

w1.16

$$\int e^{4\sin^2 x} \sin 2x dx = 2 \int e^{4\sin^2 x} \cos x \sin x dx = 2 \int \frac{e^{4t} \cdot \cos x \sin x dt}{2 \cdot \cos x \sin x}$$

$$\sin^2 x = t$$

$$dt = 2 \sin x \cos x$$

$$= 2 \int \frac{e^{4t} dt}{2} = \int e^{4t} dt =$$

$$= \frac{1}{4} e^{4\sin^2 x} + C$$

w2.16

$$\int 3^{-x} \cdot (x+2) dx = \int \frac{1}{3^x} \cdot (x+2) dx$$

$$u = x+2$$

$$du = 1$$

$$dv = \frac{1}{3^x}$$

$$v = -\frac{1}{\ln 3 \cdot 3^x}$$

$$\int \frac{1}{3^x} \cdot (x+2) dx = -(x+2) \cdot \frac{1}{\ln 3 \cdot 3^x} + \int \left( \frac{1}{\ln 3 \cdot 3^x} dx \right) = -(x+2) \cdot \frac{1}{\ln 3 \cdot 3^x} + \frac{1}{\ln 3} \cdot \frac{1}{\ln 3 \cdot 3^x}$$

$$\frac{1}{\ln 3} \int \frac{1}{3^x} dx = \frac{1}{\ln 3} \cdot \int 3^{-x} dx = \frac{1}{\ln 3} \cdot -\frac{1}{\ln 3 \cdot 3^x}$$

w3.16

$$\int \frac{24x^3 - 12x^2 + 11x - 2}{4x^2 - 2x + 1} dx = \int \frac{5x-2}{4x^2-2x+1} + 6x dx \quad \Bigg| \quad - \frac{24x^3 - 12x^2 + 11x - 2}{24x^3 - 12x^2 + 6x} \frac{4x^2 - 2x + 1}{6x} \frac{5x-2}{5x-2}$$

$$\frac{4x^2 - 2x + 1}{4x^2 - 2x + 1} = 1$$

$$(24x^3 - 12x^2 + 11x - 2) - (4x^2 - 2x + 1) =$$

$$= 24x^3 - 16x^2 + 13x - 1$$

$$\frac{5x-2}{4x^2-2x+1} dx$$

$$\int \frac{5x-2}{4x^2-2x+1} dx =$$

$$5x-2 = \frac{5}{8}(8x-2) - \frac{3}{4}$$

$$\frac{5}{8} \int \frac{8x-2}{4x^2-2x+1} dx - \int \frac{3}{4(4x^2-2x+1)} dx$$

$$\downarrow$$

$$t = 4x^2 - 2x + 1$$

$$dt = (8x-2) dx$$

$$\downarrow$$

$$\frac{5}{8} \int \frac{(8x-2)}{t \cdot (8x-2)} dt = \frac{5}{8} \int \frac{1}{t} dt = \frac{5}{8} \ln t = \frac{5}{8} \ln(4x^2 - 2x + 1)$$

$$- \int \frac{3}{4(4x^2-2x+1)} dx = -\frac{3}{4} \int \frac{1}{4x^2-2x+1} dx$$

$$\int \frac{1}{4(x^2 - \frac{x}{2} + \frac{1}{4})} dx = \frac{1}{4} \int \frac{1}{(x - \frac{1}{4})^2 + \frac{3}{16}} dx = \frac{1}{4} \int \frac{1}{(x - \frac{1}{4})^2 + \frac{3}{16}} dx = \frac{1}{4} \int \frac{1}{t^2 + \frac{3}{16}} dt$$

$$(x - \frac{1}{4})^2 = x^2 - \frac{x}{2} + \frac{1}{16}$$

$$(x - \frac{1}{4}) = t$$

$$dt = dx$$

$$= -\frac{3}{16} \cdot \frac{4 \arctan(\frac{4t}{\sqrt{3}})}{\sqrt{3}}$$

$$= \frac{\sqrt{3} \arctan(\frac{4t}{\sqrt{3}})}{4}$$

$$* = \frac{5}{8} \ln(4x^2 - 2x + 1) - \frac{\sqrt{3} \arctan(\frac{4x-1}{\sqrt{3}})}{4} + C$$



w4.16

$$\int \frac{5x^2+5x-7}{(x+1)^2(x^2+4)} dx$$

$$\sqrt{(x+1)^2} \quad \sqrt{(x^2+4)(x+1)} \quad \sqrt{x^2+4}$$

$$\frac{5x^2+5x-7}{(x+1)^2(x^2+4)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

$$= \frac{Ax+B(x^2+2x+1) + C(x^3+x^2+4x+4) + D(x^2+4)}{(x+1)^2(x^2+4)}$$

$$= \frac{Ax^3+2Ax^2+Ax+Bx^2+2Bx+B + Cx^3+Cx^2+4Cx+4C + Dx^2+4D}{(x+1)^2(x^2+4)}$$

