Типовые расчеты, вариант 24.

$\boxed{1.24}$ $\int \sin 2x \left(2-\sqrt[3]{\cos 2x}\right)dx=\int 2\sin 2x\,dx-\int \sin 2x\sqrt[3]{\cos 2x}\,dx=\left\{d2x=2dx\right\}=$

$=\frac{1}{2}\cdot 2\int \sin 2x\,d2x-\frac{1}{2}\int \sin 2x\sqrt[3]{\cos 2x}\,d2x=-\cos 2x-\frac{1}{2}\int \sin 2x\sqrt[3]{\cos 2x}\,d2x=$

$=\left\{d\cos 2x=-\sin 2x\,d2x\right\}=-\cos 2x+\frac{1}{2}\int \frac{\sin 2x\sqrt[3]{\cos 2x}\,d\cos 2x}{\sin 2x}=$

$=-\cos 2x+\frac{1}{2}\int \sqrt[3]{\cos 2x}\,d\cos 2x=-\cos 2x+\frac{1}{2}\frac{(\cos 2x)^{\frac{1}{3}+1}}{\frac{1}{3}+1}+C=$

$=-\cos 2x+\frac{3\sqrt[3]{\cos^4 2x}}{8}+C$

$\boxed{2.24}$ $\int e^{-x}(x+2)dx=\left\{\begin{matrix}u=x+2\\ du=1\\ dv=e^{-x}\\ v=-e^{-x}\end{matrix}\right\}=-e^{-x}(x+2)-\int -e^{-x}\cdot 1\,dx=$

$=-e^{-x}(x+2)+\int e^{-x}dx=-e^{-x}(x+2)-e^{-x}=-e^{-x}(x+3)+C$

$\boxed{3.24}$ $\int \frac{2x^3-12x+63}{x^2-4x+8}dx=\int \frac{2x^3dx}{x^2-4x+8}-12\int \frac{x\,dx}{x^2-4x+8}+63\int \frac{dx}{x^2-4x+8}=$

$=2\int \left(x+4+\frac{8x-32}{x^2-4x+8}\right)dx-12\int \frac{x\,dx}{x^2-4x+8}+63\int \frac{dx}{x^2-4x+8}=$

$=2\int x\,dx+8\int dx+16\int \frac{x\,dx}{x^2-4x+8}-64\int \frac{dx}{x^2-4x+8}-12\int \frac{x\,dx}{x^2-4x+8}+63\int \frac{dx}{x^2-4x+8}=$

$=2\int x\,dx+8\int dx+4\int \frac{x\,dx}{x^2-4x+8}-\int \frac{dx}{x^2-4x+8}=$

$=2\cdot\frac{x^2}{2}+8x+4\int \frac{x\,dx}{x^2-4x+8}-\int \frac{dx}{x^2-4x+8}=x^2+8x+4\int \frac{x\,dx}{(x-2)^2+4}-\int \frac{dx}{(x-2)^2+4}=$

$=\left\{\begin{matrix}t=x-2\\ x=t+2\end{matrix}\right\}=x^2+8x+4\int \frac{(t+2)dt}{t^2+4}-\int \frac{dt}{t^2+4}=x^2+8x+4\int \frac{t\,dt}{t^2+4}+8\int \frac{dt}{t^2+4}-\int \frac{dt}{t^2+4}=$

$=x^2+8x+4\int \frac{t\,dt}{t^2+4}+7\int \frac{dt}{t^2+4}=\left\{dt^2=2t\,dt\right\}=$

$=x^2+8x+4\cdot\frac{1}{2}\int \frac{dt^2}{t^2+4}+7\cdot\frac{1}{2}\operatorname{arctg}\frac{t}{2}=x^2+8x+2\ln|t^2+4|+\frac{7}{2}\operatorname{arctg}\frac{t}{2}+C=$

$=x^2+8x+2\ln|x^2-4x+8|+\frac{7}{2}\operatorname{arctg}\frac{x-2}{2}+C$

4.24 $\int \frac{x^3+9x^2-31x+35}{(x-3)^2(x^2+1)}\,dx$

$$\frac{x^3+9x^2-31x+35}{(x-3)^2(x^2+1)} = \frac{A_1}{(x-3)} + \frac{A_2}{(x-3)^2} + \frac{Bx+C}{x^2+1} =$$

$$= \frac{A_1(x-3)(x^2+1)+A_2(x^2+1)+(Bx+C)(x-3)^2}{(x-3)^2(x^2+1)} =$$

$$= \frac{A_1(x^3+x-3x^2-3)+A_2(x^2+1)+(Bx+C)(x^2-6x+9)}{(x-3)^2(x^2+1)} =$$

$$= \frac{x^3A_1+xA_1-3x^2A_1-3A_1+x^2A_2+A_2+Bx^3-6Bx^2+9Bx+Cx^2-6Cx+9C}{(x-3)^2(x^2+1)}$$

$$\begin{cases} A_1+B=1 \quad \Rightarrow \quad A_1 = 1-B \\ -3A_1+A_2-6B+C=9 \\ A_1+9B-6C=-31 \\ -3A_1+A_2+9C=35 \end{cases}$$

$$\begin{cases} -6B-8C=9-35 \\ 8B-6C=-32 \end{cases}$$

$$\begin{cases} -6B-8C=-26 \\ 8B-6C=-32 \end{cases}$$

$$\begin{cases} -3B-4C=-13 \\ 4B-3C=-16 \end{cases} \Rightarrow B=-4+\frac{3}{4}C$$

$$-3\left(-4+\frac{3}{4}C\right)-4C=-13$$

$$12-\frac{9}{4}C-4C=-13$$

$$-\frac{9-16}{4}C=-25$$

$$-25C=-100$$

$$C=4 \quad \Rightarrow \quad B=-4+\frac{3}{4}\cdot 4=-1$$

$$A_1 = 1-(-1)=2$$

$$-3\cdot 2+A_2+9\cdot 4=35$$

$$A_2=5$$

$$\int \frac{x^3+9x^2-31x+35}{(x-3)^2(x^2+1)}dx = \int\frac{2dx}{x-3} + \int\frac{5dx}{(x-3)^2} + \int\frac{-1x+4}{x^2+1}dx =$$

$$= 2\ln|x-3| + 5\cdot\frac{(x-3)^{-2+1}}{-2+1} - \int\frac{x-4}{x^2+1}dx =$$

$$= 2\ln|x-3| + 5\cdot\frac{1}{(-1)(x-3)} - \int\frac{xdx}{x^2+1} + 4\int\frac{dx}{x^2+1} = \left\{dx^2 = 2xdx\right\} =$$

$$= 2\ln|x-3| + 5\cdot\frac{(-1)}{(x-3)} - \frac{1}{2}\int\frac{dx^2}{x^2+1} + 4\,\mathrm{arctg}\,x =$$

$$= 2\ln|x-3| - \frac{5}{x-3} - \frac{1}{2}\ln|x^2+1| + 4\,\mathrm{arctg}\,x + C$$

