

TP *tramehammukun arawap. Nund yarak*

TP. 7.7

$$\int \frac{\sin 2x}{\sqrt{1+\cos 4x}} dx = \int \frac{2 \cos x \sin x}{\sqrt{1+\cos 4x}} dx = - \int \frac{d \cos^2 x}{\sqrt{1+\cos 4x}} = - \int \frac{dt}{\sqrt{1+t^2}} =$$

$$= -\ln |t + \sqrt{t^2 + 1}| = -\ln |\cos^2 x + \sqrt{\cos 4x}| + C$$

$$d \cos^2 x = -2 \sin x \cos x dx \quad \cos^2 x = t$$

$$d \cos^2 x = 2 \sin x \cos x dx \quad \cos^4 x = t^2$$

$$\text{Answer: } -\ln |\cos^2 x + \sqrt{\cos 4x}| + C$$

TP No 2.7

$$\int \arcsin 2x \cdot dx = \left\{ \begin{array}{l} u = \arcsin 2x \\ u' = \frac{2}{\sqrt{1-4x^2}} \\ v' = 1 \\ v = x \end{array} \right\} = x \arcsin 2x - \int \frac{2x}{\sqrt{1-4x^2}} dx \quad (\equiv)$$

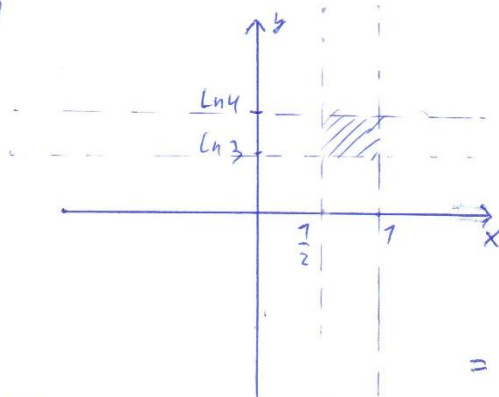
$$\int \frac{2x}{\sqrt{1-4x^2}} dx = \left\{ \begin{array}{l} 1-4x^2 = t \\ dt = -8x dx \\ -\frac{1}{4} dt = 2x dx \end{array} \right\} = -\frac{1}{4} \int \frac{dt}{\sqrt{t}} = -\frac{1}{4} \cdot 2\sqrt{t} = -\frac{1}{2} \sqrt{1-4x^2}$$

$$\equiv x \arcsin 2x + \frac{\sqrt{1-4x^2}}{2} + C$$

$$\text{Answer: } x \arcsin 2x + \frac{\sqrt{1-4x^2}}{2} + C$$

TP 123

$$\int_0^1 \int_{\ln 3}^{\ln 4} 4y e^{2xy} dx dy$$



$$0: y = \ln 3$$

$$y = \ln 4$$

$$x = \frac{1}{2}$$

$$x = 1$$

$$\int_{\ln 3}^{\ln 4} dy \int_{\frac{1}{2}}^1 4y^2 e^{2xy} dx$$

$$\int_{\frac{1}{2}}^1 4y^2 e^{2xy} dx = 4y^2 \frac{e^{2xy}}{2y} \Big|_{\frac{1}{2}}^1 = 2y(e^{2y} - e^y)$$

$$= 2y e^{2y} - 4y e^y$$

$$\int_{\ln 3}^{\ln 4} 2y e^{2y} - 2e^y = 2 \frac{e^{2y}}{2} \Big|_{\ln 3}^{\ln 4} - 2e^y \Big|_{\ln 3}^{\ln 4} = 16 - 9 - 2(4 - 3) = 7 - 8 + 6 = 5$$

Answer: 5

$$\int \frac{x^3 - 3x}{(x+2)(x+7)^2} dx \equiv$$

$$(x+2)(x+7)^2 = (x+2)(x^2+2x+7) = x^3+2x^2+x+2x^2+4x+2 = x^3+4x^2+5x+2$$

$$\frac{x^3-3x}{x^3+4x^2+5x+2} = \frac{p_n(x)}{q_m(x)}, \text{ где } n=m \Rightarrow \text{гробовое деление}$$

$$\begin{array}{r} x^3-3x \\ \underline{-(x^3+4x^2+5x+2)} \\ -4x^2-8x-2 \end{array}$$

$$\frac{x^3-3x}{x^3+4x^2+5x+2} = 1 - \frac{4x^2+8x+2}{x^3+4x^2+5x+2}$$

$$\equiv \int dx - 2 \int \frac{2x^2+4x+7}{(x+2)(x+7)^2} dx$$

$$\frac{2x^2+4x+7}{(x+2)(x+7)^2} = \frac{A}{x+2} + \frac{B}{x+7} + \frac{C}{(x+7)^2} = \frac{Ax^2+2Ax+A+Bx^2+3Bx+2B+C}{(x+2)(x+7)^2}$$

