$\boxed{1.6.}$

$$\int \frac{e^x}{\sqrt{4+e^{2x}}}\,dx = \left| \begin{matrix} t=e^x \\ \frac{dt}{e^x}=\frac{dx}{1} \end{matrix} \right\} = \int \frac{1}{\sqrt{4+t^2}}\,dt = \ln\left|t+\sqrt{t^2+4}\right| =$$

$$= \ln\left|e^x+\sqrt{e^{2x}+4}\right| = \ln\left|e^x+\sqrt{e^{2x}+4}\right| + c$$

Ответ: $\ln\left|e^x+\sqrt{e^x+4}\right| + c$

$\boxed{2.6}$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{\ln x}{\sqrt[3]{x^2}}\,dx = \int \frac{\ln x}{x^{\frac{2}{3}}}\,dx = \left\{ \begin{matrix} u=\ln x \\ dv = x^{-\frac{2}{3}} \\ du=\frac{1}{x}\,dx \\ v = 3\sqrt[3]{x} \end{matrix} \right\} = 3\sqrt[3]{x}\ln x - \int 3\sqrt[3]{x}\cdot\frac{1}{x}\,dx =$$

$$= 3\sqrt[3]{x}\ln x - 3\int\frac{\sqrt[3]{x}}{x}\,dx = 3\sqrt[3]{x}\ln x - 3\int x^{-\frac{2}{3}}\,dx = 3\sqrt[3]{x}\ln x - 3\cdot 3\sqrt[3]{x} =$$

$$= 3\sqrt[3]{x}(\ln x - 3) + c$$

Ответ: $3\sqrt[3]{x}\ln x - 9\sqrt[3]{x} + c$

$\boxed{3.6.}$

$$\int \frac{3x^2+16x+14}{x^2+4x+5}\,dx = \int\left(3+\frac{4x-1}{x^2+4x+5}\right)dx = \int 3\,dx + \int \frac{4x-1}{x^2+4x+5}\,dx =$$

$$= 3x + \int \frac{2(2x+4)-9}{x^2+4x+5}\,dx = 3x + \int \frac{2(2x+4)}{x^2+4x+5}\,dx - \int \frac{9}{x^2+4x+5}\,dx =$$

$$= 3x + 2\ln|x^2+4x+5| - 9\,\mathrm{arctg}(x+2) = 3x + 2\ln|x^2+4x+5| -$$

$$- 9\,\mathrm{arctg}(x+2) + C$$

Ответ: $3x + 2\ln|x^2+4x+5| - 9\,\mathrm{arctg}(x+2) + C$

$\boxed{4.6.}$

$$\int \frac{x^3-x^2+x-4}{(x^2-1)(x^2+1)}\,dx \ominus$$

$$\frac{x^3-x^2+x-4}{(x^2-1)(x^2+1)} = \frac{Ax+B}{x^2-1}^{\backslash x^2+1} + \frac{Cx+D}{x^2+1}^{\backslash x^2-1} = \frac{Ax^3+Ax+Bx^2+B+Cx^3-Cx+Dx^2-D}{(x^2-1)(x^2+1)} =$$

$$= \frac{x^3(A+C)+x^2(B+D)+x(A-C)+(B-D)}{(x^2-1)(x^2+1)}$$

$$\begin{cases} A+C=1 \\ B+D=-1 \\ A-C=1 \\ B-D=-4 \end{cases} \Rightarrow A=1;\ B=-2{,}5;\ C=0;\ D=1{,}5$$

$$\frac{x^3-x^2+x-4}{(x^2-1)(x^2+1)} = \frac{x-2{,}5}{(x^2-1)} + \frac{1{,}5}{(x^2+1)}$$

$$= \int \frac{x-2{,}5}{x^2-1}\,dx + \int \frac{1{,}5}{x^2+1}\,dx = \int \frac{2x-5}{2x^2-2}\,dx + \int \frac{3}{2x^2+2}\,dx =$$

$$= \frac{1}{2}\int \frac{2x-5}{x^2-1}\,dx + \frac{3}{2}\int \frac{1}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2-1} - \frac{5}{4}\int \frac{1}{x^2-1}\,dx +$$

$$+ \frac{3}{2}\int \frac{1}{x^2+1}\,dx =$$

$$= \frac{1}{2}\ln|x^2-1| - \frac{5}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{3}{2}\,\mathrm{arctg}\,x$$

Ответ: $\frac{1}{2}\ln|x^2-1| - \frac{5}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{3}{2}\,\mathrm{arctg}\,x + C$

$\boxed{5.6.}$

$$t = \frac{1}{x-4}\ ;\ x = \frac{1}{t}+4;\ dx = -\frac{dt}{t^2}$$

$$\int \frac{dx}{(x-4)^2\sqrt{5x^2-48x+108}} = \int \frac{-dt\cdot \cancel{t^2}}{\cancel{t^2}\sqrt{5(\frac{1}{t}+4)^2-48(\frac{1}{t}+4)+108}} =$$

$$= -\int \frac{dt}{\sqrt{5(\frac{1}{t^2}+\frac{8}{t}+16)-\frac{48}{t}-192+108}} = -\int \frac{dt}{\sqrt{\frac{5}{t^2}+\frac{40}{t}+80-\frac{48}{t}-84}} =$$

$$= -\int \frac{dt}{\sqrt{\frac{5+40t+80t^2-48t-84t^2}{t^2}}} = -\int \frac{t\,dt}{\sqrt{-4t^2-8t+5}} =$$

$$= -\int \frac{t\,dt}{\sqrt{9-(2t+2)^2}} = \left\{\begin{array}{l} 2t+2=u \\ t=\frac{u-2}{2} \\ dt=\frac{1}{2}du \end{array}\right\} = -\int \frac{u-2}{4\sqrt{9-u^2}}\,du =$$

$$= -\frac{1}{4}\left(\int \frac{u}{\sqrt{9-u^2}}du - 2\int \frac{du}{\sqrt{9-u^2}}\right) = +\frac{1}{4}\sqrt{9-u^2} + \frac{2}{4}\arcsin\left(\frac{u}{3}\right) =$$

$$= \frac{1}{4}\sqrt{9-(2t+2)^2} + \frac{1}{2}\arcsin\left(\frac{2t+2}{3}\right) = \frac{\sqrt{9-4t^2-8t-4}}{4} + \frac{\arcsin\left(\frac{2t+2}{3}\right)}{2} =$$

$$= \frac{\sqrt{5-8\cdot\frac{1}{x-4}-4\cdot\frac{1}{(x-4)^2}}}{4} + \frac{1}{2}\arcsin\left(\frac{2\cdot\frac{1}{x-4}+2}{3}\right) =$$

$$= \frac{1}{4}\sqrt{5-\frac{8}{x-4}-\frac{4}{(x-4)^2}} + \frac{1}{2}\arcsin\left(\frac{2x-6}{3x-12}\right) + C$$

$\boxed{6.6.}$

$$\int \frac{2x+\sqrt{4x-1}-5}{\sqrt{4x-1}+3}\,dx = \left\{ \begin{array}{l} t=\sqrt{4x-1} \\ x=\frac{t^2+1}{4} \\ dx=\frac{t}{2}\,dt \end{array} \right\} =$$

$$= \int \frac{2\cdot\frac{t^2+1}{4}+t-5}{t+3}\cdot\frac{t}{2}\,dt = \int \frac{\left(\frac{t^2+1}{2}+t-5\right)t}{2(t+3)}\,dt =$$

$$= \int \frac{t^3+2t^2-9t}{4t+12}\,dt = \int\left(\frac{1}{4}t^2-\frac{1}{4}t-\frac{6t}{4t+12}\right)dt =$$

$$= \int\left(\frac{1}{4}t^2-\frac{1}{4}t\right)dt - \int\frac{3t}{2t+6}\,dt =$$

$$= \frac{t^3}{12}-\frac{t^2}{8}-\frac{3}{2}\int\frac{t}{t+3}\,dt = \frac{t^3}{12}-\frac{t^2}{8}-\frac{3}{2}\left(t-3\ln|t+3|\right) =$$

$$= \frac{\left(\sqrt{4x-1}\right)^3}{12}-\frac{4x-1}{8}-\frac{3}{2}\sqrt{4x-1}+\frac{9}{2}\ln\left|\sqrt{4x-1}+3\right| =$$

$$= \frac{(4x-1)^{\frac{3}{2}}}{12}-\frac{4x-1}{8}-\frac{3\sqrt{4x-1}}{2}+\frac{9}{2}\ln\left|\sqrt{4x-1}+3\right|+C$$