$\boxed{5.24}$

$$\int\frac{dx}{(x-4)^2\sqrt{x^2-14x+41}} = \int\frac{dx}{(x-4)^2\sqrt{(x-4)^2-6x+25}} = \left\{\begin{matrix} t = x-4 \\ x = t+4\end{matrix}\right\} =$$

$$= \int\frac{dt}{t^2\sqrt{t^2-6(t+4)+25}} = \int\frac{dt}{t^2\sqrt{t^2-6t+1}} = \left\{\begin{matrix} u = \frac{1}{t} \\ du = -\frac{1}{t^2}\end{matrix}\right\} = -\frac{du}{\sqrt{\frac{1}{u^2}-\frac{6}{u}+1}} =$$

$$= -\int\frac{udu}{\sqrt{1-6u+u^2}} = -\int\frac{udu}{\sqrt{(u-3)^2-8}} = \left\{\begin{matrix} v = u-3 \\ u = v+3\end{matrix}\right\} = -\int\frac{(v+3)dv}{\sqrt{v^2-8}} =$$

$$= -\int\frac{vdv}{\sqrt{v^2-8}} - 3\int\frac{dv}{\sqrt{v^2-8}} = \left\{dv^2 = 2vdv\right\} = -\frac{1}{2}\int\frac{dv^2}{\sqrt{v^2-8}} - 3\int\frac{dv}{\sqrt{v^2-8}} =$$

$$= -\frac{1}{2}\int\frac{d(v^2-8)}{\sqrt{v^2-8}} - 3\ln\left|v+\sqrt{v^2-8}\right| = -\frac{1}{2}\frac{(v^2-8)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 3\ln\left|v+\sqrt{v^2-8}\right| =$$

$$= -\frac{1}{2}\cdot 2\sqrt{v^2-8} - 3\ln\left|v+\sqrt{v^2-8}\right| + C = -\sqrt{(u-3)^2-8} - 3\ln\left|u-3+\sqrt{(u-3)^2-8}\right| + C =$$

$$= -\sqrt{u^2-6u+1} - 3\ln\left|u-3+\sqrt{u^2-6u+1}\right| + C = -\sqrt{\frac{1}{t^2}-6t+1} - 3\ln\left|\frac{1}{t}-3+\sqrt{\frac{1}{t^2}-\frac{6}{t}+1}\right| + C =$$

$$= -\sqrt{\frac{1-6t+t^2}{t^2}} + 3\ln\left|\frac{1-3t}{t}+\sqrt{\frac{1-6t+t^2}{t^2}}\right| + C =$$

$$= -\frac{\sqrt{1-6(x-4)+(x-4)^2}}{x-4} + 3\ln\left|\frac{1-3(x-4)}{x-4} + \frac{\sqrt{1-6(x-4)+(x-4)^2}}{x-4}\right| + C =$$

$$= -\frac{\sqrt{x^2-14x+41}}{x-4} + 3\ln\left|\frac{13-3x+\sqrt{x^2-14x+41}}{x-4}\right| + C$$

$\underline{8.24}$ $\int\sqrt{-3x^2-8x-4}\,dx = \int\sqrt{-3(x^2+\frac{8}{3}x+\frac{4}{3})}\,dx = \int\sqrt{-3((x+\frac{4}{3})^2-\frac{4}{9})}\,dx =$

$= \left\{ t = x+\frac{4}{3} \right\} = \sqrt{3}\int\sqrt{-(t^2-\frac{4}{9})}\,dt = \sqrt{3}\int\sqrt{\frac{4}{9}-t^2}\,dt = \left\{ \begin{array}{l} t=\frac{2}{3}\sin u \\ dt=\frac{2}{3}\cos u \end{array} \right\} =$

$= \sqrt{3}\int\sqrt{\frac{4}{9}-\frac{4}{9}\sin^2 u}\cdot\frac{2}{3}\cos u\,du = \sqrt{3}\cdot\frac{2}{3}\int\frac{2}{3}\sqrt{1-\sin^2 u}\cos u\,du =$

$= \sqrt{3}\cdot\frac{4}{9}\int\sqrt{\cos^2 u}\cos u\,du = \frac{4\sqrt{3}}{9}\int\cos^2 u\,du = \frac{4\sqrt{3}}{9}\int\frac{1+\cos 2u}{2}\,du =$

$= \frac{4\sqrt{3}}{9}\cdot\frac{1}{2}\int du + \frac{4\sqrt{3}}{9}\cdot\frac{1}{2}\int\cos 2u\,du = \frac{2\sqrt{3}}{9}u + \frac{2\sqrt{3}}{9}\int\cos 2u\,du = \left\{ d2u = 2du \right\} =$

$= \frac{2\sqrt{3}}{9}u + \frac{2\sqrt{3}}{9}\cdot\frac{1}{2}\int\cos 2u\,d2u = \frac{2\sqrt{3}}{9}u + \frac{\sqrt{3}}{9}\cdot\sin 2u + C =$

$= \frac{2\sqrt{3}}{9}u + \frac{\sqrt{3}}{9}\cdot 2\sin u\cos u + C = \left\{ \begin{array}{l} \sin u = \frac{3}{2}t \\ \cos u = \frac{3}{2}\sqrt{\frac{4}{9}-t^2} \\ u = \arcsin(\frac{3}{2}t) \end{array} \right\} =$

$= \frac{2\sqrt{3}}{9}\arcsin(\frac{3}{2}t) + \frac{\sqrt{3}}{9}\cdot 2\cdot\frac{3}{2}t\cdot\frac{3}{2}\sqrt{\frac{4}{9}-t^2} + C =$

$= \frac{2\sqrt{3}}{9}\arcsin(\frac{3}{2}t) + \frac{\sqrt{3}}{2}t\sqrt{\frac{4}{9}-t^2} + C =$

$= \frac{2\sqrt{3}}{9}\arcsin(\frac{3}{2}(x+\frac{4}{3})) + \frac{\sqrt{3}}{2}(x+\frac{4}{3})\sqrt{\frac{4}{9}-(x+\frac{4}{3})^2} + C =$

$= \frac{2\sqrt{3}}{9}\arcsin(\frac{7}{2}x) + \frac{\sqrt{3}}{2}(x+\frac{4}{3})\sqrt{-x^2-\frac{8}{3}x-\frac{4}{3}} + C =$

$= \frac{2\sqrt{3}}{9}\arcsin(\frac{7}{2}x) + \frac{\sqrt{3}}{2}(x+\frac{4}{3})\sqrt{-3x^2-8x-4} + C$

$\underline{9.24}$ $\int\cos^4\frac{x}{4}\,dx = \int(\cos^2\frac{x}{4})^2\,dx = \int\left(\frac{1+\cos(2\cdot\frac{x}{4})}{2}\right)^2 dx =$

$= \int\frac{(1+\cos\frac{x}{2})^2}{4}\,dx = \frac{1}{4}\int(1+2\cos\frac{x}{2}+\cos^2\frac{x}{2})\,dx = \frac{1}{4}\int dx + \frac{2}{4}\int\cos\frac{x}{2}\,dx + \frac{1}{4}\int\cos^2\frac{x}{2}\,dx =$

$= \frac{1}{4}x + \frac{1}{2}\int\cos\frac{x}{2}\,dx + \frac{1}{4}\int\cos^2\frac{x}{2}\,dx = \left\{ d\frac{x}{2} = \frac{1}{2}dx \right\} = \frac{1}{4}x + \frac{1}{2}\cdot 2\int\cos\frac{x}{2}\,d\frac{x}{2} + \frac{1}{4}\int\frac{1+\cos(2\cdot\frac{x}{2})}{2}\,dx =$

$= \frac{1}{4}x + \sin\frac{x}{2} + \frac{1}{8}\int dx + \frac{1}{8}\int\cos x\,dx = \frac{1}{4}x + \sin\frac{x}{2} + \frac{1}{8}x + \frac{1}{8}\sin x + C =$

$= \frac{3}{8}x + \sin\frac{x}{2} + \frac{1}{8}\sin x + C$

$\boxed{\text{II}.24}$ $\displaystyle\int_{15}^{80} \frac{dx}{\sqrt{x+1}+\sqrt[4]{x+1}} = \int_{16} \frac{d(x+1)}{\sqrt{x+1}+\sqrt[4]{x+1}} = \int_{16} \frac{dt}{\sqrt{t}+\sqrt[4]{t}} = \left\{ \begin{array}{l} u=\sqrt[4]{t} \\ dt=4u^3du \end{array} \right\} =$

$\displaystyle = \int_2^3 \frac{4u^3du}{u^2+u} = \int_2^3 \frac{4u^2du}{u+1} = \left\{ \begin{array}{l} v=u+1 \\ u=v-1 \end{array} \right\} = \int_3^4 \frac{4(v-1)^2dv}{v} = 4\int_3^4 \frac{v^2-2v+1}{v}dv =$

$\displaystyle = 4\cdot\left(\int \frac{v^{\cancel{2}}dv}{\cancel{v}} - \int\frac{2\cancel{v}dv}{\cancel{v}} + \int\frac{dv}{v} \right)\Bigg|_3^4 = 4\left(\frac{v^2}{2} - 2v + \ln|v|\right)\Bigg|_3^4 =$