$$\begin{aligned} \text{при } x^2: & \begin{cases} A+B=2 \\ 2A+3B=C=4 \end{cases} \Rightarrow \begin{cases} A=2-B \\ 4-2B+3B+C=4 \\ 2-B+2B+2C=7 \end{cases} \Rightarrow \begin{cases} A=2-B \\ C=-B \\ B-2B=-7 \end{cases} \\ \text{при } x^1: & \\ \text{при } x^0: & \end{aligned}$$

$$\Rightarrow \begin{cases} A=7 \\ C=-7 \\ B=7 \end{cases}, \quad \frac{2x^2+4x+7}{(x+2)(x+7)^2} = \frac{7}{x+2} + \frac{7}{x+7} - \frac{7}{(x+7)^2}$$

$$\equiv \int dx - 2 \int \frac{dx}{x+2} - 2 \int \frac{dx}{x+7} + 2 \int \frac{dx}{(x+7)^2} = x - 2 \ln|x+2| - 2 \ln|x+7| +$$

$$+ \frac{2}{(7-2)(x+7)} + C = x - 2 \ln \left| \frac{x+2}{x+7} \right| - \frac{2}{(x+7)} + C$$

$$\text{Ответ: } x - 2 \ln \left| \frac{x+2}{x+7} \right| - \frac{2}{(x+7)} + C$$

T. P. 5. 7

$$\int \frac{dx}{(x+7)\sqrt{23x^2-2x-27}} = \left| \begin{array}{l} t = \frac{7}{x+7}; x+7 = \frac{7}{t} \\ x = \frac{7}{t} - 7; dx = -\frac{dt}{t^2} \end{array} \right| = \int \frac{dt}{t^2 \sqrt{23\left(\frac{7}{t} - 7\right)^2 - 2\left(\frac{7}{t} - 7\right) - 27}} =$$

$$= \int \frac{-\frac{dt}{t^2}}{\frac{7}{t} \sqrt{23\left(\frac{7}{t} - 7\right)^2 - 2\left(\frac{7}{t} - 7\right) - 27}} = - \int \frac{dt}{t \sqrt{23\left(\frac{7}{t} - 7\right)^2 - \frac{46}{t} + 23 - \frac{2}{t} + 2 - 27}} =$$

$$= - \int \frac{dt}{t \sqrt{\frac{23}{t^2} - \frac{48}{t} + 2}} = - \int \frac{dt}{\sqrt{2t^2 - 48t + 23}} = - \frac{1}{\sqrt{2}} \int \frac{dt}{(t-72)^2 - 72 + \frac{23}{2}} =$$

$$= - \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{(t-72)^2 - 60,5}} = - \frac{1}{\sqrt{2}} \ln \left| -72 + \sqrt{(t-72)^2 - 60,5} \right| + C =$$

$$= - \frac{1}{\sqrt{2}} \ln \left| \frac{7}{x+7} - 72 + \sqrt{\left(\frac{7}{x+7} - 72\right)^2 - 60,5} \right| + C$$

Ответ: $-\frac{1}{\sqrt{2}} \ln \left| \frac{7}{x+7} - 72 + \sqrt{\left(\frac{7}{x+7} - 72\right)^2 - 60,5} \right| + C$

Zusatz Formel

A-02-23

TP 6.7

$$\int \frac{3\sqrt{4-x} - 2\sqrt{3x+2}}{(\sqrt{3x+2} + 3\sqrt{4-x})(3x+2)^2} dx = \int \frac{3\sqrt{4-x} + 4\sqrt{3x+2} - 7\sqrt{3x+2}}{(\sqrt{3x+2} + 3\sqrt{4-x})(3x+2)^2} dx =$$

$$= \int \left(1 - \frac{3\sqrt{3x+2}}{\sqrt{3x+2} + 3\sqrt{4-x}} \right) \frac{1}{(3x+2)^2} dx = \int \left(1 - \frac{3}{1 + 3\sqrt{\frac{4-x}{3x+2}}} \right) \frac{1}{(3x+2)^2} dx =$$

$$= \left| \begin{array}{l} \sqrt{\frac{4-x}{3x+2}} = t \quad dt^2 = 2t dt \\ \frac{4-x}{3x+2} = t^2 \\ \frac{(4-x)(3x+2) - (4-x)(3x+2)'}{(3x+2)^2} dx = 2t dt \end{array} \right. \quad \left| \begin{array}{l} \frac{-3x-2-72+3x}{(3x+2)^2} dx = 2t dt \\ \frac{-74}{(3x+2)^2} dx = 2t dt \\ \frac{dx}{(3x+2)^2} = \frac{-t dt}{7} \end{array} \right| =$$

