Снегура Дана А-02-23 В-17 ТР1

1.17

$$\int \frac{1+\ln(6x+5)}{6x+5}dx = \begin{Bmatrix} 6x+5=t \\ dt=6dx \\ dx=\frac{dt}{5} \end{Bmatrix} = \int \frac{1}{6}\cdot\frac{1+\ln t}{t}dt = \begin{Bmatrix} \ln t = y \\ \frac{1}{t}dt=dy \\ dt=t\cdot dy \end{Bmatrix} =$$

$$= \frac{1}{6}\int (1+y)\,dy = \frac{1}{6}\left(\int dy + \int y\,dy\right) = \frac{1}{6}\left(y+\frac{y^2}{2}\right)+C =$$

$$= \frac{1}{6}\left(\ln(6x+5)+\frac{\ln^2(6x+5)}{2}\right)+C = \frac{1}{6}\ln(6x+5)+\frac{\ln^2(6x+5)}{12}+C$$

По задачнику математический анализ. интегралы МЭИ 2019 г.
авторы: Игнатьева Н.У., Черепова М.Ф., Синицкий А.А., Бирюков А.М., Булычева О.Н.

Снегура Дана А-02-23 В-17 ТР2

2.17

$$\int (2-3x)\cos(4x)\,dx = \left\{\begin{array}{l} u=2-3x \\ u'=-3 \\ v'=\cos 4x \\ v=\frac{1}{4}\sin 4x \end{array}\right\} = (2-3x)\cdot\frac{1}{4}\sin(4x)+\frac{3}{4}\int \sin(4x)\,dx =$$

$$=\frac{2-3x}{4}\sin(4x)-\frac{3}{16}\cos(4x)+C$$

Снеура Дана А-02-23 В-17. ТР3

3.17

$$\int \frac{10x+53}{\sqrt{x^2+10x+29}}dx = \int \frac{10(x+5)+3}{\sqrt{(x+5)^2+2^2}}dx = 10\int \frac{x+5}{\sqrt{(x+5)^2+2^2}}dx +$$

$$+\ 3\int \frac{dx}{\sqrt{(x+5)^2+2^2}} = 10\sqrt{x^2+10x+29} + 3\ln\left|x+5+\sqrt{x^2+10x+29}\right| + C$$

$$\int \frac{x+5}{\sqrt{(x+5)^2+2^2}}dx = \frac{1}{2}\int \frac{d\left((x+5)^2\right)}{\sqrt{(x+5)^2+2^2}} = \left\{t=(x+5)^2\right\} =$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t+2^2}} = \frac{1}{2}\int (t+2^2)^{-\frac{1}{2}}\, d(t+2^2) = \frac{1}{2}\cdot 2\sqrt{t+4} \overset{+C}{=}$$

$$= \sqrt{t+4} \overset{+C}{=} \sqrt{(x+5)^2+4} \overset{+C}{=} \sqrt{x^2+10x+29} + C$$

$$\int \frac{dx}{\sqrt{(x+5)^2+2^2}} = \int \frac{d(x+5)}{\sqrt{(x+5)^2+2^2}} = \ln\left|x+5+\sqrt{(x+5)^2+2^2}\right| + C =$$

$$= \ln\left|x+5+\sqrt{x^2+10x+29}\right| + C$$

Снегура Дана А-02-23 В-17 ТР4

4.17.

$$\int \frac{-x^5+25x^3+1}{x^2+5x}dx = \int \frac{x^3(5-x)(5+x)+1}{x(x+5)}dx =$$

$$= \int \frac{x^3(5-x)(5+x)}{x(x+5)}dx + \int \frac{dx}{x(x+5)} = \int x^2(5-x)dx + \int \frac{dx}{x(x+5)} =$$

$$= -\int x^3 dx + 5\int x^2 dx + \int \frac{dx}{x(x+5)} = -\frac{x^4}{4} + \frac{5x^3}{3} + \frac{1}{5}\ln|x| - \frac{1}{5}\ln|x+5| + C$$

$$\frac{1}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5} = \frac{x(A+B)+5A}{x(x+5)}$$

$$\begin{cases} A+B=0 & B=-1/5 \\ 5A=1 & A=1/5 \end{cases}$$

$$\int \frac{dx}{x(x+5)} = \frac{1}{5}\int \frac{dx}{x} - \frac{1}{5}\int \frac{dx}{x+5} = \frac{1}{5}\ln|x| - \frac{1}{5}\ln|x+5|$$

1P5

$$\int \frac{dx}{(x-5)(3x^2-32x+84)} = \int \frac{dx}{(x-5)(x-6)(x-\frac{14}{3})} \ominus$$

$$\frac{1}{(x-5)(x-6)(x-\frac{14}{3})} = \frac{A}{x-5} + \frac{B}{x-6} + \frac{C}{x-\frac{14}{3}}$$

$$\frac{A(x-6)(x-\frac{14}{3}) + B(x-5)(x-\frac{14}{3}) + C(x-5)(x-6)}{(x-5)(x-6)(x-\frac{14}{3})} =$$

$$= \frac{Ax^2 - \frac{32}{3}Ax + 28A + Bx^2 - \frac{29}{3}Bx + \frac{70}{3}B + Cx^2 - 11Cx + 30C}{(x-5)(x-6)(x-\frac{14}{3})} =$$

$$= \frac{x^2(A+B+C) + x(-\frac{32}{3}A - \frac{29}{3}B - 11C) + (28A + \frac{70}{3}B + 30C)}{(x-5)(x-6)(x-\frac{14}{3})} =$$

$$\begin{cases} A+B+C=0 \\ -A\cdot\frac{32}{3} - \frac{29}{3}B - 11C = 0 \\ 28A + \frac{70}{3}B + 30C = 1 \end{cases} \qquad \begin{cases} A = -3 \\ B = 3/4 \\ C = 9/4 \end{cases}$$

$$\ominus -3\int\frac{dx}{x-5} + \frac{3}{4}\int\frac{dx}{x-6} + \frac{9}{4}\int\frac{dx}{x-\frac{14}{3}} = -3\ln|x-5| +$$

$$+ \frac{3}{4}\ln|x-6| + \frac{9}{4}\ln|x-\frac{14}{3}| + C$$

…ецура Даша ТР8 В-17 А-02-23

$$\int e^{\sqrt{(6-x)(5+x)}} \cdot \frac{\sqrt{5+x}}{\sqrt{6-x}\cdot(5+x)^2}dx = \left\{ \sqrt{\frac{6-x}{5+x}} = t \right.$$

$$dt = \frac{1}{2\sqrt{\frac{6-x}{5+x}}}\cdot\frac{(-1)(5+x)-(6-x)\cdot 1}{(5+x)^2}dx \quad \Bigg| dx = \frac{dt\, 2\sqrt{\frac{(6-x)}{5+x}}(5+x)^2}{-(5+x)-(6-x)}$$

$$dx = \frac{dt\cdot 2\sqrt{\frac{6-x}{5+x}}(5+x)^2}{-11}\Bigg\} = \int e^t\cdot\frac{1}{t}\cdot\left(-\frac{2}{11}\right)t = -\frac{2}{11}e^t =$$

$$-\frac{2}{11}e^{\sqrt{(6-x)/(5+x)}}$$

ТР8

$$\int\sqrt{2x^2-7x-2}\,dx = 2\int\sqrt{x^2-\frac{7}{2}x-1}\,dx = 2\int\sqrt{\left(x-\frac{7}{4}\right)^2-\frac{65}{16}}\,dx =$$

$$= 2\int\sqrt{\left(x-\frac{7}{4}\right)^2-\frac{65}{16}}\,d\left(x-\frac{7}{4}\right) = \left\{ \begin{matrix} t = x-\frac{7}{4} \\ dt = dx \end{matrix} \right\} =$$

$$= 2\int\sqrt{t^2-\frac{65}{16}}\,dt = 2\left(\frac{t}{2}\sqrt{t^2-\frac{65}{16}} - \frac{65}{16}\ln\left|t+\sqrt{t^2-\frac{65}{16}}\right|\right) +$$

$$+ C = t\sqrt{t^2-\frac{65}{16}} - \frac{65}{8}\ln\left|t+\sqrt{t^2-\frac{65}{16}}\right| + C =$$

$$= \left(x-\frac{7}{4}\right)\sqrt{\left(x-\frac{7}{4}\right)^2-\frac{65}{16}} - \frac{65}{8}\ln\left|\left(x-\frac{7}{4}\right)+\sqrt{\left(x-\frac{7}{4}\right)^2-\frac{65}{16}}\right| +$$

$$+ C$$

Снегура Дана, А-02-23, В-17. ТРЗ.

