1-x}{2}\right) - 10x \cdot \left(\frac{1-x}{2}\right) - \frac{28}{2}\left(\frac{(1-x)^2}{4} - 0\right) dx =$$



4.22

$a = xi + (y + yz^2)j + (\cancel{z} - \cancel{z}y^2)k \quad S: x^2 + y^2 + z^2 = 4 \; ; \; z \geq 0$

$S: z = \sqrt{4 - x^2 - y^2}$

$\vec{n} = grad\left(z - \sqrt{4-x^2-y^2}\right) = \frac{x\vec{i}}{\sqrt{4-x^2-y^2}} + \frac{y\vec{j}}{\sqrt{4-x^2-y^2}} + \vec{k}$

$\Pi = \iint (\vec{a}, \vec{n})\Big|_{z=\varphi(x;y)} dxdy = \iint \left(\frac{x^2}{\sqrt{4-x^2-y^2}} + \frac{y(y+yz^2)}{\sqrt{4-x^2-y^2}} + (\cancel{z} - \cancel{z}y^2)\right) dxdy =$

$= \iint \frac{x^2 + y^2 + y^2(4-x^2-y^2) + (4-x^2-y^2)(1-y^2)}{\sqrt{4-x^2-y^2}} dxdy =$

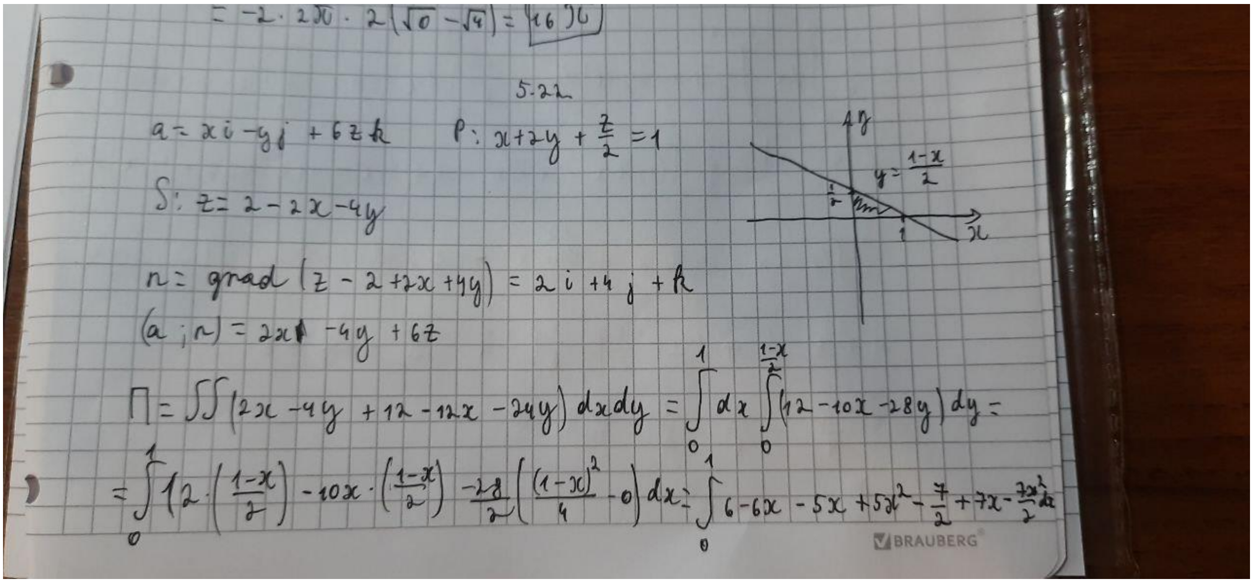
$= \iint \frac{\cancel{x^2} + \cancel{y^2} + \cancel{2y^2} - \cancel{y^2x^2} - \cancel{y^4} + 4 - \cancel{4y^2} - \cancel{x^2} + \cancel{x^2y^2} + \cancel{y^4} - \cancel{y^2}}{\sqrt{4-x^2-y^2}} dxdy =$

