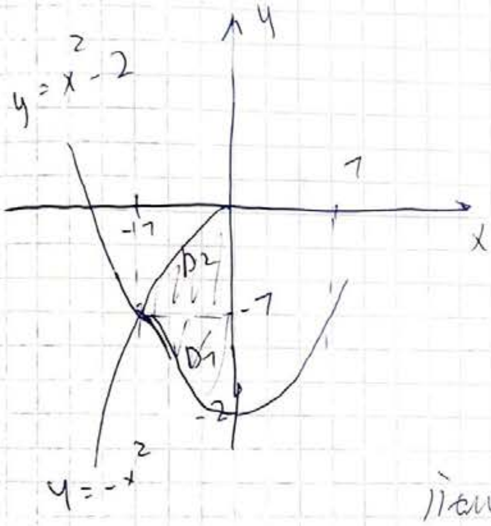
1.1. $\int\limits_{-2}^{-1} dy \int\limits_{-\sqrt{2+y}}^{0} f\,dx + \int\limits_{-1}^{0} dy \int\limits_{-\sqrt{-y}}^{0} f\,dx =$

Обл. интегрирования $D = D_1 \cup D_2$

$D_1: \begin{cases} -2 \le y \le -1 \\ -\sqrt{2+y} \le x \le 0 \end{cases}$ $\quad D_2: \begin{cases} -1 \le y \le 0 \\ -\sqrt{-y} \le x \le 0 \end{cases}$



$-\sqrt{2+y} = x \le 0$

$2 + y = x^2$;

$y = x^2 - 2$

$-\sqrt{-y} = x \le 0$

$-y = x^2$

$y = -x^2$

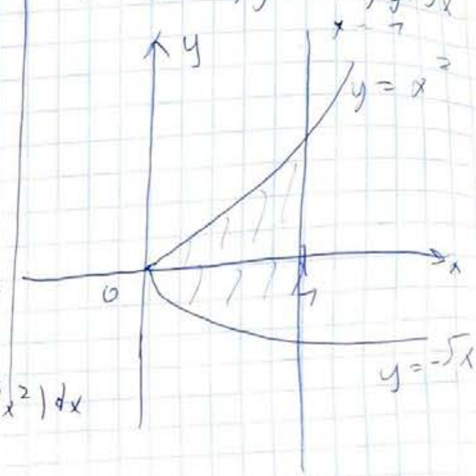
Изменим порядок

$D: \begin{cases} -1 \le x \le 0 \\ x^2 - 2 \le y \le -x^2 \end{cases}$

$= \int\limits_{-1}^{0} dx \int\limits_{x^2-2}^{-x^2} f\,dy$

2.7 $\iint\limits_D (12x^2y^2+16x^3y^3)\,dx\,dy =$

D: $x=1$; $y=x^2$; $y=-\sqrt{x}$



$= \int_0^1 dx \int_{-\sqrt{x}}^{x^2} (12x^2y^2+16x^3y^3)\,dy =$

$= \int_0^1 dx\,(4x^2y^3+4x^3y^4)\Big|_{-x^{1/2}}^{x^2} =$

$= \int_0^1 (4x^2\cdot x^6 + 4x^3x^8 + 4x^2x^{3/2} - 4x^3x^2)\,dx$

$= \int_0^1 (4x^8+4x^{11}+4x^{3,5}-4x^5)\,dx =$

$= \left(\frac{4x^9}{9}+\frac{4x^{12}}{12}+\frac{4x^{4,5}}{4,5}-\frac{4x^6}{6}\right)\Big|_0^1 =$

$= \frac{4}{9}+\frac{1}{3}+\frac{8}{9}-\frac{2}{3} = \frac{4}{3}-\frac{1}{3} = 1$

D: $\begin{cases} 0 \le x \le 1 \\ -\sqrt{x} \le y \le x^2 \end{cases}$

3.7
$\iint\limits_D y e^{xy/2}\,dx\,dy;$

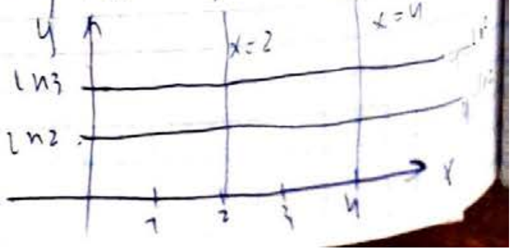
D: $y=\ln 2$; $y=\ln 3$; $x=2$; $x=4$

$\iint\limits_D y e^{xy/2}\,dx\,dy = \int_{\ln 2}^{\ln 3} dy \int_2^4 y e^{\frac{xy}{2}}\,dx = \int_{\ln 2}^{\ln 3} y\,dy\,\frac{2}{y}\cdot e^{\frac{xy}{2}}\Big|_2^4$

$= 2\int_{\ln 2}^{\ln 3} (e^{\frac{4y}{2}} - e^{\frac{2y}{2}})\,dy = 2\int_{\ln 2}^{\ln 3} (e^{2y}-e^{y})\,dy = 2\left(\frac{1}{2}e^{2y}-e^{y}\right)\Big|_{\ln 2}^{\ln 3}$

$= 2\left(\frac{1}{2}(e^{\ln 3})^2 - e^{\ln 3} - \frac{1}{2}(e^{\ln 2})^2 + e^{\ln 2}\right) = 2\left(\frac{9}{2}-3-\frac{4}{2}+2\right) =$

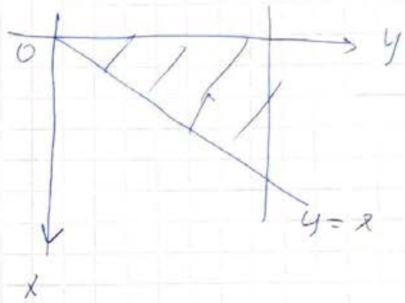
$= 5-2 = 3$



4.7 $\iiint\limits_V 2y^2 e^{xy}\, dx\, dy\, dz \ominus$ $V: \begin{cases} x=0, y=1 \\ z=0; \quad y=x \\ z=1 \end{cases}$

Область интегрирования

$V: \begin{cases} 0 \le y \le 1 \\ 0 \le x \le y \\ 0 \le z \le 1 \end{cases}$



$\ominus \int\limits_0^1 dy \int\limits_0^y 2y^2 e^{xy}\, dx \int\limits_0^1 dz =$

$= \int\limits_0^1 2y^2\, dy \int\limits_0^y e^{xy}\, dx \cdot 1 = \int\limits_0^1 2y^2 \frac{e^{xy}}{y} \Big|_0^y dy = 2\int\limits_0^1 y(e^{y^2} - e^0)\, dy$

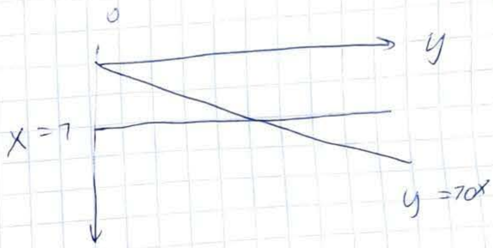
$= \int\limits_0^1 e^{y^2}\, dy^2 - \int\limits_0^1 2y\, dy = e^{y^2} \Big|_0^1 - y^2 \Big|_0^1 = e^1 - e^0 - 1^2 + 0^2 =$