$$= \frac{(A+C)x^3 + (2A+B+C+D)x^2 + (4A+2B+4C)x + B+4C+4D}{(x+1)^2(x^2+4)}$$

$$\int \frac{1}{(x+2)^2 \sqrt{8x^2+26x+11}} dx = - \int t \frac{(x+2)^2 dt}{(x+2) \sqrt{8x^2+26x+11}} =$$

$$t = \frac{1}{(x+2)} \quad dt = -\frac{1}{(x+2)^2} dx$$

$$x = -\frac{2t-1}{t}$$

$$= - \int \frac{t dt}{t \cdot \frac{1}{t} \sqrt{9t^2-6t+8}} = - \int \frac{t dt}{\sqrt{9-(3t+1)^2}}$$

$$\sqrt{9-(3t+1)^2} = \sqrt{-(3t+1)^2+9}$$

$$3t+1=u$$

$$du=3dt$$

$$t = \frac{u-1}{3}$$

$$= \frac{1}{9} \int \frac{u-1}{\sqrt{9-u^2}} du$$

$$= -\frac{1}{9} \left( \int \frac{u du}{\sqrt{9-u^2}} - \int \frac{1 du}{\sqrt{9-u^2}} \right)$$

$$= -\frac{1}{9} \sqrt{9-u^2} - \frac{\arcsin(\frac{u}{3})}{9}$$

$$= -\frac{1}{9} \sqrt{9-(3t+1)^2} - \frac{\arcsin(\frac{3t+1}{3})}{9}$$

$$= -\frac{\sqrt{9-(\frac{3}{(x+2)}+1)^2}}{9} - \frac{\arcsin(\frac{3}{(x+2)}+1)}{9} + C$$

$$8x^2+26x+11 =$$

$$= 8 \left( \frac{2t-1}{t} \right)^2 + 26 \frac{2t-1}{t} + 11$$

$$= \frac{8(4t^2-4t+1)}{t^2} + \frac{52t-26}{t} + 11$$

$$= \frac{32t^2-32t+8-52t^2+26t+11t^2}{t^2}$$

$$= \frac{-20t^2-6t+8}{t^2}$$



Ex 6.16

$$\int \frac{\sqrt{3x+2}}{(3x+2)^2 \sqrt{3x+1}} dx = \int \frac{1}{\sqrt{3x+1} (3x+2)^{\frac{3}{2}}} = \frac{1}{3} \int \frac{1}{\sqrt{t+1} (t+2)^{\frac{3}{2}}} dt =$$

$$t=3x, dt=3dx$$

$$u = \sqrt{\frac{t+1}{t+2}}$$

$$t+1 = (t+2) \cdot u^2$$

$$t = \frac{1}{u^2-1} - 2$$

$$dt = \frac{2u}{(u^2-1)^2} du$$

$$= \frac{1}{3} \int 2du = \frac{1}{3} \cdot 2u + C =$$

$$= \frac{1}{3} \cdot 2 \left( \sqrt{\frac{t+1}{t+2}} \right) + C$$

$$= \frac{2}{3} \cdot \sqrt{\frac{3x+1}{3x+2}} + C$$

8.16

$$\int \sqrt{5x^2-4x+7} dx = \int \sqrt{\left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}} dx = \int \frac{\sqrt{t^2 + \frac{31}{5}}}{\sqrt{5}} dt =$$

$$5x^2-4x+7 = \left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}$$

$$= \frac{1}{\sqrt{5}} \int \sqrt{t^2 + \frac{31}{5}} dt =$$

$$t = \sqrt{5}x - \frac{2}{\sqrt{5}}$$

$$x = \frac{\sqrt{5}t+2}{5} \quad dx = \frac{1}{\sqrt{5}} dt$$

$$\frac{1}{\sqrt{5}} \left( \frac{t}{2} \sqrt{t^2 + \frac{31}{5}} + \frac{31}{10} \ln(t + \sqrt{t^2 + \frac{31}{5}}) \right)$$

$$= \frac{1}{\sqrt{5}} \cdot \left( \frac{31 \ln(t + \sqrt{t^2 + \frac{31}{5}})}{10} + \frac{t \sqrt{t^2 + \frac{31}{5}}}{2} \right)$$

$$= \frac{1}{\sqrt{5}} \cdot \left( \frac{31 \ln\left(\sqrt{5}x - \frac{2}{\sqrt{5}} + \sqrt{\left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}}\right)}{10} + \frac{\sqrt{5}x - \frac{2}{\sqrt{5}} \sqrt{\left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}}}{2} \right)$$

+ C



w3.16

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$$\frac{4x^2 - 2x + 1}{4x^2 - 2x + 1} = 1 \quad (24x^3 - 12x^2 + 11x - 2) - (4x^2 - 2x + 1) \cdot 6x =$$

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