$= (2v^2 - 8v + 4\ln|v|)\Big|_3^4 = 2\cdot16 - 8\cdot4 + 4\ln4 - 2\cdot9 + 8\cdot3 - 4\ln3 =$

$= 32 - 32 + 4\ln4 - 18 + 24 - 4\ln3 = 6 + 4\cdot\ln4 - 4\ln3 = 6 + 4\ln4 - 4\ln3$

$\boxed{12.24}$ $\displaystyle\int_{1/3}^{1} (3x-1)\ln3x\,dx \;=$

$\displaystyle\int(3x-1)\ln3x\,dx = \left\{ \begin{array}{l} u=\ln3x \\ du=\frac{1}{x} \\ dv=3x-1 \\ v=3\cdot\frac{x^2}{2}-x \end{array} \right\} = \left(\frac{3}{2}x^2-x\right)\ln3x - \int\left(\frac{3}{2}x^2-x\right)\cdot\frac{1}{x}dx =$

$\displaystyle = \left(\frac{3}{2}x-1\right)x\ln3x - \int\left(\frac{3}{2}x-1\right)dx = \left(\frac{3}{2}x-1\right)x\cdot\ln3x - \frac{3}{2}\cdot\frac{x^2}{2} + x =$

$\displaystyle = \left(\frac{3}{2}x-1\right)x\ln3x - \frac{3}{4}x^2 + x$

$\displaystyle = \left(\left(\frac{3}{2}x-1\right)x\ln3x - \frac{3}{4}x^2+x\right)\Bigg|_{1/3}^{1} =$

$\displaystyle = \left(\frac{3}{2}-\frac{2}{2}\right)\ln3 - \frac{3}{4}\cdot\frac{1}{1} + 1 - \left(\frac{3}{2}\cdot\frac{1}{3}-1\right)\cdot\frac{1}{3}\ln3\cdot\frac{1}{3} + \frac{3}{4}\cdot\frac{1}{9} - \frac{1}{3} =$

$\displaystyle = \frac{1}{2}\ln3 + \frac{1}{4} - \left(\frac{3}{2}\cdot\frac{1}{3}-1\right)\cdot\frac{1}{3}\ln1 + \frac{1}{12} - \frac{1}{3} = \frac{1}{2}\ln3 + \frac{3+1-4}{12} = \frac{1}{2}\ln3$

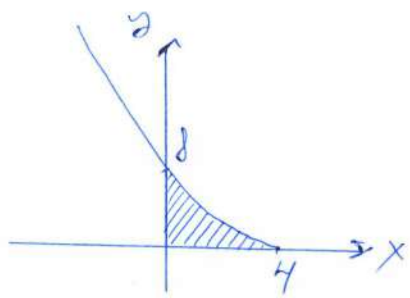
13.24) $y^2=(4-x)^3$; $x=0$

$y=\sqrt{(4-x)^3}$; $x=0$

$\sqrt{(4-x)^3}=0$

$4-x=0$

$x=4$



$x=0$: $y=\sqrt{4^3}=8$

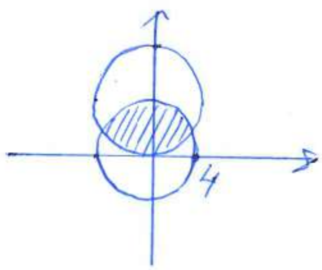
$$\int_0^4 \sqrt{(4-x)^3}\,dx \ominus$$

$$\int \sqrt{(4-x)^3}\,dx = \{d(4-x)=-dx\} = -\int \sqrt{(4-x)^3}\,d(4-x) = -\frac{(4-x)^{\frac{3}{2}+1}}{\frac{3}{2}+1} = -\frac{2}{5}\sqrt{(4-x)^5}$$

$$\ominus \left(-\frac{2}{5}\sqrt{(4-x)^5}\right)\Big|_0^4 = -\frac{2}{5}\sqrt{(4-4)^5}+\frac{2}{5}\sqrt{(4-0)^5}=\frac{2}{5}\sqrt{4^5}=\frac{2}{5}\cdot 2^5=\frac{64}{5}=12,8$$

Ответ: 12,8

14.24 $\rho=8\sin\varphi$; $\rho=4$



$$S=\frac{1}{2}\int_\alpha^\beta \rho^2(\varphi)\,d\varphi \ ; \ \alpha<\varphi<\beta$$

$$S=\frac{1}{2}\int_0^{\pi/2}(8\sin\varphi)^2d\varphi-\frac{1}{2}\int_0^{\pi/2}4^2d\varphi=$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}64\sin^2\varphi\,d\varphi-\frac{1}{2}\cdot16\cdot\int_0^{\frac{\pi}{2}}d\varphi=32\int_0^{\frac{\pi}{2}}\frac{1-\cos2\varphi}{2}d\varphi-8\int_0^{\frac{\pi}{2}}d\varphi \ominus$$

$$\int\frac{1-\cos2\varphi}{2}d\varphi=\frac{1}{2}\int d\varphi-\frac{1}{2}\int\cos2\varphi\,d\varphi=\{d2\varphi=2d\varphi\}=\frac{1}{2}\varphi-\frac{1}{2}\cdot\frac{1}{2}\int\cos2\varphi\,d2\varphi=$$

$$=\frac{1}{2}\varphi-\frac{1}{4}\sin2\varphi$$

$$\ominus\ 32\cdot\frac{1}{2}\left(\varphi-\frac{1}{2}\sin2\varphi\right)\Big|_0^{\frac{\pi}{2}}-8\varphi\Big|_0^{\frac{\pi}{2}}=16\left(\frac{\pi}{2}-\frac{1}{2}\sin2\cdot\frac{\pi}{2}-0+\frac{1}{2}\sin2\cdot0\right)-8\frac{\pi}{2}=$$

$$=16\cdot\frac{\pi}{2}-8\cdot\frac{\pi}{2}=8\pi-4\pi=4\pi$$

Ответ: 4π

$\boxed{15.24}$ $\rho = 1 + \sin\varphi$; $\frac{\pi}{6} \le \varphi \le \frac{\pi}{3}$

Длина дуги:

$$l = \int_{\alpha}^{\beta} \sqrt{\rho^2(\varphi) + (\rho'(\varphi))^2}\, d\varphi$$

$$l = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{(1+\sin\varphi)^2 + \cos^2\varphi}\, d\varphi = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 + 2\sin\varphi + \sin^2\varphi + \cos^2\varphi}\, d\varphi =$$

$$= \int_{\frac{\pi}{6}}^{\pi/3} \sqrt{2 + 2\sin\varphi}\, d\varphi = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{2} \cdot \sqrt{1+\sin\varphi}\, d\varphi =$$

$$\int \sqrt{1+\sin\varphi}\, d\varphi = \left\{ \sin\varphi = \cos\left(\frac{\pi}{2} - \varphi\right) \right\} = \int \sqrt{1 + \cos\left(\frac{\pi}{2} - \varphi\right)}\, d\varphi$$

$$= \sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1+\sin\varphi}\, d\varphi = \sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 + \cos\left(\frac{\pi}{2} - \varphi\right)}\, d\varphi = \left\{ \begin{array}{l} t = \frac{\pi}{2} - \varphi \\ dt = -d\varphi \end{array} \right\} =$$

$$= \left\{ \begin{array}{l} -\frac{\pi}{3} + \frac{\pi}{2} = \frac{\pi}{6} \\ -\frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi}{3} \end{array} \text{ - пределы остаются те же} \right\} = -\sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1+\cos t}\, dt = \left\{ \frac{1+\cos t}{2} = \cos^2\frac{t}{2} \right\} =$$

$$= -\sqrt{2} \int_{\frac{\pi}{6}}^{\pi/3} \sqrt{2\cos^2\frac{t}{2}}\, dt = -\sqrt{2}\cdot\sqrt{2} \int_{\frac{\pi}{6}}^{\pi/3} \sqrt{\cos^2\frac{t}{2}}\, dt = -2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos\frac{t}{2}\, dt = \left\{ d\frac{t}{2} = \frac{1}{2} dt \right\} =$$