$= e - 2$

5.1 $\iiint x\,dx\,dy\,dz$

$$V:\begin{cases} y=10x;\ y=0;\ x=1 \\ z=xy;\ z=0\end{cases}$$

$$V:\begin{cases} 0\le x\le 1 \\ 0\le y\le 10x \\ 0\le z\le xy\end{cases}$$

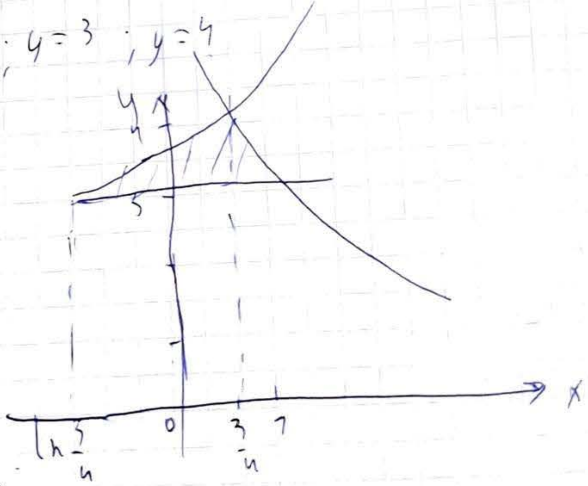


$$=\int_0^1 x\,dx\int_0^{10x} xy\,dy=$$

$$=\int_0^1 x^2\,dx\int_0^{10x} y\,\cancel{dy}=y\,dy=\int_0^1 x^2\,\frac{y^2}{2}\Big|_0^{10x}dx=$$

$$=\int_0^1 \frac{100}{2}x^4\,dx=50\,\frac{x^5}{5}\Big|_0^1\ 10\cdot 1^5=10$$

6.1 $y=\frac{3}{x}$ $y=4e^x$; $y=3$; $y=4$



$$S=\int_3^4 dy\cdot\Big(-\int_{\frac{3}{y}}^{\ln\frac{y}{4}}dx\Big)=$$

$$=-\int_3^4\Big(\ln\Big(\frac{y}{4}\Big)-\Big(\frac{3}{y}\Big)\Big)dy=$$

$$=-\int_3^4\Big(\ln\frac{1}{4}+\ln y-\frac{3}{y}\Big)dy=$$

$$=\int \ln y\,dy=\left|\begin{array}{l} u=\ln y\\ dv=dy\\ v=y\\ du=\frac{1}{y}dy\end{array}\right|=y\ln y-\int dy=y(\ln y-1)$$

$$= -\Big[y\cdot\ln\frac{1}{4}+y(\ln|y|-1)-3\ln y\Big]\Big|_3^4=$$

$-(4\ln\frac{1}{4}-3\ln\frac{1}{4}+4(\ln 4-1)-3(\ln 3-1)-3\ln 4+$

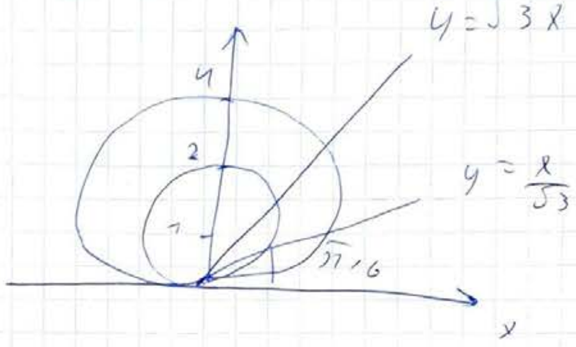
$+3\ln 3) = -(-\ln 4+4\ln 4-4-3\ln 3+3-3\ln 4+3\ln 3)=1$

7.7

$y^2-2y+x^2=0;$

$y^2-4y+x^2=0;$

$y=\frac{x}{\sqrt{3}}$; $y=\sqrt{3}x$;



$y^2-2y+1+x^2=1$

$(y-1)^2+x^2=1^2$ - ур. окр.

$C_1(0;1)$ с радиусом $R=1$

$y^2-4y+4+x^2=4 \Rightarrow x^2+(y-2)^2=2^2$ - окр.

с центром $C_2(0;2)$ и радиусом $R=2$

Ур. в поляр. коорд.

$x=\rho\cos\theta$, $y=\rho\sin\theta$; $x^2+y^2=\rho^2$. уравнение

окружности $\rho^2=2\rho\sin\theta$ или $\rho=2\sin\theta$

$\rho^2=4\rho\sin\theta$ или $\rho=4\sin\theta$

$\rho\sin\theta=\frac{\rho\cos\theta}{\sqrt{3}}$; $\operatorname{tg}\theta=\frac{1}{\sqrt{3}}$; $\theta=\frac{\pi}{6}$

$\rho\sin\theta=\sqrt{3}\rho\cos\theta$; $\operatorname{tg}\theta=\sqrt{3}$; $\theta=\frac{\pi}{3}$

$D: \begin{cases} \pi/6 \le \theta \le \pi/3 \\ 2\sin\theta \le \rho \le 4\sin\theta \end{cases}$

Площадь $S=\iint\limits_D dxdy = \int\limits_{\pi/6}^{\pi/3} d\theta \int\limits_{2\sin\theta}^{4\sin\theta} \rho\, d\rho = \int\limits_{\pi/6}^{\pi/3} \frac{\rho^2}{2}\Big|_{2\sin\theta}^{4\sin\theta} d\theta$

$$= \int_{\pi/6}^{\pi/3} \frac{16\sin^2\theta - 4\sin^2\theta}{2}\, d\theta = 6\int_{\pi/6}^{\pi/3} \sin^2\theta\, d\theta =$$

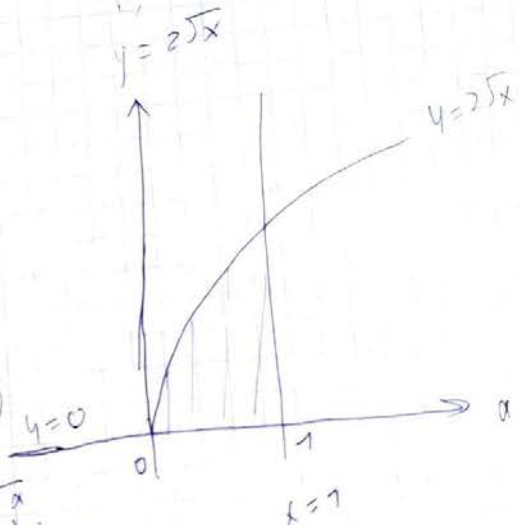
$$= 6\int_{\pi/6}^{\pi/3} \frac{1-\cos 2\theta}{2}\, d\theta = 3\left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_{\pi/6}^{\pi/3} =$$

$$= 3\left(\frac{\pi}{3} - \frac{\sin\frac{2\pi}{3}}{2} - \frac{\pi}{6} + \frac{\sin\frac{\pi}{3}}{2}\right) = 3\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = \frac{3\pi}{6} = \frac{\pi}{2}$$

