Хатюхин Евгений **А-02-23**

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1.22

$$\int (8-3x)\cdot\cos(5x)\,\mathrm{d}x = \left|\begin{array}{l} u = 8-3x \\ \mathrm{d}u = -3\cdot\mathrm{d}\,x \\ \mathrm{d}v = \cos(5x)\cdot\mathrm{d}x \\ v = \displaystyle\int \cos(5x)\cdot\mathrm{d}x = \left|\begin{array}{l} t = 5x \\ \mathrm{d}t = 5\cdot\mathrm{d}x \end{array}\right| = \frac{\cos t\cdot\mathrm{d}t}{5} = \frac{1}{5}\int \cos t\cdot\mathrm{d}t = \frac{\sin(5x)}{5} \end{array}\right|$$

$$= (8-3x)\cdot\frac{\sin(5x)}{5} + \int \frac{\sin(5x)}{5}\cdot 3\mathrm{d}x = \left|\begin{array}{l} t = 5x \\ \mathrm{d}t = 5\mathrm{d}x \end{array}\right| = (8-3x)\cdot\frac{\sin(5x)}{5} + \frac{3}{5}\int \sin t\cdot\frac{\mathrm{d}t}{5} = (8-3x)\cdot\frac{\sin(5x)}{5} +$$

$$+\frac{3}{25}\cdot(-\cos(5x) + \mathrm{C} = \frac{(8-3x)\cdot\sin(5x)}{5} - \frac{3\cdot\cos(5x)}{25} + \mathrm{C}$$

2.22

$$\int \sqrt{x}\cdot\operatorname{arctg}\sqrt{x}\mathrm{d}x = \left|\begin{array}{l} u = \operatorname{arctg}\sqrt{x} \\ \mathrm{d}u = \dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}} \\ \mathrm{d}v = \sqrt{x}\mathrm{d}x \\ v = \displaystyle\int x^{\frac{1}{2}}\mathrm{d}x = \frac{2\cdot x\sqrt{x}}{3} \end{array}\right| = \frac{2\cdot x\sqrt{x}}{3}\cdot\operatorname{arctg}\sqrt{x} - \int \frac{\cancel{2}\cdot x\cancel{\sqrt{x}}}{3}\cdot\frac{1}{(1+x)\cdot\cancel{2}\cancel{\sqrt{x}}}\cdot\mathrm{d}x =$$

$$\frac{2\cdot x\sqrt{x}}{3}\cdot\operatorname{arctg}\sqrt{x} - \frac{1}{3}\int \frac{x}{(1+x)}\cdot\mathrm{d}x = \left|\begin{array}{l} 1+x = t \\ \mathrm{d}x = \mathrm{d}t \\ x = t-1 \end{array}\right| = \frac{2\cdot x\sqrt{x}}{3}\cdot\operatorname{arctg}\sqrt{x} - \frac{1}{3}\int \frac{t-1}{t}\mathrm{d}t = \frac{2\cdot x\sqrt{x}}{3}\cdot\operatorname{arctg}\sqrt{x} -$$

$$-\frac{1}{3}\left(\int \mathrm{d}t - \int \frac{\mathrm{d}t}{t}\right) = \frac{2\cdot x\sqrt{x}}{3}\cdot\operatorname{arctg}\sqrt{x} - \frac{1}{3}(1+x-\ln(1+x)) + \mathrm{C} = \frac{2\cdot x\sqrt{x}}{3}\cdot\operatorname{arctg}\sqrt{x} - \frac{x}{3} + \frac{\ln(|1+x|)}{3} +$$

$$+\mathrm{C} = \frac{2x\sqrt{x}\operatorname{arctg}\sqrt{x} - x + \ln(|1+x|)}{3} + \mathrm{C}$$

3.22

$$\int \frac{23-10x}{\sqrt{-x^2+6x-5}}\mathrm{d}x = 5\cdot\int \frac{-2x+\frac{23}{5}}{\sqrt{-x^2+6x-5}}\mathrm{d}x = 5\cdot\int \frac{-2x+6-\frac{7}{5}}{\sqrt{-x^2+6x-5}}\mathrm{d}x = 5\cdot\Bigg(\left(\int \frac{-2x+6}{\sqrt{-x^2+6x-5}}\mathrm{d}x\right)^{*} - \frac{7}{5}\times$$

$$\times\left(\int \frac{\mathrm{d}x}{-x^2+6x-5}\right)^{**}\Bigg) \ominus$$

$$* = \begin{vmatrix} -x^2+6x-5 = t \\ (-2x+6x)\mathrm{d}x = \mathrm{d}t \end{vmatrix} = \int \frac{\mathrm{d}t}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{-x^2+6x-5} + C$$