9.17

$$\int \sin^5 2x \cos^2 2x\, dx = \frac{1}{2}\int \cos^2(2x)(1-\cos^2(2x))^2 \sin(2x)\, d2x =$$

$$= \left\{ \begin{array}{l} v = \cos(2x) \\ dv = -\sin(2x)\, d2x \end{array} \right\} = \frac{1}{2}\int -v^2(1-v^2)^2 dv =$$

$$= \frac{1}{2}\int (-v^2 - v^6 + 2v^4)\, dv = \frac{1}{2}\left(-\frac{v^3}{3} - \frac{v^7}{7} + 2\cdot\frac{v^5}{5}\right) =$$

$$= -\frac{v^3}{6} - \frac{v^7}{14} + \frac{v^5}{5} = -\frac{\cos^3(2x)}{6} - \frac{\cos^7(2x)}{14} + \frac{\cos^5(2x)}{5}$$

Снегура Даша, А-02-23, В-17 ТР10

10.17.

$$\int\frac{dx}{3\sin x-\cos x+1}=\int\frac{\frac{2dt}{1+t^2}}{3\cdot\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}+1}=$$

$$=\int\frac{2dt}{6t-1+t^2+1+t^2}=\int\frac{dt}{t^2+3t}=\int\frac{dt}{t(3+t)}=$$

$$\frac{1}{t(3+t)}=\frac{A_1}{t}+\frac{A_2}{t+3}=\frac{A_1t+3A_1+A_2t}{t(t-3)}$$

$$\begin{cases}A_1+A_2=0\\3A_1=1\end{cases}\quad\begin{cases}A_2=-\frac{1}{3}\\A_1=\frac{1}{3}\end{cases}$$

$$\frac{1}{t(t+3)}=\frac{1}{3t}-\frac{1}{3(t+3)}$$

$$=\int\frac{dt}{3t}-\int\frac{dt}{3(t+3)}=\frac{1}{3}\cdot\ln|t|-\frac{1}{3}\ln|t+3|=$$

$$=\frac{1}{3}\ln\left|\operatorname{tg}\frac{x}{2}\right|-\frac{1}{3}\ln\left|\operatorname{tg}\frac{x}{2}+3\right|$$

$\operatorname{tg}\frac{x}{2}=t$ $\quad\cos x=\frac{1-t^2}{1+t^2}$ замена

$\sin x=\frac{2t}{1+t^2}$ $\quad dx=\frac{2dt}{1+t^2}$

ТР 11 . В-17

$$\int_0^1 \frac{\sqrt{\operatorname{arctg} x}+1}{1+x^2}dx = \int_0^1 (\sqrt{\operatorname{arg tg} x}+1)\,d(\operatorname{arctg} x) =$$

$$= \frac{2(\operatorname{arctg} x)^{3/2}}{3}\Bigg|_0^1 + \operatorname{arctg} x\Bigg|_0^1 = \frac{2}{3}\left(\frac{\Pi}{4}\right)^{3/2} + \frac{\Pi}{4}$$

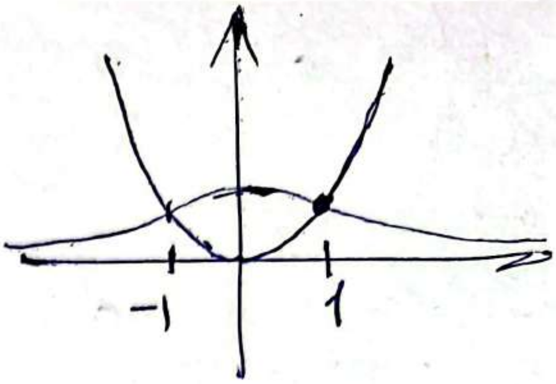
ТР 12 . В-17

$$\int_0^{1/3} x \operatorname{arctg} 3x\,dx = \left\{\begin{array}{l} v' = x \\ v = \frac{1}{2}x^2 \\ u = \operatorname{arctg} 3x \\ u' = \frac{3}{1+9x^2} \end{array}\right] = \frac{x^2 \operatorname{arctg}(3x)}{2}\Bigg|_0^{1/3} -$$

$$- \int_0^{1/3} \frac{3x^2}{2(9x^2+1)}dx = \frac{x^2 \operatorname{arg tg}(3x)}{2}\Bigg|_0^{1/3} -$$

$$- \frac{x}{6}\Bigg|_0^{1/3} - \frac{\operatorname{arg tg}(3x)}{18}\Bigg|_0^{1/3} = \frac{\Pi}{36} - \frac{1}{18}$$

TP 13. B-17



$$y=\frac{1}{1+x^2},\ y=\frac{x^2}{2}$$

$$S=\int_{-1}^{1}dx\int_{\frac{x^2}{2}}^{\frac{1}{1+x^2}}dy$$

$$\int_{-1}^{1}\left(\frac{1}{1+x^2}-\frac{x^2}{2}\right)dx=\int_{-1}^{1}\frac{1}{1+x^2}dx-\frac{1}{2}\int_{-1}^{1}x^2dx=$$

$$=\frac{\pi}{2}-\frac{1}{3}$$

TP 14. B-17 $\qquad S=2S_1=19\pi$

$$\rho=3-\sin\varphi$$

$$S_1=\frac{1}{2}\int_{\alpha}^{\beta}\rho^2(\varphi)d\varphi=\frac{1}{2}\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}}(9+\sin^2\varphi-6\sin\varphi)d\varphi=$$

$$=9\varphi\Big|_{\frac{3\pi}{2}}^{\frac{3\pi}{2}}+\frac{\varphi}{2}\Big|_{\frac{3\pi}{2}}^{\frac{3\pi}{2}}-\frac{1}{4}\sin(2\varphi)\Big|_{\frac{3\pi}{2}}^{\frac{3\pi}{2}}+6\cos\varphi\Big|_{\frac{3\pi}{2}}^{\frac{3\pi}{2}}=$$

$$=9\left(\frac{-\pi}{2}+\frac{3\pi}{2}\right)+\frac{1}{2}\left(\frac{-\pi}{2}+\frac{3\pi}{2}\right)+\frac{1}{4}\left(\cancel{\sin\pi-\sin3\pi}\right)^{0}+$$

$$+6\left(\cancel{\cos\frac{\pi}{2}-\cos\frac{3\pi}{2}}\right)^{0}=9(-\pi)+\frac{-1}{2}(-\pi)=\frac{19}{2}\pi$$

ТР 15. В. 17

$$\begin{cases} x = 6t^5 \\ y = 5t(1 - t^8) \end{cases}$$ от (.) $A(0;0)$ до (.) $B(6,0)$

$$l = \int_{\alpha}^{\beta} \sqrt{x'^2(t) + y'^2(t)}\, dt$$

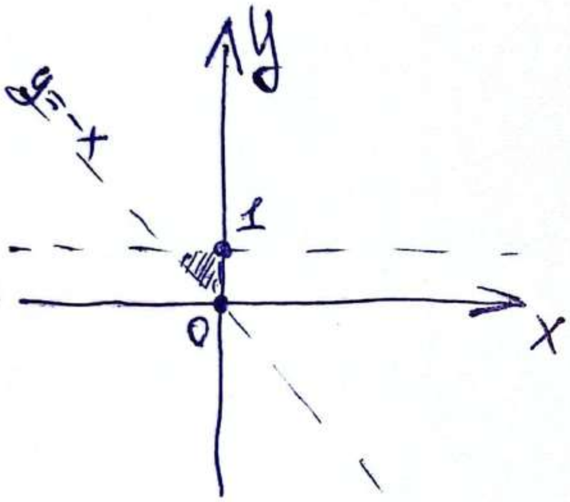
$$l = \int_0^1 \sqrt{30^2 t^8 + 25 + 45^2 t^{16} - 450 t^8}\, dt =$$

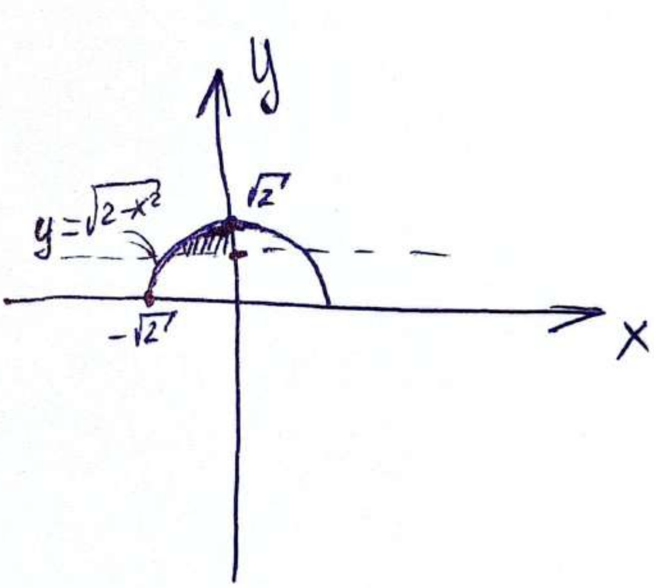
$$= \int_0^1 \sqrt{2025 t^{16} + 450 t^8 + 25}\, dt = 5t^9 \Big|_0^1 + 5t \Big|_0^1 = 10$$

неура Даша, А-02-23, В-17, ТР1

$$\int_0^1 dy \int_{-y}^{0} f\,dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^{0} f\,dx = \int_{-1}^{0} dx \int_{-x} f\,dy +$$