8.1 $\quad x=1;\ y=0;\ y^2=4x;\ (y\geq 0);\ \mu = 7x^2+y$

Пл D: $\quad y = 2\sqrt{x}$

Область интегрирования



$$D: \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 2\sqrt{x} \end{cases}$$

Масса пластины

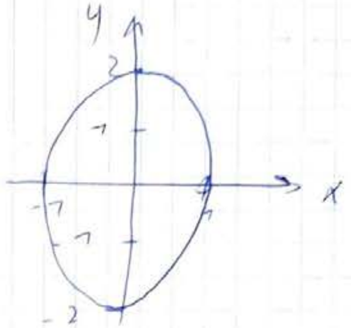
$$M = \iint_D \mu(x,y)\,dx\,dy = \int_0^1 dx \int_0^{2\sqrt{x}} (7x^2+y)\,dy = \int_0^1 \left(7x^2y + \frac{y^2}{2}\right)\Big|_0^{2\sqrt{x}} dx =$$

$$= \int_0^1 \left(7x^2 \cdot 2\sqrt{x} + \frac{4x}{2}\right) dx = \int_0^1 \left(14x^{2,5} + 2x\right) dx = \left(\frac{14x^{3,5}}{3,5} \ldots\right.$$

$$= 4\cdot 1^{3,5} + 1 = 5$$

9.7 $D: x^2 + \frac{y^2}{4} \le 1$, $\mu = y^2$

полярн. коорд.



$x = \rho\cos\theta$ $\left(\begin{matrix} a=1 \\ b=2 \end{matrix}\right)$

$y = 2\rho\sin\theta$

$dx\,dy = a\,b\,\rho\,d\rho\,d\theta = 2\rho\,d\rho\,d\theta$

Уравнение эллипса $\rho^2\cos^2\theta + \frac{4\rho^2\sin^2\theta}{4} = 1$

$\rho = 1$

Обл. инт. $D: \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le \rho \le 1 \end{cases}$

площадь $S = \iint\limits_D dx\,dy = \int_0^{2\pi} d\theta \int_0^1 2\rho\,d\rho = \int_0^{2\pi} \rho^2 \Big|_0^1 d\theta = 2\pi$

$M = \iint\limits_D \mu\,dx\,dy = \int_0^{2\pi} d\theta \int_0^1 4\rho^2\sin^2\theta \cdot 2\rho\,d\rho = 8\int_0^{2\pi} \sin^2\theta\,d\theta$

$\int_0^1 \rho^3\,d\rho = 8\int_0^{2\pi} \frac{1-\cos 2\theta}{2}\,d\theta \cdot \frac{\rho^4}{4}\Big|_0^1 = \left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_0^{2\pi} \cdot 1 =$

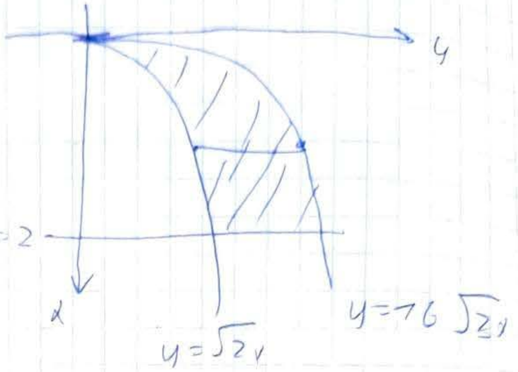
$= \left(2\pi - \frac{\sin 4\pi}{2}\right) = 2\pi$

10.1

$y = 16\sqrt{2x}$; $y = \sqrt{2x}$

$z = 0$, $x + z = 2$

odr. [illegible]



$$D \begin{cases} 0 \le x \le 2 \\ \sqrt{2x} \le y \le 16\sqrt{2x} \\ 0 \le z \le 2-x \end{cases}$$

Odredi mera

$$V = \int_0^2 dx \int_{\sqrt{2x}}^{16\sqrt{2x}} dy \int_0^{2-x} dz = \int_0^2 (16\sqrt{2x} - \sqrt{2x})\cdot(2-x)\,dx =$$

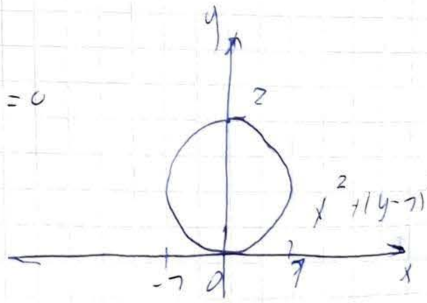
$$= 15\sqrt{2}\int_0^2 (2\sqrt{x} - x\sqrt{x})\,dx = 15\sqrt{2}\left(2\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2}\right)\Big|_0^2 =$$

$$= 15\sqrt{2}\left(\frac{4}{3}\cdot 2\sqrt{2} - \frac{2}{5}\cdot 2^2\sqrt{2}\right) = 30\left(\frac{8}{3} - \frac{8}{5}\right) = 8\cdot 30\left(\frac{5-3}{15}\right) =$$

$$= 16 \cdot 2 = 32$$

11.1

$$x^2 + y^2 = 2y \ ; \ z = \frac{5}{4} - x^2 \ ; \ z = 0$$



$1)\ x^2 + (y^2 - 2y + 1) = 1$

$x^2 + (y-1)^2 = 1$

$V_1 =$

$$V = \int_0^{\pi} d\varphi \int_0^{2\sin\varphi} \rho\, d\rho \int_0^{5/4 - \rho^2\cos^2\varphi} dz =$$

$$= \int_0^{\pi} d\varphi \int_0^{2\sin\varphi} \rho\left(\frac{5}{4} - \rho^2\cos^2\varphi\right) d\rho = \int_0^{\pi} d\varphi \int_0^{2\sin\varphi} \left(\frac{5}{4}\rho - \rho^3\cos^2\varphi\right) d\rho =$$

$$= \int_0^{\pi} d\varphi \cdot \left(\frac{5}{4}\frac{\rho^2}{2} - \frac{\rho^4}{4}\cos^2\varphi\right)\Big|_0^{2\sin\varphi} = \int_0^{\pi}\left(\frac{5}{8}\rho^2 - \frac{\rho^4}{4}\cos^2\varphi\right)\Big|_0^{2\sin\varphi} d\varphi$$

$$= \int_0^{\pi} d\varphi \left(\frac{5}{8}\cdot 4\sin^2\varphi - \underline{16\sin^4}\varphi\cos^2\varphi\right) = \int_0^{\pi} d\varphi\left(\frac{5}{2}\sin^2\varphi - 4\sin^4\varphi \cdot \cos^2\varphi\right) = \int_0^{\pi} d\varphi \sin^2\varphi\left(\frac{5}{2} - 4\sin^2\varphi\cdot\cos^2\varphi\right) = \int_0^{\pi}\frac{1}{2}(1 - \cos 2\varphi)$$