$$** = \int \frac{\mathrm{d}x}{\sqrt{-(x^2-6x+5)}} = \int \frac{\mathrm{d}x}{\sqrt{-((x-3)^2-4)}} = \begin{vmatrix} x-3 = u \\ \mathrm{d}x = \mathrm{d}u \end{vmatrix} = \int \frac{\mathrm{d}u}{\sqrt{4-u^2}} = \arcsin\frac{u}{2} + C = \arcsin\frac{x-3}{2} + C$$

$$\ominus\, 5\cdot(2\sqrt{-x^2+6x-5} - \frac{7}{5}\cdot\arcsin\frac{x-3}{2} + C) = 10\sqrt{-x^2+6x-5} - 7\cdot\arcsin\frac{x-3}{2} + C$$

4.22

$$\int \frac{x^3+6x^2+13x-6}{(x+2)(x-2)^3}\mathrm{d}x \circledequal$$

$$\frac{x^3+6x^2+13x-6}{(x+2)(x-2)^3} = \frac{A^{(x-2)^3}}{x+2} + \frac{B^{(x+2)(x-2)^2}}{x-2} + \frac{C^{(x+2)(x-2)}}{(x-2)^2} + \frac{D^{(x+2)}}{(x-2)^3} =$$

$$= \frac{A(x-2)^3+B(x+2)(x-2)^2+C(x+2)(x-2)+D(x+2)}{(x+2)(x-2)^3}$$

$$x^3+6x^2+13x-6 = A(x-2)^3+B(x+2)(x-2)^2+C(x+2)(x-2)+D(x+2)$$

$$x=2: 8+24+26-6=4D \Rightarrow D=\frac{52}{4}=13$$

$$x=-2: -8+24-26-6=-64A \Rightarrow A=\frac{16}{64}=\frac{1}{4}$$

$$x=0: -6=-\overset{2}{\cancel{8}}\cdot\frac{1}{\cancel{4}}+8B-4C+26$$

$$x=1: 14=-\frac{1}{4}+3B-3C+39$$

$$\begin{cases} 3B-3C=-24\frac{3}{4} \\ 8B-4C=-30 \end{cases}$$

$$\begin{cases} B-C=-\frac{\overset{33}{\cancel{99}}}{4\cdot\cancel{3}} \\ 4B-2C=-15 \end{cases}$$

$$\begin{cases} B=C-\frac{33}{4} \\ 4C-33-2C=-15 \end{cases}$$

$$\begin{cases} C=9 \\ B=\frac{3}{4} \end{cases}$$

$$\circledequal \frac{1}{4}\int\frac{\mathrm{d}x}{x+2}+\frac{3}{4}\int\frac{\mathrm{d}x}{x-2}+9\int\frac{\mathrm{d}x}{(x-2)^2}+13\int\frac{\mathrm{d}x}{(x-2)^3} = \begin{vmatrix} x+2=t; x-2=u \\ \mathrm{d}x=\mathrm{d}t; \mathrm{d}x=\mathrm{d}u \end{vmatrix} = \frac{1}{4}\int\frac{\mathrm{d}t}{t}+\frac{3}{4}\int\frac{\mathrm{d}u}{u}+$$

$$+9\int\frac{\mathrm{d}u}{u^2}+13\int\frac{\mathrm{d}u}{u^3} = \frac{\ln t}{4}+\frac{3\ln u}{4}-\frac{9}{u}-\frac{26}{u^2}+C = \frac{\ln(|x+2|)+3\ln(|x-2|)}{4}-\frac{9}{x-2}-\frac{26}{(x-2)^2}+C$$

5.22

$$\int \frac{\mathrm{d}x}{(x-3)\sqrt{11x^2-54x+67}} = \begin{vmatrix} x-3=t \\ x=t+3 \\ \mathrm{d}x=\mathrm{d}t \end{vmatrix} = \int \frac{\mathrm{d}t}{t\sqrt{11(t^2+6t+9)-54(t+3)+67}} =$$

$$= \int \frac{\mathrm{d}t}{t\sqrt{11t^2+66t+99-54t-162+67}} = \int \frac{\mathrm{d}t}{t\sqrt{11t^2+12t+4}} = \int \frac{\mathrm{d}t}{t^2\sqrt{11+\frac{12}{t}+\frac{4}{t^2}}} = \begin{vmatrix} \frac{1}{t}=u \\ -\frac{\mathrm{d}t}{t^2}=\mathrm{d}u \end{vmatrix} =$$

