ТР Математический анализ. Интегралы

ТР. 7.7

$$\int \frac{\sin 2x}{\sqrt{1+\cos 4x}}dx = \int \frac{2\cos x \sin x}{\sqrt{1+\cos 4x}}dx = -\int \frac{d\cos^2 x}{\sqrt{1+\cos 4x}} = -\int \frac{dt}{\sqrt{1+t^2}} =$$

$$= -\ln|t+\sqrt{t^2+1}| = -\ln|\cos^2 x + \sqrt{\cos 4x} + 1| + C$$

$d\cos^2 x = -2\sin x \cos x\, dx \qquad \cos^2 x = t$

$-d\cos^2 x = 2\sin x \cos x\, dx \qquad \cos^4 x = t^2$

Ответ: $-\ln|\cos^2 x + \sqrt{\cos 4x} + 1| + C$

ТР № 2.7

$$\int \arcsin 2x \cdot dx = \left\{\begin{array}{l} u = \arcsin 2x \\ u' = \frac{2}{\sqrt{1-4x^2}} \\ v' = 1 \\ v = x \end{array}\right\} = x\arcsin 2x - \int \frac{2x}{\sqrt{1-4x^2}}dx =$$

$$\int \frac{2x}{\sqrt{1-4x^2}}dx = \left\{\begin{array}{l} 1-4x^2 = t \\ dt = -8x\,dx \\ -\frac{1}{4}dt = 2x\,dx \end{array}\right\} = -\frac{1}{4}\int \frac{dt}{\sqrt{t}} = -\frac{1}{4}\cdot 2\sqrt{t} = -\frac{1}{2}\sqrt{1-4x^2}$$

$$= x\arcsin 2x + \frac{\sqrt{1-4x^2}}{2} + C$$

Ответ: $x\arcsin 2x + \frac{\sqrt{1-4x^2}}{2} + C$

ТР №3

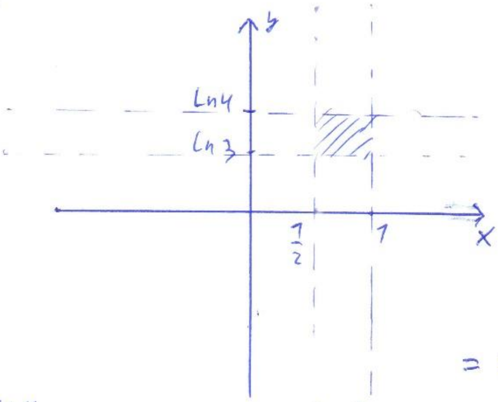
$$\iint\limits_D 4y\, e^{2xy}\, dx\, dy$$

$D:$ $y = \ln 3$

$y = \ln 4$

$x = \frac{1}{2}$

$x = 1$



$$\int\limits_{\ln 3}^{\ln 4} dy \int\limits_{\frac{1}{2}}^{1} 4y^2 e^{2xy}\, dx$$

$$\int\limits_{\frac{1}{2}}^{1} 4y^2 e^{2xy}\, dx = 4y^2\, \frac{e^{2xy}}{2y}\Bigg|_{\frac{1}{2}}^{1} = 2y(e^{2y} - e^{y}) =$$

$$= 2y\, e^{2y} - 4y\, e^{y}$$

$$\int\limits_{\ln 3}^{\ln 4} 2y\, e^{2y} - 2e^{y} = 2\,\frac{e^{2y}}{2}\Bigg|_{\ln 3}^{\ln 4} - 2\,e^{y}\Bigg|_{\ln 3}^{\ln 4} = 16 - 9 - 2(4-3) = 7 - 8 + 6 = 5$$

Ответ: 5

ТР № 4.7

$$\int \frac{x^3-3x}{(x+2)(x+1)^2} =$$

$$(x+2)(x+1)^2 = (x+2)(x^2+2x+1) = x^3+2x^2+x+2x^2+4x+2 = x^3+4x^2+5x+2$$

$$\frac{x^3-3x}{x^3+4x^2+5x+2} = \frac{P_n(x)}{Q_m(x)},$$ где $n=m \Rightarrow$ дробь неправильная

$$\cancel{\frac{x^3-3x}{x^3+4x^2+5x+2}}$$

$$\begin{array}{r|l} x^3-3x & x^3+4x^2+5x+2 \\ \underline{-\;x^3+4x^2+5x+2} & 1 \\ \cancel{-4x^2+5x+2} & \\ -4x^2-8x-2 & \end{array}$$

$$\frac{x^3-3x}{x^3+4x^2+5x+2} = 1 - \frac{4x^2+8x+2}{x^3+4x^2+5x+2}$$

$$= \int dx - 2\int \frac{2x^2+4x+1}{(x+2)(x+1)^2} \overset{*}{=}$$

$$\frac{2x^2+4x+1}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} = \frac{Ax^2+2Ax+A+Bx^2+3Bx+2B+Cx+2C}{(x+2)(x+1)^2}$$