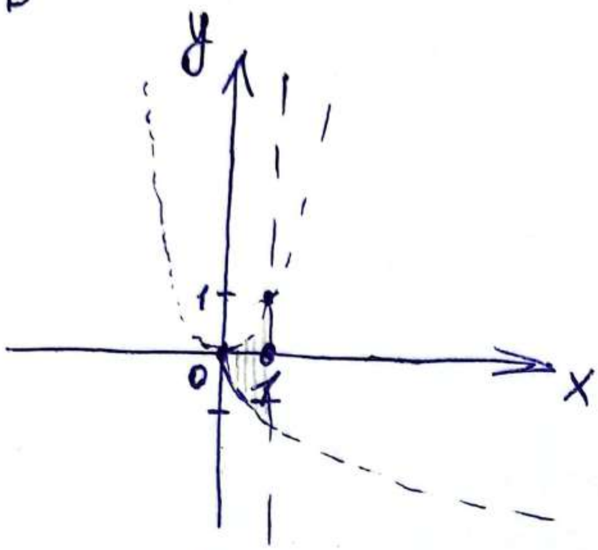
$$+ \int_1^{\sqrt{2-x^2}} dy \int_{\sqrt{2}}^{0} f\,dx$$





ТР2

$$\iint\limits_D (24xy-48x^3y^3)\,dx\,dy, \quad D: x=1,\ y=\sqrt{x},\ y=x^2$$



$$\int_0^1 dx \int_{-\sqrt{x}}^{x^2} (24xy-48x^3y^3)\,dy = 3$$

$$\int_{-\sqrt{x}}^{x^2} (24xy-48x^3y^3)\,dy = \int_{-\sqrt{x}}^{x^2} (24xy)\,dy - \int_{-\sqrt{x}}^{x^2} (48x^3y^3)\,dy =$$

$$= 24x\cdot\frac{y^2}{2}\Big|_{-\sqrt{x}}^{x^2} - 48x^3\,\frac{y^4}{4}\Big|_{-\sqrt{x}}^{x^2} = 12x(x^4-x) - 12x^3(x^8-x^2) =$$

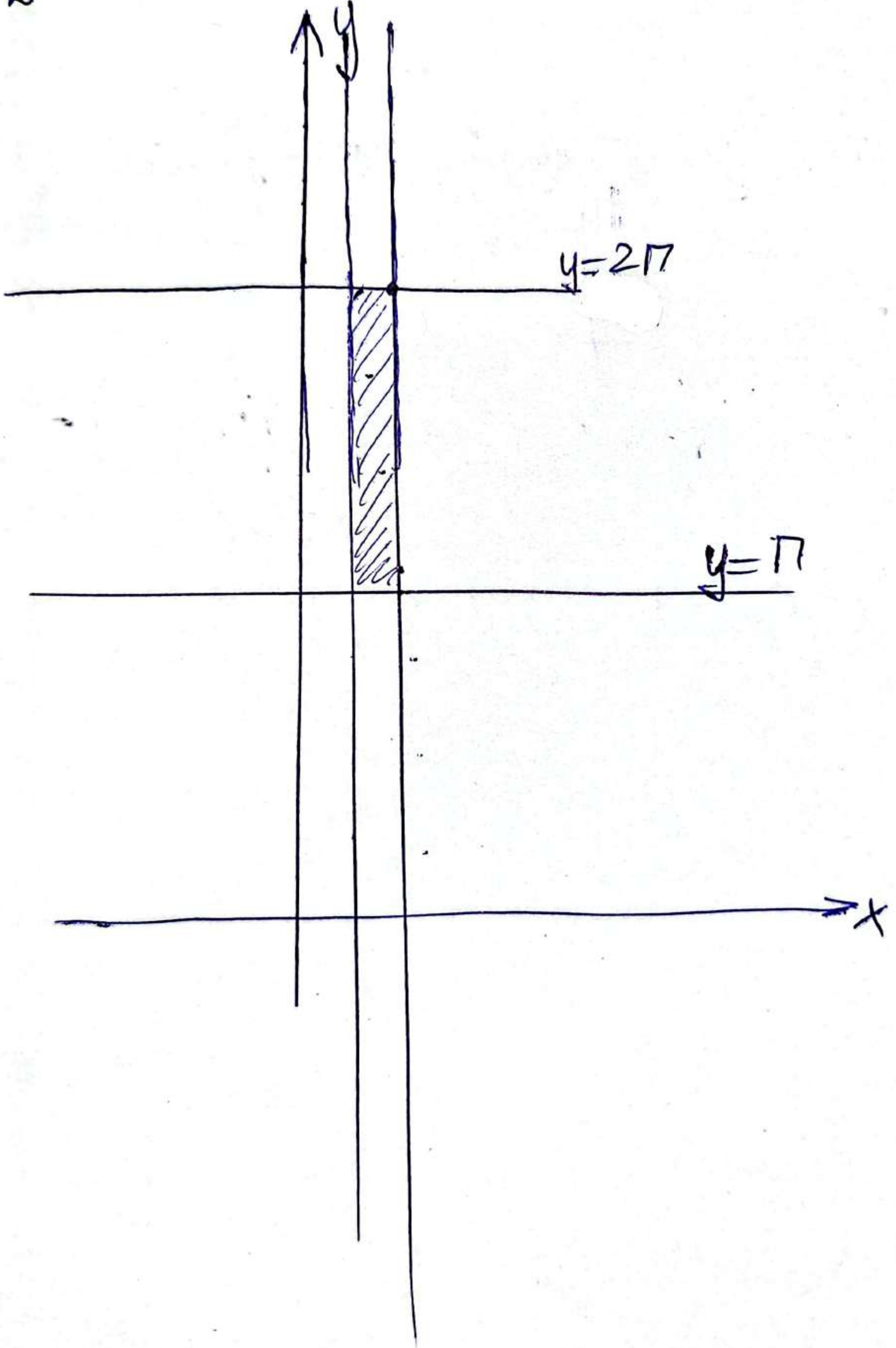
$$= 12x^5 - 12x^2 - 12x^{11} + 12x^5 = 24x^5 - 12x^2 - 12x^{11}$$

$$\int_0^1 (24x^5-12x^2-12x^{11})\,dx = \int_0^1 24x^5\,dx - \int_0^1 12x^2\,dx - \int_0^1 12x^{11}\,dx =$$

$$= 24\,\frac{x^6}{6}\Big|_0^1 - 12\,\frac{x^3}{3}\Big|_0^1 - 12\,\frac{x^{12}}{12}\Big|_0^1 = 8(1) - 4(1) - 1(1) = 3$$

№3

$$\iint_D y \sin xy \, dx\,dy \; ; D: y=\pi, y=2\pi, x=\frac{1}{2}, x=1$$



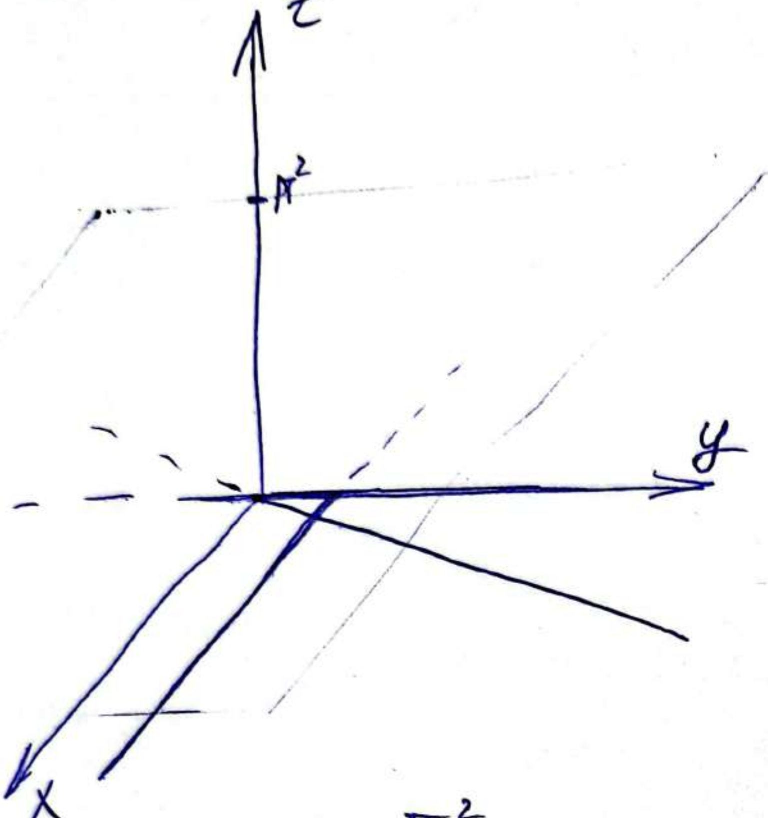
$$\int_{\pi}^{2\pi} dy \int_{1/2}^{1} (y \sin xy)\,dx = \pi\left(\cos\left(\tfrac{1}{2}\right) - \cos(1)\right)$$