$$= \int \left(1 - \frac{3}{3t+1} \right) \cdot \frac{-t dt}{7} = -\frac{1}{7} \int \left(1 - \frac{3}{3t+1} \right) \cdot t dt = -\frac{1}{7} \int \left(t - \frac{3t}{3t+1} \right) dt =$$

$$= -\frac{1}{7} \int \left(t - 1 + \frac{1}{3t+1} \right) dt = -\frac{1}{7} \int t dt + \frac{1}{7} \int dt - \frac{1}{7} \int \frac{dt}{3t+1} = -\frac{t^2}{14} + \frac{t}{7} - \frac{1}{7} \cdot \frac{1}{3} \ln \left| \frac{4-x}{3x+2} \right| =$$

$$= -\frac{t^2}{14} + \frac{t}{7} - \frac{1}{21} \ln \left| t + \frac{1}{3} \right| + C = -\frac{4-x}{14(3x+2)} + \frac{1\sqrt{4-x}}{7\sqrt{3x+2}} - \frac{1}{21} \ln \left| \sqrt{\frac{4-x}{3x+2}} + \frac{1}{3} \right| + C$$

TP n 8.7

$$\int \sqrt{5x^2 + 7x + 8} dx = \sqrt{5} \int \sqrt{x^2 + \frac{7}{5}x + \frac{8}{5}} dx = \sqrt{5} \int \sqrt{\left(x + \frac{7}{10}\right)^2 + \frac{8}{5} - \frac{49}{100}} dx$$

$$= \sqrt{5} \int \sqrt{(x + 0,3)^2 + 7,57} dx = \sqrt{5} \int \sqrt{(x + 0,3)^2 + 7,57} d(x + 0,3) =$$

$$= \sqrt{5} \int \sqrt{u^2 + 7,57} du = \frac{\sqrt{5}}{2} u \sqrt{u^2 + 7,57} + \frac{7,57 \cdot \sqrt{5}}{2} \ln|u + \sqrt{u^2 + 7,57}| + C =$$

$$= \left(\frac{\sqrt{5}}{2} x + \frac{0,3\sqrt{5}}{2} \right) \sqrt{(x + 0,3)^2 + 7,57} + \frac{7,57 \cdot \sqrt{5}}{2} \cdot \ln|x + 0,3 + \sqrt{(x + 0,3)^2 + 7,57}| + C$$

$$\text{Antw.: } \left(\frac{\sqrt{5}}{2} x + \frac{0,3\sqrt{5}}{2} \right) \sqrt{(x + 0,3)^2 + 7,57} + \frac{7,57 \cdot \sqrt{5}}{2} \cdot \ln|x + 0,3 + \sqrt{(x + 0,3)^2 + 7,57}| + C$$

n 9.7

$$\int \sin 6x \sin 3x \cos 5x dx = \int \sin 3x \frac{\sin 7x + \sin x}{2} dx = \frac{1}{2} \int \sin 3x \sin 7x + \sin 3x \sin x dx = \frac{1}{2} \int \frac{\cos 8x - \cos 14x}{2} dx + \frac{1}{2} \int \frac{\cos 2x - \cos 4x}{2} dx =$$

$$= \frac{1}{4} \int \cos 8x dx - \frac{1}{4} \int \cos 14x dx + \frac{1}{4} \int \cos 2x dx - \frac{1}{4} \int \cos 4x dx =$$

$$= \frac{\sin 8x}{32} - \frac{\sin 14x}{56} + \frac{\sin 2x}{8} - \frac{\sin 4x}{16} + C$$

$$\text{Antw.: } \frac{\sin 8x}{32} - \frac{\sin 14x}{56} + \frac{\sin 2x}{8} - \frac{\sin 4x}{16} + C$$

TP 10.7

$$\int \frac{dx}{7+9\sin x} = \left| \begin{array}{l} \sin x = \frac{2t}{1+t^2} ; t = \tan \frac{x}{2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right| = \int \frac{2dt}{1+t^2} : \left(7 + \frac{2t}{1+t^2} \right) =$$

$$= \int \frac{2 dt}{1+t^2} \cdot \frac{1+t^2}{t^2+2t+7} = 2 \int \frac{dt}{(t+7)^2} = \frac{2}{-7(t+7)} + C = -\frac{2}{t \frac{x}{2} + 7} + C$$

