

w1.16

$$\int e^{4\sin^2 x} \sin 2x dx = 2 \int e^{4\sin^2 x} \cos x \sin x dx = 2 \int \frac{e^{4t} \cdot \cos x \sin x dt}{2 \cdot \cos x \sin x}$$

$$\sin^2 x = t$$

$$dt = 2 \sin x \cos x$$

$$= 2 \int \frac{e^{4t} dt}{2} = \int e^{4t} dt =$$

$$= \frac{1}{4} e^{4\sin^2 x} + C$$

w2.16

$$\int 3^{-x} \cdot (x+2) dx = \int \frac{1}{3^x} \cdot (x+2) dx$$

$$u = x+2$$

$$du = 1$$

$$dv = \frac{1}{3^x}$$

$$v = -\frac{1}{\ln 3 \cdot 3^x}$$

$$\int \frac{1}{3^x} \cdot (x+2) dx = -(x+2) \cdot \frac{1}{\ln 3 \cdot 3^x} + \int \left(\frac{1}{\ln 3 \cdot 3^x} dx \right) = -(x+2) \cdot \frac{1}{\ln 3 \cdot 3^x} + \frac{1}{\ln 3} \cdot \frac{1}{\ln 3 \cdot 3^x}$$

$$\frac{1}{\ln 3} \int \frac{1}{3^x} dx = \frac{1}{\ln 3} \cdot \int 3^{-x} dx = \frac{1}{\ln 3} \cdot -\frac{1}{\ln 3 \cdot 3^x}$$

w3.16

$$\int \frac{24x^3 - 12x^2 + 11x - 2}{4x^2 - 2x + 1} dx = \int \frac{5x-2}{4x^2-2x+1} + 6x dx \quad \Bigg| \quad - \frac{24x^3 - 12x^2 + 11x - 2}{24x^3 - 12x^2 + 6x} \frac{4x^2 - 2x + 1}{6x} \frac{5x-2}{5x-2}$$

$$\frac{4x^2 - 2x + 1}{4x^2 - 2x + 1} = 1$$

$$(24x^3 - 12x^2 + 11x - 2) - (4x^2 - 2x + 1) =$$

$$= 24x^3 - 16x^2 + 13x - 1$$

$$\frac{5x-2}{4x^2-2x+1} dx$$

$$\int \frac{5x-2}{4x^2-2x+1} dx =$$

$$5x-2 = \frac{5}{8}(8x-2) - \frac{3}{4}$$

$$\frac{5}{8} \int \frac{8x-2}{4x^2-2x+1} dx - \int \frac{3}{4(4x^2-2x+1)} dx$$

$$\downarrow$$

$$t = 4x^2 - 2x + 1$$

$$dt = (8x-2) dx$$

$$\downarrow$$

$$\frac{5}{8} \int \frac{(8x-2)}{t \cdot (8x-2)} dt = \frac{5}{8} \int \frac{1}{t} dt = \frac{5}{8} \ln t = \frac{5}{8} \ln(4x^2 - 2x + 1)$$

$$- \int \frac{3}{4(4x^2-2x+1)} dx = -\frac{3}{4} \int \frac{1}{4x^2-2x+1} dx$$

$$\int \frac{1}{4(x^2 - \frac{x}{2} + \frac{1}{4})} dx = \frac{1}{4} \int \frac{1}{(x - \frac{1}{4})^2 + \frac{3}{16}} dx = \frac{1}{4} \int \frac{1}{(x - \frac{1}{4})^2 + \frac{3}{16}} dx = \frac{1}{4} \int \frac{1}{t^2 + \frac{3}{16}} dt$$

$$(x - \frac{1}{4})^2 = x^2 - \frac{x}{2} + \frac{1}{16}$$

$$(x - \frac{1}{4}) = t$$

$$dt = dx$$

$$= -\frac{3}{16} \cdot \frac{4 \arctan(\frac{4t}{\sqrt{3}})}{\sqrt{3}}$$

$$= \frac{\sqrt{3} \arctan(\frac{4t}{\sqrt{3}})}{4}$$

$$* = \frac{5}{8} \ln(4x^2 - 2x + 1) - \frac{\sqrt{3} \arctan(\frac{4x-1}{\sqrt{3}})}{4} + C$$

w4.16

$$\int \frac{5x^2+5x-7}{(x+1)^2(x^2+4)} dx$$

$$\sqrt{(x+1)^2} \quad \sqrt{(x^2+4)(x+1)} \quad \sqrt{x^2+4}$$

$$\frac{5x^2+5x-7}{(x+1)^2(x^2+4)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