$$= -\int \frac{\mathrm{d}u}{\sqrt{4u^2+12u+11}} = -\int \frac{\mathrm{d}u}{\sqrt{4(u^2+3u+\frac{11}{4})}} = -\frac{1}{2}\int \frac{\mathrm{d}u}{\sqrt{(u+\frac{3}{2})^2+\frac{1}{2}}} =$$

$$= -\frac{1}{2}\ln\left|u+\frac{3}{2}+\sqrt{\left(u+\frac{3}{2}\right)^2+\frac{1}{2}}\right| + C = -\frac{1}{2}\ln\left|\frac{1}{x-3}+\frac{3}{2}+\sqrt{\left(\frac{1}{x-3}+\frac{3}{2}\right)^2+\frac{1}{2}}\right| + C$$

6.22

$$\int \frac{\sqrt{3x-2}}{(3x-2)^2\sqrt{3x+5}}\mathrm{d}x = \begin{vmatrix} 3x-2=t \\ 3x=t+2 \\ 3\mathrm{d}x=\mathrm{d}t \end{vmatrix} = \frac{1}{3}\int \frac{\sqrt{t}}{t^2\sqrt{t+7}}\mathrm{d}t = \frac{1}{3}\int \frac{\mathrm{d}t\cancel{\sqrt{t}}}{t^2\cancel{\sqrt{t}}\sqrt{1+\frac{7}{t}}} = \begin{vmatrix} \frac{1}{t}=u \\ -\frac{\mathrm{d}t}{t^2}=\mathrm{d}u \end{vmatrix} =$$

$$= -\frac{1}{3}\int \frac{\mathrm{d}u}{\sqrt{1+7u}} = \begin{vmatrix} 1+7u=v \\ 7\mathrm{d}u=\mathrm{d}v \end{vmatrix} = -\frac{1}{21}\int \frac{\mathrm{d}v}{\sqrt{v}} = -\frac{2}{21}\cdot\sqrt{v}+C = \begin{bmatrix} v=1+7u= \\ =1+\frac{7}{t}= \\ =1+\frac{7}{3x-2} \end{bmatrix} = -\frac{2}{21}\sqrt{1+\frac{7}{3x-2}}+C$$

8.22

$$\int\sqrt{5x^2-x+10}\mathrm{d}x = \sqrt{5}\int\sqrt{x^2+\frac{1}{5}x+2}\mathrm{d}x = \sqrt{5}\int\sqrt{\left(x-\frac{1}{10}\right)^2+\frac{199}{100}}\mathrm{d}x =$$

$$= \sqrt{5}\cdot\frac{10x-1}{20}\sqrt{x^2-\frac{1}{5}x+2}+\frac{199}{200}\ln\left|x-\frac{1}{10}+\sqrt{x^2-\frac{1}{5}x+2}\right|+C$$

9.22

$$\int \frac{\mathrm{d}x}{\sqrt{\cos^5 x \cdot \sin^3 x}} = \int \frac{\mathrm{d}x}{\cos^2 x\sqrt{\cos x \cdot \sin x} \cdot \sin x} = \int \frac{\mathrm{d}x}{\cos^2 x\sqrt{\cos^2 x \cdot \operatorname{tg} x} \cdot \cos x \cdot \operatorname{tg} x} = \int \frac{\operatorname{tg}^2 x + 1}{\cos^2 x\sqrt{\operatorname{tg} x} \cdot \operatorname{tg} x}\mathrm{d}x =$$

$$= \left| \begin{array}{l} \operatorname{tg} x = t \\ \dfrac{\mathrm{d}x}{\cos^2 x} = \mathrm{d}t \end{array} \right| = \int \frac{(t^2+1)}{t \cdot \sqrt{t}}\mathrm{d}t = \int \frac{\cancelto{\sqrt{t}}{t^2}}{\cancel{\sqrt{t}} \cdot \cancel{t}}\mathrm{d}t + \int \frac{\mathrm{d}t}{t^{\frac{3}{2}}} = \int \sqrt{t}\mathrm{d}t + \int \frac{\mathrm{d}t}{t^{\frac{3}{2}}} = \frac{2}{3} \cdot t^{\frac{3}{2}} - 2 \cdot \frac{1}{\sqrt{t}} + C = \frac{2t^2+6}{3\sqrt{t}} + C =$$