Obtemos: $-\frac{2}{t \frac{x}{2} + 7} + C$

Курелл Рини

A-02-23

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin^4 x \, dx &= \int_0^{\frac{\pi}{2}} \frac{(1 - \cos^2 x)^2}{4} \, dx = \frac{1}{4} \int_0^{\frac{\pi}{2}} dx - \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos^2 x \, dx + \frac{1}{4} \int_0^{\frac{\pi}{2}} \cos^4 x \, dx = \frac{\pi}{8} - \frac{1}{2} \left(\sin \frac{\pi}{2} - \sin 0 \right) + \\ &+ \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2x}{2} \, dx = \frac{\pi}{8} - \frac{1}{2} + \frac{1}{8} \int_0^{\frac{\pi}{2}} dx + \frac{1}{8} \int_0^{\frac{\pi}{2}} \cos 2x \, dx = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{16} + \frac{1}{16} \left(\sin \frac{\pi}{2} - \sin 0 \right) + \\ &+ \frac{1}{16} \int_0^{\frac{\pi}{2}} \cos 4x \, dx = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{16} + \frac{1}{16} \left(\sin \frac{\pi}{2} - \sin 0 \right) + \frac{1}{16} \left(\sin \frac{\pi}{2} - \sin 0 \right) = \frac{3\pi}{16} - \frac{1}{2} \end{aligned}$$

интегралы
интегрируем

$$0 \leq x \leq \frac{\pi}{2} \Leftrightarrow 0 \leq 2x \leq \pi$$

$$\Rightarrow \frac{3\pi}{16} - \frac{1}{2} \quad \text{Ответ: } \frac{3\pi}{16} - \frac{1}{2}$$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} x \sin^2 \frac{x}{2} \, dx &= \left[\begin{array}{l} \frac{x}{2} = u \\ x = 2u \\ du = \frac{1}{2} dx \end{array} \right] \begin{array}{l} dx = 2du \\ 0 \leq x \leq \frac{\pi}{2} \\ 0 \leq \frac{x}{2} \leq \frac{\pi}{4} \end{array} = \int_0^{\frac{\pi}{4}} 4u \sin^2 u \, du = 4 \int_0^{\frac{\pi}{4}} u \left(\frac{1 - \cos 2u}{2} \right) du = \\ &= 2 \int_0^{\frac{\pi}{4}} u \, du - 2 \int_0^{\frac{\pi}{4}} u \cos 2u \, du = 2 \left[\frac{u^2}{2} \right]_0^{\frac{\pi}{4}} - 2 \left(\frac{u \sin 2u}{2} + \frac{\cos 2u}{4} \right) \Big|_0^{\frac{\pi}{4}} = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} + \\ &+ \frac{\cos \frac{\pi}{2}}{2} - \frac{\cos 0}{4} = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} \\ \text{Ответ: } \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} \end{aligned}$$

N. 13. 7

$$y = 3^x \sqrt{3^x - 7} \quad y = 0; \quad x = \log_3 5$$

$$y = 0 \quad 3\sqrt{2}$$

$$x = 0 \quad 7 \log_3 5$$

$$3^{\log_3 5} \sqrt{3^{\log_3 5} - 7} = 5 \sqrt{5-7} = 10$$

$$\int_0^{\log_3 5} 3^x \sqrt{3^x - 7} dx = \left| \frac{d(3^x - 7) = 3^x \ln 3 dx}{dx = \frac{d(3^x - 7)}{3^x \ln 3}} \right| =$$

$$0 \leq x \leq \log_3 5$$

$$7 \leq 3^x \leq 5$$

$$0 \leq 3^x - 7 \leq 4$$

$$3^x - 7 = u$$

$$= \int_0^4 \frac{\sqrt{u} du}{\ln 3} = \frac{1}{\ln 3} \int_0^4 \sqrt{u} du =$$

$$= \frac{1}{\ln 3} \frac{2}{3} u^{\frac{3}{2}} \Big|_0^4 = \frac{2}{3 \ln 3} \sqrt{4^3} = \frac{16}{3 \ln 3}$$