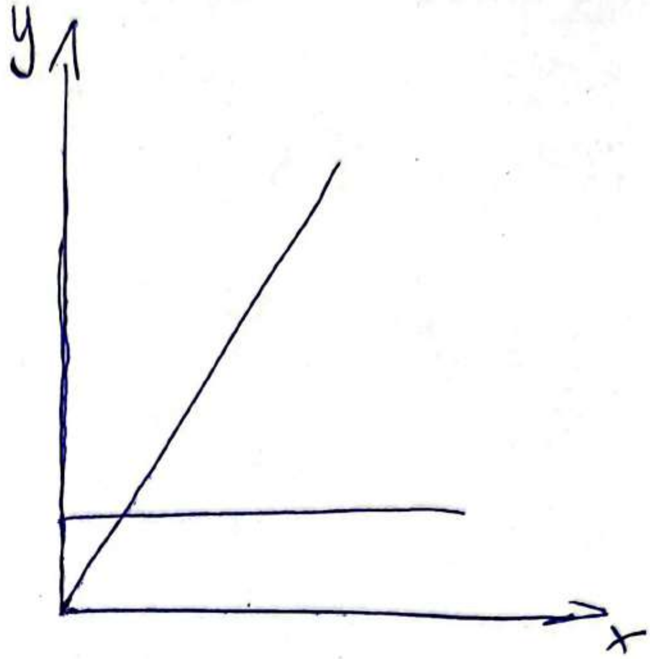
$$\int_{1/2}^{1} (y \sin xy)\,dx = y \cdot (-\cos xy)\frac{1}{y}\Big|_{1/2}^{1} = \cos\left(\tfrac{1}{2}\right) - \cos(1)$$

$$\int_{\pi}^{2\pi} \left(\cos\left(\tfrac{1}{2}\right) - \cos(1)\right) dy = \left(\cos\left(\tfrac{1}{2}\right) - \cos(1)\right)(2\pi - \pi) =$$

$$= \pi\left(\cos\left(\tfrac{1}{2}\right) - \cos(1)\right)$$

ТР 4. В-17.





$$V=\int_0^1 dy \int_0^{y/2} dx \int_0^{\pi^2} (y^2\cos(\pi x y))dz$$

$$\int_0^{\pi^2} y^2\cos(\pi x y)\,dz = y^2\cos(\pi x y)(\pi^2-0)=y^2\pi^2\cos(\pi x y)$$

$$\int_0^{y/2} (y^2\pi^2\cos(\pi x y))\,dx = y\pi\int_0^{\frac{\pi y^2}{2}} \cos(\pi x y)\,d(\pi x y) =$$

$$= y\pi\sin\left(\frac{\pi y^2}{2}\right)$$

$$\int_0^1 \left(y\pi\sin\left(\frac{\pi y^2}{2}\right)\right)dy = 1$$

TP5 B-17

$$\iiint\limits_V \left(\frac{10}{3}x+\frac{5}{3}\right)dx\,dy\,dz \quad V:\ y=9x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0$$

$$\int_0^1 dx\int_0^{9x} dy\int_0^{\sqrt{xy}}\left(\frac{10}{3}x+\frac{5}{3}\right)dz$$

$$\int_0^{\sqrt{xy}}\left(\frac{10}{3}x+\frac{5}{3}\right)dz=\left(\frac{10}{3}x+\frac{5}{3}\right)\cdot z\Big|_0^{\sqrt{xy}}=\left(\frac{10}{3}x+\frac{5}{3}\right)\cdot\sqrt{xy}$$

$$\int_0^{9x}\left(\left(\frac{10}{3}x+\frac{5}{3}\right)\sqrt{xy}\right)dy=\frac{10}{3}\int_0^{9x}x\sqrt{xy}\,dy+\frac{5}{3}\int_0^{9x}\sqrt{xy}\,dy=$$

$$=\frac{20}{9x^3}\cdot\left(x^3y\right)^{3/2}\Big|_0^{9x^4}+\frac{10}{9x}(xy)^{3/2}\Big|_0^{9x^2}=\frac{20}{9x^3}\left(9x^4\right)^{3/2}+\frac{10}{9x}\left(9x^2\right)^{3/2}$$

$$\int_0^1\left(\frac{20}{9x^3}\left(9x^4\right)^{3/2}+\frac{10}{9x}\left(9x^2\right)^{3/2}\right)dx=25$$

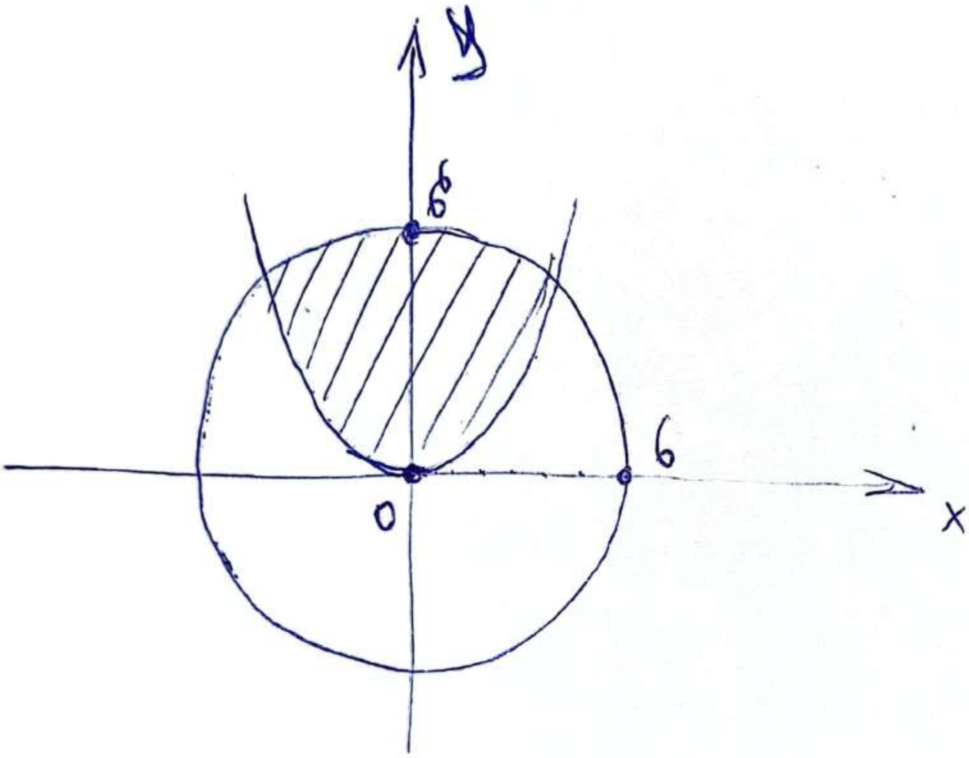
6.17

$x^2+y^2=36$ $\qquad S=2S_1$

$3\sqrt{2}\,y=x^2$

$$S_1=\int_0^{3\sqrt{2}}\left(\sqrt{36-x^2}-\frac{x^2}{3\sqrt{2}}\right)dx=\frac{3\sqrt{2}}{2}\sqrt{36-9\cdot 2}+18\arcsin\frac{3\sqrt{2}}{6}-$$

$$-18\arcsin 0-\frac{1}{9\sqrt{2}}\left(3\sqrt{2}\right)^3=9+\frac{9\pi}{2}-6=3+\frac{9\pi}{2}$$



$S=6+9\pi$

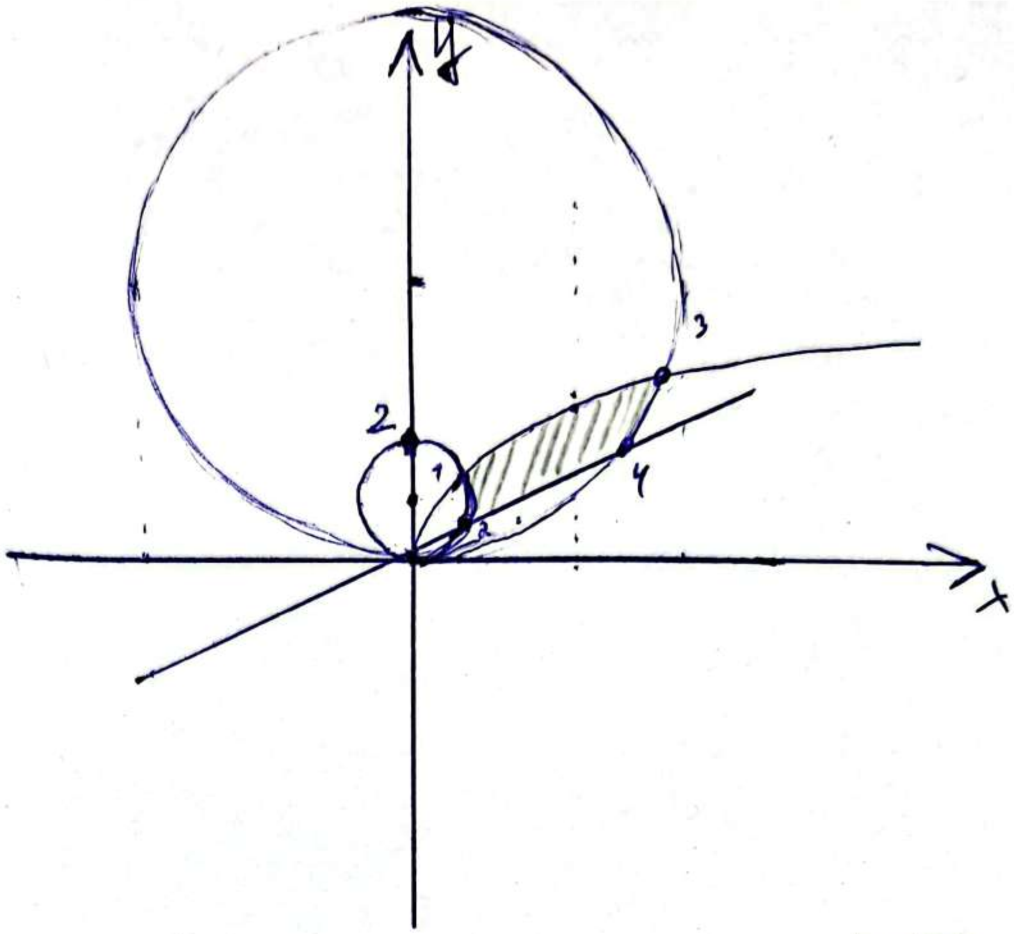
№7.17

$y^2-2y+x^2=0$

$y^2-10y+x^2=0$

$y=\frac{x}{\sqrt{3}}$

$y=\sqrt{3x}$



$$S=\int\limits_{0,822}^{4,858}\left(\sqrt{3x}-\frac{x}{\sqrt{3}}\right)dx=\frac{2x^{3/2}}{\sqrt{3}}\Bigg|_{0,822}^{4,858}-\frac{x^2}{2\sqrt{3}}\Bigg|_{0,822}^{4,858}=$$

$$=\frac{2429\sqrt{2429}}{4\sqrt{3}\,625\sqrt{5}}-\frac{137\sqrt{3}\sqrt{411}}{4\cdot 625\sqrt{5}}-\frac{71639}{6250\sqrt{3}}$$