Отв.:

$\boxed{8.6.}$

$$\int \sqrt{5x^2+13x+7}\,dx = \sqrt{5}\int\sqrt{x^2+\frac{13x}{5}+\frac{7}{5}}\,dx =$$

$$= \sqrt{5}\int\sqrt{x^2+\frac{13x}{5}+\frac{169}{100}-\frac{29}{100}}\,dx = \sqrt{5}\int\sqrt{\left(x+\frac{13}{10}\right)^2-\frac{29}{100}}\,dx =$$

$t = x+\frac{13}{10}$; $dt = dx$

$$= \sqrt{5}\int\sqrt{t^2-\frac{29}{100}}\,dt = \left\{\begin{array}{l} u=\sqrt{t^2-\frac{29}{100}} \\ dv = 1 \\ v = t \\ du = \frac{\cancel{2}t}{\cancel{2}\sqrt{t^2-\frac{29}{100}}}\,dt \end{array}\right\} =$$

$$= \sqrt{5}\,t\sqrt{t^2-\frac{29}{100}} - \sqrt{5}\int\frac{t^2\,dt}{\sqrt{t^2-\frac{29}{100}}} = \sqrt{5}\,t\sqrt{t^2-\frac{29}{100}} - \sqrt{5}\int\frac{t^2\,dt}{\sqrt{t^2-\frac{29}{100}}} =$$

$$= \sqrt{5}\,t\sqrt{t^2-\frac{29}{100}} - \sqrt{5}\left(\frac{29\ln\left|\sqrt{100t^2-29}+10t\right|}{200} + \frac{t\sqrt{100t^2-29}}{20}\right) =$$

$$= \sqrt{5}\left(x+\frac{13}{10}\right)\sqrt{x^2+\frac{13x}{5}+\frac{169}{100}-\frac{29}{100}} - \frac{\sqrt{5}\cdot 29\ln\left|\sqrt{100\left(x^2+\frac{13x}{5}+\frac{169}{100}\right)-29}+10\left(x+\frac{13}{10}\right)\right|}{20}$$

$$- \frac{\sqrt{5}\left(x+\frac{13}{10}\right)\sqrt{100\left(x^2+\frac{13x}{5}+\frac{169}{100}\right)-29}}{20} =$$

$$= \left(x+\frac{13}{10}\right)\sqrt{5x^2+13x+7} - \frac{29\sqrt{5}}{20}\ln\left|\sqrt{100x^2+260x+169-29}\right.+$$

$$\left.+10x+13\right| - \frac{\sqrt{5}}{20}\left(x+\frac{13}{10}\right)\sqrt{100x^2+260x+140} =$$

$$= \left(x+\frac{13}{10}\right)\sqrt{5x^2+13x+7} - \frac{29\sqrt{5}}{20}\ln\left|\sqrt{20(5x^2+13x+7)}+10x+13\right| -$$

$$- \frac{\left(x+\frac{13}{10}\right)}{20}\sqrt{5}\sqrt{20(5x^2+13x+7)} =$$

$$= \left(x+\frac{13}{10}\right)\sqrt{5x^2+13x+7} - \frac{29\sqrt{5}}{20}\ln\left|\sqrt{20(5x^2+13x+7}+10x+13\right| -$$

$$- \frac{\left(x+\frac{13}{10}\right)}{2\cancel{0}}\cdot \cancel{10}\sqrt{5x^2+13x+7} = \left(x+\frac{13}{10}\right)\sqrt{5x^2+13x+7}\cdot\frac{1}{2} - \frac{29\sqrt{5}}{20}\ln\left|\sqrt{20(5x^2+13x+7)}+\right.$$

$$\left.+10x+13\right| =$$

$$= \frac{1}{2}\left(x+\frac{13}{10}\right)\sqrt{5x^2+13x+7} - \frac{29\sqrt{5}}{20}\ln\left|\sqrt{20(5x^2+13x+7)}+10x+13\right| + C$$

Ответ

$\boxed{9.6.}$

$$\int \frac{dx}{\cos^3\frac{x}{4}\sin\frac{x}{4}} = \left\{ \begin{array}{l} u=\frac{x}{4} \\ dx=4du \end{array} \right\} = \int \frac{4}{\cos^3 u \sin u}\, du =$$

$$= 4\int \frac{\cos u}{(1-\sin^2 u)^2 \sin u}\, du = \left\{ \begin{array}{l} v=\sin u \\ dv=\cos u\, du \end{array} \right\} = 4\int \frac{dv}{v(1-v^2)^2} =$$

$$= \left\{ \begin{array}{l} 1-v^2=t \\ dt=-2v\,dv \\ dv=\frac{dt}{-2v} \end{array} \right\} = 4\int \frac{1}{2t^3-2t^2}\, dt = 2\int \frac{1}{t^2(t-1)}\, dt =$$

$$= 2\left(-\ln|t| - \frac{t-1}{t} + \ln|t-1|\right) = 2\ln|t-1| - 2\ln|t| - \frac{2(t-1)}{t} =$$

$$= 2\ln|\cancel{1}-v^2\cancel{-1}| - 2\ln|1-v^2| - \frac{2(\cancel{1}-v^2\cancel{-1})}{1-v^2} = 2\ln v^2 - 2\ln|1-v^2| + \frac{2v^2}{1-v^2} =$$

$$= 2\ln(\sin^2 u) - 2\ln|1-\sin^2 u| + \frac{2\sin^2 u}{1-\sin^2 u} = 2\ln(\sin^2 u) - 2\ln(\cos^2 u) + 2tg^2 u =$$

$$= 2\ln\left(\sin^2\frac{x}{4}\right) - 2\ln\left(\cos^2\frac{x}{4}\right) + 2tg^2\frac{x}{4} = 2\ln\frac{\sin^2\frac{x}{4}}{\cos^2\frac{x}{4}} + 2tg^2\frac{x}{4} =$$

$$= 2\ln\left(tg^2\frac{x}{4}\right) + 2tg^2\frac{x}{4} + C$$

Ответ: $2\ln\left(tg^2\frac{x}{4}\right) + 2tg^2\frac{x}{4} + C$

$\boxed{1.6.}$

$$\int_0^{\frac{1}{\sqrt{2}}} dy \int_0^{\arcsin y} f\,dx + \int_{\frac{1}{\sqrt{2}}}^{1} dy \int_0^{\arccos y} f\,dx \;=$$

Область инт-я $D = D_1 \cup D_2$; $D_1 : \begin{cases} 0 \le y \le \frac{1}{\sqrt{2}} \\ 0 \le x \le \arcsin y \end{cases}$

$D_2 : \begin{cases} \frac{1}{\sqrt{2}} \le y \le 1 \\ 0 \le x \le \arccos y \end{cases}$