при x^2: при x^1: при x^0:
$$\begin{cases} A+B=2 \\ 2A+3B=C=4 \\ A+2B+2C=1 \end{cases} \Leftrightarrow \begin{cases} A=2-B \\ 4-2B+3B+C=4 \\ 2-B+2B+2C=1 \end{cases} \Leftrightarrow \begin{cases} A=2-B \\ C=-B \\ B-2B=-1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} A=1 \\ C=-1 \\ B=1 \end{cases} \quad \frac{2x^2+4x+1}{(x+2)(x+1)^2} = \frac{1}{(x+2)} + \frac{1}{(x+1)} - \frac{1}{(x+2)^2}$$

$$\overset{*}{=} \int dx - 2\int\frac{dx}{x+2} - 2\int\frac{dx}{x+1} + 2\int\frac{dx}{(x+1)^2} = x - 2\ln|x+2| - 2\ln|x+1| +$$

$$+ \frac{2}{(1-2)(x+1)} + C = x - 2\ln\left|\frac{x+2}{x+1}\right| - \frac{2}{(x+1)} + C$$

Ответ: $x - 2\ln\left|\frac{x+2}{x+1}\right| - \frac{2}{(x+1)} + C$

Т.Р №5.7

$$\int \frac{dx}{(x+1)\sqrt{23x^2-2x-21}} = \left| \begin{matrix} t=\frac{1}{(x+1)};\ x+1=\frac{1}{t} \\ x=\frac{1}{t}-1;\ dx=-\frac{dt}{t^2} \end{matrix} \right| = \cancel{\int \frac{dt}{\ }}$$

$$= \int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{23\left(\frac{1}{t}-1\right)^2-2\left(\frac{1}{t}-1\right)-21}} = -\int \frac{dt}{t\sqrt{23\frac{1}{t^2}-\frac{46}{t}+23-\frac{2}{t}+2-21}} =$$

$$= -\int \frac{dt}{t\sqrt{\frac{23}{t^2}-\frac{48}{t}+2}} = -\int \frac{dt}{\sqrt{2t^2-48t+23}} = -\frac{1}{\sqrt{2}}\int \frac{dt}{(t-12)^2-72+\frac{23}{2}} =$$

$$= -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{(t-12)^2-60{,}5}} = -\frac{1}{\sqrt{2}}\ln\left|-12+\sqrt{(t-12)^2-60{,}5}\right|+C =$$

$$= -\frac{1}{\sqrt{2}}\ln\left|\frac{1}{(x+1)}-12+\sqrt{\left(\frac{1}{(x+1)}-12\right)^2-60{,}5}\right|+C$$

Ответ: $-\frac{1}{\sqrt{2}}\ln\left|\frac{1}{(x+1)}-12+\sqrt{\left(\frac{1}{(x+1)}-12\right)^2-60{,}5}\right|+C$

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ТР №6.7

$$\int \frac{3\sqrt{4-x}-2\sqrt{3x+2}}{(\sqrt{3x+2}+3\sqrt{4-x})(3x+2)^2}dx = \int \frac{3\sqrt{4-x}+\sqrt{3x+2}-3\sqrt{3x+2}}{(\sqrt{3x+2}+3\sqrt{4-x})\ (3x+2)^2}dx =$$

$$= \int\left(1-\frac{3\sqrt{3x+2}}{\sqrt{3x+2}+3\sqrt{4-x}}\right)\frac{1}{(3x+2)^2}dx = \int\left(1-\frac{3}{1+3\sqrt{\frac{4-x}{3x+2}}}\right)\frac{1}{(3x+2)^2}dx =$$

$$= \left| \begin{array}{ll} \sqrt{\frac{4-x}{3x+2}} = t \quad dt^2 = 2t\,dt & \frac{-3x-2-12+3x}{(3x+2)^2}\,dx = 2t\,dt \\ \frac{4-x}{3x+2} = t^2 & \frac{-14}{(3x+2)^2}\,dx = 2t\,dt \\ \frac{(4-x)'(3x+2)-(4-x)(3x+2)'}{(3x+2)^2}\,dx = 2t\,dt & \frac{dx}{(3x+2)^2} = \frac{-t\,dt}{7} \end{array} \right| =$$