8.17

$$M = \iint\limits_{D} (7x^2+2y)\,dx\,dy = \int_0^1 dx \int_0^{\sqrt{4x}} (7x^2+2y)\,dy$$

$$\int_0^{\sqrt{4x}} (7x^2+2y)\,dy = 7x^2\cdot y\Big|_0^{\sqrt{4x}} + y^2\Big|_0^{\sqrt{4x}} =$$

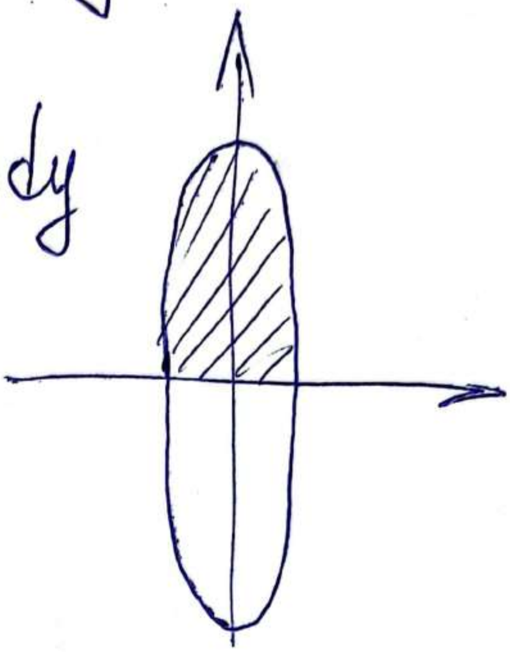
$$= 7x^2\sqrt{4x} + 4x = 14\sqrt{x^5} + 4x$$

$$\int_0^1 (14\sqrt{x^5}+4x)\,dx = 4x^{7/2}\Big|_0^1 + 2x^2\Big|_0^1 = 6$$

✓9.17

$x^2+\frac{y^2}{25}\le 1 \quad y\geqslant 0 \quad \mu=7x^4y$

$$\iint\limits_D (7x^4y)\,dx\,dy=\int_{-1}^{1}dx\int_0^{5\sqrt{1-x^2}}(7x^4y)\,dy$$



$$\int_0^{5\sqrt{1-x^2}}(7x^4y)\,dy=\frac{7x^4}{2}\,y^2\Big|_0^{5\sqrt{1-x^2}}=$$

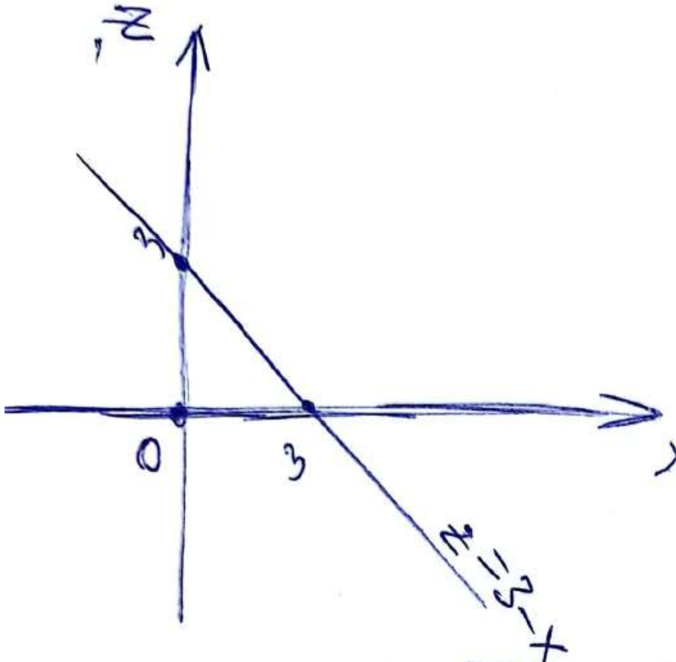
$$=\frac{7\cdot 25}{2}x^4(1-x^2)=\frac{7\cdot 25}{2}(x^4-x^6)$$

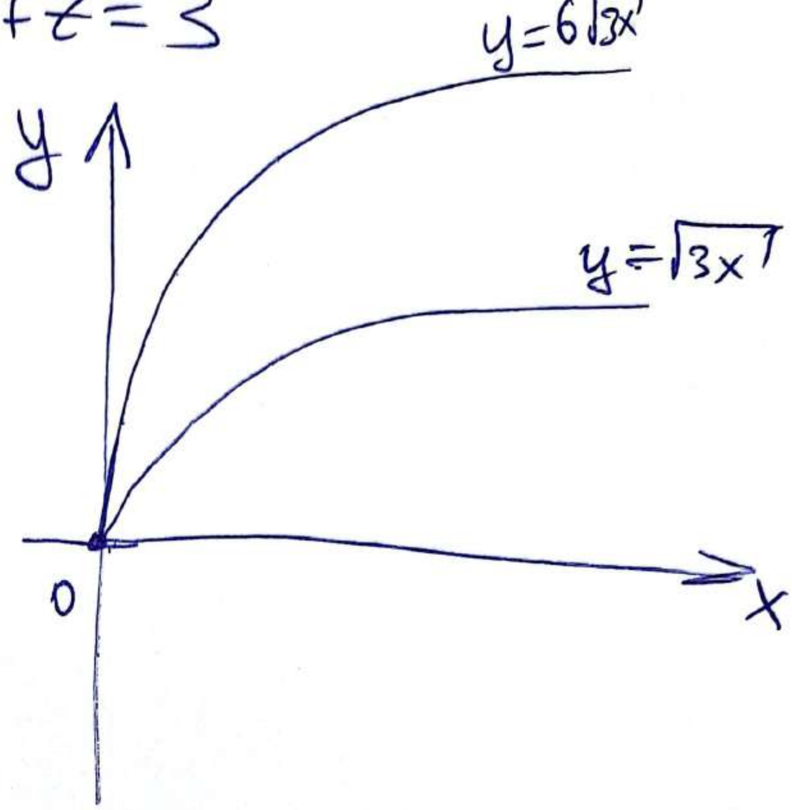
$$\int_{-1}^{1}\frac{7\cdot 25}{2}(x^4-x^6)\,dx=\frac{7\cdot 25}{2}\left(\frac{x^5}{5}\Big|_{-1}^{1}-\frac{x^7}{7}\Big|_{-1}^{1}\right)=$$

$$=\frac{7\cdot 25}{2}\left(\frac{2}{5}-\frac{2}{7}\right)=10$$

10.17

$y=6\sqrt{3x};\ y=\sqrt{3x};\ z=0;\ x+z=3$





$$V=\int_0^3 dz\int_0^{3-z}dx\int_{\sqrt{3x}}^{6\sqrt{3x}}dy=36$$

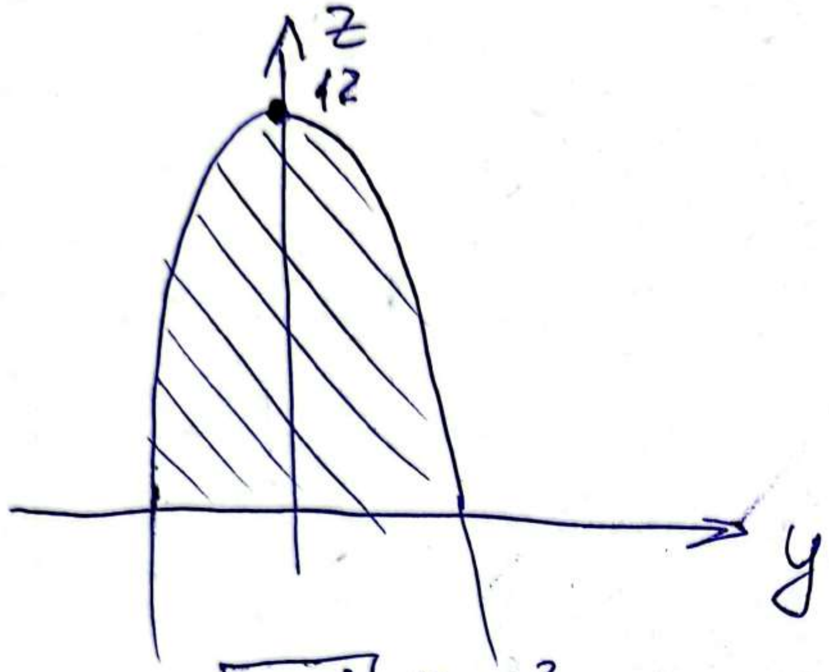
$$\int_0^{3-z}\left(6\sqrt{3x}-\sqrt{3x}\right)dx=5\sqrt{3}\int_0^{3-z}\sqrt{x}\,dx=\frac{10}{\sqrt{3}}(3-z)^{3/2}$$