$\left(\frac{5}{2} - \sin^2 2\varphi\right) \quad \frac{1}{2}\int_0^{\pi}\left(\frac{5}{2} - \sin^2 2\varphi - \frac{5}{2}\cos 2\varphi +\right.$

$\left.+ \cos 2\varphi \sin^2 2\varphi\right) d\varphi = \frac{1}{2}\left[\frac{5}{2}\varphi\Big|_0^{\pi} - \frac{1}{2}\int_0^{\pi}(1-\cos 4\varphi)d\varphi\right.$

$\left.- \frac{5}{2}\cdot\frac{1}{2}\cdot\sin 2\varphi\Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi}\sin^2 2\varphi \, d(\sin 2\varphi)\right] = \frac{1}{2}\left[\frac{5}{2}\varphi\Big|_0^{\pi}\right.$

$\left.- \frac{1}{2}\varphi\Big|_0^{\pi} + \frac{1}{2}\cdot\frac{1}{4}\sin 4\varphi\Big|_0^{\pi} - \frac{5}{4}\sin 2\varphi\Big|_0^{\pi} + \frac{1}{2}\frac{\sin^3 2\varphi}{3}\Big|_0^{\pi}\right.$

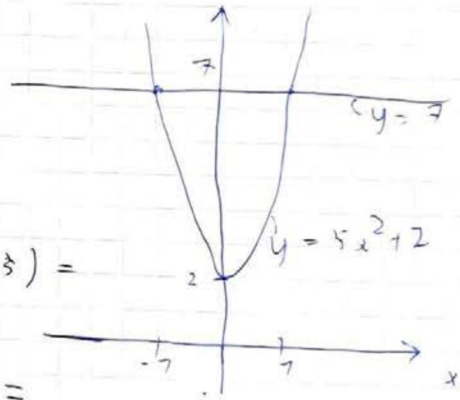
$= \frac{1}{2}\left[\frac{5}{2}\pi - \frac{1}{2}\pi\right] = \pi$

12.1

$y = 5x^2+2; \; y = 7; \; z = 3y^2 - x^2 - 2; \; z = 3y^2 - x^2 - 5$

1) $y = 5x^2+2$

x	0	1
y	2	7



$V = \int_{-1}^{1} dx \int_{5x^2+2}^{7} dy \int_{3y^2-x^2-5}^{3y^2-x^2-2} dz =$

$= \int_{-1}^{1} dx \int_{5x^2+2}^{7} dy\,(3y^2 - x^2 - 2 - 3y^2 + x^2 + 5) =$

$= \int_{-1}^{1} dx \int_{5x^2+2}^{7} 3\,dy = 3\int_{-1}^{1} dx\,(7 - 5x^2 - 2) =$

$= 3\int_{-1}^{1}(5-5x^2)dx = 15\int_{-1}^{1}(1-x^2)\,dx = 15\cdot\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} =$

$= 15\cdot\left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right) = 15\cdot\frac{4}{3} = 20$

13.1

$z = \sqrt{9-x^2-y^2}$; $\frac{9z}{2} = x^2+y^2$

$$\begin{cases} z = \sqrt{9-x^2-y^2} \\ \frac{9z}{2} = x^2+y^2 \\ z \geq 0 \end{cases} \rightarrow z = \sqrt{9-\frac{9z}{2}}$$

$$z^2 = 9 - \frac{9z}{2}$$

$$z^2 + \frac{9z}{2} - 9 = 0$$

$$D = \frac{81}{4} + 36 = \frac{225}{4}$$

$$z = \frac{-\frac{9}{2} \pm \frac{15}{2}}{2} ; \quad z_1 = -6; \quad z_2 = \frac{3}{2}$$

$z \Rightarrow z \geq 0$, то $z = \frac{3}{2}$ и

$$x^2+y^2 = \frac{9}{2} \cdot \frac{3}{2} = \frac{27}{4}$$

$$V = \int_0^{2\pi} d\varphi \int_0^{\sqrt{27}/2} \rho\, d\rho \int_{\frac{2}{9}\rho^2}^{\sqrt{9-\rho^2}} dz = \int_0^{2\pi} d\varphi \int_0^{\sqrt{27}/2} \rho\left(\sqrt{9-\rho^2} - \frac{2}{9}\rho^2\right)$$

$$= \int_0^{2\pi} d\varphi \left[\int_0^{\sqrt{27}/2} - d\rho\, \rho\sqrt{9-\rho^2} - \int_0^{\sqrt{27}/2} \frac{2}{9}\rho^3 d\rho \right] = \int_0^{2\pi} d\varphi \left[\frac{1}{2}\int_0 -\sqrt{9-\rho^2} \cdot d(9-\rho^2) - \frac{2}{9}\cdot\frac{\rho^4}{4}\Big|_0^{\sqrt{27}/2} \right] = \int_0^{2\pi} d\varphi \left[-\frac{1}{2}(9-\rho^2)^{3/2}\cdot\frac{2}{3}\Big|_0^{\frac{\sqrt{27}}{2}} - \right.$$

$$\left. - \frac{1}{18}\rho^4\Big|_0^{\frac{\sqrt{27}}{2}} \right] = \int_0^{2\pi} d\varphi \left[-\frac{1}{3}\left(9-\frac{27}{4}\right)^{\frac{3}{2}} - \frac{1}{18}\cdot\frac{27\cdot27}{16} - -\frac{1}{3}(9)^{\frac{3}{2}} \right] =$$

$$= \int_0^{2\pi} d\varphi \left[-\frac{27}{8} \cdot \frac{1}{3} - \frac{1}{6}\cdot\frac{243}{16} + \frac{1}{3}\,27 \right] = \int_0^{2\pi} d\varphi \left[-\frac{9}{8} - \frac{243}{96} + 9 \right] =$$

$$= \frac{-9\cdot12 - 243 + 864}{96}\cdot 2\pi = \frac{-108 - 243 + 864}{48}\pi = \frac{513}{48}\pi = \frac{171}{16}\pi$$

14 1

$z = 2 - 12(x^2 + y^2)$

$z = 24x + 2$

$V = \iint\limits_S |z_1 - z_2|\, dx\, dy$, де z_1, z_2 задан. поверх.

$$\begin{cases} z = 2 - 12(x^2 + y^2) \\ z = 24x + 2 \end{cases} \Rightarrow 2 - 12(x^2 + y^2) = 24x + 2,$$

$(x+1)^2 + y^2 = 1$ $\quad S$ = круг.

$R = 1$ ц. у $(-1, 0)$

$$V = \iint\limits_S (2 - 12(x^2 + y^2) - 24x - 2)\, dS = -12 \iint\limits_S (x^2 + y^2 + 2x)\, dS$$

перейд. до полярн. коорд.