$$= \frac{2\operatorname{tg}^2 x - 6}{3\sqrt{\operatorname{tg} x}} + C$$

10.22

$$\int \frac{\mathrm{d}x}{3\sin^2 x - 7\cos^2 x - 2} = \int \frac{\mathrm{d}x}{3\sin^2 x + 3\cos^2 x - 10\cos^2 x - 2} = \int \frac{\mathrm{d}x}{1 - 10\cos^2 x} = \int \frac{\mathrm{d}x}{\cos^2 x\left(\dfrac{1}{\cos^2 x} - 10\right)} =$$

$$= \int \frac{\mathrm{d}x}{\cos^2 x\left(\operatorname{tg}^2 x - 9\right)} = \left| \begin{array}{l} \operatorname{tg} x = t \\ \dfrac{\mathrm{d}x}{\cos^2 x} = \mathrm{d}t \end{array} \right| = \int \frac{\mathrm{d}t}{t^2 - 9} = \frac{1}{6}\ln\left|\frac{t-3}{t+3}\right| + C = \frac{1}{6}\ln\left|\frac{\operatorname{tg} x - 3}{\operatorname{tg} x + 3}\right| + C$$

11.22

$$\int_0^{\pi/4} \frac{\sqrt{\sin x}}{\sqrt{\cos^9 x}}\mathrm{d}x = \int_0^{\pi/4} \frac{\sqrt{\sin x}}{\cos^4 x \cdot \sqrt{\cos x}}\mathrm{d}x = \int_0^{\pi/4} \frac{\sqrt{\operatorname{tg} x}(1 + \operatorname{tg}^2 x)}{\cos^2 x}\mathrm{d}x = \left| \begin{array}{l} \operatorname{tg} x = t \\ \dfrac{\mathrm{d}x}{\cos^2 x} = \mathrm{d}t \\ t \in [0;1] \end{array} \right| = \int_0^1 \sqrt{t} \cdot (1 + t^2)\mathrm{d}t =$$

$$= \int_0^1 t^{\frac{1}{2}}\,\mathrm{d}t + \int_0^1 t^{\frac{5}{2}}\,\mathrm{d}t = \frac{2}{3} \cdot t^{\frac{3}{2}}\Big|_0^1 + \frac{2}{7}t^{\frac{7}{2}}\Big|_0^1 = \frac{2}{3} + \frac{2}{7} = \frac{20}{21}$$

12.22

$$\int_0^1 2^x\left(x^2-2x\right)\mathrm{d}x=\left(\int_0^1 2^x\cdot x^2\mathrm{d}x\right)^{*}-2\left(\int_0^1 x\cdot 2^x\mathrm{d}x\right)^{**}\ominus$$

$$* = \int_0^1 2^x\cdot x^2\mathrm{d}x=\begin{vmatrix} u=x^2 & \mathrm{d}v=2^x \\ \mathrm{d}u=2x & v=\dfrac{2^x}{\ln 2}\end{vmatrix}=\left.\frac{x^2\cdot 2^x}{\ln 2}\right|_0^1-\frac{2}{\ln 2}\left(\int_0^1 x\cdot 2^x\mathrm{d}x\right)^{***}$$

$$*** = ** = \int_0^1 x\cdot 2^x\mathrm{d}x=\begin{vmatrix} u=x & \mathrm{d}v=2^x \\ \mathrm{d}u=1 & v=\dfrac{2^x}{\ln 2}\end{vmatrix}=\left.\frac{x\cdot 2^x}{\ln 2}\right|_0^1-\frac{1}{\ln 2}\int_0^1 2^x\mathrm{d}x=\left.\frac{x\cdot 2^x}{\ln 2}\right|_0^1-\frac{1}{\ln 2}\left(\left.\frac{2^x}{\ln 2}\right|_0^1\right)$$

$$\ominus\left.\frac{x^2\cdot 2^x}{\ln 2}\right|_0^1-\frac{2}{\ln 2}\left(\left.\frac{x\cdot 2^x}{\ln 2}\right|_0^1-\frac{1}{\ln 2}\left(\left.\frac{2^x}{\ln 2}\right|_0^1\right)\right)-2\cdot\left(\left.\frac{x\cdot 2^x}{\ln 2}\right|_0^1-\frac{1}{\ln 2}\left(\left.\frac{2^x}{\ln 2}\right|_0^1\right)\right)=$$

$$=\frac{1\cdot 2}{\ln 2}-\frac{2}{\ln 2}\left(\frac{1\cdot 2}{\ln 2}-\frac{1}{\ln^2 2}\left(2^1-2^0\right)\right)-2\cdot\left(\frac{1\cdot 2}{\ln 2}-\frac{1}{\ln^2 2}\left(2^1-2^0\right)\right)=$$