$x = \arcsin y \;| \Rightarrow y = \sin x$

$x = \arccos y \;| \Rightarrow y = \cos x$

$$D : \begin{cases} 0 \le x \le \frac{\pi}{4} \\ \sin x \le y \le \cos x \end{cases}$$

$$= \iint_D f\,dx\,dy = \int_0^{\frac{\pi}{4}} dx \int_{\sin x}^{\cos x} f\,dy$$

Ответ: $\iint_D f\,dx\,dy = \int_0^{\frac{\pi}{4}} dx \int_{\sin x}^{\cos x} f\,dy$

$\boxed{2.6.}$

$$\iint\limits_D (18x^2y^2 + 32x^3y^3)\,dx\,dy = \quad D:\ x=1,\ y=\sqrt[3]{x},\ y=-x^2$$

$$= \int_0^1 dx \int_{-x^2}^{\sqrt[3]{x}} (18x^2y^2 + 32x^3y^3)\,dy = \int_0^1 (18x^2y^2 + 32x^3y^3)\Big|_{-x^2}^{\sqrt[3]{x}} dx =$$

$$= \int_0^1 \left(6x^2(\sqrt[3]{x})^3 + 8x^3(\sqrt[3]{x})^4 - 6x^2(-x^2)^3 - 8x^3(-x^2)^4\right) dx =$$

$$= \int_0^1 \left(6x^3 + 8x^{\frac{13}{3}} + 6x^8 - 8x^{11}\right) dx = \left(\frac{6x^4}{4} + \frac{3\cdot 8x^{\frac{16}{3}}}{16} + \frac{6x^9}{9} - \frac{8x^{12}}{12}\right)\Big|_0^1 =$$

$$= \frac{\cancel{6}^{\,3}}{\cancel{4}_{\,2}} + \frac{\cancel{24}^{\,3}}{\cancel{16}_{\,2}} - \frac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}} + \frac{6}{9} = \frac{3}{2} + \frac{3}{2} + \cancel{\frac{2}{3}} - \cancel{\frac{2}{3}} = 3$$

Отв.: 3.

$\boxed{3.6.}$

$$\iint_D y^2 \cos\frac{xy}{2}\,dx\,dy \qquad D:\ x=0,\ y=\sqrt{\frac{\pi}{2}}\ ;\ y=\frac{x}{2}$$

$$\int_0^{\sqrt{\frac{\pi}{2}}} dy \int_0^{2y} \left(y^2\cos\frac{xy}{2}\right)dx = \int_0^{\sqrt{\frac{\pi}{2}}} y^{\cancel{2}}\sin\frac{\cancel{2}y^2}{\cancel{2}}\cdot\frac{2}{\cancel{y}} = \int_0^{\sqrt{\frac{\pi}{2}}} 2y\sin y^2\,dy =$$

$$= -\cos y^2\Big|_0^{\sqrt{\frac{\pi}{2}}} = -\cancelto{0}{\cos\frac{\pi}{2}} + \cos 0^\circ = 1$$

Отв.: 1.

$\boxed{4.6.}$

$$\iiint_V y^2 z\cos(xyz)\,dx\,dy\,dz \qquad V: \begin{cases} x=1,\ y=2\pi,\ z=2 \\ x=0,\ y=1,\ z=0 \end{cases}$$

$$\int_0^{\pi} dy \int_0^{2} dz \int_0^{1} y^2 z\cos(xyz)\,dx =$$

$$\int_0^1 y^2 z\cos(xyz)\,dx = y\sin(xyz)\Big|_0^1 = y\sin(yz)$$

$$\int_0^2 y\sin(yz)\,dz = -\cos(yz)\Big|_0^2 = -\cos(2y)+\cos 0^\circ = -\cos(2y)+1$$

$$\int_0^{\pi}(1-\cos(2y))\,dy = \int_0^{\pi} dy - \int_0^{\pi}\cos(2y) = \pi - \left(\frac{1}{2}\sin(2y)\right)\Big|_0^{\pi} = \pi$$

Ответ: π.

$\boxed{5.6.}$

$$\iiint\limits_V (27+54y^3)\,dx\,dy\,dz \qquad V:\ y=x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0$$

$$\int_0^1 dx \int_0^x (27+54y^3)\,dy \int_0^{\sqrt{xy}} dz = \int_0^1 dx \int_0^x (27+54y^3)\sqrt{xy}\,dy =$$

$$= \int_0^1 dx \left(\sqrt{x}\int_0^x 27\sqrt{y}\,dy + \sqrt{x}\int_0^x 54y^3\sqrt{y}\,dy\right) = \int_0^1 \sqrt{x}\left(18x^{\frac{3}{2}}+12x^{\frac{9}{2}}\right)dx =$$

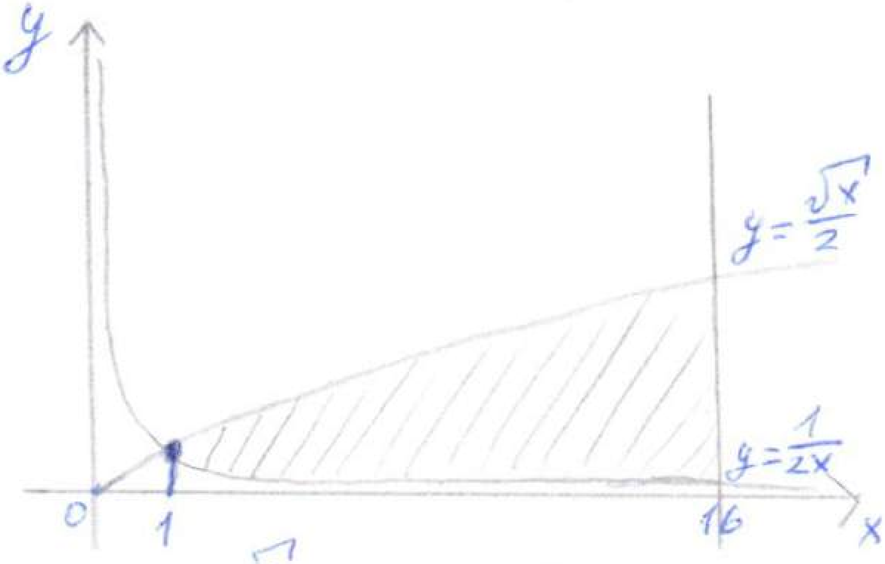
$$= \int_0^1 (18x^2+12x^5)\,dx = \int_0^1 18x^2dx + \int_0^1 12x^5dx = \frac{18x^3}{3}\Big|_0^1 + \frac{12x^6}{6}\Big|_0^1 =$$

$$= \frac{18}{3}+\frac{12}{6} = 6+2=8$$

Ответ: 8.

6.6

$y=\frac{\sqrt{x}}{2}$; $y=\frac{1}{2x}$; $x=16$



Пересечение $\frac{\sqrt{x}}{2}=\frac{1}{2x}$; $x\sqrt{x}=1$; $x=1$. Область интегрирования

$D:\begin{cases}1\le x\le 16\\ \frac{1}{2x}\le y\le\frac{\sqrt{x}}{2}\end{cases}$. Площадь $S=\iint dx\,dy=\int_1^{16}dx\int_{\frac{1}{2x}}^{\frac{\sqrt{x}}{2}}dy=\int_1^{16}\left(\frac{\sqrt{x}}{2}-\frac{1}{2x}\right)dx=$

$=\left(\frac{x^{\frac{3}{2}}}{3}-\frac{1}{2}\ln x\right)\Big|_1^{16}=\frac{(\sqrt{16})^3}{3}-\frac{1}{2}\ln 16-\left(\frac{1}{3}-\frac{1}{2}\ln 1\right)=\frac{64}{3}-\frac{1}{2}\ln 2^4-\frac{1}{3}=\frac{63}{3}-\frac{1}{2}4\ln 2=$