$$= -2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2 \cdot \cos\frac{t}{2}\, d\frac{t}{2} = -4\sin\frac{t}{2} \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -4\left(\sin\frac{\pi}{6} - \sin\frac{\pi}{12}\right) = -4 \cdot \frac{1}{2} + 4\sin\frac{\pi}{12} =$$

$$= -2 + 4\sin\frac{\pi}{12}$$

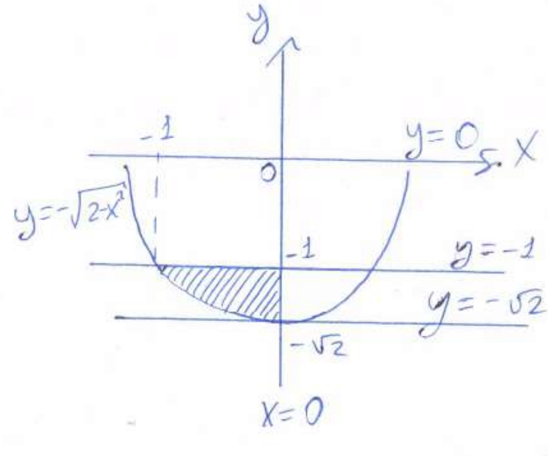
Ответ: $-2 + 4\sin\frac{\pi}{12}$

КРАТНЫЕ ИНТЕГРАЛЫ

$\boxed{1.24}$ $$\int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^{0} f\,dx + \int_{-1}^{0} dy \int_{y}^{0} f\,dx$$

Выпишем область D_1:

$$\begin{cases} -\sqrt{2} \le y \le -1 \\ -\sqrt{2-y^2} \le x \le 0 \end{cases}$$



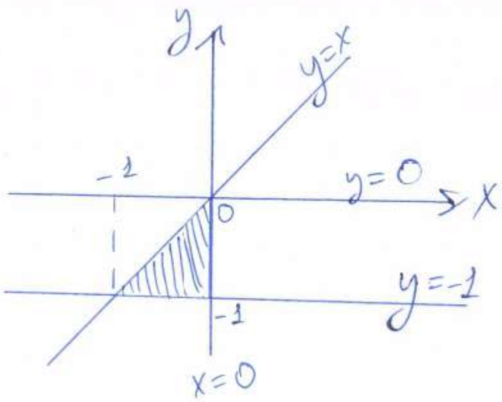
Построим область D_1.
Из рисунка следует:

$y=-1: \quad x = -\sqrt{2-1} = -1$

$$\begin{cases} -1 \le x \le 0 \\ -\sqrt{2-x^2} \le y \le -1 \end{cases} - D_1'$$

Выпишем область D_2:

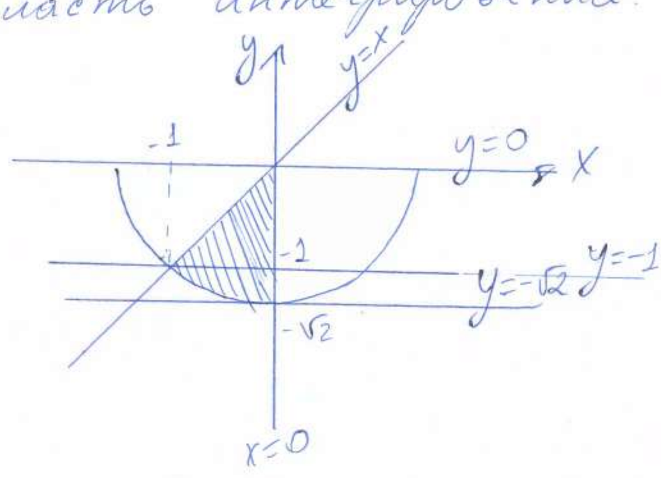
$$\begin{cases} -1 \le y \le 0 \\ y \le x \le 0 \end{cases}$$



Построим область D_2.
Из рисунка следует:

$$\begin{cases} -1 \le x \le 0 \\ -1 \le y \le x \end{cases} - D_2'$$

Область интегрирования:



$$D: \begin{cases} -1 \le x \le 0 \\ -\sqrt{2-x^2} \le y \le x \end{cases}$$

После изменения порядка интегрирования:

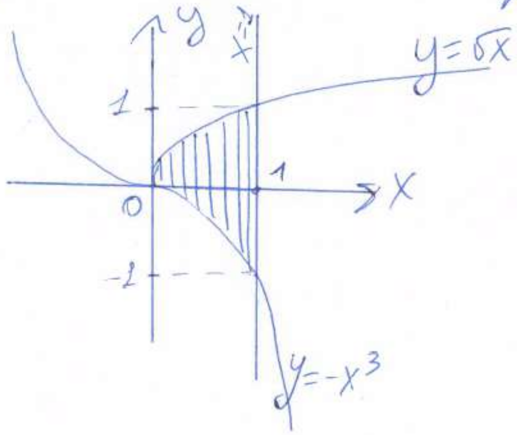
$$\int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^{0} f\,dx + \int_{-1}^{0} dy \int_{y}^{0} f\,dx =$$

$$= \int_{-1}^{0} dx \int_{-\sqrt{2-x^2}}^{x} f\,dy$$

Ответ: $\int_{-1}^{0} dx \int_{-\sqrt{2-x^2}}^{x} f\,dy$

$\boxed{2.24}$ $\iint\limits_D (4xy + 176x^3y^3)\,dx\,dy$; D: $x=1$, $y=\sqrt{x}$, $y=-x^3$

Построим область интегрирования D



$D: \begin{cases} 0 \le x \le 1 \\ -x^3 \le y \le \sqrt{x} \end{cases}$

$$\int_0^1 dx \int_{-x^3}^{\sqrt{x}} (4xy + 176x^3y^3)\,dy = \int_0^1 dx \left(4x \cdot \frac{y^2}{2} + 176x^3 \cdot \frac{y^4}{4}\right)\Bigg|_{-x^3}^{\sqrt{x}} =$$

$$= \int_0^1 \left(2x \cdot (\sqrt{x})^2 + 176x^3 \cdot \frac{(\sqrt{x})^4}{4} - 2x \cdot (-x^3)^2 - 44x^3 \cdot (-x^3)^4\right) dx =$$

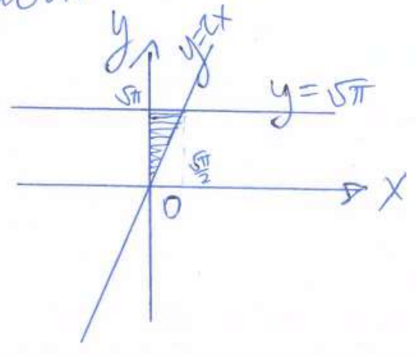
$$= \int_0^1 (2x^2 + 44x^5 - 2x^7 - 44x^{15})\,dx = \left(2 \cdot \frac{x^3}{3} + 44 \cdot \frac{x^6}{6} - 2 \cdot \frac{x^8}{8} - 44 \cdot \frac{x^{16}}{16}\right)\Bigg|_0^1 =$$

$$= \frac{2}{3} + \frac{44}{6} - \frac{2}{8} - \frac{44}{16} = \frac{2}{3} + \frac{22}{3} - \frac{1}{4} - \frac{11}{4} = \frac{24}{3} - \frac{12}{4} = 8 - 3 = 5$$

Ответ: 5

$\boxed{3.24}$ $\iint\limits_D y^2 \cos xy\,dx\,dy$; D: $x=0$, $y=\sqrt{\pi}$; $y=2x$