$$= \int\left(1-\frac{3}{3t+1}\right)\cdot\frac{-t\,dt}{7} = -\frac{1}{7}\int\left(1-\frac{3}{3t+1}\right)\cdot t\,dt = -\frac{1}{7}\int\left(t-\frac{3t}{3t+1}\right)dt =$$

$$= -\frac{1}{7}\int\left(t-1+\frac{1}{3t+1}\right)dt = -\frac{1}{7}\int t\,dt + \frac{1}{7}\int dt - \frac{1}{7}\int\frac{dt}{3t+1} = -\frac{t^2}{14}+\frac{t}{7}-\frac{1}{7}\cdot\frac{1}{3}\int\frac{dt}{t+\frac{1}{3}} =$$

$$= -\frac{t^2}{14}+\frac{t}{7}-\frac{1}{21}\ln\left|t+\frac{1}{3}\right|+C = -\frac{4-x}{14(3x+2)}+\frac{1\sqrt{4-x}}{7\sqrt{3x+2}}-\frac{1}{21}\ln\left|\sqrt{\frac{4-x}{3x+2}}+\frac{1}{3}\right|+C$$

ТР №8.7

$$\int \sqrt{5x^2+3x+8}\,dx = \sqrt{5}\int \sqrt{x^2+\frac{3}{5}x+\frac{8}{5}}\,dx = \sqrt{5}\int \sqrt{\left(x+\frac{3}{10}\right)^2+\frac{8}{5}-\frac{9}{100}}\,dx =$$

$$= \sqrt{5}\int \sqrt{(x+0,3)^2+1,51}\,dx = \sqrt{5}\int \sqrt{(x+0,3)^2+1,51}\,d(x+0,3) =$$

$$= \sqrt{5}\int \sqrt{u^2+1,51}\,du = \frac{\sqrt{5}}{2}u\sqrt{u^2+1,51} + \frac{1,51\cdot\sqrt{5}}{2}\ln\left|u+\sqrt{u^2+1,51}\right| + C =$$

$$= \left(\frac{\sqrt{5}}{2}x + \frac{0,3\sqrt{5}}{2}\right)\sqrt{(x+0,3)^2+1,51} + \frac{1,51\cdot\sqrt{5}}{2}\cdot\ln\left|x+0,3+\sqrt{(x+0,3)^2+1,51}\right| + C$$

Ответ: $\left(\frac{\sqrt{5}}{2}x + \frac{0,3\sqrt{5}}{2}\right)\sqrt{(x+0,3)^2+1,51} + \frac{1,51\cdot\sqrt{5}}{2}\cdot\ln\left|x+0,3+\sqrt{(x+0,3)^2+1,51}\right| + C$

№9.7

$$\int \sin 6x \sin 3x \cos 5x = \int \sin 3x\,\frac{\sin 11x+\sin x}{2}\,dx = \frac{1}{2}\int \sin 3x \sin 11x +$$

$$+ \sin 3x \sin x = \frac{1}{2}\int \frac{\cos 8x - \cos 14x}{2}\,dx + \frac{1}{2}\int \frac{\cos 2x - \cos 4x}{2}\,dx =$$

$$= \frac{1}{4}\int \cos 8x\,dx - \frac{1}{4}\int \cos 14x\,dx + \frac{1}{4}\int \cos 2x\,dx - \frac{1}{4}\int \cos 4x\,dx =$$

$$= \frac{\sin 8x}{32} - \frac{\sin 14x}{56} + \frac{\sin 2x}{8} - \frac{\sin 4x}{16} + C$$

Ответ: $\frac{\sin 8x}{32} - \frac{\sin 14x}{56} + \frac{\sin 2x}{8} - \frac{\sin 4x}{16} + C$

ТР № 10. 7

$$\int \frac{dx}{1+\sin x} = \left| \begin{array}{l} \sin x = \frac{2t}{1+t^2}\ ;\ t = \operatorname{tg}\frac{x}{2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right| = \int \frac{2dt}{1+t^2} : \left(1^{1+t^2} + \frac{2t}{1+t^2}\right) =$$

$$= \int \frac{2\,dt}{\cancel{1+t^2}} \cdot \frac{\cancel{1+t^2}}{t^2+2t+1} = 2\int \frac{dt}{(t+1)^2} = \frac{2}{-1(t+1)} + C = -\frac{2}{\operatorname{tg}\frac{x}{2}+1} + C$$