$x = -1 + \rho\cos\theta$

$y = \rho\sin\theta$

$$V = -12 \int_0^1 d\rho \int_0^{2\pi} (1 - 2 + \rho\cos\theta - 2\rho\cos\theta + \rho^2 + 1)\rho\, d\theta =$$

$$= -12 \int_0^1 d\rho \int_0^{2\pi} (\rho^2 - 1)\rho\, d\theta = -24\pi \int_0^1 ((\rho^2 - 1)\rho)\, d\rho = -24\pi$$

$$\left(\frac{\rho^4}{4} - \frac{\rho^2}{2}\right)\Big|_0^1 = 6\pi$$

15.1

$$1 \le x^2+y^2+z^2 \le 49$$

$$-\sqrt{\frac{x^2+y^2}{35}} \le z \le \sqrt{\frac{x^2+y^2}{3}}$$

$$-x \le y \le 0$$

сферич. сист. коорд

$$x = r\cos\varphi\cos\theta$$

$$y = r\sin\varphi\cos\theta$$

$$z = r\sin\theta$$

Тогда тело ст.:

$$\begin{cases} 1 \le r \le 7 \\ -\dfrac{r\cos\theta}{\sqrt{35}} \le r\sin\theta \le \dfrac{r\cos\theta}{\sqrt{3}} \\ -r\cos\varphi\cos\theta \le r\sin\varphi\cos\theta \le 0 \end{cases} \Rightarrow \begin{cases} 1 \le r \le 7 \\ -\dfrac{1}{\sqrt{35}} \le \operatorname{tg}\theta \le \dfrac{1}{\sqrt{3}} \\ -\cos\varphi \le \sin\varphi \le 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 1 \le r \le 7 \\ -\operatorname{arctg}\dfrac{1}{\sqrt{35}} \le \theta \le \operatorname{arctg}\dfrac{1}{\sqrt{3}} \\ -\dfrac{\pi}{4} \le \varphi \le 0 \end{cases}$$

$$V = \int_{-\pi/4}^{0} d\varphi \int_a^b d\theta \int_1^7 dr, \quad a = -\operatorname{arctg}\frac{1}{\sqrt{35}}, \quad b = \operatorname{arctg}\frac{1}{\sqrt{3}}$$

$$I = r^2\cos\theta$$

$$V = \int_{-\pi/4}^{0} d\varphi \int_a^b d\theta \int_1^7 r^2\cos\theta\, dr = \frac{\pi}{4}(\sin b - \sin a)\left(\frac{343}{3} - \frac{1}{3}\right) = \frac{57\pi}{2}(\sin b - \sin a)$$

$$\sin a = -\sqrt{\frac{\operatorname{tg}^2 a}{\operatorname{tg}^2 a + 1}} = -\frac{1}{2}$$

$$\sin b = \sqrt{\frac{\operatorname{tg}^2 b}{\operatorname{tg}^2 b + 1}} = \frac{1}{6}$$

$$V = \frac{57\pi}{2}\left(\frac{1}{6} + \frac{1}{2}\right) = 19\pi$$

16 7

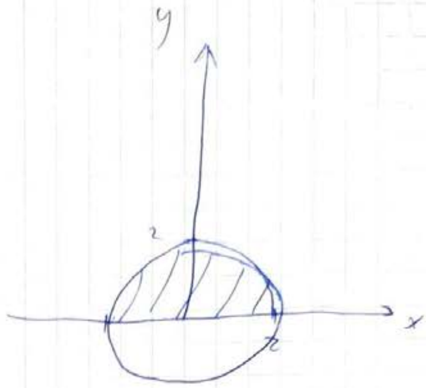
$64(x^2+y^2)=z^2$

$x^2+y^2=4$

$y=0 \quad (y \geq 0)$

$z=0 \quad (z \geq 0)$

$\mu = 5(x^2+y^2)/4$



$0 \leq \varphi \leq \pi$

$0 \leq \rho \leq 2$

$0 \leq z \leq 8\rho$

$$m = \int_0^{\pi} d\varphi \int_0^2 d\rho \int_0^{8\rho} \rho \, dz \cdot \frac{5\rho^2}{4} = \pi \cdot \int_0^2 d\rho \cdot \frac{5\rho^3}{4} \cdot 8\rho =$$

$$= \pi \cdot \frac{10\rho^5}{5} \Big|_0^2 = 2\pi \cdot 32 = 64\pi$$

1.1. $\int \frac{2x+5}{\sqrt{x+3}}\,dx \quad \left| \begin{array}{l} t = x+3 \\ dt = dx \end{array} \right| \quad = \int \frac{2t-1}{\sqrt{t}}\,dt$

$= \int \frac{2t}{\sqrt{t}}\,dt - \int \frac{1}{\sqrt{t}}\,dt = 2 \cdot \int \sqrt{t}\,dt - \int \frac{1}{\sqrt{t}}\,dt = 2 \cdot \frac{2t^{\frac{3}{2}}}{3} - 2\sqrt{t} + C$

$= \frac{4t\sqrt{t}}{3} - 2\sqrt{t} + C = \frac{4(x+3)\sqrt{x+3}}{3} - 2\sqrt{x+3} + C$

2.1. $\int \ln|x^2+2|\,dx = \left\{ \begin{array}{ll} u = \ln|x^2+2| & v' = 1 \\ u' = \frac{2x}{x^2+2} & v = x \end{array} \right\}$

$x\ln|x^2+2| - \int \frac{2x^2}{x^2+2}\,dx = x\ln|x^2+2| - \int 2\,dx - \int x^2\,dx$

$= x\ln|x^2+2| - 2x - \frac{x^3}{3}$

2.1. $\int \ln|x^2+2|\,dx \quad \left\{ \begin{array}{ll} u = \ln|x^2+2| & v' = 1 \\ u' = \frac{2x}{x^2+2} & v = x \end{array} \right\}$

$x\ln|x^2+2| - \int \frac{2x^2}{x^2+2}\,dx = x\ln|x^2+2| - 2\int dx + 4\int \frac{1}{x^2+2}\,dx$

$= x\ln|x^2+2| - 2x + \frac{4}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C$

$$2.1 \int \frac{x^3+x^2+2x+1}{x^2+x+1}\,dx = \int \left(\frac{x+1}{x^2+x+1} + x \right) dx$$

$$\int \frac{x+1}{x^2+x+1}\,dx + \int x\,dx = \int \frac{x+1}{x^2+x+1}\,dx + \frac{x^2}{2} + C \;(=)$$

$$= \int \frac{x+1}{x^2+x+1}\,dx \quad \left| \; x+1 = \frac{1}{2}(2x+1) + \frac{1}{2} \right|$$

$$\frac{1}{2}\int \frac{2x+1}{x^2+x+1}\,dx + \int \frac{1}{2(x^2+x+1)}\,dx = \frac{1}{2} \cdot \ln|x^2+x+1|$$

$$+ \int \frac{1}{2(x^2+x+1)}\,dx + C$$

$$\int \frac{1\,dx}{2(x^2+x+1)} = \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}\,dx \quad \left| \; t = x + \frac{1}{2} \quad dx = dt \right|$$

$$\frac{1}{2}\int \frac{1}{t^2+\frac{3}{4}}\,dt = \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg}\left(\frac{t}{\frac{\sqrt{3}}{2}}\right) + C = \frac{\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$(=) \; \frac{1}{2}\ln|x^2+x+1| + \frac{\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{x^2}{2} + C$$