Построим область интегрирования



$D: \begin{cases} 0 \le y \le \sqrt{\pi} \\ 0 \le x \le \frac{1}{2}y \end{cases}$

$$\int_0^{\sqrt{\pi}} dy \int_0^{\frac{1}{2}y} y^2 \cos xy\,dx = \int_0^{\sqrt{\pi}} y^2 dy \int_0^{\frac{1}{2}y} \cos xy\,dx = \{dxy = y\,dx\} =$$

$$= \int_0 y^2 dy \int_0 \frac{\cos xy}{y}\, dxy = \int_0 y^2 \cdot \frac{1}{y}\, dy \int_0 \cos xy\, dxy =$$

$$= \int_0^{\sqrt{\pi}} y\, dy\, (\sin xy)\Big|_0^{\frac{1}{2}y} = \int_0^{\sqrt{\pi}} y\left(\sin\left(\tfrac{1}{2}y\cdot y\right) - \sin(0\cdot y)\right) dy =$$

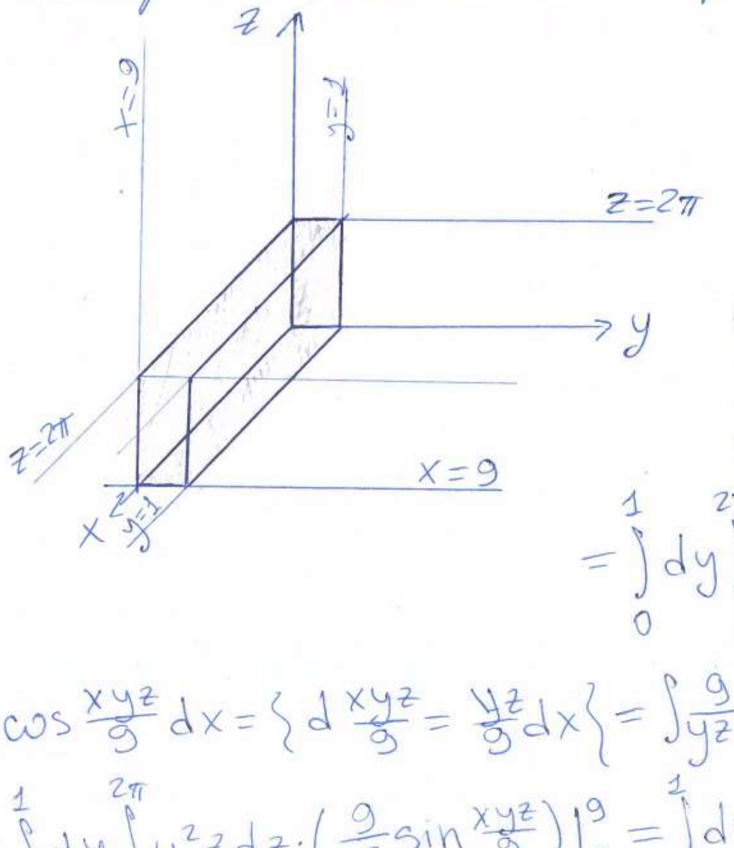
$$= \int_0^{\sqrt{\pi}} y \sin\frac{y^2}{2}\, dy = \left\{ d\frac{y^2}{2} = y\,dy \right\} = \int_0^{\pi/2} \sin\frac{y^2}{2}\, d\frac{y^2}{2} = \left(-\cos\frac{y^2}{2}\right)\Big|_0^{\frac{\pi}{2}} =$$

$$= -\cos\frac{\pi^2}{8} + \cos 0^\circ = 1$$

Ответ: 1

$\boxed{4.24}$ $\iiint\limits_V y^2 z \cos\frac{xyz}{9}\, dx\,dy\,dz$; $V: \begin{cases} x=9, & y=1, & z=2\pi \\ x=0, & y=0, & z=0 \end{cases}$

Построим область интегрирования



$$\begin{cases} 0 \le x \le 9 \\ 0 \le y \le 1 \\ 0 \le z \le 2\pi \end{cases}$$

$$\int_0^1 dy \int_0^{2\pi} dz \int_0^9 y^2 z \cos\frac{xyz}{9}\, dx =$$

$$= \int_0^1 dy \int_0^{2\pi} y^2 z\, dz \int_0^9 \cos\frac{xyz}{9}\, dx =$$

$$\int \cos\frac{xyz}{9}\, dx = \left\{ d\frac{xyz}{9} = \frac{yz}{9}\, dx \right\} = \int \frac{9}{yz} \cos\frac{xyz}{9}\, d\frac{xyz}{9} = \frac{9}{yz} \sin\frac{xyz}{9}$$

$$= \int_0^1 dy \int_0^{2\pi} y^2 z\, dz \cdot \left(\frac{9}{yz} \sin\frac{xyz}{9}\right)\Big|_0^9 = \int_0^1 dy \int_0^{2\pi} 9y\left(\sin\frac{9yz}{9} - \sin 0\right) dz =$$

$$= 9\int_0^1 y\, dy \int_0^{2\pi} \sin yz\, dz =$$

$$\int \sin yz\, dz = \{ dyz = y\,dz \} = \int \frac{\sin yz}{y}\, dyz = \frac{1}{y}\int \sin yz\, dyz = \frac{1}{y}(-\cos yz)$$

$$\Rightarrow 9\int\limits_0 y\,dy\left(\frac{1}{y}(-\cos yz)\right)\Big|_0^{2\pi} = 9\int\limits_0 y\cdot\frac{1}{y}\left(-\cos(y\cdot 2\pi)+\cos(y\cdot 0)\right)dy =$$

$$= 9\int\limits_0^1\left(-\cos(2\pi y)+1\right)dy = \left\{\int(-\cos 2\pi y+1)\,dy = -\frac{1}{2\pi}\sin 2\pi y + y\right\} =$$

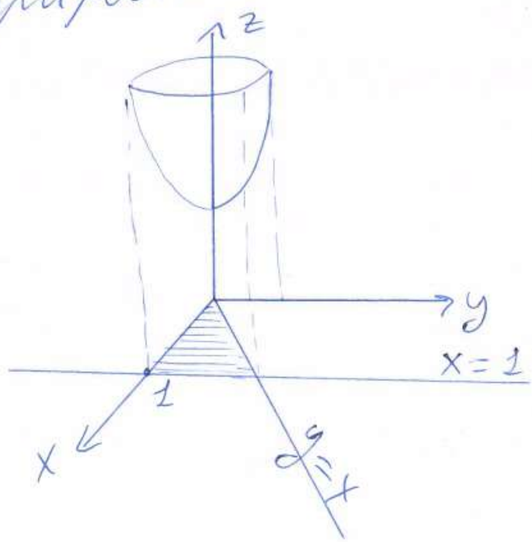
$$= 9\left(-\frac{1}{2\pi}\sin 2\pi y + y\right)\Big|_0^1 = 9\left(-\frac{1}{2\pi}\sin 2\pi\cdot 1 + 1 + \frac{1}{2\pi}\sin 2\pi\cdot 0 - 0\right) =$$

$$= 9\cdot 1 = 9$$

Ответ: 9

$\boxed{5.24}$ $\iiint\limits_V (x+y)\,dx\,dy\,dz$; $V: \begin{cases} y=x;\ y=0;\ x=1 \\ z=30x^2+60y^2;\ z=0 \end{cases}$

Построим область интегрирования



$$\begin{cases} 0\le x\le 1 \\ 0\le y\le x \\ 0\le z\le 30x^2+60y^2 \end{cases}$$

$$\int\limits_0 dx\int\limits_0^x dy\int\limits_0^{30x^2+60y^2}(x+y)\,dz =$$

$$= \int\limits_0^1 dx\int\limits_0^x (x+y)\,dy\int\limits_0^{30x^2+60y^2} dz =$$

$$= \int\limits_0^1 dx\int\limits_0^x (x+y)(30x^2+60y^2)\,dy = 30\int\limits_0^1 dx\int\limits_0^x (x+y)(x^2+2y^2)\,dy =$$

$$= 30\int\limits_0^1 dx\int\limits_0^x (x^3+2xy^2+x^2y+2y^3)\,dy = 30\int\limits_0^1 dx\left(x^3y+2x\cdot\frac{y^3}{3}+\frac{x^2y^2}{2}+2\cdot\frac{y^4}{4}\right)\Big|_0^x =$$

$$= 30\int\limits_0^1\left(x^4+\frac{2}{3}x^4+\frac{x^4}{2}+\frac{x^4}{2}\right)dx = 30\cdot\frac{8}{3}\int\limits_0^1 x^4\,dx = 80\cdot\frac{x^5}{5}\Big|_0^1 = \frac{80}{5} = 16$$