Ответ: $-\dfrac{2}{\operatorname{tg}\frac{x}{2}+1} + C$

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№ 11.7

$$\int_0^{\frac{\pi}{2}} \sin^4 \frac{x}{2}\,dx = \int_0^{\frac{\pi}{2}} \frac{(1-\cos x)^2}{4}\,dx = \frac{1}{4}\int_0^{\frac{\pi}{2}} dx - \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos x\,dx + \frac{1}{4}\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = \frac{\pi}{8} - \frac{1}{2}(\sin\frac{\pi}{2} - \underbrace{\sin 0}_{0}) +$$

$$+\frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2}\,dx = \frac{\pi}{8} - \frac{1}{2} + \frac{1}{8}\int_0^{\frac{\pi}{2}} dx + \frac{1}{8}\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 2x\,d2x = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{16} + \frac{1}{16}\cancel{(\sin\frac{\pi}{2} -}$$

$$\cancel{-\sin 0)} = \cancel{\frac{3\pi}{16} - \frac{1}{2} + \frac{1}{16} = \frac{3\pi}{16} - \frac{1}{16}} = \frac{3\pi}{16} - \frac{1}{2} + \frac{1}{16}\int_0^{\pi} \cos u\,du = \frac{3\pi}{16} - \frac{1}{2} + \frac{1}{16}(\underbrace{\sin\pi - \sin 0}_{0}) =$$

смена пределов интегрирования

$0 \le x \le \frac{\pi}{2} \iff 0 \le 2x \le \pi$

$$= \frac{3\pi}{16} - \frac{1}{2}$$

Ответ: $\frac{3\pi}{16} - \frac{1}{2}$

№ 12.7

$$\int_0^{\frac{\pi}{2}} x \sin^2\frac{x}{2}\,dx = \left| \begin{array}{ll} \frac{x}{2} = u & dx = 2du \\ x = 2u & 0 \le x \le \frac{\pi}{2} \\ du = \frac{1}{2}dx & 0 \le \frac{x}{2} \le \frac{\pi}{4} \end{array} \right| = \int_0^{\frac{\pi}{4}} 4u \sin^2 u\,du = 4\int_0^{\frac{\pi}{4}} u\left(\frac{1-\cos 2u}{2}\right)du =$$

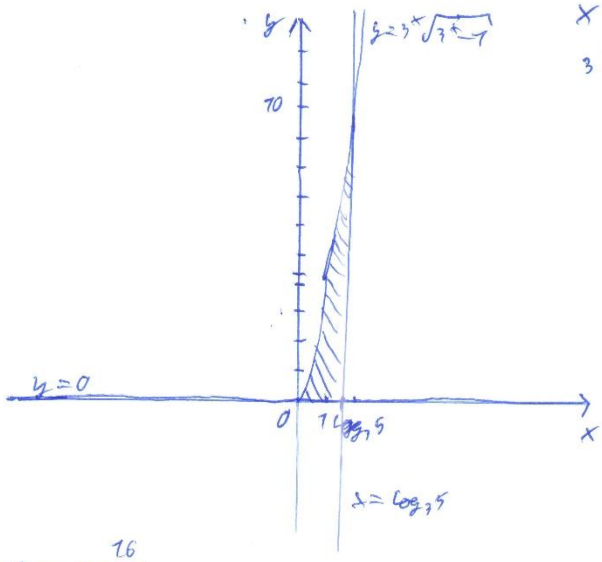
$$= 2\int_0^{\frac{\pi}{4}} u\,du - 2\int_0^{\frac{\pi}{4}} u\cos 2u\,du = 2\,\frac{u^2}{2}\Bigg|_0^{\frac{\pi}{4}} - 2\left(\frac{u\sin 2u}{2} + \frac{\cos 2u}{4}\right)\Bigg|_0^{\frac{\pi}{4}} = \frac{\pi^2}{16} - \frac{\pi}{4}\sin\frac{\pi}{2} +$$

$$+ \underbrace{\frac{\cos\frac{\pi}{2}}{2}}_{0} - \underbrace{0\sin 0}_{0} - \frac{\cos 0}{2}\Big) = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}$$

Ответ: $\frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}$

№ 13.7

$y = 3^x\sqrt{3^x-1}$ $y = 0;$ $x = \log_3 5$



y	0	$3\sqrt{2}$	
x	0	1	$\log_3 5$

$3^{\log_3 5}\sqrt{3^{\log_3 5}-1} = 5\sqrt{5-1} = 10$

$$\int_0^{\log_3 5} 3^x\sqrt{3^x-1}\,dx = \left| \begin{array}{l} d(3^x-1) = 3^x\ln 3\,dx \\ dx = \dfrac{d(3^x-1)}{3^x\ln 3} \\ 0 \le x \le \log_3 5 \\ 1 \le 3^x \le 5 \\ 0 \le 3^x - 1 \le 4 \\ 3^x-1=u \end{array} \right| =$$