$$= \frac{Ax+B(x^2+2x+1) + C(x^3+x^2+4x+4) + D(x^2+4)}{(x+1)^2(x^2+4)}$$

$$= \frac{Ax^3+2Ax^2+Ax+Bx^2+2Bx+B + Cx^3+Cx^2+4Cx+4C + Dx^2+4D}{(x+1)^2(x^2+4)}$$

$$= \frac{(A+C)x^3 + (2A+B+C+D)x^2 + (4A+2B+4C)x + B+4C+4D}{(x+1)^2(x^2+4)}$$

$$\int \frac{1}{(x+2)^2 \sqrt{8x^2+26x+11}} dx = - \int t \frac{(x+2)^2 dt}{(x+2) \sqrt{8x^2+26x+11}} =$$

$$t = \frac{1}{(x+2)} \quad dt = -\frac{1}{(x+2)^2} dx$$

$$x = -\frac{2t-1}{t}$$

$$= - \int \frac{t dt}{t \cdot \frac{1}{t} \sqrt{9t^2-6t+8}} = - \int \frac{t dt}{\sqrt{9-(3t+1)^2}}$$

$$\sqrt{9-(3t+1)^2} = \sqrt{-(3t+1)^2+9}$$

$$3t+1=u$$

$$du = 3dt$$

$$t = \frac{u-1}{3}$$

$$= \frac{1}{9} \int \frac{u-1}{\sqrt{9-u^2}} du$$

$$= -\frac{1}{9} \left(\int \frac{u du}{\sqrt{9-u^2}} - \int \frac{1 du}{\sqrt{9-u^2}} \right)$$

$$= -\frac{1}{9} \sqrt{9-u^2} - \frac{\arcsin\left(\frac{u}{3}\right)}{9}$$

$$= -\frac{1}{9} \sqrt{9-(3t+1)^2} - \frac{\arcsin\left(\frac{3t+1}{3}\right)}{9}$$

$$= -\frac{\sqrt{9-\left(\frac{3}{(x+2)}+1\right)^2}}{9} - \frac{\arcsin\left(\frac{3}{(x+2)}+1\right)}{9} + C$$

$$8x^2+26x+11 =$$

$$= 8\left(\frac{2t-1}{t}\right)^2 - 26\frac{2t-1}{t} + 11$$

$$= \frac{8(4t^2-4t+1)}{t^2} - \frac{52t-26}{t} + 11$$

$$= \frac{32t^2-32t+8-52t^2+26t+11t^2}{t^2}$$

$$= \frac{-20t^2-6t+8}{t^2}$$

Ex 6.16

$$\int \frac{\sqrt{3x+2}}{(3x+2)^2 \sqrt{3x+1}} dx = \int \frac{1}{\sqrt{3x+1} (3x+2)^{\frac{3}{2}}} = \frac{1}{3} \int \frac{1}{\sqrt{t+1} (t+2)^{\frac{3}{2}}} dt =$$

$$t=3x, dt=3dx$$

$$u = \sqrt{\frac{t+1}{t+2}}$$

$$t+1 = (t+2) \cdot u^2$$

$$t = \frac{1}{u^2-1} - 2$$

$$dt = \frac{2u}{(u^2-1)^2} du$$

$$= \frac{1}{3} \int 2du = \frac{1}{3} \cdot 2u + C =$$

$$= \frac{1}{3} \cdot 2 \left(\sqrt{\frac{t+1}{t+2}} \right) + C$$

$$= \frac{2}{3} \cdot \sqrt{\frac{3x+1}{3x+2}} + C$$

8.16

$$\int \sqrt{5x^2-4x+7} dx = \int \sqrt{\left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}} dx = \int \frac{\sqrt{t^2 + \frac{31}{5}}}{\sqrt{5}} dt =$$

$$5x^2-4x+7 = \left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}$$

$$= \frac{1}{\sqrt{5}} \int \sqrt{t^2 + \frac{31}{5}} dt =$$

$$t = \sqrt{5}x - \frac{2}{\sqrt{5}}$$

$$x = \frac{\sqrt{5}t+2}{5} \quad dx = \frac{1}{\sqrt{5}} dt$$

$$\frac{1}{\sqrt{5}} \left(\frac{t}{2} \sqrt{t^2 + \frac{31}{5}} + \frac{31}{10} \ln(t + \sqrt{t^2 + \frac{31}{5}}) \right)$$

$$= \frac{1}{\sqrt{5}} \cdot \left(\frac{31 \ln(t + \sqrt{t^2 + \frac{31}{5}})}{10} + \frac{t \sqrt{t^2 + \frac{31}{5}}}{2} \right)$$

$$= \frac{1}{\sqrt{5}} \cdot \left(\frac{31 \ln\left(\sqrt{5}x - \frac{2}{\sqrt{5}} + \sqrt{\left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}}\right)}{10} + \frac{\sqrt{5}x - \frac{2}{\sqrt{5}} \sqrt{\left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}}}{2} \right)$$