$4 - \rho^2 = t$

$-2\rho d\rho = dt$

$= \iint \frac{4}{\sqrt{4-x^2y^2}} dxdy = 4\int_0^{2\pi} d\varphi \int_0^2 \frac{\rho d\rho}{\sqrt{4-\rho^2}} \overset{''}{=} 4\left(-\frac{1}{2}\right)\int_0^{2\pi} d\varphi \int_4^0 \frac{dt}{\sqrt{t}} =$

$= -2 \cdot 2\pi \cdot 2(\sqrt{0} - \sqrt{4}) = \boxed{16\pi}$



$= -2 \cdot 2\pi \cdot 2(\sqrt{0} - \sqrt{4}) = \boxed{16\pi}$

5.22

$a = xi - yj + 6zk \qquad P: x + 2y + \frac{z}{2} = 1$

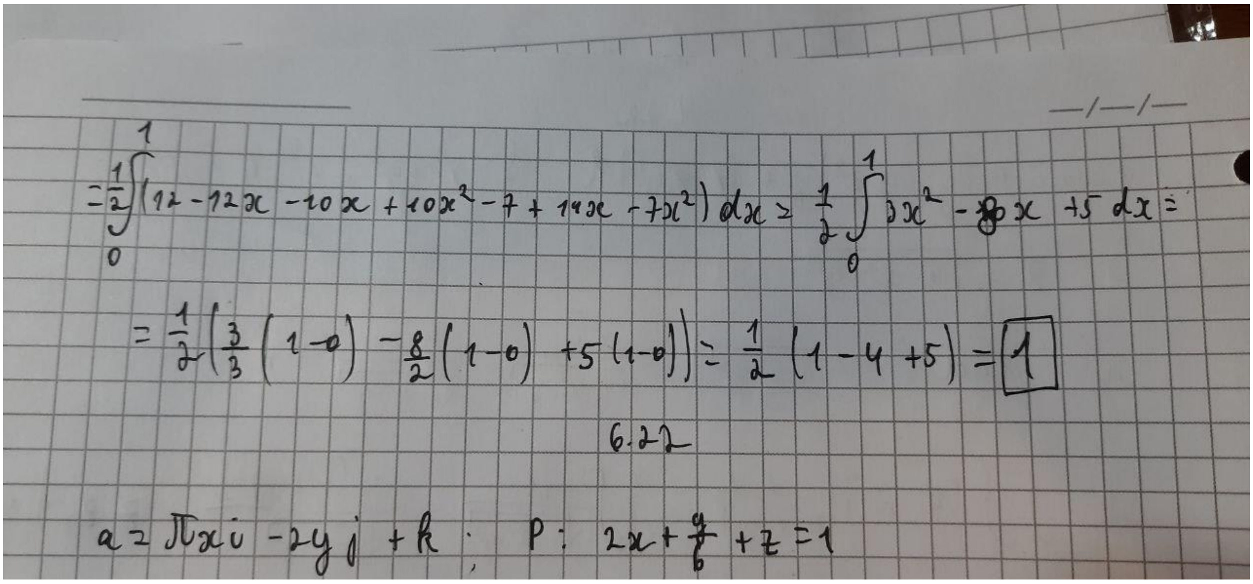
$S: z = 2 - 2x - 4y$

$n = grad(z - 2 + 2x + 4y) = 2i + 4j + k$

$(a; n) = 2x - 4y + 6z$

$\Pi = \iint (2x - 4y + 12 - 12x - 24y)dxdy = \int_0^1 dx \int_0^{\frac{1-x}{2}} (12 - 10x - 28y)dy =$

$= \int_0^1 \left(12\cdot\left(\frac{1-x}{2}\right) - 10x\cdot\left(\frac{1-x}{2}\right) - \frac{28}{2}\left(\frac{(1-x)^2}{4} - 0\right)\right)dx = \int_0^1 6 - 6x - 5x + 5x^2 - \frac{7}{2} + 7x - \frac{7x^2}{2} dx$



$= \frac{1}{2}\int_0^1 (12 - 12x - 10x + 10x^2 - 7 + 14x - 7x^2)dx = \frac{1}{2}\int_0^1 3x^2 - 8x + 5 dx =$