$=21-2\ln 2$

Ответ: $21-2\ln 2$

$\boxed{7.6.}$ $x^2-4x+y^2=0;\ x^2-8x+y^2=0;\ y=0,\ y=x$

$x^2-4x+4+y^2=4$

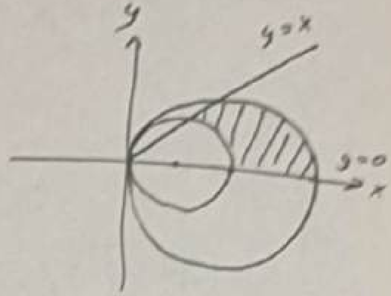
$(x-2)^2+y^2=2^2$

окр. в т. $(2;0)$, $R=2$

$x^2-8x+16+y^2=16$

$(x-4)^2+y^2=4^2$

окр. в т. $(4;0)$, $R=4$



$x=\rho\cos\theta;\ y=\rho\sin\theta$

$dx\,dy=\rho\,d\rho\,d\theta;\ x^2+y^2=\rho^2$

$$\left.\begin{aligned}&\rho^2-4\rho\cos\theta=0\\&\rho^2-8\rho\cos\theta=0\\&\rho\sin\theta=0\\&\rho\sin\theta=\rho\cos\theta\end{aligned}\right\}\Rightarrow\begin{aligned}&\rho=4\cos\theta\\&\rho=8\cos\theta\\&\theta=0\\&\operatorname{tg}\theta=1\Rightarrow\theta=\frac{\pi}{4}\end{aligned}$$

$$D:\begin{cases}0\le\theta\le\frac{\pi}{4}\\4\cos\theta\le\rho\le8\cos\theta\end{cases}$$

$$S=\iint_D dx\,dy=\int_0^{\frac{\pi}{4}}d\theta\int_{4\cos\theta}^{8\cos\theta}\rho\,d\rho=\int_0^{\frac{\pi}{4}}\left.\frac{\rho^2}{2}\right|_{4\cos\theta}^{8\cos\theta}d\theta=\frac{1}{2}\int_0^{\frac{\pi}{4}}(64\cos^2\theta-16\cos^2\theta)\,d\theta=$$

$$=24\int_0^{\frac{\pi}{4}}\cos^2\theta\,d\theta=24\int_0^{\frac{\pi}{4}}\frac{1+\cos2\theta}{2}\,d\theta=12\int_0^{\frac{\pi}{4}}(1+\cos2\theta)\,d\theta=\cancel{12(\sin\tfrac{\pi}{4}-\sin0)}$$

$$=12\left(\theta+\frac{\sin2\theta}{2}\right)\Big|_0^{\frac{\pi}{4}}=12\left(\frac{\pi}{4}+\frac{\sin\frac{\pi}{2}}{2}-\cancel{\frac{\sin0}{2}}\right)=\cancel{12\tfrac{1}{2}}=12\cdot\frac{\pi}{4}+12\cdot\frac{1}{2}=$$

$=3\pi+6$ Ответ: $3\pi+6$.

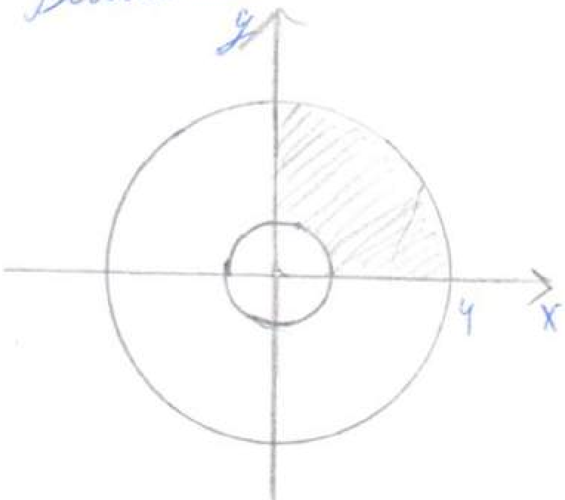
(8.6) Пластинка D задана ограничивающими её плоскостями, m – поверхностная плотность. Найти массу пластинки.

$D: x^2+y^2=1, \quad x^2+y^2=16,$

$x=0, y=0 \quad (x\geq 0, y\geq 0);$

$\mu=\dfrac{(x+y)}{(x^2+y^2)}$

Решение



$1=\rho\cos\theta \qquad y=\rho\sin\theta$

$0\leq\theta\leq\frac{\pi}{2}$

$1\leq\rho\leq 4$

$\mu=\dfrac{x+y}{x^2+y^2}=\dfrac{\cos\theta+\sin\theta}{\rho}$

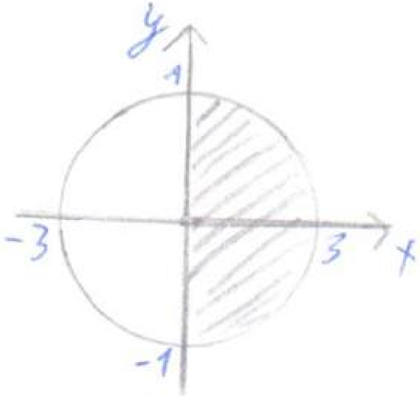
$M=\int_0^{\frac{\pi}{2}} d\theta\int_1^4\left(\dfrac{\cos\theta+\sin\theta}{\rho}\right)\cdot\rho\, d\rho=$

$=3\left(\int_0^{\frac{\pi}{2}}\cos\theta\, d\theta+\int_0^{\frac{\pi}{2}}\sin\theta\, d\theta\right)=3\left(\sin\theta\Big|_0^{\frac{\pi}{2}}-\cos\theta\Big|_0^{\frac{\pi}{2}}\right)=3\cdot 2=6$

Ответ: 6

(9.6) Пластинка D задана неравенствами, μ - поверхностная плотность. Найти массу пластинки.

$$D: \begin{cases} \frac{x^2}{9} + y^2 \leq 1 \\ x \geq 0 \end{cases} \qquad \mu = 7xy^6$$



Решение: обобщ. полярн. координаты

$$\begin{cases} x = 3\rho\cos\varphi \\ y = \rho\sin\varphi \end{cases}$$

$$m = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^1 3\rho\, 7(3\rho\cos\varphi)(\rho\sin\varphi)^6 d\rho =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^1 3\rho\, 21\rho^7 \cos\varphi \sin^6\varphi\, d\rho = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \cdot \int_0^1 3\rho \cdot 21\rho^7 \cos\varphi \sin^8\varphi\, d\rho =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \frac{21\rho^9}{9}\Big|_0^1 \cos\varphi \sin^8\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d(\sin\varphi) \sin^8\varphi \cdot \frac{21}{8} = \frac{\cancel{8}\cdot\cancel{21}}{3\cdot\cancel{8}} \cdot \frac{\sin^7\varphi}{\cancel{7}}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 3\frac{1}{3}(1+1) =$$

$$= \frac{8\cdot 2}{8} = 2$$

Ответ: 2

$\boxed{10.6.}$

$V - ?$

$x = \frac{5\sqrt{y}}{2}$, $x = \frac{5y}{6}$, $z = 0$, $z = \frac{5(3+\sqrt{y})}{6}$

$$V = \iiint dx\,dy\,dz = \int_0^9 dy \int_{\frac{5y}{6}}^{\frac{5\sqrt{y}}{2}} dx \int_0^{\frac{5(3+\sqrt{y})}{6}} dz = \int_0^9 dy \int_{\frac{5}{6}y}^{\frac{5}{2}\sqrt{y}} \frac{5}{6}(3+\sqrt{y})\,dx =$$

$$= \int_0^9 \frac{5}{6}(3+\sqrt{y})\left(\frac{5\sqrt{y}}{2} - \frac{5y}{6}\right)dy = \int_0^9 \left(\frac{5}{2} + \frac{5\sqrt{y}}{6}\right)\left(\frac{5\sqrt{y}}{2} - \frac{5y}{6}\right)dy =$$

$$= \int_0^9 \frac{25}{4}\sqrt{y}\,dy - \cancel{\int_0^9 \frac{25}{12}y\,dy} + \cancel{\int_0^9 \frac{25}{12}y\,dy} - \int_0^9 \frac{25}{36}y^{\frac{3}{2}} = \frac{25}{6}y^{\frac{3}{2}}\Big|_0^9 - \frac{5}{18}y^{\frac{5}{2}}\Big|_0^9 =$$

$$= \frac{25}{\cancel{6}_{2}} \cdot \cancel{9}^{3} \cdot 3 - \frac{5}{\cancel{18}_{\cancel{6}_{2}}} \cdot \cancel{81}^{27} \cdot \cancel{3} = \frac{9}{2} \cdot 25 - \frac{5}{2} \cdot 27 = \frac{225 - 135}{2} = 45$$

Ответ: 45.