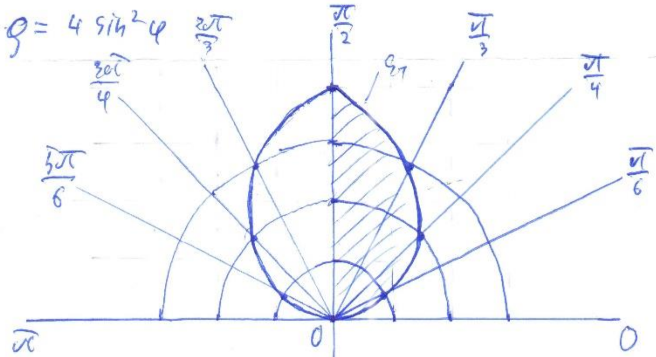
$$= \int_0^4 \frac{\sqrt{u}\,du}{\ln 3} = \frac{1}{\ln 3}\int_0^4 \sqrt{u}\,du =$$

$$= \frac{1}{\ln 3}\,\frac{2}{3}\,u^{\frac{3}{2}}\Big|_0^4 = \frac{2}{3\ln 3}\sqrt{4^3} = \frac{16}{3\ln 3}$$

Ответ: $\frac{16}{3\ln 3}$

№14.7

$\rho = 4\sin^2\varphi$



φ	0	$\frac{\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
ρ	0	4	1	2	3

$4\sin^2\varphi = 4$

$4\sin^2\frac{\pi}{6} = 4\cdot\frac{1}{4} = 1$ $4\sin^2\frac{\pi}{3} = 4\cdot\frac{3}{4} = 3$

$4\sin^2\frac{\pi}{4} = 4\cdot\frac{1}{2} = 2$

Фигура симметрична поэтому $S = 2S_1$ $0 \le \varphi \le \frac{\pi}{2}$ $\frac{1}{2}\int_\alpha^\beta \rho^2(\varphi)\,d\varphi = S_1$

$$2\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} 16\sin^4\varphi\,d\varphi = 16\int_0^{\frac{\pi}{2}}\sin^4\varphi\,d\varphi = 16\int_0^{\frac{\pi}{2}}\frac{(1-\cos 2\varphi)^2}{4}\,d\varphi = 4\int_0^{\frac{\pi}{2}}d\varphi - 4\int_0^{\frac{\pi}{2}}2\cos 2\varphi\,d\varphi +$$

$$+4\int_0^{\frac{\pi}{2}}(\cos 2\varphi)^2\,d\varphi = 2\pi - 2\left(\frac{\sin\pi}{2} - \frac{\sin 0}{2}\right) + 4\int_0^{\frac{\pi}{2}}\frac{1+\cos 4\varphi}{2}\,d\varphi = 2\pi + 2\int_0^{\frac{\pi}{2}}(1+\cos 4\varphi)\,d\varphi =$$

$$= 2\pi + \pi + 2\left(\frac{\sin\pi}{4} - \frac{\sin 0}{4}\right) = 3\pi$$

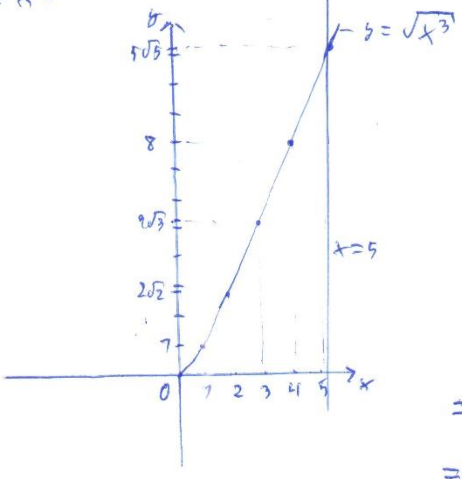
Ответ: 3π

ТР №19.7

$y^2 = x^3$, $x=5$

$y=\sqrt{x^3} = x^{\frac{3}{2}}$

y	0	1	$2\sqrt{2}$	$3\sqrt{3}$	8	$5\sqrt{5}$
x	0	1	2	3	4	5



$$l=\int_a^b \sqrt{1+(f'(x))^2}$$

$$y' = (x^{\frac{3}{2}})' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

$$l=\int_0^5 \sqrt{1+\frac{9}{4}x} = \left|\begin{array}{l} d(\underbrace{1+\frac{9}{4}x}_{=u}) = \frac{9}{4}dx \\ 0\le x\le 5 \\ 0\le \frac{9}{4}x \le \frac{45}{4} \\ 1\le \frac{9}{4}x+1 \le \frac{49}{4} \end{array}\right| =$$