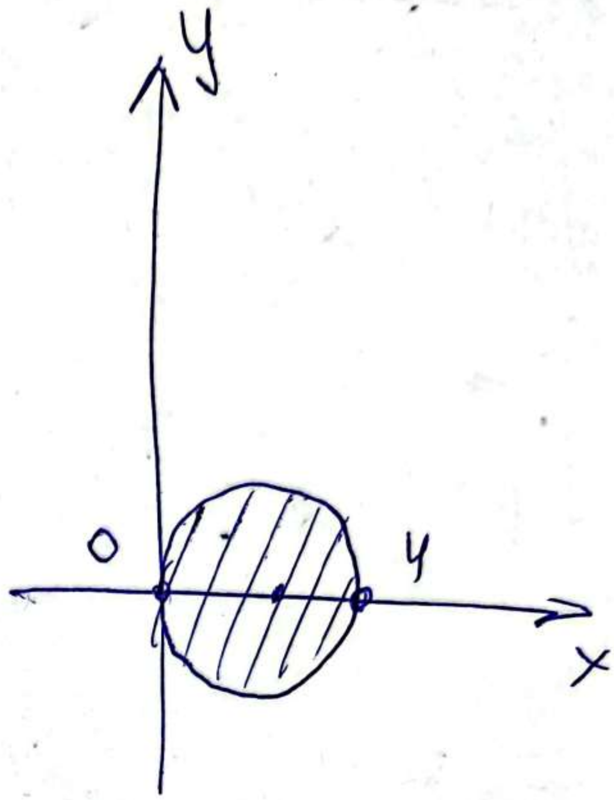
$$\frac{10}{\sqrt{3}}\int_0^3(3-z)^{3/2}dz=\left\{\begin{array}{l}3-z=t\\ -dz=dt\\ z=3\quad t=0\\ z=0\quad t=3\end{array}\right\}=\frac{10}{\sqrt{3}}\int_0^3 t^{3/2}dt=$$

$$=\frac{4}{\sqrt{3}}\cdot 3^{5/2}=36$$

№11.17

$x^2+y^2=4x$, $z=12-y^2$, $z=0$





$$S=\int_0^4 dx\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}dy\int_0^{12-y^2}dz=\frac{140\pi}{3}$$

$$\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}(12-y^2)\,dy=\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}12\,dy-\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}y^2\,dy=12y\Big|_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}-$$

$$-\frac{y^3}{3}\Big|_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}=12\cdot2\sqrt{4x-x^2}-\frac{2}{3}\sqrt{4x-x^2}=$$

$$=\frac{70}{3}\sqrt{4x-x^2}$$

$$\frac{70}{3}\int_0^4\left(\sqrt{4x-x^2}\right)dx=\frac{70}{3}\cdot2\pi$$

$$\int_0^4 \sqrt{4x-x^2}\,dx = \left\{\begin{matrix} x-2=t \\ x=t+2 \\ dx=dt \end{matrix}\right\} = \int_{-2}^{2} \sqrt{4-t^2}\,dt =$$

$x=4 \quad t=2$

$x=0 \quad t=-2$

$$= \left(\frac{t}{2}\sqrt{4-t^2} + \frac{4}{2}\arcsin\frac{t}{2}\right)\Bigg|_{-2}^{2} =$$

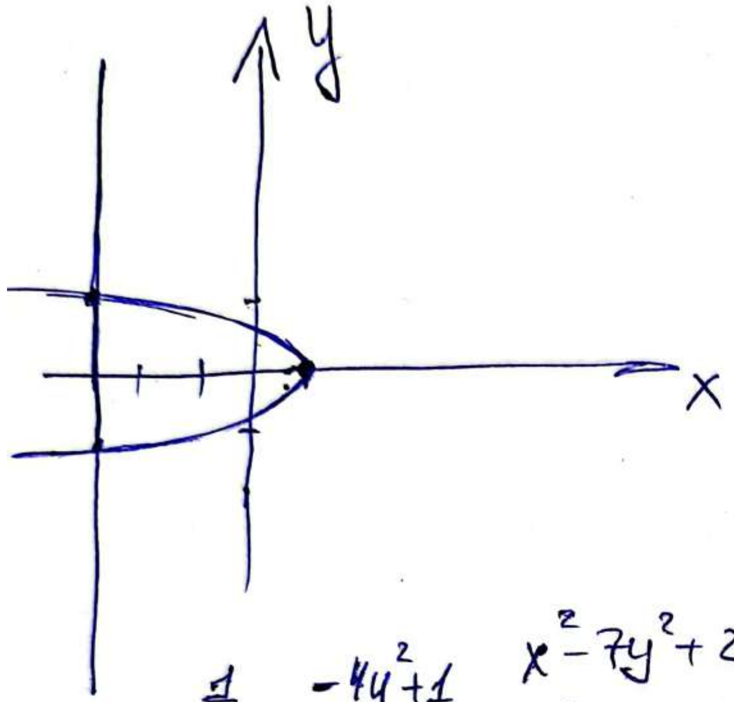
$$= \sqrt{0} + 2\arcsin(1) + \sqrt{0} - 2\arcsin(-1) =$$

$$= \frac{2\pi}{2} + \frac{2\pi}{2} = 2\pi$$

ТР 11 лист 2

ТР-12 В-17

$x=-4y^2+1,\ x=-3,\ z=x^2-7y^2-1,\ z=x^2-7y^2+2$



$$V=\int_{-1}^{1}dy\int_{-3}^{-4y^2+1}dx\int_{x^2-7y^2-1}^{x^2-7y^2+2}dz=16$$

$$\int_{x^2-7y^2-1}^{x^2-7y^2+2}dz=x^2-7y^2+2-x^2+7y^2+1=3$$

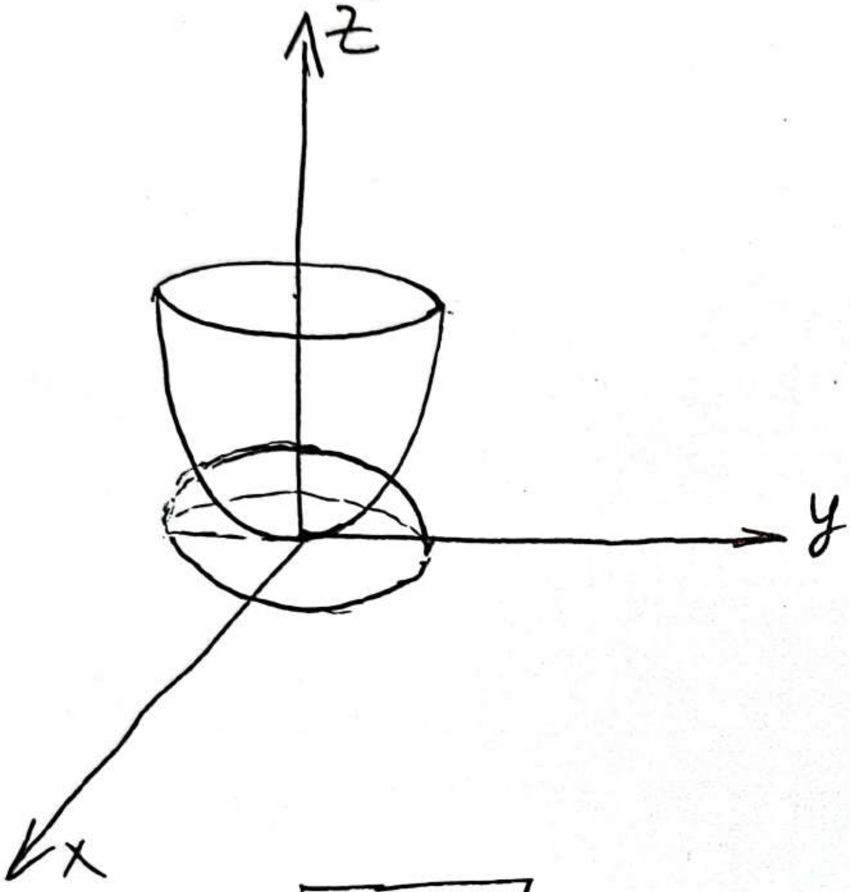
$$\int_{-3}^{-4y^2+1}3\,dx=3(-4y^2+1+3)=-12y^2+12$$

$$\int_{-1}^{1}(-12y^2+12)\,dy=-\overset{4}{\cancel{12}}\frac{y^3}{\cancel{3}}\Big|_{-1}^{1}+12y\Big|_{-1}^{1}=-4(1+1)+12(1+1)$$