4.7. $\frac{x^3+6x^2+74x+2}{(x+2)^2(x^2+4)}dx$

$$\frac{x^3+6x^2+74x+2}{(x+2)^2(x^2+4)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{(x^2+4)} =$$

$$\frac{A(x+2)\cdot(x^2+4) + B(x+2)(x^2+4) + (Cx+D)(x+2)^2}{(x+2)^2(x^2+4)}$$

$$= \frac{x^3(A+C) + x^2(2A+B+D+4C) + x(4A+4D+4C) + (8A+4B+4D)}{(x+2)^2(x^2+4)}$$

$$\begin{cases} A+C=7 \\ 2A+B+D+4C=6 \\ 4A+4D+4C=74 \\ 8A+4B+4D=2 \end{cases} \Rightarrow \quad A=-\frac{3}{8},\ B=-\frac{5}{4},\ C=\frac{77}{8},\ D=\frac{5}{2}$$

$$-\frac{3}{8}\int\frac{dx}{(x+2)} - \frac{5}{4}\int\frac{dx}{(x+2)^2} + \int\frac{77x+20}{8(x^2+4)}dx = -\frac{3}{8}\ln|x+2| - \frac{5}{4x+8} +$$

$$+\frac{77}{16}\ln|x^2+4| + \frac{5}{4}\operatorname{arctg}\frac{x}{2}$$

5.1. $\displaystyle\int \frac{dx}{(x+2)^2\sqrt{3x^2+16x+21}}$ $\left|\begin{array}{l} \frac{1}{x+2} = t \Rightarrow x = \frac{1}{t} - 2 \\ dx = -\frac{1}{t^2}\,dt \end{array}\right.$

$$= \int \frac{-dt}{t^2}\cdot t^2 \cdot \frac{1}{\sqrt{3\left(\frac{1}{t}-2\right)^2+16\left(\frac{1}{t}-2\right)+21}} =$$

$$= -\int \frac{dt}{\sqrt{3\left(\frac{1}{t^2}-\frac{4}{t}+4\right)+\frac{16}{t}-32+21}} = -\int \frac{dt}{\sqrt{\frac{3}{t^2}-\frac{12}{t}+12+\frac{16}{t}-11}} =$$

$$= -\int \frac{dt}{\sqrt{\frac{3}{t^2}+\frac{4}{t}+1}} = -\int \frac{t\,dt}{\sqrt{t^2+4t+3}} = -\int \frac{t\,dt}{\sqrt{(t+2)^2-1}} \quad \left|\begin{array}{l} t+2=u \\ t=u-2 \\ dt=du \end{array}\right.$$

$$-\int \frac{u-2}{\sqrt{u^2-1}}\,du = -\int \frac{u}{\sqrt{u^2-1}}\,du + 2\int \frac{1}{\sqrt{u^2-1}} =$$

$$\cancel{\frac{u}{\sqrt{u^2-1}}\,du} \qquad -\sqrt{u^2-1} + 2\ln\left|u+\sqrt{u^2-1}\right| + C =$$

$$= -\sqrt{(t+2)^2-1} + 2\ln\left|t+2+\sqrt{(t+2)^2-1}\right| + C = -\sqrt{\left(\frac{1}{x+2}+2\right)^2-1} +$$

$$+ 2\ln\left|\frac{1}{x+2}+2+\sqrt{\left(\frac{1}{x+2}+2\right)^2-1}\right| + C$$

6.1. $\displaystyle\int \frac{3\sqrt{2-x}-\sqrt{x+3}}{(\sqrt{x+3}+5\sqrt{2-x})(x+3)^2}dx = \int \frac{3\sqrt{2-x}-\sqrt{x+3}}{\sqrt{2-x}(5x^2+30x+45)+\sqrt{x+3}(x^2+6x+9)}dx$

$$= \int \frac{8\sqrt{2-x}}{\sqrt{2-x}(5x^2+30x+45)+\sqrt{x+3}(x^2+6x+9)} - \int \frac{1}{x^2+6x+9}dx$$

$$\int \frac{\sqrt{2-x}\,dx}{\sqrt{2-x}(5x^2+30x+45)+\sqrt{x+3}(x^2+6x+9)} = \left| \begin{array}{l} u = 2-x \quad x = \\ dx = -du \\ x = 2-u \end{array}\right.;$$

$$-\int \frac{\sqrt{u}\,du}{(30(2-u)+5(2-u)^2+45)\sqrt{u}+(6(2-u)+(2-u)^2+9)\sqrt{5-u}}$$

$$\left| \begin{array}{ll} v=\sqrt{u} & u=v^2 \\ & du = 2v\,dv \end{array}\right|$$

$$-\int \frac{2v^2\,dv}{v(5(2-v^2)^2+30(2-v^2)+45)+\sqrt{5-v^2}((2-v^2)^2+6(2-v^2)+9)}$$

$$= -2\int\left(\frac{5v^3}{26v^6-265v^4+706v^2+725} - \frac{1}{\sqrt{5-v^2}(650v^2-725)+\sqrt{5-v^2}(25v^2-125)}\right)dv =$$

$$= -\frac{\ln\left(\left|\frac{1}{\sqrt{5-v^2}}\right|\left|5\sqrt{5-v^2}+25v\right|\right)}{625} + \frac{\ln\left|5\sqrt{x+3}\left|\frac{1}{\sqrt{x+3}}\right|+25\sqrt{2-x}\right|}{625}$$

$$\left|\frac{1}{\sqrt{x+3}}\right| + \frac{\ln\left(\left|\frac{1}{\sqrt{5-v^2}}\right|\left|5\sqrt{5-v^2}-25v\right|\right)}{625} - \frac{\sqrt{5-v^2}((v^2-5)\ln(|}{}$$

$$\frac{26v^2-5))+(5-v^2)(\ln(|v^2-5|)-725)-10v^2+30v}{\sqrt{5-v^2}(625v^2-3125)}$$