$$=\frac{4}{9}\int_1^{\frac{49}{4}} \sqrt{u}\,du = \frac{4}{9}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_1^{\frac{49}{4}} =$$

$$=\frac{8}{27}\left(\sqrt{\left(\frac{49}{4}\right)^3}-\sqrt{1^3}\right) = \frac{8}{27}\left(\sqrt{\frac{7^6}{2^6}}-1\right) =$$

$$=\frac{8}{27}\left(\frac{7^3}{2^3}-1\right) = \frac{343}{27}-\frac{8}{27} = \frac{335}{27}$$

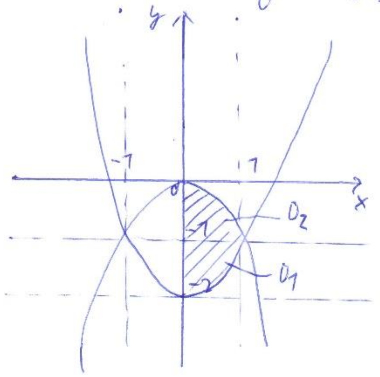
Ответ: $\frac{335}{27}$

Кратные интегралы ТР № 1

$$\int_{-2}^{-1} dy \int_{0}^{\sqrt{2+y}} f\,dx + \int_{-1}^{0} dy \int_{0}^{\sqrt{-y}} f\,dx =$$

область интегрирования: $D = D_1 \cup D_2$

$$D_1 = \begin{cases} -2 \le y \le -1 \\ 0 \le x \le \sqrt{2+y} \end{cases} ; \quad D_2 = \begin{cases} -1 \le y \le 0 \\ \cancel{\sqrt{-y}}\ 0 \le x \le \cancel{0}\ \sqrt{-y} \end{cases}$$



$0 < x = \sqrt{2+y}$

$x^2 = 2+y$

$y = x^2 - 2$

$\cancel{\sqrt{-y} \le x < 0}$

$\cancel{-y = x^2}$

$\cancel{y = -x^2}$

$0 < x = \sqrt{-y}$

$x^2 = -y$

$y = -x^2$

$-x^2 = x^2 - 2$

$2x^2 = 2$

$x = \pm 1$

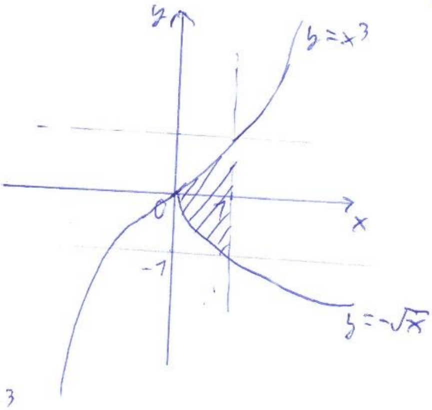
$$D: \begin{cases} 0 \le x \le 1 \\ x^2 - 2 \le y \le -x^2 \end{cases}$$

$$= \int_{0}^{1} dx \int_{x^2-2}^{-x^2} f\,dy$$

Ответ: $\int_{0}^{1} dx \int_{x^2-2}^{-x^2} f\,dy$

$$\iint\limits_D (18x^2y^2 + 32x^3y^3)\,dx\,dy$$ ТР №2

$$D: \begin{cases} x = 1 \\ y = x^3 \\ y = -\sqrt{x} \end{cases}$$



$$\int_0^1 dx \int_{-\sqrt{x}}^{x^3} (18x^2y^2 + 32x^3y^3)\,dy$$

$$\int_{-\sqrt{x}}^{x^3} (18x^2y^2 + 32x^3y^3)\,dy = 18x^2 \frac{y^3}{3}\Big|_{-\sqrt{x}}^{x^3} + 32x^3 \frac{y^4}{4}\Big|_{-\sqrt{x}}^{x^3} = 18x^2\left(\frac{x^9}{3} - \frac{-x^{\frac{3}{2}}}{2}\right) +$$

$$+ 32x^3\left(\frac{x^{12}}{4} - \frac{x^2}{4}\right) = 6x^{11} + 6x^{\frac{7}{2}} + 8x^{15} - 8x^5$$

$$\int_0^1 (6x^{11} + 6x^{\frac{7}{2}} + 8x^{15} - 8x^5)\,dx = 6\frac{x^{12}}{12}\Big|_0^1 + 6\frac{x^{\frac{9}{2}}}{\frac{9}{2}}\Big|_0^1 + 8\frac{x^{16}}{16}\Big|_0^1 - 8\frac{x^6}{6}\Big|_0^1 =$$

$$= \frac{1}{2} + \frac{4}{3} + \frac{1}{2} - \frac{4}{3} = \cancel{2 + \frac{8}{3} = \frac{14}{3}} = 1$$

Ответ: $\cancel{\frac{14}{3}}$ 1.