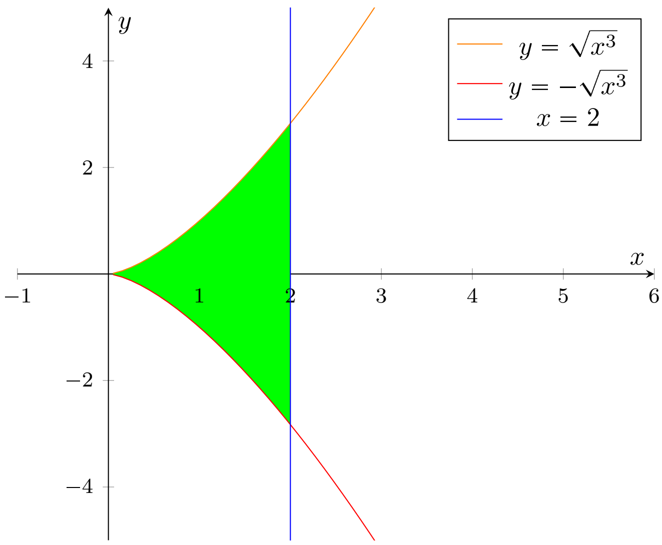
$$=\frac{2}{\ln 2}-\frac{2}{\ln 2}\left(\frac{2}{\ln 2}-\frac{1}{\ln^2 2}\right)-2\cdot\left(\frac{2}{\ln 2}-\frac{1}{\ln^2 2}\right)=\frac{2}{\ln 2}-\frac{4}{\ln^2 2}+\frac{2}{\ln^3 2}-\frac{4}{\ln 2}+\frac{2}{\ln^2 2}=$$

$$=\frac{2}{\ln^3 2}-\frac{2}{\ln^2 2}-\frac{2}{\ln 2}$$

13.22

$$S=\int\limits_0^2 \mathrm{d}x \int\limits_{-\sqrt{x^3}}^{\sqrt{x^3}} 1\mathrm{d}y = \int\limits_0^2(\sqrt{x^3}+\sqrt{x^3})\mathrm{d}x =$$

$$=2\int\limits_0^2 x^{\frac{3}{2}}\mathrm{d}x = 2\cdot\frac{2}{5}\cdot(x^2\sqrt{x})\Big|_0^2 = \frac{4}{5}\cdot 4\sqrt{2}=\frac{16\sqrt{2}}{5}$$



15.22

$$y=e^x+x,\ \frac{\ln 8}{2}\leqslant x\leqslant\frac{\ln 15}{2}$$

$$y' = e^x$$

$$y'^2=e^{2x}$$

$$l=\int\limits_{\frac{\ln 8}{2}}^{\frac{\ln 15}{2}}\sqrt{1+e^{2x}}\mathrm{d}x=\begin{vmatrix}2x=t\\ 2\mathrm{d}x=\mathrm{d}t\\ t\in[\ln 8;\ln 15]\end{vmatrix}=\frac{1}{2}\int\limits_{\ln 8}^{\ln 15}\sqrt{1+e^t}\mathrm{d}t=\int\limits_{\ln 8}^{\ln 15}\frac{e^t\cdot\sqrt{1+e^t}}{e^t}\mathrm{d}t=\begin{vmatrix}1+e^t=u\\ e^t\mathrm{d}t=\mathrm{d}u\\ u\in[9;16]\end{vmatrix}=\frac{1}{2}\int\limits_9^{16}\frac{\sqrt{u}}{u-1}=$$

$$=\begin{vmatrix}u=v^2\\ \mathrm{d}u=2v\mathrm{d}v\\ v\in[3;4]\end{vmatrix}=\frac{2}{2}\int\limits_3^4\frac{v^2}{v^2-1}\mathrm{d}v=\left|\begin{array}{l|l} v^2 & v^2-1 \\ \cline{2-2} v^2-1 & 1 \\ \quad 1 & \end{array}\right|=\int\limits_3^4 1+\frac{1}{v^2-1}\mathrm{d}v=\int\limits_3^4 1\mathrm{d}v+\int\limits_3^4\frac{\mathrm{d}v}{v^2-1}=v\Big|_3^4+\frac{1}{2}\times$$

$$\times\ln\left|\frac{x-1}{x+1}\right|\Bigg|_3^4=(4-3)+\frac{1}{2}\left(\ln\left(\frac{3}{5}\right)-\ln\left(\frac{2}{4}\right)\right)=1+\frac{1}{2}\ln\left(\frac{6}{5}\right)$$