$\boxed{11.6.}$

$$x^2+y^2=6\sqrt{2}\,y \qquad z=0 \quad (z\geq 0)$$

$$z=x^2+y^2-36$$

$$x^2+(y-3\sqrt{2})^2=18$$

$$x=\rho\cos\theta$$

$$y=\rho\sin\theta$$

$$z=x^2+y^2-36$$

$$z=\rho^2-36$$

$z\geq 0$

$$\rho^2-36\geq 0 \;\Rightarrow\; \rho\leq -6\;;\;\rho\geq 6$$

$$x^2+y^2=6\sqrt{2}\,y$$

$$\rho^{\cancel{2}}=6\sqrt{2}\,\cancel{\rho}\sin\theta \;:\cancel{\rho}$$

$$\rho=6\sqrt{2}\,\cancel{\rho}\sin\theta$$

$$6\sqrt{2}\sin\theta\geq 6$$

$$\sin\theta\geq\frac{\sqrt{2}}{2} \;\Rightarrow\; \frac{\pi}{4}\leq\theta\leq\frac{3\pi}{4}$$

$$V=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}d\theta\int_{6}^{6\sqrt{2}\sin\theta}\rho\,d\rho\int_{0}^{\rho^2-36}dz=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}d\theta\int_{0}^{6\sqrt{2}\sin\theta}\rho(\rho^2-36)\,d\rho=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left(\frac{1}{4}\rho^4-18\rho^2\right)\Big|_{6}^{6\sqrt{2}\sin\theta}d\theta=$$

$$= 1296\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^4\theta\, d\theta + 324\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\theta - 1296\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\theta\, d\theta =$$

$$= 162 - 324\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\theta\, d\theta + 324\cdot\frac{\pi}{2} = -162\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1-\cos 2\theta)\, d\theta + 162\pi + 162 =$$

$$= 162 - 162\left(\theta\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} - \frac{1}{2}\sin 2\theta\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\right) + 162\pi = -81\pi + 162\pi = 81\pi$$

Ответ: 81π.

$\boxed{12.6.}$ $y = 5x^2 - 1$, $y = -3x^2 + 1$, $z = -2 + \sqrt{3x^2+y^2}$

$z = -5 + \sqrt{3x^2+y^2}$

$5x^2 - 1 = -3x^2 + 1$

$8x^2 = 2$

$x = \pm\frac{1}{2}$

$$D: \begin{cases} -\frac{1}{2} \le x \le \frac{1}{2} \\ 5x^2 - 1 \le y \le -3x^2 + 1 \\ -5 + \sqrt{3x^2+y^2} \le z \le -2 + \sqrt{3x^2+y^2} \end{cases}$$

$$V = \iiint\limits_D dx\,dy\,dz = \int\limits_{-\frac{1}{2}}^{\frac{1}{2}} dx \int\limits_{5x^2-1}^{-3x^2+1} dy \int\limits_{-5+\sqrt{3x^2+y^2}}^{-2+\sqrt{3x^2+y^2}} dz = \int\limits_{-\frac{1}{2}}^{\frac{1}{2}} dx \int\limits_{5x^2-1}^{-3x^2+1} (-2 + \cancel{\sqrt{3x^2+y^2}} + 5 - \cancel{\sqrt{3x^2+y^2}})\, dy =$$

$$= 3\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} dx \int\limits_{5x^2-1}^{-3x^2+1} dy = 3\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} (-3x^2 \cancel{+1}^{+1} - 5x^2 + 1)\, dx = 3\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} (2 - 8x^2)\, dx =$$

$$= 6\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} (1 - 4x^2)\, dx = 6\left(x - \frac{4x^3}{3}\right)\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = 6\left(\frac{1}{2} - \frac{4\cdot 1}{8\cdot 3} + \frac{1}{2} - \frac{4}{8\cdot 3}\right) =$$

$$= 6\left(1-\frac{8}{24}\right) = 6\cdot\frac{2}{3} = 4$$

Ответ: 4.

$\boxed{13.6.}$ $z = 3\sqrt{x^2+y^2}$; $z = 10 - x^2 - y^2$

$$\sqrt{x^2+y^2} = \frac{z}{3} \qquad z = 10 - \frac{z^2}{9} \;\Big|\Rightarrow z^2+9z-90=0$$

$$z_1 = 6, \quad z_2 = -15$$

$$V: \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le \rho \le 2 \\ 3\rho \le z \le 10-\rho^2 \end{cases} \qquad x^2+y^2 = 10-6 = 4$$

$$dx\,dy\,dz = \rho\,d\rho\,d\theta\,dz$$

$$V = \iiint\limits_V dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^2 \rho\,d\rho \int_{3\rho}^{10-\rho^2} dz = \int_0^{2\pi} d\theta \int_0^{2\pi} (10-\rho^2-3\rho)\rho\,d\rho =$$

$$= \int_0^{2\pi} d\theta \int_0^2 (10\rho - \rho^3 - 3\rho^2)\,d\rho = \int_0^{2\pi} \left(\frac{10\rho^2}{2} - \frac{\rho^4}{4} - \frac{3\rho^3}{3}\right)\Bigg|_0^2 d\theta =$$

$$= \int_0^{2\pi} \left(\frac{10\cdot\cancel{4}^{2}}{\cancel{2}} - \frac{\cancel{16}^{4}}{\cancel{4}} - \frac{\cancel{3}\cdot 8}{\cancel{3}}\right) d\theta = \int_0^{2\pi} 8\,d\theta = 8\theta\Big|_0^{2\pi} = 16\pi$$

Ответ: 16π

$\boxed{14.6.}$

$$z = 28[(x+1)^2+y^2]+3 \qquad V - ?$$

$$z = 56x+59$$

$$28((x+1)^2+y^2)+3 = 56x+59$$

$$28x^2+\cancel{56x}+28+28y^2+3 = \cancel{56x}+59$$

$$28x^2+28y^2 = 28$$

$$x^2+y^2 = 1$$

$$x = \rho\cos\theta \qquad y = \rho\sin\theta \qquad z = z$$

$$V = \int_0^{2\pi} d\theta \int_0^1 \rho\, d\rho \int_{28((\rho\cos\theta+1)^2+\rho\sin\theta)+3}^{56\rho\cos\theta+59} dz = 28\int_0^{2\pi} d\theta \int_0^1 \rho(1-\rho^2)\, d\rho =$$

$$= 28\int_0^{2\pi} d\theta \int_0^1 (\rho-\rho^3)\, d\rho = 28\int_0^{2\pi} \left(\frac{\rho^2}{2}-\frac{\rho^4}{4}\right)\Big|_0^1 d\theta = 28\cdot 2\pi\cdot\frac{1}{4} = 14\pi$$

Ответ: 14π.