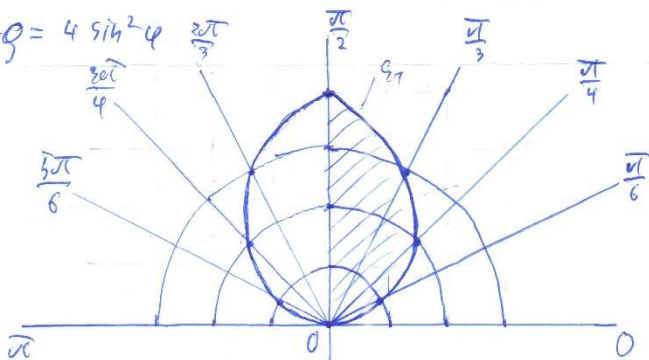
Ответ: $\frac{16}{3 \ln 3}$

N. 14. 7

$$\rho = 4 \sin^2 \varphi \quad \frac{2\pi}{3}$$

$$\frac{2\pi}{4}$$

$$\frac{5\pi}{6}$$



$$\varphi = 0 \quad \frac{\pi}{2} \quad \frac{\pi}{4} \quad \frac{\pi}{3}$$

$$\rho = 0 \quad 4 \quad 1 \quad 2 \quad 3$$

$$4 \sin^2 \varphi = 4$$

$$4 \sin^2 \frac{\pi}{6} = 4 \cdot \frac{1}{4} = 1 \quad 4 \sin^2 \frac{\pi}{3} = 4 \cdot \frac{3}{4} = 3$$

$$4 \sin^2 \frac{\pi}{4} = 4 \cdot \frac{1}{2} = 2$$

периметр кривой $\rho = 2r_1$ $0 \leq \varphi \leq \frac{\pi}{2}$ $\frac{1}{2} \int_0^{\pi} \rho^2(\varphi) d\varphi = S_1$

$$2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} 16 \sin^4 \varphi d\varphi = 16 \int_0^{\frac{\pi}{2}} \sin^4 \varphi d\varphi = 16 \int_0^{\frac{\pi}{2}} \frac{(1 - \cos 2\varphi)^2}{4} d\varphi = 4 \int_0^{\frac{\pi}{2}} d\varphi - 4 \int_0^{\frac{\pi}{2}} 2 \cos 2\varphi d\varphi +$$

$$+ 4 \int_0^{\frac{\pi}{2}} (\cos 2\varphi)^2 d\varphi = 2\pi - 2 \left(\frac{\sin 2\varphi}{2} - \frac{\sin 4\varphi}{4} \right) + 4 \int_0^{\frac{\pi}{2}} \frac{1 + \cos 4\varphi}{2} d\varphi = 2\pi + 2 \int_0^{\frac{\pi}{2}} (1 + \cos 4\varphi) d\varphi =$$

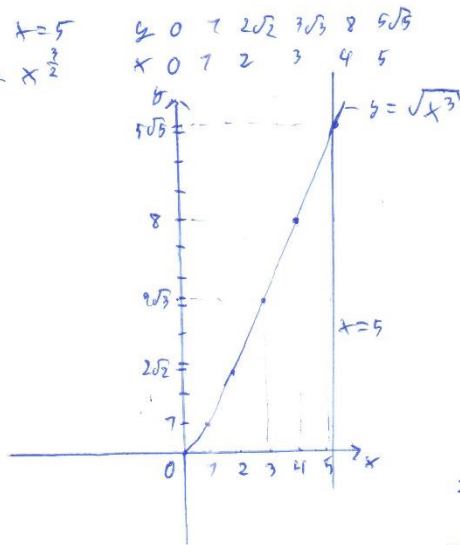
$$= 2\pi + 2\pi + 2 \left(\frac{\sin 4\varphi}{4} - \frac{\sin 8\varphi}{8} \right) = 3\pi$$

Ответ: 3π

TP 19.7

$$y^2 = x^3 \Rightarrow x = 5$$

$$y = \sqrt{x^3} = x^{\frac{3}{2}}$$



Answer: $\frac{37\sqrt{5}}{27}$

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$f' = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x}$$

$$L = \int_0^5 \sqrt{1 + \frac{9}{4}x} dx = \left| d\left(1 + \frac{9}{4}x\right) = \frac{9}{4}dx \right| =$$

$$0 \leq x \leq 5$$

$$0 \leq \frac{9}{4}x \leq \frac{45}{4}$$

$$1 \leq \frac{9}{4}x + 1 \leq \frac{49}{4}$$

$$= \frac{4}{9} \int_1^{\frac{49}{4}} \sqrt{u} du = \frac{4}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^{\frac{49}{4}} =$$

$$= \frac{8}{27} \left(\sqrt{\left(\frac{49}{4}\right)^3} - \sqrt{1} \right) = \frac{8}{27} \left(\sqrt{\frac{7^6}{2^6}} - 1 \right) =$$

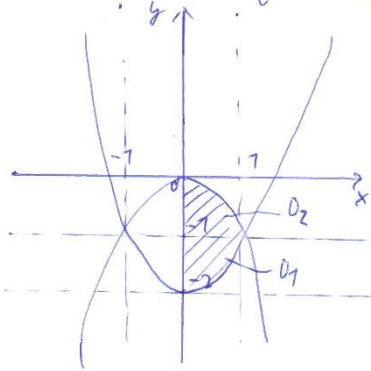
$$= \frac{8}{27} \left(\frac{7^3}{2^3} - 1 \right) = \frac{343}{27} - \frac{8}{27} = \frac{335}{27}$$