$= \frac{1}{2}\left(\frac{3}{3}(1-0) - \frac{8}{2}(1-0) + 5(1-0)\right) = \frac{1}{2}(1 - 4 + 5) = \boxed{1}$

6.22

$a = \pi xi - 2yj + k \; ; \; P: 2x + \frac{y}{6} + z = 1$

6.22

$a = \pi x i - 2y j + k$; $P: 2x + \frac{y}{6} + z = 1$

$\bar{N} = \{2; \frac{1}{6}; 1\}$; $|N| = \sqrt{4 + \frac{1}{36} + 1} = \frac{\sqrt{181}}{6.}$

$n = \{\frac{12}{\sqrt{181}}; \frac{1}{\sqrt{181}}; \frac{6}{\sqrt{181}}\}$; $z = 1 - 2x - \frac{y}{6.}$

$(a \cdot n) = \frac{12\pi x - 2y + 6}{\sqrt{181}}$ [illegible] $= \frac{\sqrt{181}}{6.} = \sqrt{1 + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}}$

$\Pi = \iint \frac{12\pi x - 2y + 6}{\sqrt{181}} dS = \iint \frac{12\pi x - 2y + 6}{\sqrt{181}} \cdot \frac{\sqrt{181}}{6} dx\, dy =$

$= \frac{1}{6}\int_0^{\frac{1}{2}} dx \int_0^{6-12x} (12\pi x - 2y + 6) dy = \frac{1}{6}\int_0^{\frac{1}{2}} dx \Big|_0^{6-12x} (12\pi x(6-12x) - \frac{2}{2}(36 - 144x + 144x^2) +$

$+ 36 - 72x) dx = \frac{1}{6}\int_0^{\frac{1}{2}} (72\pi x - 144\pi x^2 \cancel{-36} + 144x - 144x^2 \cancel{+36} - 72x) dx =$

$= \frac{1}{6}\left(\frac{72\pi}{2} \cdot \frac{1}{4} - \frac{144\pi}{3} \cdot \frac{1}{8} + \frac{72}{2} \cdot \frac{1}{4} - \frac{144}{3} \cdot \frac{1}{8}\right) =$

$\frac{1}{6}(9\pi - 6\pi + 9 - 6) = \boxed{\frac{\pi}{2} + \frac{1}{2}}$

7.22

$a = (\overset{a_x}{\sin z + 2x}) i + (\overset{a_y}{\sin x - 3y}) j + (\overset{a_z}{\sin y + 2z}) k$

$S: x^2 + y^2 = z^2$; $z = 3$; $z = 6.$

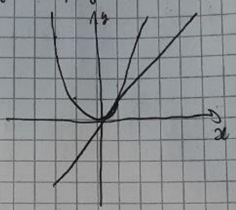
$\frac{\partial a_x}{\partial x} = 2$; $\frac{\partial a_y}{\partial y} = -3$; $\frac{\partial a_z}{\partial z} = 2 \Rightarrow div\, a = 2 - 3 + 2 = 1$

$\Pi = \underset{V}{\iiint} div\, a\, dx\, dy\, dz = \underset{V}{\iiint} dx\, dy\, dz. = \frac{1}{3}(\pi \cdot 6^2 \cdot 6 - \pi \cdot 3^2 \cdot 3) =$

$= \boxed{63\pi.}$

8.22

$$\vec a = \overset{a_x}{17x}\, i + \overset{a_y}{7y}\, j + \overset{a_z}{11z}\, k \quad ; \quad S: \; x^2+y^2=z, \; z=2(x^2+y^2), \; y=x^2 ; \; y=x.$$



$$\operatorname{div} a = 17+7+11 = 35.$$

$$\Pi = \iiint_V \operatorname{div} a \, dx\, dy\, dz =$$

$$= 35\int_0^1 dx \int_{x^2}^{x} dy \int_{x^2+y^2}^{2(x^2+y^2)} dz =$$

$$= 35\int_0^1 dx \int_{x^2}^{x} (x^2+y^2)\, dy = 35\int_0^1 x^2(x-x^2) + \frac{1}{3}(x^3-x^6)\, dx =$$

$$= 35\int_0^1 \left(x^3 - x^4 + \frac{x^3}{3} - \frac{x^6}{3}\right) dx = 35\left(\frac{1}{4}\cdot 1 - \overset{\frac{3}{35}}{\frac{1}{5}} + \frac{1}{12} - \frac{1}{21}\right) = \boxed{3}$$