ТР № 3.7

$$\int \frac{6x-5}{\sqrt{-x^2+2x+3}}dx = -\int \frac{6x-5}{\sqrt{x^2-2x-3}}dx = -\int \frac{6x-5}{\sqrt{(x-1)^2-4}}dx = \begin{cases} u = x-1 \\ du = dx \\ x = u+1 \end{cases} =$$

$$= -\int \frac{6u+1}{\sqrt{u^2-4}}du = -6\int \frac{u\,du}{\sqrt{u^2-4}} - \int \frac{du}{\sqrt{u^2-4}} = -6\int \frac{u\,du}{\sqrt{u^2-4}} + \arcsin\left(\frac{u}{2}\right) \ominus$$

$$\int \frac{u\,du}{\sqrt{u^2-4}} = \begin{cases} z = u^2-4 \\ dz = 2u\,du \\ \frac{dz}{2} = u\,du \end{cases} = \frac{1}{2}\int \frac{dz}{\sqrt{z}} = \frac{1}{2}\sqrt{z}\cdot 2 + C = \sqrt{u^2-4} + C$$

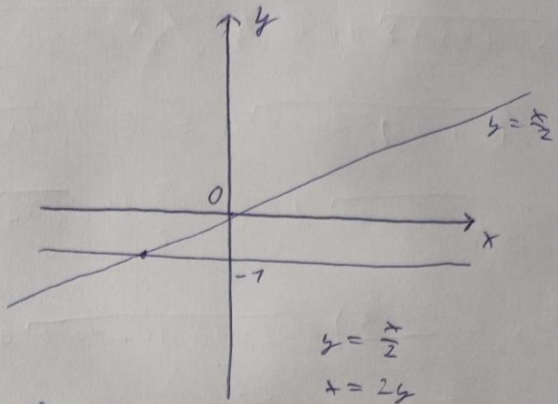
$$\ominus\ -6\sqrt{u^2-4} + \arcsin\left(\frac{u}{2}\right) + C = \arcsin\left(\frac{x-1}{2}\right) - 6\sqrt{x^2-2x-3} + C$$

Ответ: $\arcsin\left(\frac{x-1}{2}\right) - 6\sqrt{x^2-2x-3} + C$

ТР № 4.7

$$\iiint\limits_V y^2 \cos\left(\frac{\pi}{4} x y\right) dx\,dy\,dz$$

$$V = \begin{cases} x=0\ ;\ y=-1\ ;\ y=\frac{x}{2} \\ z=0\ ;\ z=-\pi^2 \end{cases}$$



$$\int\limits_{-1}^{0} dy \int\limits_{0}^{2y} dx \int\limits_{-\pi^2}^{0} y^2 \cos\left(\frac{\pi}{4} x y\right) dz =$$

$y = \frac{x}{2}$

$x = 2y$

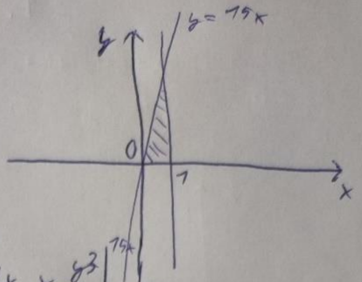
$$= \int\limits_{-1}^{0} dy \int\limits_{0}^{2y} dx\ y^2 \cos\left(\frac{\pi}{4} x y\right) z \Big|_{-\pi^2}^{0} = \int\limits_{-1}^{0} dy \int\limits_{0}^{2y} -\pi^2 y^2 \cos\left(\frac{\pi}{4} x y\right) dx = \cancel{\int\limits_{-1}^{0} dy}$$

$$= \int\limits_{-1}^{0} dy\ -\pi^2 y^2\ \frac{\sin\left(\frac{\pi}{4} x y\right)}{\frac{\pi}{4} \cancel{y}} \Bigg|_{0}^{2y} = -4\pi \cancel{y} \int\limits_{-1}^{0} dy\ y\ \sin\left(\frac{\pi}{2} y^2\right) = \left| \begin{matrix} t = \frac{y^2}{2} \\ dt = y\,dy \end{matrix} \right| =$$

$$= -4\pi \int\limits_{-1}^{0} \sin(\pi t)\,dt = 4\pi\ \frac{\cos(t\pi)}{\pi} \Bigg|_{-1}^{0} = \frac{4\pi}{\pi} \cdot \cos\left(\frac{y^2 \pi}{2}\right) \Bigg|_{-1}^{0} = 4(1-0) = 4$$