Криволинейные интегралы ТР № 7

$$\int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f dx \oplus$$

область интегрирования: $D = D_1 \cup D_2$

$$D_1 = \begin{cases} -2 \leq y \leq -1 \\ 0 \leq x \leq \sqrt{2+y} \end{cases}; \quad D_2 = \begin{cases} -1 \leq y \leq 0 \\ \sqrt{-y} \leq x \leq 0 \end{cases}$$



$$0 \leq x = \sqrt{2+y}$$

$$x^2 = 2+y$$

$$y = x^2 - 2$$

$$\sqrt{-y} \leq x \leq 0$$

$$-y = x^2$$

$$y = -x^2$$

$$0 \leq x = \sqrt{-y}$$

$$x^2 = -y$$

$$y = -x^2$$

$$-x^2 = x^2 - 2$$

$$2x^2 = 2$$

$$x = \pm 1$$

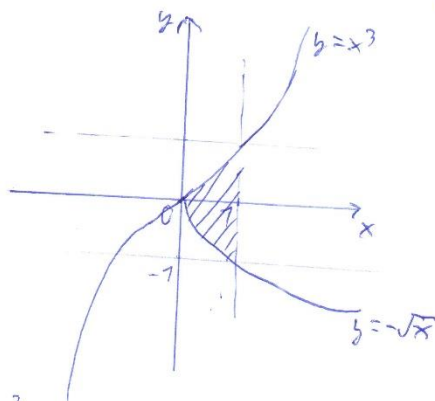
$$D: \begin{cases} 0 \leq x \leq 1 \\ x^2 - 2 \leq y \leq -x^2 \end{cases}$$

$$\oplus \int_0^1 dx \int_{x^2-2}^{-x^2} f dy$$

$$\text{Answer: } \int_0^1 dx \int_{x^2-2}^{-x^2} f dy$$

$$\iint_D (78x^2y^2 + 32x^3y^3) dx dy \quad \text{TP 12.2}$$

$$D: \begin{cases} x=7 \\ y=x^3 \\ y=-\sqrt{x} \end{cases}$$



$$\int_0^7 dx \int_{-\sqrt{x}}^{x^3} (78x^2y^2 + 32x^3y^3) dy$$

$$\int_{-\sqrt{x}}^{x^3} (78x^2y^2 + 32x^3y^3) dy = 78x^2 \left. \frac{y^3}{3} \right|_{-\sqrt{x}}^{x^3} + 32x^3 \left. \frac{y^4}{4} \right|_{-\sqrt{x}}^{x^3} = 78x^2 \left(\frac{x^9}{3} - \frac{-x^{\frac{3}{2}}}{2} \right) + 32x^3 \left(\frac{x^{\frac{7}{2}}}{4} - \frac{-x^2}{4} \right) = 6x^{11} + 6x^{\frac{7}{2}} + 8x^{15} - 8x^5$$

$$\int_0^7 (6x^{11} + 6x^{\frac{7}{2}} + 8x^{15} - 8x^5) dx = 6 \left. \frac{x^{12}}{12} \right|_0^7 + 6 \left. \frac{x^{\frac{9}{2}}}{\frac{9}{2}} \right|_0^7 + 8 \left. \frac{x^{16}}{16} \right|_0^7 - 8 \left. \frac{x^6}{6} \right|_0^7 =$$

$$= \frac{7}{2} + \frac{4}{3} + \frac{7}{2} - \frac{4}{3} = 7$$

Intem: $\frac{7}{3}$ 7.

TP No 3.7

$$\int \frac{6x-5}{\sqrt{-x^2+2x+3}} dx = - \int \frac{6x-5}{\sqrt{x^2-2x-3}} dx = - \int \frac{6x-5}{\sqrt{(x-1)^2-4}} dx = \left. \begin{array}{l} u = x-1 \\ du = dx \\ x = u+1 \end{array} \right\} =$$

$$= - \int \frac{6u+1}{\sqrt{u^2-4}} du = -6 \int \frac{u du}{\sqrt{u^2-4}} - \int \frac{du}{\sqrt{u^2-4}} = -6 \int \frac{u du}{\sqrt{u^2-4}} + \operatorname{arcsinh}\left(\frac{u}{2}\right) \oplus$$

$$\int \frac{u du}{\sqrt{u^2-4}} = \left\{ \begin{array}{l} z = u^2-4 \\ dz = 2u du \\ \frac{dz}{2} = u du \end{array} \right\} = \frac{1}{2} \int \frac{dz}{\sqrt{z}} = \frac{1}{2} \sqrt{z} \cdot 2 + C = \sqrt{u^2-4} + C$$

$$\oplus -6 \sqrt{u^2-4} + \operatorname{arcsinh}\left(\frac{u}{2}\right) + C = \operatorname{arcsinh}\left(\frac{x-1}{2}\right) - 6 \sqrt{x^2-2x-3} + C$$