$\boxed{15.6.}$ $25 \le x^2+y^2+z^2 \le 100, \quad z \geqslant -\sqrt{\frac{x^2+y^2}{99}}, \quad \cancel{0 \le y \le -\frac{x}{\sqrt{3}}}$

$\sqrt{3}x \le y \le -\sqrt{3}x$

$$\left.\begin{aligned} x &= \rho \sin\theta \cos\varphi \\ y &= \rho \sin\theta \sin\varphi \\ z &= \rho\cos\theta \end{aligned}\right| \Rightarrow \quad 25 \le \rho^2\sin^2\theta\cos^2\varphi + \rho^2\sin^2\theta\sin^2\varphi + \rho^2\cos^2\theta \le 100$$

$$5 \le \rho \le 10$$

$$\rho\cos\theta \le -\sqrt{\frac{\rho^2\sin^2\theta}{99}}$$

$$V = 2\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} d\varphi \int_0^{\operatorname{arcctg}(-\frac{1}{3\sqrt{\pi}})} d\theta \int_5^{10} \rho^2 \sin\theta \, d\rho = \frac{2}{3}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} d\varphi \int_0^{\operatorname{arcctg}(-\frac{1}{3\sqrt{\pi}})} d\theta \sin\theta (1000-125) =$$

$$= \frac{1750}{3}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} d\varphi \int \sin\theta \Big|_0^{\operatorname{arcctg}\frac{1}{3\sqrt{\pi}}} = \left(\frac{2\pi}{3} - \frac{\pi}{3}\right) \cancel{\frac{1750\pi}{3}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} d\varphi \int \sin\theta \Big|_0^{\operatorname{arcctg}\frac{1}{3\pi}}} =$$

$$= \left(\frac{2\pi}{3} - \frac{\pi}{3}\right) \cdot \frac{1750\pi}{3}\left(\cos \operatorname{arcctg}\frac{1}{3\sqrt{\pi}} - \cos 0\right) =$$

$$= \frac{1750\pi}{9}\left(\sqrt{\frac{\operatorname{ctg}^2 \operatorname{arcctg}(-\frac{1}{3\sqrt{\pi}})}{1+\operatorname{ctg}^2\operatorname{arcctg}(-\frac{1}{3\sqrt{\pi}})}} - \cos 0\right) = \frac{1750\pi}{9} \cdot \frac{9}{10} = 175\pi$$

Ответ: 175π

$\boxed{16.6.}$

$$36(x^2+y^2)=z^2, \quad x^2+y^2=1$$

$$x=0, \quad z=0 \; (x\geqslant 0, \; z\geqslant 0)$$

$$\mu=\frac{5}{6}(x^2+y^2)$$

$$x=\rho\cos\theta \qquad y=\rho\sin\theta \qquad z=z$$

$$M=2\cdot\frac{5}{6}\int_0^{\frac{\pi}{2}}d\theta\int_0^1\rho^4\,d\rho\int_0^6 dz=\frac{5}{3}\int_0^{\frac{\pi}{2}}d\theta\int_0^1 6\rho^4\,d\rho=$$

$$=\overset{2}{\cancel{10}}\cdot\frac{1}{\cancel{5}}\int_0^{\frac{\pi}{2}}\rho^5\Big|_0^1\,d\theta=2\cdot\frac{\pi}{2}=\pi$$

Ответ: π

Капитонов М.
А-02-23

$\boxed{1.6.}$

$u = x\sqrt{y} - yz^2$, $S: x^2+y^2=4z$, $M=(2,1,-1)$

$F = x^2+y^2-4z=0$, тогда $F'_x = 2x$; $F'_y = 2y$; $F'_z = -4$

$\vec{N}\{2x; 2y; -4\}$

В точке М: $\vec{N}\{4; 2; -4\}$, тогда

Возьмём вектор $\vec{N_1} = -\vec{N} = \{-4; -2; 4\}$

$|\vec{N_1}| = \sqrt{16+4+16} = 6$

Единичный вектор $\vec{n} = \frac{\vec{N_1}}{|\vec{N_1}|} = \{-\frac{4}{6}; -\frac{2}{6}; +\frac{4}{6}\} = \{-\frac{2}{3}; -\frac{1}{3}; \frac{2}{3}\}$

$\frac{\partial u}{\partial x} = \sqrt{y}$; $\frac{\partial u}{\partial y} = \frac{x}{2\sqrt{y}} - z^2$; $\frac{\partial u}{\partial z} = -2yz$

в т. М: $\frac{\partial u}{\partial x} = 1$; $\frac{\partial u}{\partial y} = 0$; $\frac{\partial u}{\partial z} = 2$

Производные по напр.

$\frac{\partial u}{\partial N_1} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma = 1\cdot(-\frac{2}{3}) + \cancel{0\cdot(-\frac{1}{3})} + 2\cdot\frac{2}{3} =$

$= \frac{2}{3}$

Отв.: $\frac{2}{3}$.

$\boxed{2.6.}$ $\quad v = 3\sqrt{2}\,x^2 - \dfrac{y^2}{\sqrt{2}} + 3\sqrt{2}\,z^2 \; ; \; u = \dfrac{z^2}{xy^2} \; ; \; M\left(\dfrac{1}{3}; 2; \sqrt{\dfrac{2}{3}}\right)$

$$\frac{\partial v}{\partial x}\Big|_M = 6\sqrt{2}\,x\Big|_M = 2\sqrt{2}$$

$$\frac{\partial u}{\partial x}\Big|_M = -\frac{z^2}{x^2y^2}\Big|_M = -\frac{2\cdot 9}{3\cdot 4\cdot 2} = -\frac{3}{2}$$

$$\frac{\partial v}{\partial y}\Big|_M = -\sqrt{2}\,y\Big|_M = -2\sqrt{2}$$

$$\frac{\partial u}{\partial y}\Big|_M = -\frac{2z^2}{xy^3}\Big|_M = -\frac{2\cdot 2\cdot 3}{3\cdot 8\cdot 2} = -\frac{1}{2}$$

$$\frac{\partial v}{\partial z}\Big|_M = 6\sqrt{2}\,z = \frac{6\sqrt{2}\cdot\sqrt{2}}{\sqrt{3}} = \frac{12}{\sqrt{3}}$$

$$\frac{\partial u}{\partial z}\Big|_M = \frac{2z}{xy^2} = \frac{2\sqrt{2}\cdot 3\sqrt{3}}{\sqrt{3}\cdot 4\sqrt{2}} = \sqrt{\frac{3}{2}}$$

$$\alpha = \arccos \frac{2\sqrt{2}\left(-\frac{3}{2}\right) + 2\sqrt{2}\left(\frac{1}{2}\right) - \frac{2\cdot 6\sqrt{2}\cdot\sqrt{3}\cdot 12}{\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{3}}}{\sqrt{8+8+48}\cdot\sqrt{\frac{9}{4}+\frac{1}{4}+\frac{18}{12}}} = \cancel{\arccos\frac{-3\sqrt{2}+\sqrt{2}-6\sqrt{3}}{2\cdot 2\sqrt{22}}} =$$

$$= \cancel{\arccos\frac{-2\sqrt{2}-6\sqrt{3}}{4\sqrt{22}}} \quad \arccos\frac{-3\sqrt{2}+\sqrt{2}-6\sqrt{2}}{\sqrt{64}\cdot\sqrt{4}} = \arccos\frac{-8\sqrt{2}}{8\cdot 2} =$$

$$= \frac{3\pi}{4}$$

Отв.: $\dfrac{3\pi}{4}$.