9.22

$$a = (x^2+xy)\,i + (y^2+yz)\,j + (z^2+xz)\,k \qquad S = \begin{matrix} x^2+y^2+z^2=1 \\ x^2+y^2=z^2 \end{matrix} \qquad z \geq 0$$

$$z = \frac{1}{\sqrt{2}} \;;\; x^2+y^2 = \frac{1}{2} \;,\; R = \frac{1}{\sqrt{2}}$$

$$\operatorname{div} a = 2x+y+2y+z+2z+x = 3x+3y+3z.$$

$$\Pi = 3\iiint_V (x+y+z)\, dx\, dy\, dz =$$

$$= 3\int_0^{2\pi} d\theta \int_0^{\frac{\pi}{4}} \sin\varphi\, d\varphi \int_0^1 (\rho\sin\varphi\cos\theta + \rho\sin\varphi\sin\theta + \rho\cos\varphi)\,\rho^2 d\rho =$$

$$= 3\int_0^{\frac{\pi}{4}} \sin\varphi\, d\varphi \int_0^{2\pi} (\sin\varphi\cos\theta + \sin\varphi\sin\theta + \cos\varphi)\cdot\frac{1}{4}\, d\theta =$$

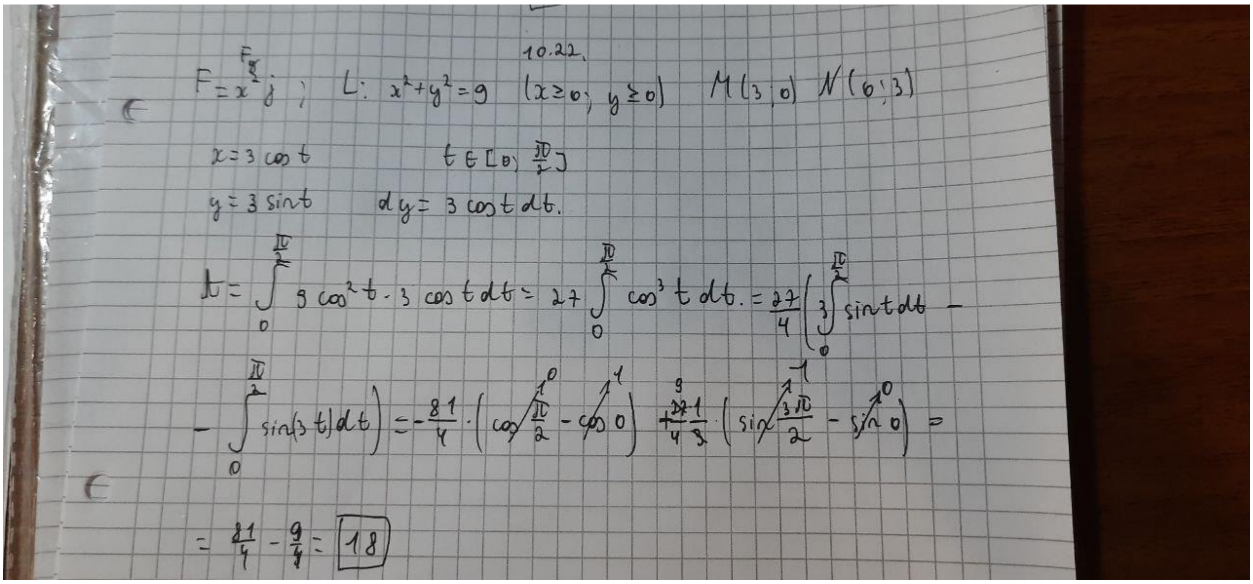
$$= \frac{3}{4}\int_0^{\frac{\pi}{4}} (\sin\varphi \left(\sin\varphi(\overset{0}{\cancel{\sin 2\pi}} - \overset{0}{\cancel{\sin 0}}) - \sin\varphi(\overset{1}{\cancel{\cos 2\pi}} - \overset{1}{\cancel{\cos 0}}) + 2\pi\cos\varphi \right) d\varphi =$$