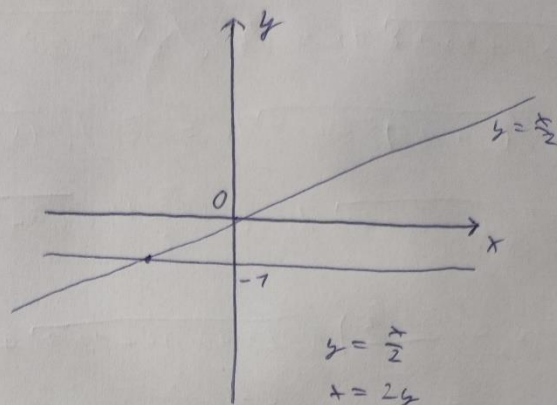
$$\text{Omgem: } \operatorname{arcsinh}\left(\frac{x-1}{2}\right) - 6 \sqrt{x^2-2x-3} + C$$

TP 4.7

$$\iiint_V y^2 \cos\left(\frac{\sqrt{\pi}}{4} x y\right) dx dy dz$$

$$V = \begin{cases} x=0; & y=-1; & y=\frac{x}{2} \\ z=0; & z=-\sqrt{1^2} \end{cases}$$

$$\int_{-1}^0 dy \int_0^{2y} dx \int_{-\sqrt{1^2}}^0 y^2 \cos\left(\frac{\sqrt{\pi}}{4} x y\right) dz =$$



$$= \int_{-1}^0 dy \int_0^{2y} dx y^2 \cos\left(\frac{\sqrt{\pi}}{4} x y\right) z \Big|_{-\sqrt{1^2}}^0 = \int_{-1}^0 dy \int_0^{2y} -\pi^2 y^2 \cos\left(\frac{\sqrt{\pi}}{4} x y\right) dx = \int_{-1}^0 dy$$

$$= \int_{-1}^0 dy -\pi^2 y^2 z \frac{\sin\left(\frac{\sqrt{\pi}}{4} x y\right)}{\frac{\sqrt{\pi}}{4} y} \Big|_0^{2y} = -4\pi y \int_{-1}^0 dy y \sin\left(\frac{\sqrt{\pi}}{2} y^2\right) = \left| t = \frac{y^2}{2} \right|_{-1}^0 =$$

$$= -4\pi \int_{-1}^0 \sin(\sqrt{\pi} t) dt = 4\pi \frac{\cos(t\sqrt{\pi})}{\sqrt{\pi}} \Big|_{-1}^0 = \frac{4\pi}{\sqrt{\pi}} \cos\left(\frac{y^2\sqrt{\pi}}{2}\right) \Big|_{-1}^0 = 4(1-0) = 4$$

Answer: 4

T.P. ~ 5.7

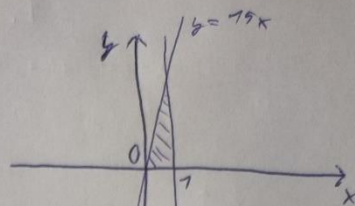
$$\int \int \int_V y \, dx \, dy \, dz$$

$$V: \begin{cases} y = 15x, y=0, x=7 \\ z=x^2, z=0 \end{cases}$$

$$\int_0^7 dx \int_0^{15x} dy \int_0^{x^2} dz = \int_0^7 dx \int_0^{15x} y \, dy \cdot x^2 = \int_0^7 dx \cdot x \cdot \frac{y^2}{2} \Big|_0^{15x} =$$

$$\textcircled{3} \int_0^7 \frac{15^2 x^4}{2} dx = \frac{15^2 x^5}{10} \Big|_0^7 = 225$$

Ans: 225



T.P. ~ 6.7

$$x = 5 - y^2; x = -4y$$

$$y^2 = 5 - x; y = -\frac{x}{4}$$

$$y = \sqrt{5 - x}$$

$$S = \int_{-7}^5 dy \int_{-4y}^{5-y^2} dx = \int_{-7}^5 (5 - y^2 + 4y) dy =$$

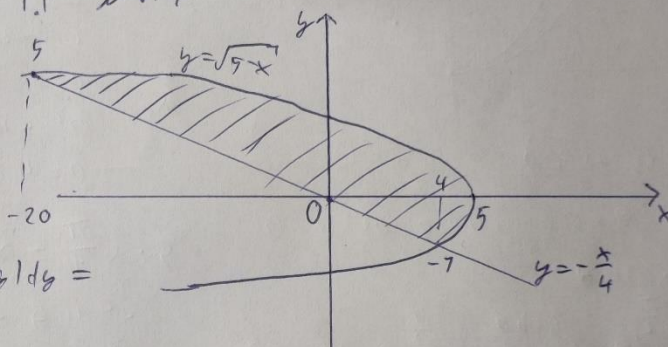
$$= \left(5y - \frac{y^3}{3} + \frac{4y^2}{2} \right) \Big|_{-7}^5 =$$