+ C

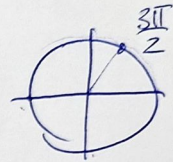
w14.16

$$\rho = 2 \sin \frac{2\varphi}{3}$$

$$2 \sin \frac{2\varphi}{3} \geq 0 \quad \sin \frac{2\varphi}{3} \geq 0$$

$$0 \leq \frac{2\varphi}{3} \leq \pi$$

$$0 \leq 2\varphi \leq 3\pi \quad 0 \leq \varphi \leq \frac{3\pi}{2}$$



$$S = \frac{1}{2} \int_0^{\frac{3\pi}{2}} 4 \sin^2 \frac{2\varphi}{3} d\varphi$$

$$\int 4 \sin^2 \frac{2\varphi}{3} d\varphi = 4 \int \frac{3 \sin^2 \frac{2\varphi}{3}}{2} d\varphi = 6 \int \sin^2 t dt =$$

$$S = \frac{1}{2} \left(2\varphi - 3 \sin \frac{4\varphi}{3} \right) \Big|_0^{\frac{3\pi}{2}} = 6 \int \frac{1 - \cos 2t}{2} dt = 6 \left(\frac{1}{2} \left(\frac{2\varphi}{3} \right) - \frac{\sin \frac{4\varphi}{3}}{2} \right)$$

$$S = \frac{1}{2} \left(3\pi - 3 \sin \frac{4 \cdot 3\pi}{2 \cdot 3} \right) - (0)$$

$$S = \frac{3\pi}{2} - \frac{3}{2} \sin 2\pi \quad S = \frac{3\pi}{2}$$

w/5.16

$$x = \frac{2}{3}y^{3/2}, 0 \leq y \leq 3$$

$$f'(y) = \frac{2}{3} \cdot \frac{3}{2} \cdot \sqrt{y}$$

$$S = \int_0^3 \sqrt{1 + (f'(y))^2} dy = \int_0^3 \sqrt{1 + y} dy = \left(\frac{2}{3} (1+y)^{3/2} \right) \Big|_0^3 = \frac{2}{3} (8 - 1) = \frac{14}{3}$$

$$\int \sqrt{1+y} dy = \int \sqrt{t} dt = \frac{2}{3} \cdot t^{3/2} = \frac{2}{3} \cdot (1+y)^{3/2} = \frac{2y\sqrt{1+y}}{3} + \frac{2\sqrt{1+y}}{3}$$

$1+y=t$
 $dt=dy$

$$\begin{aligned} \frac{2}{3} (1+y)^{3/2} \Big|_0^3 &= \frac{2}{3} \sqrt{4^3} - \frac{2}{3} \sqrt{1} = \\ &= \frac{16}{3} - \frac{2}{3} = \frac{14}{3} \end{aligned}$$

$$\left(\frac{2y\sqrt{1+y}}{3} + \frac{2\sqrt{1+y}}{3} \right) \Big|_0^3 = \frac{6 \cdot \sqrt{4}}{3} + \frac{2 \cdot \sqrt{4}}{3} = \frac{12+4}{3} = \frac{16}{3}$$