$$= \frac{3}{4}\int_0^{\frac{\pi}{4}} 2\pi \sin\varphi\cos\varphi\, d\varphi = \frac{3\pi}{2}\int_0^{\frac{\pi}{4}} \sin\varphi\cos\varphi = \frac{3\pi}{4}\int_0^{\frac{\pi}{4}} \sin 2\varphi\, d\varphi =$$

$$= -\frac{3\pi}{8}\cdot\left(\overset{0}{\cancel{\cos\frac{\pi}{2}}} - \overset{1}{\cancel{\cos 0}}\right) = \boxed{+\frac{3\pi}{8}}$$

10.22.

F_y



10.22.

$F = x^{2}j$; $L: x^2+y^2=9$ $(x \geq 0;\ y \geq 0)$ $M(3;0)$ $N(0;3)$

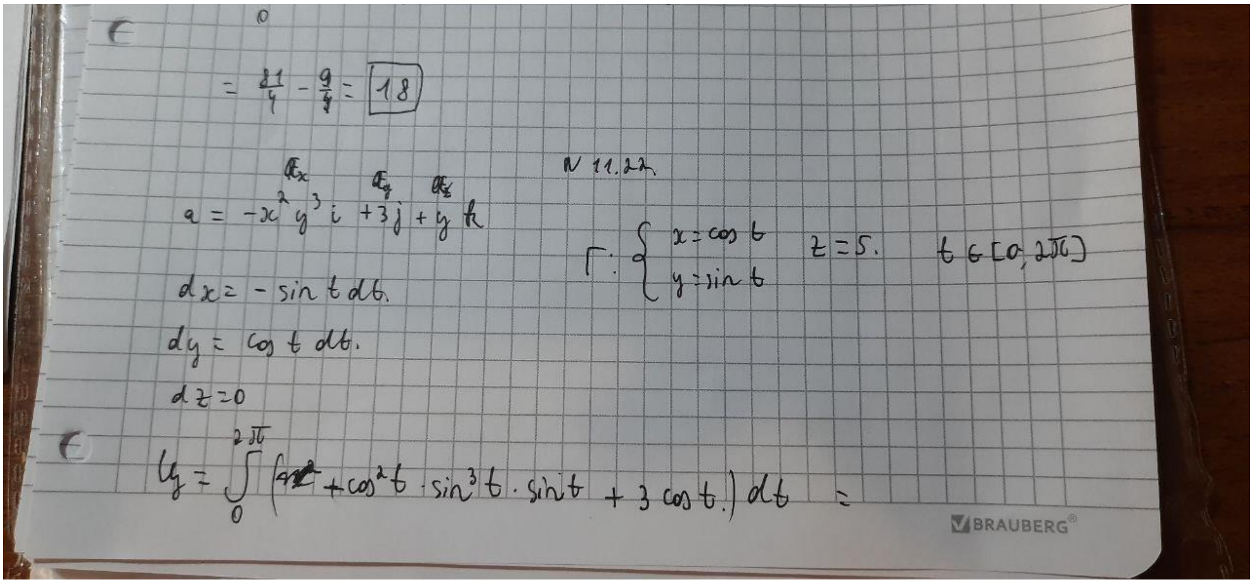
$x = 3\cos t$ $t \in [0; \frac{\pi}{2}]$

$y = 3\sin t$ $dy = 3\cos t\, dt.$

$$A = \int_0^{\frac{\pi}{2}} 9\cos^2 t \cdot 3\cos t\, dt = 27\int_0^{\frac{\pi}{2}} \cos^3 t\, dt. = \frac{27}{4}\left(3\int_0^{\frac{\pi}{2}} \sin t\, dt -\right.$$

$$\left.-\int_0^{\frac{\pi}{2}} \sin(3t)\, dt\right) = -\frac{81}{4}\cdot\left(\overset{0}{\cancel{\cos\frac{\pi}{2}}} - \overset{1}{\cancel{\cos 0}}\right) + \frac{\overset{9}{\cancel{27}}}{4}\cdot\frac{1}{\cancel{3}}\cdot\left(\overset{-1}{\cancel{\sin\frac{3\pi}{2}}} - \overset{0}{\cancel{\sin 0}}\right) =$$

$$= \frac{81}{4} - \frac{9}{4} = \boxed{18}$$



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$$= \frac{81}{4} - \frac{9}{4} = \boxed{18}$$