$\boxed{3.6.}$

$\vec{a} = 3x\vec{i} + 6z\vec{k}$

$a = a_x i + a_y j + a_z k$ $\qquad \frac{dx}{a_x} = \frac{dy}{a_y} = \frac{dz}{a_z}$

$$\left.\begin{aligned} a_x &= 3x \\ a_y &= 0 \\ a_z &= 6z \end{aligned}\right\} \Rightarrow \frac{dx}{3x} = \frac{dy}{0} = \frac{dz}{6z} \Rightarrow \begin{cases} \frac{\partial x}{x} = \frac{\partial z}{2z} \\ \partial y = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y = const \\ \int \frac{dx}{x} = \frac{1}{2}\int \frac{dz}{z} \end{cases} \Rightarrow \begin{cases} y = c \\ \ln x = \frac{1}{2}\ln z + c_1 \end{cases}$$

Ответ: $y = c$; $z = cx^2$

$\boxed{4.6.}$ $a = (x-y)i + (x+y)j + z^2 k$; $S: x^2 + y^2 = 1$; $z = 0$; $z = 2$

$F = x^2 + y^2 - 1$

$\frac{\partial F}{\partial x} = 2x$

$\vec{N}_i(2x, 2y, 0)$ $\qquad \vec{N}(x, y, 0)$ $\qquad |\vec{N}| = \sqrt{\underbrace{x^2+y^2}_{1}+0} = 1$

$\frac{\partial F}{\partial y} = 2y$

$\frac{\partial F}{\partial z} = 0$

$$\vec{n} = \frac{\vec{N}}{|\vec{N}|} = \left(\frac{x}{1}; \frac{y}{1}; 0\right) \Rightarrow \vec{n} = (x, y, 0)$$

$$(\vec{a}, \vec{n}) = (x-y)x + (x+y)y + z^2 \cdot 0 = x^2 \cancel{- xy} \cancel{+ xy} + y^2 = 1$$

$$\Pi = \iint_S (\vec{a}, \vec{n})\, d\sigma = \iint_S d\sigma = |S| = 2\pi R H = 2\pi \cdot 1 \cdot 2 = 4\pi$$

Ответ: 4π.

$\boxed{5.6.}$ $a = xi + yj + zk$; $p:\ \frac{x}{2} + y + z = 1$

$\vec{N} = (\frac{1}{2}; 1; 1)$, $|\vec{N}| = \sqrt{\frac{1}{4} + 1 + 1} = \frac{3}{2}$

$\vec{n} = \frac{N}{|N|} = (\frac{1}{3}; \frac{2}{3}; \frac{2}{3})$

$(\vec{a}, \vec{n}) = \frac{x}{3} + \frac{2y}{3} + \frac{2z}{3} = \frac{x + 2y + 2z}{3}$

$$\Pi = \iint_\sigma (\vec{a}, \vec{n})\, d\sigma = \frac{1}{3} \iint_\sigma (x + 2y + 2z)\, d\sigma$$

$z = 1 - \frac{x}{2} - y$

$z'_x = -\frac{1}{2}$; $z'_y = -1$; тогда

$$\sqrt{1 + (z'_x)^2 + (z'_y)^2} = \sqrt{1 + \frac{1}{4} + 1} = \frac{3}{2}$$

$$\Pi = \frac{1}{3} \iint_D (\cancel{x} + \cancel{2y} + 2 \cancel{- x} \cancel{- 2y}) \frac{3}{2}\, dx\, dy = \frac{1}{2} \iint_D dx\, dy \ (2)$$

$D: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 1 - \frac{x}{2} \end{cases}$

$$\Pi = \int_0^2 dx \int_0^{1-\frac{x}{2}} dy = \int_0^2 (1 - \frac{x}{2})\, dx = (x - \frac{x^2}{4})\Big|_0^2 = 2 - 1 = 1$$

Ответ: 1.

$\boxed{6.6.}$ $a = i + 5yj + 11\pi z k$; $p: x + y + \frac{z}{3} = 1$

$z = 3 - 3x - 3y$

$z'_x = -3$; $z'_y = -3$

$\vec{N} = (1, 1, \frac{1}{3})$

$|\vec{N}| = \sqrt{1 + 1 + \frac{1}{9}} = \frac{\sqrt{19}}{3}$

$n = \frac{N}{|N|} = \left(\frac{3}{\sqrt{19}}; \frac{3}{\sqrt{19}}; \frac{1}{\sqrt{19}}\right)$

$(\vec{a}, \vec{n}) = \frac{3}{\sqrt{19}} + \frac{5y \cdot 3}{\sqrt{19}} + \frac{11\pi z}{\sqrt{19}} = \frac{3 + 15y + 11\pi z}{\sqrt{19}}$

$\Pi = \iint\limits_{\sigma} \frac{3 + 15y + 11\pi z}{\sqrt{19}} d\sigma = \frac{1}{\cancel{\sqrt{19}}} \iint\limits_{D} (3 + 15y + 11\pi(3 - 3x - 3y))\cancel{\sqrt{19}}\, dx\, dy =$

$= \int\limits_0^1 dx \int\limits_0^{1-x} (3 + 15y + 33\pi - 33\pi x - 33\pi y)\, dy = \int\limits_0^1 (3 + 15 \cdot \frac{y^2}{2} + 33\pi y - 33\pi x y -$

$- \frac{33\pi}{2} y^2)\Big|_0^{1-x} = \int\limits_0^1 (3 - 3x + \frac{15}{2}(1 - 2x + x^2) - 33\pi + 33\pi x - 33\pi x + 33\pi x^2 - \frac{33\pi}{2} \cdot$

$\cdot (1 - 2x + x^2))dx = \int\limits_0^1 (\frac{21}{2} x - \frac{33\pi}{2} x - 9x^2 + \frac{33\pi}{2} x^2 + \frac{5}{2} x^3 + \frac{11\pi}{x} x^2)\Big|_0^1 =$

$= \frac{21}{2} - \cancel{\frac{33\pi}{2}} - 9 + \cancel{\frac{33\pi}{2}} + \frac{5}{2} + \frac{11\pi}{2} = 4 + \frac{11\pi}{2}$

Ответ: $4 + \frac{11\pi}{2}$.

$\boxed{7.6.}$ $\bar{a} = (6x - \cos y)\bar{i} - (e^x + z)\bar{j} - (2y + 3z)\bar{k}$

$S: x^2 + y^2 = z^2 \; ; \; z = 1; \; z = 2$

$a_x = 6x - \cos y$

$a_y = -(e^x + z)$ $\qquad \frac{\partial a_x}{\partial x} = 6 \; ; \; \frac{\partial a_y}{\partial y} = 0; \; \frac{\partial a_z}{\partial z} = -3$

$a_z = -(2y + 3z)$

$\operatorname{div} \bar{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = 6 + 0 - 3 = 3$

$\Pi = \iint_\sigma (\bar{a}, \bar{n}) \, d\sigma = \iiint_V \operatorname{div} \bar{a} \, dx\,dy\,dz = \iiint_V 3 \, dx\,dy\,dz = 3 \iiint_V dx\,dy\,dz = 3V$ ⊜

$V_k = \frac{1}{3} \pi R^2 \cdot L \Rightarrow V = V_{б.кон} - V_{м.кон} = \frac{1}{3}\pi \cdot 2^2 \cdot 2 - \frac{1}{3}\pi \cdot 1^2 \cdot 1 =$

$= \frac{1}{3}\pi (8 - 1) = \frac{7}{3}\pi$

⊜ $\cancel{3} \cdot \frac{7}{\cancel{3}}\pi = 7\pi$

Ответ: 7π.