w13.16

$$y = \sqrt{x}, y = x^3$$

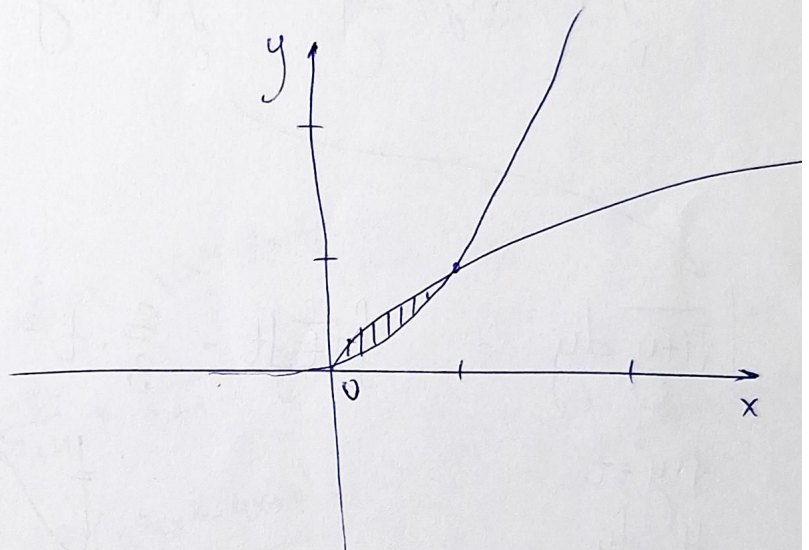
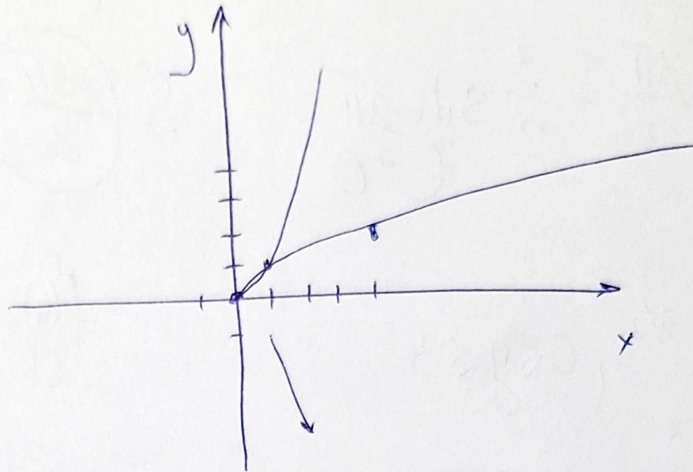
$$S = \int_0^1 \sqrt{x} - x^3 dx =$$

$$= \int_0^1 \sqrt{x} dx - \int_0^1 x^3 dx =$$

$$= \frac{2}{3} \cdot x^{\frac{3}{2}} \Big|_0^1 - \frac{1}{4} x^4 \Big|_0^1 =$$

$$= \frac{2}{3} \cdot 1^{\frac{3}{2}} - 0 - \frac{1}{4} 1^4 - 0 =$$

$$= \frac{2}{3} - \frac{1}{4} = \left(\frac{5}{12} \right)$$



$$\int \frac{dx}{2\sin^2 x + \cos^2 x + 1} \quad \text{w10.16} = \int \frac{dx}{3 - \cos^2 x} = \int \frac{1}{3t^2 + 2} dt = \frac{1}{3} \int \frac{1}{t^2 + \frac{2}{3}} =$$

$$= \frac{1}{3} \frac{\sqrt{3} \operatorname{arctg}\left(\frac{\sqrt{3}t}{\sqrt{2}}\right)}{\sqrt{2}}$$

$$2\sin^2 x + \cos^2 x + 1 = \cos^2 x + 2(1 - \cos^2 x) + 1 = \cos^2 x + 2 - 2\cos^2 x + 1 =$$

$$= 3 - \cos^2 x$$

$$t = \operatorname{tg}(x)$$

$$dt = \frac{1}{\cos^2 x} dx$$

$$\cos^2 x = \frac{1}{t^2 + 1}$$

$$\frac{\operatorname{arctg}\left(\frac{\sqrt{3} \operatorname{tg} x}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{3}} + C$$

W12.16

$$\int_0^1 \arccos \frac{x}{2} dx \xrightarrow{*} \int 2 \arccos t dt$$

$$\frac{x}{2} = t \quad dt = \frac{1}{2} dx$$

$$u = \arccos t$$

$$du = -\frac{1}{\sqrt{1-t^2}} dt$$

$$dv = 1 dt$$

$$v = t$$

$$\int 2 \arccos t dt = 2 \arccos t \cdot t + \int \frac{t}{\sqrt{1-t^2}} dt = 2 \arccos t \cdot t - \sqrt{1-t^2}$$

$$= 2 \arccos \frac{x}{2} \cdot \frac{x}{2} - \sqrt{1 - \left(\frac{x}{2}\right)^2}$$

$$2 \arccos t \cdot t - \sqrt{1-t^2} = 2 \arccos \frac{x}{2} \cdot \frac{x}{2} - \sqrt{4-x^2}$$

$$* = \left. \arccos \frac{x}{2} \cdot x - \sqrt{4-x^2} \right|_0^1 = \arccos \frac{1}{2} \cdot 1 - \sqrt{3} - \arccos 0 \cdot 0 - \sqrt{4}$$