N 11.22.

$$a = \overset{a_x}{-x^2y^3 i} + \overset{a_y}{3j} + \overset{a_z}{y k}$$

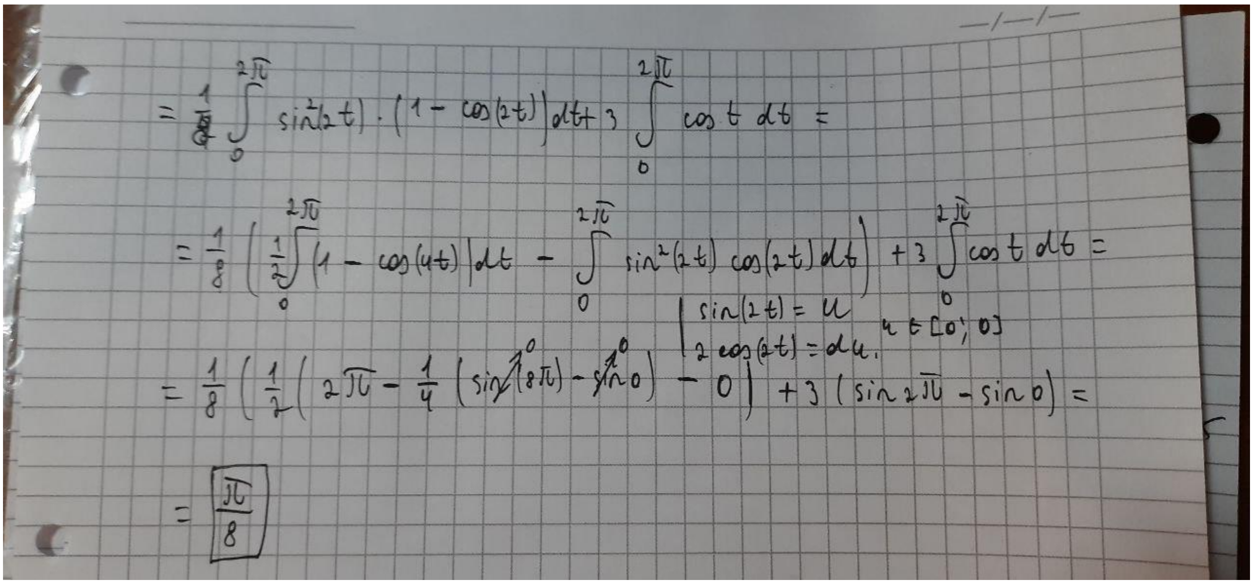
$$\Gamma: \begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad z = 5. \quad t \in [0, 2\pi]$$

$dx = -\sin t\, dt.$

$dy = \cos t\, dt.$

$dz = 0$

$$Ц = \int_0^{2\pi} \left(\cancel{-x^2} + \cos^2 t \cdot \sin^3 t \cdot \sin t + 3\cos t.\right) dt \quad =$$