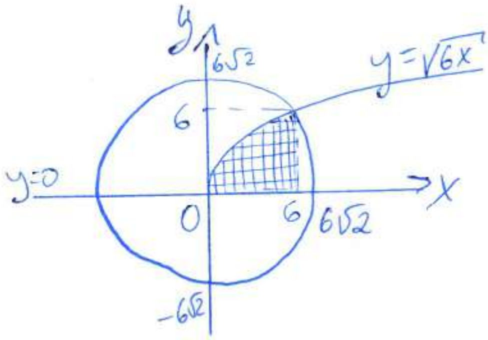
Ответ: 16

$\boxed{6.24}$ $x=\sqrt{72-y^2}$, $6x=y^2$, $y=0$

$x^2+y^2=72$

$y=\sqrt{6x}$

$y=0$



$\sqrt{6x}=\sqrt{72-x^2}$

$6x=72-x^2$

$x^2+6x-72=0$

$\frac{D}{4}=9+72=81$

$x=-3\pm 9$

$x=-12$, $x=6$

$x=6$: $y=6$.

$$D: \begin{cases} 0\le y\le 6 \\ \frac{y^2}{6}\le x\le\sqrt{72-y^2} \end{cases}$$

$$\int_0^6 dy\int_{\frac{y^2}{6}}^{\sqrt{72-y^2}} dx=\int_0^6\left(\sqrt{72-y^2}-\frac{y^2}{6}\right)dy=\int_0^6\sqrt{72-y^2}\,dy-\int_0^6\frac{y^2}{6}dy=$$

$$=\int_0^6\sqrt{72-y^2}\,dy-\frac{y^3}{18}\Big|_0^6=\int_0^6\sqrt{72-y^2}\,dy-\frac{6^3}{18}=\int_0^6\sqrt{72-y^2}\,dy-12 \;=$$

$$\int\sqrt{72-y^2}\,dy=\begin{Bmatrix} y=\sqrt{72}\sin t \\ dy=\sqrt{72}\cos t\,dt \end{Bmatrix}=\int\sqrt{72-72\sin^2 t}\cdot\sqrt{72}\cos t\,dt=\int 72\cos^2 t\,dt$$

$$=\;72\int_0^{\pi/4}\cos^2 t\,dt-12=72\int_0^{\frac{\pi}{4}}\frac{1+\cos 2t}{2}dt-12=36\int_0^{\frac{\pi}{4}}(1+\cos 2t)\,dt-12=$$

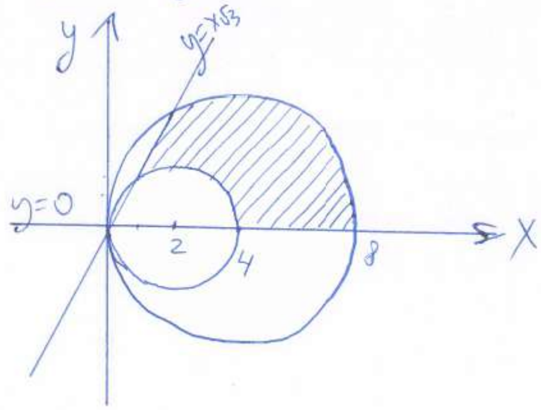
$$=36\cdot\left(t+\frac{\sin 2t}{2}\right)\Big|_0^{\frac{\pi}{4}}-12=36\cdot\left(\frac{\pi}{4}+\frac{1}{2}\sin\frac{\pi}{2}\right)-12=36\left(\frac{\pi}{4}+\frac{1}{2}\right)-12=9\pi+6$$

Ответ: $9\pi+6$

7.24 $x^2-4x+y^2=0$ $\qquad (x-2)^2+y^2=4$

$x^2-8x+y^2=0$ $\qquad (x-4)^2+y^2=16$

$y=0,\ y=x\sqrt{3}$ $\qquad y=0,\ y=\sqrt{3}x$



Перейдём к полярным координатам:

$x=\rho\cos\varphi,\ y=\rho\sin\varphi,\ dxdy=\rho d\rho d\varphi,\ x^2+y^2=\rho^2$

- Уравнение окружностей:

$\rho^2=4\rho\cos\varphi;\ \rho=4\cos\varphi$

$\rho^2=8\rho\cos\varphi;\ \rho=8\cos\varphi$

- Уравнения прямых:

$y=\rho\sin\varphi=0,\quad \varphi=0$

$y=\sqrt{3}x \Rightarrow \rho\sin\varphi=\sqrt{3}\rho\cos\varphi \Rightarrow tg\,\varphi=\sqrt{3} \Rightarrow \varphi=\frac{\pi}{3}$

Область интегрирования:

$$D:\begin{cases}0\le\varphi\le\frac{\pi}{3}\\ 4\cos\varphi\le\rho\le 8\cos\varphi\end{cases}$$

Площадь фигуры: $S=\iint\limits_D dxdy$

$$S=\int_0^{\frac{\pi}{3}}d\varphi\int_{4\cos\varphi}^{8\cos\varphi}\rho d\rho=\int_0^{\pi/3}\left(\frac{\rho^2}{2}\Bigg|_{4\cos\varphi}^{8\cos\varphi}\right)d\varphi=24\int_0^{\frac{\pi}{3}}\cos^2\varphi\,d\varphi=24\int_0^{\pi/3}\frac{1+\cos 2\varphi}{2}d\varphi=$$

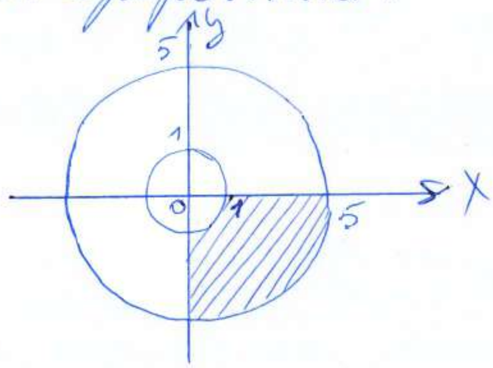
$$=12\int_0^{\pi/3}(1+\cos 2\varphi)d\varphi=12\left(\varphi+\frac{\sin 2\varphi}{2}\right)\Bigg|_0^{\pi/3}=12\left(\frac{\pi}{3}+\frac{1}{2}\sin\frac{2\pi}{3}\right)=4\pi+3\sqrt{3}$$

Ответ: $4\pi+3\sqrt{3}$

$\boxed{8.24}$ $D: x^2+y^2=1,\ x^2+y^2=25,\ x=0,\ y=0\quad (x\geq 0, y\geq 0)$.

$\mu = \dfrac{x-4y}{x^2+y^2}$ – поверхностная плотность

Построим область интегрирования.



Перейдём в полярные координаты:

$x=\rho\cos\varphi,\ y=\rho\sin\varphi,$

$dxdy=\rho d\rho d\varphi,\ x^2+y^2=\rho^2$

$D: \begin{cases} -\frac{\pi}{2}\leq\varphi\leq 0 \\ 1\leq\rho\leq 5 \end{cases}$

$\mu = \dfrac{\rho\cos\varphi-4\rho\sin\varphi}{\rho^2} = \dfrac{\cos\varphi-4\sin\varphi}{\rho}$

Рассчитаем массу пластины

$m=\iint\limits_D \mu dxdy = \int\limits_{-\frac{\pi}{2}}^{0} d\varphi\int\limits_1^5 \mu\rho d\rho = \int\limits_{-\frac{\pi}{2}}^{0} d\varphi\int\limits_1^5 \frac{\cos\varphi-4\sin\varphi}{\rho}\cdot\rho d\rho =$

$=\int\limits_{-\frac{\pi}{2}}^{0} d\varphi\int\limits_1^5(\cos\varphi-4\sin\varphi)d\rho = \int\limits_{-\frac{\pi}{2}}^{0}(\cos\varphi-4\sin\varphi)\, d\varphi\int\limits_1^5 d\rho = \int\limits_{-\frac{\pi}{2}}^{0}(5-1)(\cos\varphi-4\sin\varphi)d\varphi=$

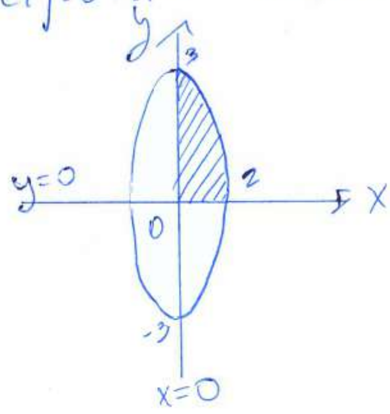
$=4(\sin\varphi+4\cos\varphi)\Big|_{-\frac{\pi}{2}}^{0} = 4(\sin 0+4\cos 0-\sin(-\frac{\pi}{2})+4\cos(-\frac{\pi}{2})) = 4(4+1)=20$

Ответ: 20.