одн. замена

$$\int \frac{v^3}{26v^6 - 265v^4 + 700v^2 - 125}\, dv \quad \left| \begin{array}{l} t = v^2 \\ \frac{1}{2}\, dt = v\, dv \end{array} \quad v^2 = t \right.$$

$$\int \frac{t}{52t^3 - 530t^2 + 1400t - 250}\, dt = \int \frac{t\, dt}{2(26t^3 - 265t^2 + 700t - 125)}$$

$$= \frac{1}{2} \int \frac{t}{(t-5)^2 (26t-5)}\, dt = \frac{1}{2} \int \frac{t}{26 (t-5)^2 (t - \frac{5}{26})}\, dt$$

$$\frac{1}{(t-5)^2 (t - \frac{5}{26})} = \frac{A}{t - \frac{5}{26}} + \frac{B}{t-5} + \frac{C}{(t-5)^2}$$

$$\left\{ A = \frac{26}{3125} \quad B = -\frac{26}{3125} \quad C = \frac{26}{25} \right.$$

$$\frac{1}{52} \int \frac{26}{3125(t - \frac{5}{26})} - \frac{26}{3125(t-5)} + \frac{26}{25(t-5)^2}\, dt =$$

$$= \frac{\ln(|26t-5|)}{6250} - \frac{1}{50(t-5)} - \frac{\ln(|t-5|)}{6250}$$

$$t = v^2$$

$$\frac{\ln|26v^2-5|}{6250} - \frac{\ln(|v^2-5|)}{6250} - \frac{1}{50(v^2-5)}$$

$$\int \frac{1\, dv}{\sqrt{5-v^2}\,(650v^2-125)} \quad \left| \begin{array}{l} \cancel{t = \arcsin\left(\frac{v}{\sqrt{5}}\right)} \\ \cancel{5\cos^2 t = 5 - v^2} \end{array} \right. \quad \begin{array}{l} \cancel{v = \sqrt{5}\sin t} \\ \cancel{dv = \sqrt{5}\cos t\, dt} \end{array}$$

$$= \frac{\ln\left(\left|\frac{1}{\sqrt{5-v^2}}\right| \left|5\sqrt{5-v^2} - 25v\right|\right)}{1250} - \frac{\ln\left(\left|\frac{1}{\sqrt{5-v^2}}\right| \left|5\sqrt{5-v^2} + 25v\right|\right)}{1250}$$

$$\int \frac{7}{\sqrt{5-v^2}(125v^2-725)}\,dv \quad \Big|\ t = \arcsin\left(\frac{v}{\sqrt5}\right) \quad v=\sqrt5\sin t \quad 5\cos^2 t = 5-v^2 \quad dv = \sqrt5\cos t\,dt$$

$$\int \frac{7}{725\sin^2 t - 725}\,dt = \int \frac{7}{725(\sin^2 t - 7)}\,dt = \frac{7}{725}\int -\frac{7}{\cos^2 t}\,dt$$

$$= -\frac{7}{725}\cdot tg\,t \cdot \Big|\ t = \arcsin\left(\frac{v}{\sqrt5}\right) \quad v=\sqrt5\sin t\Big|$$

$$= -\frac{v}{725\sqrt{5-v^2}}$$

$$\int \frac{7}{x^2+6x+9}\,dx = \int \frac{7}{(x+3)^2}\,dx = -\frac{7}{x+3}$$

$$= \frac{-\ln\left(\left|\frac{7}{\sqrt{5-v^2}}\right|\left|5\sqrt{5-v^2}+23v\right|\right)}{625} + \frac{\ln\left(\left|\frac{7}{\sqrt{5-v^2}}\right|\left|5\sqrt{5-v^2}-23v\right|\right)}{625}$$

$$-\frac{\sqrt{5-v^2}(7v^2-5)\ln(|126v^2-5|)+(5-v^2)(\ln(|v^2-5|)-725)-70v^3\ \frac{550}{5}}{\sqrt{5-v^2}(625v^2-3725)}$$

обр. замена: $v = \sqrt{u} \quad u = 2-x$

Ответ: $\frac{8\ln\left(\left|\frac{7}{\sqrt{x+3}}\right|\left|\sqrt{x+3}-5\sqrt{2-x}\right|\right)}{625} - \frac{8\ln\left(\sqrt{x+3}\left|\frac{7}{x+3}\right| + 5\sqrt{2-x}\left|\frac{7}{\sqrt{x+3}}\right|\right)}{625} + \frac{\sqrt{x+3}\left((-8x-24)\ln(|26x-47|)+(8x+24)\ln(|x+3|)-7000\right)+\sqrt{2-x}\,(80x+240}{\sqrt{x+3}\,(625+7875)} + \frac{7}{x+3} + C$

8.1 $\int \sqrt{-2x^2+3x+3}\,dx = \int \sqrt{\frac{33}{8} - \left(\sqrt{2}x - \frac{3}{2\sqrt{2}}\right)^2}\,dx$

$\left| u = \sqrt{2}x - \frac{3}{2\sqrt{2}} \qquad x = \frac{2\sqrt{2}u+3}{4} \qquad dx = \frac{1}{\sqrt{2}}du \right|$

$\int \frac{\sqrt{\frac{33}{8} - u^2}}{\sqrt{2}} \cdot du = \int \frac{\sqrt{33-8u^2}}{4}\,du = \frac{1}{4}\int 2\sqrt{2}\sqrt{\frac{33}{8} - u^2}\,du$

$\left| \begin{array}{ll} t = \arcsin\left(\frac{2\sqrt{2}u}{\sqrt{33}}\right) & u = \frac{\sqrt{33}\sin t}{2\sqrt{2}} \\ \frac{33\cos^2 t}{8} = \frac{33}{8} - u^2 & du = \frac{\sqrt{33}\cos t}{2\sqrt{2}}\,dt \end{array} \right|$

$\frac{1}{4}\int \frac{33\cos^2(t)}{2\sqrt{2}}\,dt = \frac{33}{8\sqrt{2}}\int \frac{\cos 2t + 1}{2}\,dt =$

$= \frac{33}{8\sqrt{2}}\left(\frac{1}{2}\int \cos 2t\,dt + \frac{1}{2}\int dt\right) = \frac{33\sin 2t}{32\sqrt{2}} + \frac{33t}{16\sqrt{2}}$

$\left| t = \arcsin\left(\frac{2\sqrt{2}u}{\sqrt{33}}\right) \right| \quad \frac{33\arcsin\left(\frac{2\sqrt{2}u}{\sqrt{33}}\right)}{16\sqrt{2}} + \frac{u\sqrt{33-8u^2}}{8}$

$\left| u = \sqrt{2}x - \frac{3}{2\sqrt{2}} \right| \quad \frac{33\arcsin\left(\frac{4x-3}{\sqrt{33}}\right)}{16\sqrt{2}} + \frac{x\sqrt{-16x^2+24x+24}}{4\sqrt{2}} -$

$- \frac{3\sqrt{-16x^2+24x+24}}{16\sqrt{2}} + C$

9-1 $\int \sin^2(2x)\cos^4(2x)\,dx$ $\Big|\; u = 2x \quad x = \frac{u}{2} \quad dx = \frac{1}{2}du$

$\int \frac{\cos^4 u \sin^2 u}{2}\,du$ $\Big|\; \cos^2 u = 1+\cos 2u \quad \sin^2 u = \frac{1-\cos 2u}{2}$