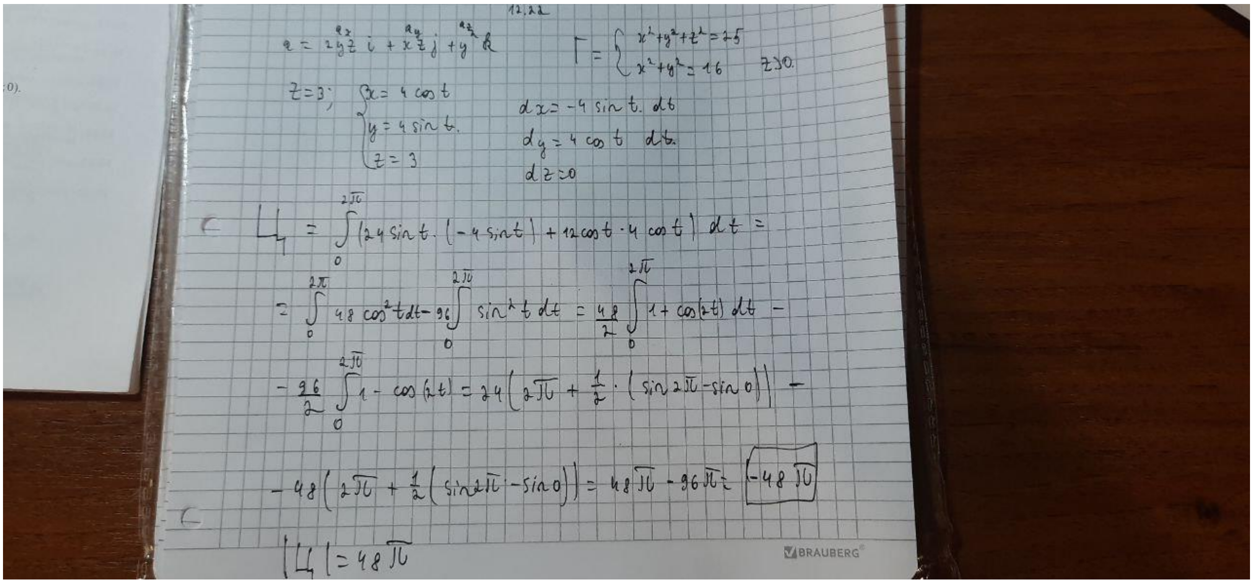
$$= \frac{1}{8}\int_0^{2\pi} \sin^2(2t)\cdot\left(1-\cos(2t)\right)dt + 3\int_0^{2\pi}\cos t\,dt =$$

$$= \frac{1}{8}\left(\frac{1}{2}\int_0^{2\pi}\left(1-\cos(4t)\right)dt - \int_0^{2\pi}\sin^2(2t)\cos(2t)\,dt\right) + 3\int_0^{2\pi}\cos t\,dt =$$

$$\left|\begin{array}{l}\sin(2t) = u\\ 2\cos(2t) = du.\end{array}\right. \quad u \in [0;\,0]$$

$$= \frac{1}{8}\left(\frac{1}{2}\left(2\pi - \frac{1}{4}\left(\overset{0}{\cancel{\sin(8\pi)}} - \overset{0}{\cancel{\sin 0}}\right)\right) - 0\right) + 3\left(\sin 2\pi - \sin 0\right) =$$

$$= \boxed{\frac{\pi}{8}}$$



:0).

12.22

$$a = \overset{a_x}{2yz}\,i + \overset{a_y}{xz}\,j + \overset{a_z}{y^2}\,k \qquad \Gamma = \begin{cases} x^2+y^2+z^2=25\\ x^2+y^2=16\end{cases} \quad z>0.$$

$$z=3; \quad \begin{cases} x = 4\cos t\\ y = 4\sin t.\\ z = 3\end{cases} \qquad \begin{array}{l} dx = -4\sin t.\,dt\\ dy = 4\cos t\;dt.\\ dz = 0\end{array}$$

$$L_4 = \int_0^{2\pi}\left(24\sin t.\left(-4\sin t\right) + 12\cos t\cdot 4\cos t\right)dt =$$

$$= \int_0^{2\pi} 48\cos^2 t\,dt - 96\int_0^{2\pi}\sin^2 t\,dt = \frac{48}{2}\int_0^{2\pi} 1+\cos(2t)\,dt -$$

$$- \frac{96}{2}\int_0^{2\pi} 1-\cos(2t) = 24\left(2\pi + \frac{1}{2}\cdot\left(\sin 2\pi - \sin 0\right)\right) -$$

$$- 48\left(2\pi + \frac{1}{2}\left(\sin 2\pi - \sin 0\right)\right) = 48\pi - 96\pi = \boxed{-48\pi}$$

$$|L_4| = 48\pi$$

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