$\boxed{9.24}$ $D: \dfrac{x^2}{4}+\dfrac{y^2}{9}\leq 1,\ x\geq 0,\ y\geq 0;$

$\mu = x^5y$ – поверхностная плотность.

Построим область интегрирования



Перейдём к полярным координатам

$x=2\rho\cos\varphi,\ y=3\rho\sin\varphi$

$\dfrac{x^2}{4}+\dfrac{y^2}{9}=1 \Rightarrow \rho=1.$

$\mu=2^5\rho^5\cos^5\varphi\cdot 3\rho\sin\varphi=32\cdot 3\rho^6\cos^5\varphi\sin\varphi$

$\mu=96\rho^6\cos^5\varphi\sin\varphi$

Масса пластинки:

$$m=\int_0^{\frac{\pi}{2}} d\varphi \int_0^1 6\rho\cdot 96\rho^6\cdot\cos^5\varphi\sin\varphi\, d\rho=\int_0^{\frac{\pi}{2}}\cos^5\varphi\cdot\sin\varphi\, d\varphi\cdot 72=$$

$$=72\int_0^{\pi/2}\cos^5\varphi\cdot\sin\varphi\, d\varphi =$$

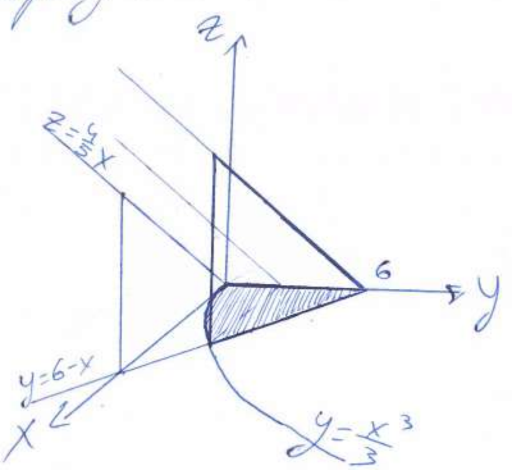
$$\int\cos^5\varphi\cdot\sin\varphi\, d\varphi=\{d\cos\varphi=-\sin\varphi d\varphi\}=-\int\cos^5\varphi\, d\cos\varphi=-\frac{\cos^6\varphi}{6}$$

$$= 72\left(-\frac{\cos^6\varphi}{6}\right)\Big|_0^{\frac{\pi}{2}}=72\left(-\frac{1}{6}\cos^6\frac{\pi}{2}+\frac{1}{6}\cos^6 0^\circ\right)=72\cdot\frac{1}{6}=12$$

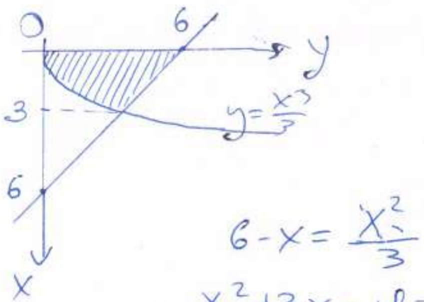
Ответ: 12

$\boxed{10.24}$ $x+y=6$, $x=\sqrt{3y}$, $z=\frac{4x}{5}$, $z=0$

Изобразим тело:



Проекция на ось Oxy:



$$6-x=\frac{x^2}{3}$$
$$x^2+3x-18=0$$
$$x=3$$

Область интегрирования:

$$V:\begin{cases}0\le x\le 3\\ \frac{x^2}{3}\le y\le 6-x\\ 0\le z\le\frac{4}{5}x\end{cases}$$

Найдём объём тела: $V=\iiint dx\,dy\,dz$

$$V=\int_0^3 dx\int_{\frac{x^2}{3}}^{6-x}dy\int_0^{\frac{4}{5}x}dz=\int_0^3 dx\int_{\frac{x^2}{3}}^{6-x}\frac{4}{5}x\,dy=\frac{4}{5}\int_0^3 x\,dx\int_{\frac{x^2}{3}}^{6-x}dy=\frac{4}{5}\int_0^3 x\,dx\cdot y\Big|_{\frac{x^2}{3}}^{6-x}=$$

$$=\frac{4}{5}\int_0^3 x\left(6-x-\frac{x^2}{3}\right)dx=\frac{4}{5}\left(6\cdot\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{12}\right)\Big|_0^3=\frac{4}{5}\left(27-9-\frac{27}{4}\right)=\frac{4}{5}\cdot 9\cdot\frac{5}{4}=9$$

Ответ: 9

$\boxed{11.24}$ $x^2+y^2=9x$, $x^2+y^2=12x$, $z=\sqrt{x^2+y^2}$, $z=0$, $y=0$ $(y\geq 0)$

Рассмотрим цилиндр 1:

$x^2+y^2=9x$

$x^2-9x+y^2=0$

$x^2+y^2-9x+(\frac{9}{2})^2=(\frac{9}{2})^2$

$(x-\frac{9}{2})^2+y^2=(\frac{9}{2})^2$

Рассмотрим цилиндр 2:

$x^2+y^2=12x$

$(x-6)^2+y^2=36$

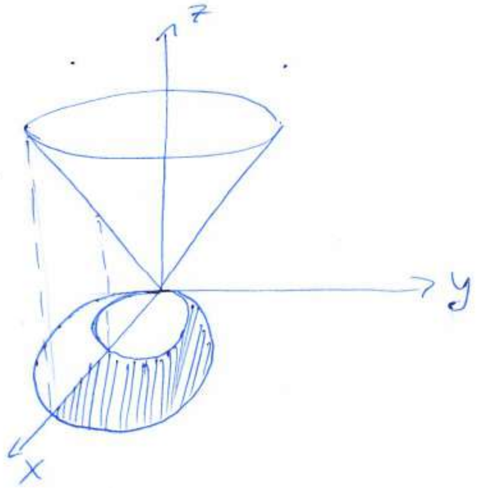
$z=\sqrt{x^2+y^2}$ – конус.