$$=-8+24=16$$

ТР13 В-17

$z=\sqrt{144-x^2-y^2}$, $18z=x^2+y^2$



$$\int_{-12}^{12} dx \int_{-\sqrt{144-x^2}}^{\sqrt{144-x^2}} dy \int_{\frac{x^2+y^2}{18}}^{\sqrt{144-x^2-y^2}} dz$$

$$\int_{\frac{x^2+y^2}{18}}^{\sqrt{144-x^2-y^2}} dz = -\frac{y^2}{18}+\sqrt{-y^2-x^2+144}-\frac{x^2}{18}$$

$$\int_{-\sqrt{144-x^2}}^{\sqrt{144-x^2}} \left(-\frac{y^2}{18}+\sqrt{-y^2-x^2+144}-\frac{x^2}{18}\right) dy = -\frac{\sqrt{144-x^2}\,(4x^2+288)}{54}-$$

$$-\frac{\pi x^2}{2}+72\pi$$

$$\int_{-12}^{12}\left(-\frac{\sqrt{144-x^2}\,(4x^2+288)}{54}-\frac{\pi x^2}{2}+72\pi\right)dx =$$

$$= 576\pi$$

ТР 14. В-17

$z=-2(x^2+y^2)-1$, $z=4y-1$

$-2(x^2+y^2)-1=4y-1$

$x^2+(y+1)^2=1$

$\rho^2=-2\rho\sin\theta$ или $\rho=-2\sin\theta$

$\pi\le\theta\le 2\pi$

$0\le\rho\le-2\sin\theta$

$4y-1\le z\le-2(x^2+y^2)-1$

$$V=\int_{\pi}^{2\pi}d\theta\int_0^{-2\sin\theta}\rho\,d\rho\int_{4y-1}^{-2(x^2+y^2)-1}dz=\int_{\pi}^{2\pi}d\theta\int_0^{-2\sin\theta}\rho(-2\rho^2-4\rho\sin\theta)d\rho=$$

$$=-\int_{\pi}^{2\pi}d\theta\int_0^{-2\sin\theta}(2\rho^3+4\rho^2\sin\theta)d\rho=-\int_{\pi}^{2\pi}d\theta\left(\frac{\rho^4}{2}+\frac{4}{3}\rho^3\sin\theta\right)\Bigg|_0^{-2\sin\theta}=$$

$$=-\int_{\pi}^{2\pi}\left(8\sin^4\theta-\frac{32}{3}\sin^4\theta\right)d\theta=\frac{8}{3}\int_{\pi}^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)^2d\theta=$$

$$=\frac{2}{3}\int_{\pi}^{2\pi}\left(1-2\cos 2\theta+\frac{1+\cos 4\theta}{2}\right)d\theta=$$

$$=\frac{2}{3}\int_{\pi}^{2\pi}\frac{3}{2}-2\cos 2\theta+\frac{1}{2}\cos 4\theta)d\theta=$$

$$=\frac{2}{3}\cdot\frac{3}{2}\left(\theta-\sin 2\theta+\frac{\sin 4\theta}{8}\right)\Bigg|_{\pi}^{2\pi}=\pi$$

ТР 15, В-17

$$9\le x^2+y^2+z^2\le 81$$

$$-\sqrt{\frac{x^2+y^2}{3}}\le z\le\sqrt{\frac{x^2+y^2}{35}}$$

$$0\le y\le -x$$

$$V=\int_{-9}^{-3}dx\int_{0}^{-x}dy\int_{-\sqrt{\frac{x^2+y^2}{3}}}^{\sqrt{\frac{x^2+y^2}{35}}}dz$$

$$\int_0^{-x}\left(\sqrt{\frac{x^2+y^2}{35}}+\sqrt{\frac{x^2+y^2}{3}}\right)dy=\frac{(\sqrt{35}+\sqrt{3})x^2\ln((\sqrt{2}-1)x^2)}{2\sqrt{3}\sqrt{35}}+$$

$$+\frac{(-2\sqrt{35}-2\sqrt{3})x^2\ln(x)}{2\sqrt{3}\sqrt{35}}+\frac{(-\sqrt{2}\sqrt{35}-\sqrt{2}\sqrt{3})x^2}{2\sqrt{3}\sqrt{35}}$$

$$\int_{-9}^{-3}\left(\frac{(\sqrt{35}+\sqrt{3})x^2\ln((\sqrt{2}-1)x^2)}{2\sqrt{3}\sqrt{35}}+\frac{(-2\sqrt{35}-2\sqrt{3})x^2\ln(x)}{2\sqrt{3}\sqrt{35}}+\right.$$

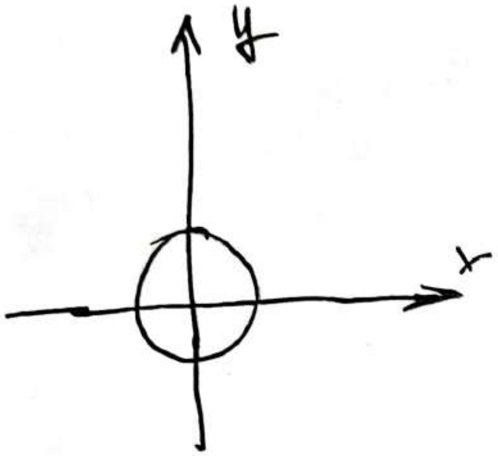
$$\left.+\frac{(-\sqrt{2}\sqrt{35}-\sqrt{2}\sqrt{3})x^2}{2\sqrt{3}\sqrt{35}}\right)dx=\frac{(13\cdot 3\sqrt{3}\sqrt{35}+117)\ln(\sqrt{2}-1)}{\sqrt{35}}+$$

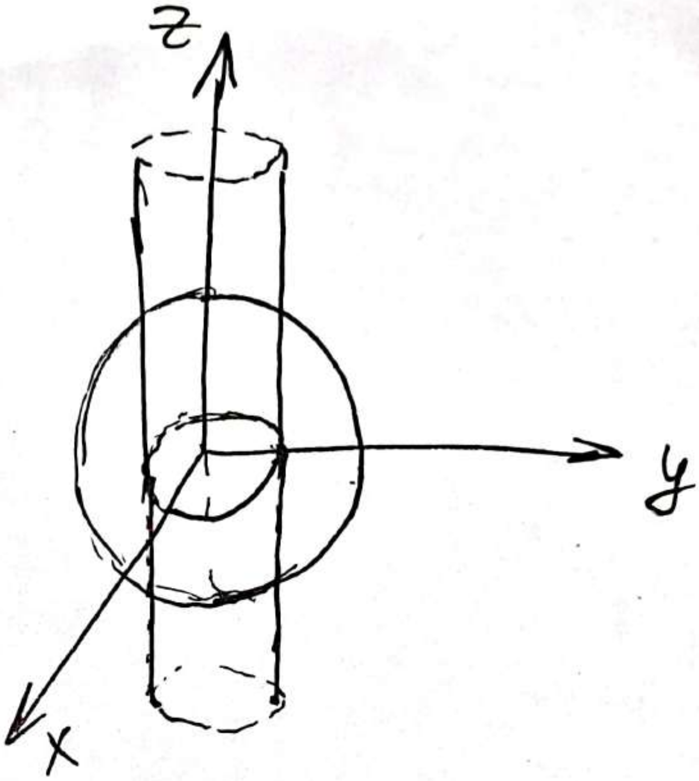
$$+\cancel{\frac{(-3\sqrt{3}\sqrt{35}-9)\ln(-3)}{\sqrt{35}}}+\cancel{\frac{(81\sqrt{3}\sqrt{35}+243)\ln(-9)}{\sqrt{35}}}-\cancel{\frac{78}{\sqrt{35}}}-\cancel{26\sqrt{3}}+$$

$$+\cancel{\frac{(3\sqrt{3}\sqrt{35}+9)\ln(-3)}{\sqrt{35}}}+\cancel{\frac{(-81\sqrt{3}\sqrt{35}-243)\ln(-9)}{\sqrt{35}}}+\cancel{\frac{78}{\sqrt{35}}}+\cancel{26\sqrt{3}}-$$

$$-\frac{117\sqrt{2}}{\sqrt{35}}-13\sqrt{2}\,3\sqrt{3}=\frac{(13\cdot 3\sqrt{3}\sqrt{35}+117)\ln(\sqrt{2}-1)}{\sqrt{35}}-\frac{117\sqrt{2}}{\sqrt{35}}-13\cdot 2\sqrt{2}\sqrt{3}$$

ТР16, В-17





$$x^2+y^2+z^2=4;\ x^2+y^2=1\quad (x^2+y^2\le 1);\ \mu=6|z|$$

$$m=\int_{-1}^{1}dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}dy\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}6|z|\,dz=21\pi$$

$$\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}6|z|\,dz=-6y^2-6x^2+24$$

$$\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}(-6y^2-6x^2+24)\,dy=(44-8x^2)\sqrt{1-x^2}$$

$$\int_{-1}^{1}(44-8x^2)\sqrt{1-x^2}\,dx=21\pi$$

ТР-1 В-17

$u = x^2y - \sqrt{xy+z^2}$

$\overline{l} = 2j - 2k$

$M(1,5,-2)$

$\overrightarrow{\text{grad} u} = \left(2yx - \frac{1}{2\sqrt{xy+z^2}} \cdot y\right)\vec{i} + \left(x^2 - \frac{1}{2\sqrt{xy+z^2}} \cdot x\right)\vec{j} +$