$\boxed{8.6.}$ $a = xi - (x+2y)j + yk$

$$S: \begin{cases} x^2+y^2=1, \quad z=0 \\ x+2y+3z=6 \end{cases}$$

$$\Pi = \iiint_V div\,\bar{a}\, dx\,dy\,dz$$

$\partial a_x = x \quad ; \quad \frac{\partial a_x}{\partial x} = 1$

$\partial a_y = -(x+2y) \; ; \quad \frac{\partial a_y}{\partial y} = -2$

$\partial a_z = y \quad ; \quad \frac{\partial a_z}{\partial z} = 0$

$$div\,\bar{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = 1-2+0 = -1$$

$$\Pi = -\iiint_V dx\,dy\,dz = \begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} = -\int_0^{2\pi} d\varphi \int_0^1 r\,dr \int_0^{2-\frac{r\cos\varphi}{3}-\frac{2r\sin\varphi}{3}} dz =$$

$$= -\int_0^{2\pi} d\varphi \int_0^1 r\left(2 - \frac{r\cos\varphi}{3} - \frac{2r\sin\varphi}{3}\right) dr = -\int_0^{2\pi} d\varphi \int_0^1 \left(2r - \frac{r^2\cos\varphi}{3} - \frac{2r^2\sin\varphi}{3}\right) dr =$$

$$= -\int_0^{2\pi} d\varphi \left(r^2 - \frac{r^3\cos\varphi}{9} - \frac{2r^3\sin\varphi}{9}\right)\Big|_0^1 = -\int_0^{2\pi}\left(1 - \frac{\cos\varphi}{9} - \frac{2\sin\varphi}{9}\right) d\varphi =$$

$$= -\left(\varphi - \frac{\sin\varphi}{9} + \frac{2}{9}\cos\varphi\right)\Big|_0^{2\pi} = -\left(2\pi - \cancel{\frac{\sin 2\pi}{9}}^{\,0} + \frac{2}{9}\underbrace{\cos 2\pi}_{1}\right) +$$

$$+ \left(\cancel{0} - \cancel{\frac{\sin 0^\circ}{9}} + \frac{2}{9}\underbrace{\cos 0^\circ}_{1}\right) = -2\pi - \frac{2}{9} + \frac{2}{9} = -2\pi \quad \text{Ответ: } -2\pi.$$

$\boxed{9.6.}$ $a = 3xz\,i - 2x\,j + y\,k$ $\qquad S: \begin{cases} x+y+z=2, \; x=1 \\ x=0, \; y=0, \; z=0 \end{cases}$

$$\frac{\partial a_x}{\partial x} = 3z \; ; \quad \frac{\partial a_y}{\partial y} = 0 \; ; \quad \frac{\partial a_z}{\partial z} = 0$$

$$\operatorname{div} a = 3z$$

$$\Pi = 3 \iiint_V z\,dx\,dy\,dz = 3\int_0^1 dy \int_0^1 dx \int_0^{2-x-y} z\,dz + 3\int_1^2 dy \int_0^{2-y} dx \int_0^{2-x-y} z\,dz =$$

$$= 3\int_0^1 dy \int_0^{1} \frac{(2-x-y)^2}{2}\,dx + 3\int_1^2 dy \int_0^{2-y} \frac{(2-x-y)^2}{2}\,dx = \frac{3}{2}\int_0^1 dy \int_0^1 (2-x-y)^2 dx +$$

$$+ \frac{3}{2}\int_1^2 dy \int_0^{2-y} (2-x-y)^2 dx = \cancel{\frac{3}{2}\int_0^1 \frac{(1-y)^3}{-3}\,dy + \frac{3}{2}\int_1^2 \frac{(2-2+y-y)^3}{-3}\,dy} =$$

$$= \cancel{\frac{3}{2}(1-y)^4 \cdot \frac{1}{12} \Big|_0^1 = \frac{3}{2}(1-1)^4 \cdot \frac{1}{12} - \frac{3}{2}(1-0)^4 \cdot \frac{1}{12} = -\frac{3}{2} \cdot \frac{1}{12} = -\frac{1}{8}}$$

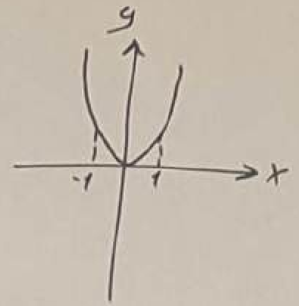
$$= \int_0^1 \left(-\frac{5}{2} + \frac{3}{2}y + \frac{3}{2}(2-y)^2\right) dy - \int_1^2 \left(4 - 6y + 3y^2 - \frac{1}{2}y^3\right) dy =$$

$$= \left(-\frac{5}{2}y + \frac{3}{2}\cdot\frac{y^2}{2} - \frac{1}{2}(2-y)^3\right)\Big|_0^1 - \left(4y - 6\frac{y^2}{2} + y^3 - \frac{1}{2}\cdot\frac{y^4}{4}\right)\Big|_1^2 =$$

$$= -\frac{5}{2} + \frac{3}{4} - \frac{1}{2} + \frac{1}{2}\cdot 8 - \left(4\cdot 2 - 3\cdot 4 + 8 - \frac{16}{8}\right) + \left(4 - 3 + 1 - \frac{1}{8}\right) =$$

$$= -\frac{5}{2} + \frac{3}{4} - \frac{1}{2} + \cancel{4} - \cancel{8} + 12 - 8 + \cancel{2} + \cancel{4} - \cancel{3} + \cancel{1} - \frac{1}{8} = 4 - \frac{5}{2} - \frac{1}{2} + \frac{3}{4} - \frac{1}{8} = \frac{7}{4} - \frac{1}{8} = \frac{13}{8}$$

Ответ: $\frac{13}{8}$



$\boxed{10.6.}$ $F=(x+y)i+(x-y)j$; $L: y=x^2$; $M(-1,1)$; $N(1,1)$

$$A=\int_L (F_x dx + F_y dy) = \int_{-1}^{1} (x+x^2+(x-x^2)\cdot 2x)\,dx = \int_{-1}^{1}(x+x^2\cdot 2x^2-2x^3)\,dx =$$

$$= \int_{-1}^{1}(x+3x^2-2x^3)\,dx = \left(\frac{x^2}{2}+x^3-\frac{x^4}{2}\right)\Big|_{-1}^{1} = \left(\frac{1}{2}+1-\frac{1}{2}\right)-\left(\frac{1}{2}-1-\frac{1}{2}\right) =$$

$= 2$

Ответ: 2.

$\boxed{11.6.}$ $a=2yi-3xj+xk$ $\quad \Gamma: \begin{cases} x=2\cos t, \ y=2\sin t \\ z=2-2\cos t-2\sin t \end{cases}$

$dx=-2\sin t\cdot dt$; $dy=2\cos t\,dt$; $dz=(2\sin t-2\cos t)\,dt$

$a_x=4\sin t$; $a_y=-6\cos t$; $a_z=2\cos t$

$$\int a_x dx+a_y dy+a_z dz = \int_0^{2\pi}(-8\sin^2 t-12\cos^2 t+4\sin t\cos t-4\cos^2 t)\,dt =$$

$$= -4\int_0^{2\pi}(1-\cos 2t)\,dt - 8\int_0^{2\pi}(1+\cos 2t)\,dt - 4\int_0^{2\pi}\sin t\cos t\,dt =$$

$= -24\pi$

Ответ: -24π.

$\boxed{12.6.}$ $a = yi - xj + z^2k$

$\Gamma: \begin{cases} z = 3(x^2+y^2)+1 \\ z = 4 \end{cases}$

$Ц = \oint_\Gamma \bar{a}\cdot\bar{n}\cdot dS = \iint_\sigma \bar{n}\cdot rot\,\bar{a}\,d\sigma$

$rot\,\bar{a} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} = \bar{i}\left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right) + \bar{j}\left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right) + \bar{k}\left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)$

$= (0-0)i + (0-0)j + (-1-1)k = -2k$

$\vec{n} = (-z_x'; -z_y';$

$\bar{n}\cdot rot\,\bar{a} = -2$

$R = 4$; $4 = 3(x^2+y^2)+1$

$x^2+y^2 = 1$

$Ц = \iint_\sigma -2d\sigma = -2\pi R^2 = -2\pi$

$\Downarrow$

$|Ц| = 2\pi$

Ответ: 2π.