$\frac{1}{2}\int \frac{(1-\cos 2u)(\cos 2u+1)^2}{8}\,du =$

$= \Big|\; v = 2u \quad u = \frac{v}{2} \quad du = \frac{1}{2}dv \;\Big| = \frac{1}{16}\int \frac{(1-\cos v)(\cos v+1)^2}{2}\,dv$

$= \frac{1}{32}\int (\cos v+1)^2 - \cos v(\cos v+1)^2\,dv$

$\int (\cos v+1)^2\,dv = \int (\cos^2 v + 2\cos v + 1)\,dv =$

$= \frac{\sin 2v}{4} + 2\sin v + \frac{3v}{2}$

$\int \cos v(\cos v+1)^2\,dv = \int (\cos^3 v + 2\cos^2 v + \cos v)\,dv$

$= \sin v - \frac{\sin^3 v}{3} + v + \frac{\sin 2v}{2} + \sin v$

обратная замена $v = 2u \quad u = 2x$

по итогу получилось равное

$\frac{\sin 8x}{128} + \frac{\sin^4 x}{16} + -\frac{3\,tg^5 2x - 8\,tg^3 2x - 9\,tg\,2x}{48(tg^2(2x)+1)^3} + \frac{x}{16} + C$

10.1 $\int \frac{1}{4-\sin x-4\cos x}\,dx$ $\left|\begin{array}{l} u=tg\,\frac{x}{2} \\ du=\frac{1}{2\cos^2\frac{x}{2}}\,dx \end{array}\right|$ $\begin{array}{l} \sin x=\frac{2u}{u^2+1} \\ \cos x=\frac{1-u^2}{u^2+1} \end{array}$

$\int \frac{1}{4u^2-u}\,du = \int \frac{1}{u(4u-1)}\,du$ $\left|\begin{array}{l} v=\frac{u}{4u-1} \\ u=(4u-1)v \\ u=\frac{1}{4(4v-1)}+\frac{1}{4} \\ du=-\frac{1}{(4v-1)^2}\,dv \end{array}\right.$

$\int -\frac{1}{v}\,dv = -\ln(|v|)+C$

обр. замена $v=\frac{u}{4u-1}$ $u=tg(\frac{x}{2})$

$\ln(|4\,tg\,\frac{x}{2}-1|)-\ln(|tg(\frac{x}{2})|)+C$

17. $\int_0^{\frac{\pi}{2}} \frac{\cos x\, dx}{4+\sqrt{\sin x}} = \left| t = 4+\sqrt{\sin x} \right| = \int_t^t \frac{2t-8}{t}\,dt =$

$= \int_t 2\,dt - \int_t \frac{8}{t}\,dt = 2t\Big|_t - 8\ln t\Big|_t = 2(4+\sqrt{\sin x})\Big|_0^{\frac{\pi}{2}} -$

$- 8\ln(4+\sqrt{\sin x})\Big|_0^{\frac{\pi}{2}} = 2(4+\sqrt{1}) - 2(4+0) - 8\ln(4+1) +$

$+ 8\ln(4) = 2\cdot 5 - 8\ln 5 + 8\ln 4 - 8 =$

$2 - 8(\ln 5 - \ln 4) = 2 - 8\ln\frac{5}{4}$

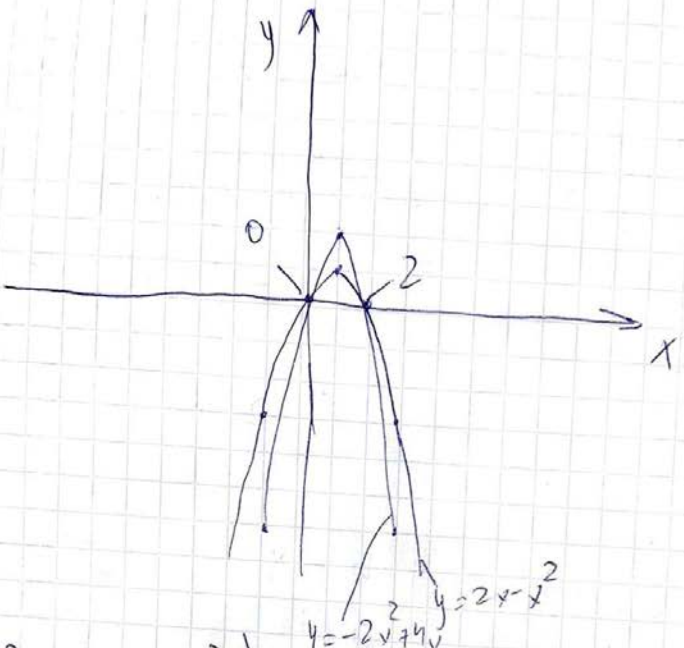
127. $\int_0^{\frac{\pi}{2}} x\cdot\cos^2(x)\,dx = \left| \begin{matrix} u = x & v' = \cos^2 x \\ u' = 1 & v = \frac{1}{2}x + \frac{\sin 2x}{4} \end{matrix} \right|$

$= x\left(\frac{1}{2}x + \frac{\sin 2x}{4}\right)\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}\left(\frac{1}{2}x + \frac{\sin 2x}{4}\right)dx =$

$= \left(\frac{x^2}{2} + \frac{x\sin 2x}{4} - \frac{x^2}{4} + \frac{\cos 2x}{8}\right)\Big|_0^{\frac{\pi}{2}} = \left(\frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}\right)\Big|_0^{\frac{\pi}{2}}$

$= \frac{\pi^2}{16} + 0 - \frac{1}{8} - 0 - 0 - \frac{1}{8} = \frac{\pi^2}{16} - \frac{1}{4}$

13.7 $y=2x-x^2$, $y=-2x^2+4x$



$$\int_0^2 \left((-2x^2+4)-(2x-x^2)\right)dx = \int_0^2 (-x^2-2x+4)\,dx =$$

$$= -\int_0^2 (x^2+2x-4)\,dx = -\frac{x^3}{3}\Big|_0^2 - \frac{2x^2}{2}\Big|_0^2 + 4x\Big|_0^2 = -\frac{8}{3}-4+8 =$$

$$= \cancel{\frac{3}{4}}\ \frac{5}{4}$$

$\text{№ }1. \quad \rho = 3\cos 2\varphi \qquad S = \frac{1}{2}\int_a^b \rho^2(\varphi)\,d\varphi$

$\frac{1}{2}\cdot 9\int \cos^2 2\varphi\, d\varphi = \left|\begin{matrix} 2\varphi = t \\ 2d\varphi = dt \end{matrix}\right| = \frac{1}{2}\cdot 9\cdot\frac{1}{2}\int \cos^2 t\, dt =$

$= \frac{9}{4}\int \frac{1+\cos 2t}{2}\, dt = \frac{9}{8}\int dt + \frac{9}{8}\int \cos 2t\, dt =$

$= \frac{9}{8} t\Big|_a^b + \frac{9}{8}\cdot\frac{\sin 2t}{2}\Big|_a^b = \frac{9}{8} b - \frac{9}{8} a + \frac{9}{16}\sin(2b) - \frac{9}{16}\sin(2a)$