$+ \left(\frac{1}{2\sqrt{xy+z^2}} \cdot 2z\right)\vec{k}$

$\overrightarrow{\text{grad} u} = \left(2yx - \frac{y}{2\sqrt{xy+z^2}}\right)\vec{i} + \left(x^2 - \frac{x}{2\sqrt{xy+z^2}}\right)\vec{j} +$

$+ \left(\frac{z}{\sqrt{xy+z^2}}\right)\vec{k}$

$l_0 = \frac{l}{|l|} = \frac{2j-2k}{\sqrt{4+4}} = \frac{2j-2k}{4\sqrt{2}} = \frac{1}{2\sqrt{2}}\vec{j} - \frac{1}{2\sqrt{2}}\vec{k}$

$(\overrightarrow{\text{grad} u}, \vec{l_0}) = \left(x^2 - \frac{x}{2\sqrt{xy+z^2}}\right) \cdot \frac{1}{2\sqrt{2}} + \left(\frac{z}{\sqrt{xy+z^2}}\right) \cdot \left(-\frac{1}{2\sqrt{2}}\right) =$

$= \frac{2x^2\sqrt{xy+z^2} - x}{4\sqrt{2}\sqrt{xy+z^2}} - \frac{z}{2\sqrt{2}\sqrt{xy+z^2}} = \frac{2x^2\sqrt{xy+z^2} - x - 2z}{2\sqrt{2}\sqrt{xy+z^2}} =$

$= \frac{2\sqrt{5+4} - 1 + 4}{2\sqrt{2}\sqrt{9}} = \frac{3}{2\sqrt{2}}$

ТР-2 В-17

$$V=\frac{6}{x}+\frac{2}{y}-\frac{3\sqrt{3}}{2\sqrt{2}z},\ u=\frac{y^2z^3}{x^2},\ M\left(\sqrt{2},\sqrt{2},\frac{\sqrt{3}}{2}\right)$$

$$\overrightarrow{\text{grad}\,v}=(6\cdot(-1)x^{-2})\vec{i}+(2(-1)y^{-2})\vec{j}+\left(\frac{-3\sqrt{3}}{2\sqrt{2}}(-1)z^{-2}\right)\vec{k}=$$

$$=\left(-\frac{6}{x^2}\right)\vec{i}+\left(-\frac{2}{y^2}\right)\vec{j}+\left(\frac{3\sqrt{3}}{2\sqrt{2}z^2}\right)\vec{k}$$

$$\cos\alpha=\frac{(\text{grad}\,u,\text{grad}\,v)}{|\text{grad}\,u|\cdot|\text{grad}\,v|}$$

$$\overrightarrow{\text{grad}\,u}=(y^2z^3(-2)x^{-3})\vec{i}+\left(\frac{z^3}{x^2}\cdot 2y\right)\vec{j}+\left(\frac{y^2}{x^2}\cdot 3z^2\right)\vec{k}=$$

$$=\left(-\frac{2y^2z^3}{x^3}\right)\vec{i}+\left(\frac{2yz^3}{x^2}\right)\vec{j}+\left(\frac{3y^2z^2}{x^2}\right)\vec{k}$$

$$(\text{grad}\,u,\text{grad}\,v)=\frac{2\cdot 6y^2z^3}{x^2x^3}-\frac{2\cdot 2yz^3}{x^2y^2}+\frac{3y^2z^2 3\sqrt{3}}{x^2 2\sqrt{2}z^2}=$$

$$=\frac{18y^2z^3}{x^5}-\frac{4z^3}{x^2y}+\frac{9\sqrt{3}y^2}{2\sqrt{2}x^2}$$

$$|\text{grad}\,u|=\sqrt{\frac{4y^4z^6}{x^6}+\frac{4y^2z^6}{x^4}+\frac{9y^4z^4}{x^4}}$$

$$|\text{grad}\,v|=\sqrt{\frac{36}{x^4}+\frac{4}{y^4}+\frac{9\cdot 3}{4\cdot 2z^4}}$$

$$|\text{grad}\,u|=\frac{3}{2}\sqrt{3}$$

$$|\text{grad}\,v|=4$$

$$(\text{grad}\,u,\text{grad}\,v)=\frac{\sqrt{3^7}}{\sqrt{2^7}}-\frac{\sqrt{3^3}}{\sqrt{2^5}}+\frac{\sqrt{3^5}}{\sqrt{2^3}}$$

$$\cos \theta = \frac{9}{2^4\sqrt{2}} - \frac{1}{8\sqrt{2}} + \frac{3}{4\sqrt{2}}$$

ТР-3 В-17

$\vec{a} = y\vec{j} + 4z\vec{k}$

$$\frac{dx}{0} = \frac{dy}{1y} = \frac{dz}{4z} \Leftrightarrow \begin{cases} dx = 0 \Rightarrow x = C_0 \\ \dfrac{dy}{y} = \dfrac{dz}{4z} \\ \ln y = \dfrac{1}{4}\ln z + \ln C \end{cases}$$

$$y = z^{\frac{1}{4}} C$$

$$y = \sqrt[4]{z}\, C$$

ТР-4 В-17

$a = xyz\,i - x^2 z\,j + 3k$ $\quad S: x^2+y^2=z^2\ (z \geq 0)$

$P: z = 2$

$x^2+y^2-z^2=0$

$\operatorname{grad} S\ \{2x; 2y; -2z\}$

$\sqrt{4x^2+4y^2+4z^2}$

$\vec{n_0} = \dfrac{\{2x; 2y; -2z\}}{\sqrt{4x^2+4y^2+4z^2}} = \dfrac{\{x; y; -z\}}{\sqrt{x^2+y^2+z^2}}$

$(\vec{a}, \vec{n_0}) = \dfrac{\cancel{x^2yz} - \cancel{x^2yz} - 3z}{\sqrt{x^2+y^2+z^2}} = \dfrac{-3z}{\sqrt{x^2+y^2+z^2}}$

$\cos\gamma = \dfrac{-z}{\sqrt{x^2+y^2+z^2}}$

$\iint\limits_{K} \dfrac{-3z}{z}\,dx\,dy = \iint\limits_{K} -3\,dx\,dy = \int\limits_{-2}^{2} dx \int\limits_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} -3\,dy = \int\limits_{-2}^{2} -6\sqrt{4-x^2}\,dx =$

$= -12\pi$

ТР-5 В-17

$a = xi + yj + zk$, $P: 2x + \frac{y}{2} + z = 1$

$2x + \frac{y}{2} + z - 1 = 0$

$\operatorname{grad} S \{2; \frac{1}{2}; 1\}$

$\sqrt{4 + \frac{1}{4} + 1} = \frac{\sqrt{21}}{2}$

$\vec{n_0} = \frac{\{2; \frac{1}{2}; 1\} \cdot 2}{\sqrt{21}} = \frac{\{4; 1; 2\}}{\sqrt{21}}$

$(\vec{a}, \vec{n_0}) = \frac{4x + y + 2z}{\sqrt{21}}$

$\cos \gamma = \frac{2}{\sqrt{21}}$

$\iint\limits_K \frac{4x + y + 2z}{2} dx\,dy = \iint\limits_K (2x + \frac{1}{2}y + z) dx\,dy =$

$= \int_0^{1/2} dx \int_0^{2-4x} (2x + \frac{1}{2}y + z) dy = \int_0^{1/2} dx \int_0^{2-4x} dy = \int_0^{1/2} (2-4x) dx =$

$= 1/2$

ТР-6. В-17.

$a = \pi y j + (1-2z)k$, $P: \frac{x}{4} + \frac{y}{3} + z = 1$

$\frac{x}{4} + \frac{y}{3} + z - 1 = 0$

$\text{drad}\, S \left\{\frac{1}{4}; \frac{1}{3}; 1\right\}$

$\sqrt{\frac{1}{16} + \frac{1}{9} + 1} = \frac{13}{12}$

$\vec{n_0} = \frac{\left\{\frac{1}{4}; \frac{1}{3}; 1\right\} \cdot 12}{13} = \frac{\{3; 4; 12\}}{13}$

$(\vec{a}, \vec{n_0}) = \frac{4\pi y + 12(1-2z)}{13}$

$\cos \gamma = \frac{12}{13}$

$\iint\limits_K \frac{4\pi y + 12(1-2z)}{12} dxdy = \iint\limits_K \left(\frac{1}{3}\pi y + 1 - 2z\right) dxdy =$

$= \iint\limits_K \left(\frac{1}{3}\pi y + 1 - 2\left(1 - \frac{x}{4} - \frac{y}{3}\right)\right) dxdy = \iint\limits_K \left(\left(\frac{1}{3}\pi + \frac{2}{3}\right)y + \frac{1}{2}x - 1\right) dxdy$

$\int_0^4 dx \int_0^{3-\frac{3}{4}x} \left(\left(\frac{1}{3}\pi + \frac{2}{3}\right)y + \frac{1}{2}x - 1\right) dy = \int_0^4 \left(-\frac{(12\pi - 48)x}{32} + \frac{3\pi^3}{32} - \right.$

$\left. - \frac{9\pi^2}{16} + \frac{3\pi}{4}\right) dx = \frac{3\pi^3}{8} - \frac{9\pi^2}{4} + 12$