$$= 25 - \frac{125}{3} + \frac{4 \cdot 25}{2} - \left(-5 + \frac{7}{3} + 2 \right) =$$

$$= 25 + 50 - \frac{125}{3} + 5 - \frac{7}{3} - 2 = 78 -$$

$$- \frac{126}{3} = 78 - 42 = 36$$

Ans: S = 36



$$\sqrt{5-x} = -\frac{x}{4}$$

$$\frac{x^2}{16} = 5 - x$$

$$x^2 + 16x - 80 = 0$$

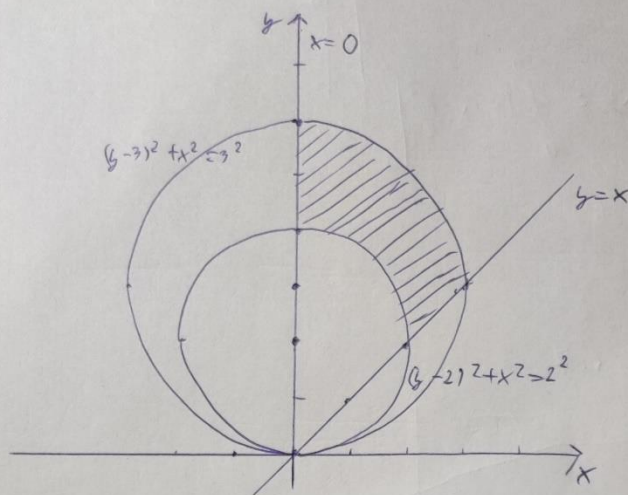
$$0 = 256 + 4 \cdot 80 = 576 = 24^2$$

$$x_1 = \frac{-16 + 24}{2} = 4$$

$$x_2 = \frac{-16 - 24}{2} = -20$$

T.P. 17.7

$$\begin{cases} y^2 - 4y + x^2 = 0 & y^2 - 4y + 4 + x^2 = 4 & (y-2)^2 + x^2 = 2^2 & \begin{cases} x = \rho \cos \theta \\ y = \rho \sin \theta \end{cases} \\ y^2 - 6y + x^2 = 0 & y^2 - 6y + 9 + x^2 = 9 & (y-3)^2 + x^2 = 3^2 \\ y = x, x = 0 \end{cases}$$



$$y = x$$

$$\rho \sin \theta = \rho \cos \theta$$

$$\tan \theta = 1$$

$$\theta = \frac{\pi}{4}$$

$$x = 0$$

$$\rho \cos \theta = 0$$

$$\begin{cases} \rho = 0 \\ \cos \theta = 0 \end{cases} \Rightarrow \theta = \frac{\pi}{2}$$

$$y^2 - 4y + x^2 = 0$$

$$\rho^2 \sin^2 \theta - 4\rho \sin \theta + \rho^2 \cos^2 \theta = 0$$

$$\rho^2 - 4\rho \sin \theta = 0$$

$$\rho = 4 \sin \theta$$

$$\rho: \begin{cases} \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \end{cases}$$

$$4 \sin \theta \leq \rho \leq 6 \sin \theta$$

$$S = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{4 \sin \theta}^{6 \sin \theta} \rho \, d\rho = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left. \frac{\rho^2}{2} \right|_{4 \sin \theta}^{6 \sin \theta} d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{36 \sin^2 \theta}{2} - \frac{16 \sin^2 \theta}{2} \right) d\theta =$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 10 \sin^2 \theta \, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{10(1 - \cos 2\theta)}{2} d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (5 - 5 \cos 2\theta) d\theta =$$

$$= 5\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \frac{5 \sin 2\theta}{2} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 5 \left(\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2} (\sin \pi - \sin \frac{\pi}{2}) \right) = 5 \left(\frac{\pi}{4} - \frac{1}{2} (0 - 1) \right) =$$

$$= 5 \left(\frac{\pi}{4} + \frac{1}{2} \right) = \frac{5\sqrt{1}}{4} + \frac{5}{2}$$

$$\text{Omfang: } \frac{5\sqrt{1}}{4} + \frac{5}{2}$$