В цилиндрических координатах:

$\rho^2=9\rho\cos\varphi \Rightarrow \rho=9\cos\varphi$

$\rho^2=12\rho\cos\varphi \Rightarrow \rho=12\cos\varphi$

$z=\rho$



Область интегрирования:

$$V:\begin{cases} 0\leq\varphi\leq\frac{\pi}{2} \\ 9\cos\varphi\leq\rho\leq 12\cos\varphi \\ 0\leq z\leq\rho \end{cases}$$

Найдём объём тела $V=\iiint\limits_V dx\,dy\,dz$

$$V=\int_0^{\pi/2}d\varphi\int_{9\cos\varphi}^{12\cos\varphi}\rho\,d\rho\int_0^{\rho}dz=\int_0^{\frac{\pi}{2}}d\varphi\int_{9\cos\varphi}^{12\cos\varphi}\rho^2d\rho=\int_0^{\frac{\pi}{2}}d\varphi\cdot\frac{\rho^3}{3}\Big|_{9\cos\varphi}^{12\cos\varphi}=$$

$$=\int_0^{\frac{\pi}{2}}\frac{12^3\cos^3\varphi-9^3\cos^3\varphi}{3}d\varphi=\frac{12^3-9^3}{3}\int_0^{\pi/2}\cos^3\varphi\,d\varphi=9\cdot 37\int_0^{\frac{\pi}{2}}(1-\sin^2\varphi)\,d\sin\varphi=$$

$$=9\cdot 37\left(\sin\varphi-\frac{\sin^3\varphi}{3}\right)\Big|_0^{\frac{\pi}{2}}=333\left(\sin\frac{\pi}{2}-\frac{1}{3}\sin^3\frac{\pi}{2}-\sin 0^\circ+\frac{1}{3}\sin 0^\circ\right)=$$

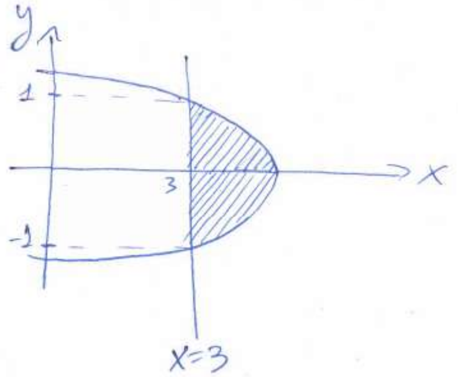
$$=333\cdot\left(1-\frac{1}{3}\right)=333\cdot\frac{2}{3}=222$$

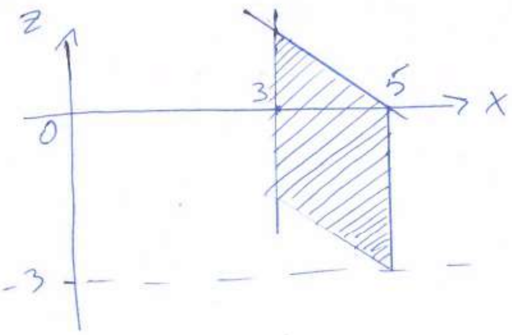
Ответ: 222

12.24 $x=-2y^2+5$, $x=3$, $z=5-\sqrt{x^2+25y^2}$, $z=2-\sqrt{x^2+25y^2}$

$\left.\begin{array}{l} z=5-\sqrt{x^2+25y^2} \\ z=2-\sqrt{x^2+25y^2} \end{array}\right\}$ - эллиптические конусы

Изобразим область интегрирования в проекциях на оси Oxy и Oxz:





Область интегрирования: $\begin{cases} 3 \le x \le -2y^2+5 \\ -1 \le y \le 1 \\ 2-\sqrt{x^2+25y^2} \le z \le 5-\sqrt{x^2+25y^2} \end{cases}$

Найдём объём тела:

$$V=\int_{-1}^{1} dy \int_{3}^{-2y^2+5} dx \int_{2-\sqrt{x^2+25y^2}}^{5-\sqrt{x^2+25y^2}} dz = \int_{-1}^{1} dy \int_{3}^{-2y^2+5} 3\,dx = 3\int_{-1}^{1}(-2y^2+2)\,dy = 6\left(-\frac{y^3}{3}+y\right)\Big|_{-1}^{1} =$$

$$=6\cdot\left(-\frac{1}{3}+1+\frac{(-1)}{3}+1\right)=6\left(-\frac{2}{3}+\frac{6}{3}\right)=6\cdot\frac{4}{3}=8$$

Ответ: 8.

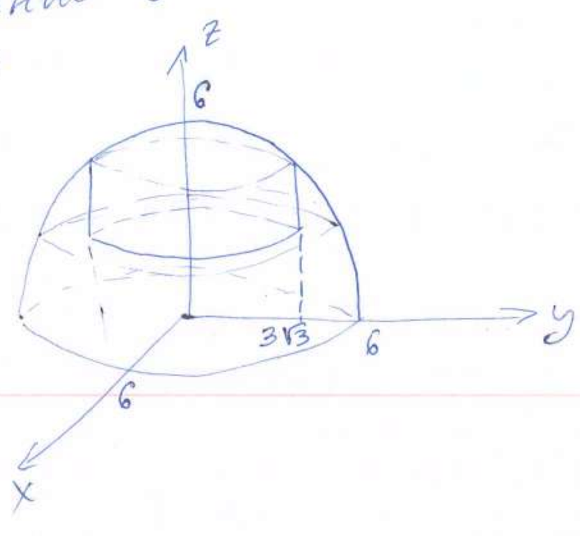
$\boxed{13.24}$ $z=\sqrt{36-x^2-y^2}$, $z=2$, $x^2+y^2=27$ (внутри цилиндра)

$z=\sqrt{36-x^2-y^2}$ - сфера $z^2+x^2+y^2=36$

$x^2+y^2=27$ - цилиндр

Напишем область интегрирования в цилиндрических областях:

$\begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le \rho \le 3\sqrt{3} \\ 2 \le z \le \sqrt{36-\rho^2} \end{cases}$



Найдём объём тела

$$V=\int_0^{2\pi} d\varphi \int_0^{3\sqrt{3}} \rho\, d\rho \int_2^{\sqrt{36-\rho^2}} dz = \int_0^{2\pi} d\varphi \int_0^{3\sqrt{3}} \rho\left(\sqrt{36-\rho^2}-2\right) d\rho =$$

$$= \varphi\Big|_0^{2\pi} \cdot \int_0^{3\sqrt{3}} \rho\left(\sqrt{36-\rho^2}-2\right)d\rho = 2\pi\left(\int_0^{3\sqrt{3}} \sqrt{36-\rho^2}\,\rho\,d\rho - 2\cdot\frac{1}{2}\rho^2\Big|_0^{3\sqrt{3}}\right) =$$

$$= \left\{\begin{matrix} t=36-\rho^2 \\ dt=-2\rho\,d\rho \end{matrix}\right\} = 2\pi\left(-\frac{1}{3}\int_{36}^{9}\sqrt{t}\,dt - \left(3\sqrt{3}\right)^2\right) = 2\pi\left(\left(-\frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_{36}^{9} - 27\right) =$$

$$= 2\pi\left(-\frac{1}{3}(27-216)-27\right) = 2\pi\cdot 36 = 72\pi$$

Ответ: 72π

$\boxed{14.24}$ $z = 2-4[(x-1)^2+y^2]$, $z = 8x-6$

Найдём пересечение графиков:

$$2-4[(x-1)^2+y^2] = 8x-6$$

$$4x^2+4y^2=4$$

$$x^2+y^2=1 \Rightarrow \rho = 1$$

Область интегрирования в цилиндрических координатах:

$$V: \begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le \rho \le 1 \\ 8x-6 \le z \le 2-4[(x-1)^2+y^2] \end{cases} \Rightarrow V: \begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le \rho \le 1 \\ 8\rho\cos\varphi - 6 \le z \le -2-4\rho^2+8\rho\cos\varphi \end{cases}$$

Найдём объём тела:

$$V = \int_0^{2\pi} d\varphi \int_0^1 \rho\, d\rho \int_{8\rho\cos\varphi-6}^{8\rho\cos\varphi-2-4\rho^2} dz = \int_0^{2\pi} d\varphi \int_0^1 \rho(8\rho\cos\varphi - 2 - 4\rho^2 - 8\rho\cos\varphi + 6)\, d\rho =$$

$$= \int_0^{2\pi} d\varphi \int_0^1 4(1-\rho^2)\rho\, d\rho = 4\int_0^{2\pi} d\varphi \int_0^1 (\rho-\rho^3)\, d\rho = 4\int_0^{2\pi} d\varphi \left(\frac{\rho^2}{2} - \frac{\rho^4}{4}\right)\Big|_0^1 =$$

$$= 4\int_0^{2\pi}\left(\frac{1}{2}-\frac{1}{4}\right) d\varphi = 4\int_0^{2\pi} \frac{1}{4}\, d\varphi = 4\,\frac{1}{4}\,\varphi\Big|_0^{2\pi} = 4\cdot 2\pi \cdot \frac{1}{4} = 2\pi$$

Ответ: 2π.