Ответ: 4

Т.Р №5.7



$$\iiint_V y\,dx\,dy\,dz$$

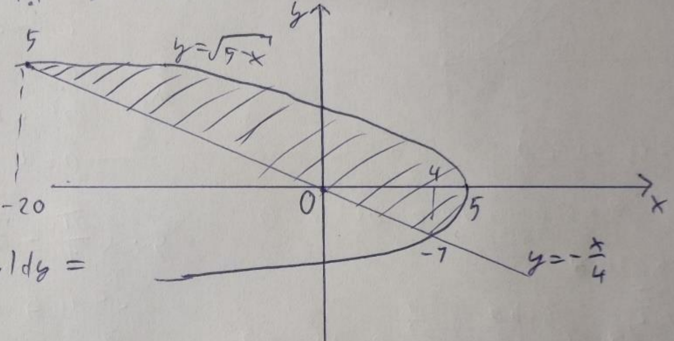
$$V: \begin{cases} y = 15x,\ y = 0,\ x = 1 \\ z = xy,\ z = 0 \end{cases}$$

$$\int_0^1 dx \int_0^{15x} y\,dy \int_0^{xy} dz = \int_0^1 dx \int_0^{15x} y\,dy\ xy = \int_0^1 dx \cdot x\frac{y^3}{3}\Big|_0^{15x} =$$

$$= \int_0^1 \frac{15^3 x^4}{3}\,dx = \frac{15^3 x^5}{15}\Big|_0^1 = 225$$

Ответ: 225

Т.Р №6.7



$x = 5 - y^2$; $x = -4y$

$y^2 = 5 - x$ $\quad y = -\frac{x}{4}$

$y = \sqrt{5-x}$

$$S = \int_{-1}^{5} dy \int_{-4y}^{5-y^2} dx = \int_{-1}^{5} (5 - y^2 + 4y)\,dy =$$

$$= \left(5y - \frac{y^3}{3} + \frac{4y^2}{2}\right)\Big|_{-1}^{5} =$$

$$= 25 - \frac{125}{3} + \frac{4\cdot 25}{2} - \left(-5 + \frac{1}{3} + 2\right) =$$

$$= 25 + 50 - \frac{125}{3} + 5 - \frac{1}{3} - 2 = 78 -$$

$$- \frac{126}{3} = 78 - 42 = 36$$

$$\left[\begin{array}{l} \begin{cases} x_1 = -20 \\ y_1 = 5 \end{cases} \\ \begin{cases} x_2 = 4 \\ y_2 = -1 \end{cases} \end{array}\right.$$

$$\sqrt{5-x} = -\frac{x}{4}$$

$$\frac{x^2}{16} = 5 - x$$

$$x^2 + 16x - 80 = 0$$

$$D = 256 + 4\cdot 80 = 576 = 24^2$$

$$x_1 = \frac{-16+24}{2} = 4$$

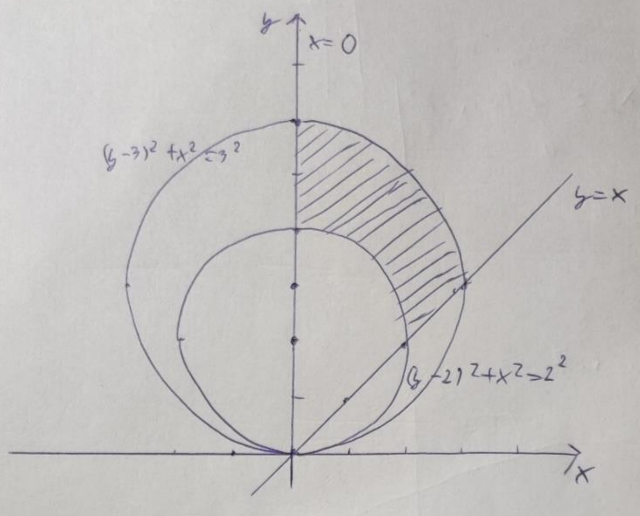
$$x_2 = \frac{-16-24}{2} = -20$$

Ответ: $S = 36$

Т. Р. №7.7

$$\begin{cases} y^2-4y+x^2=0 \\ y^2-6y+x^2=0 \\ y=x,\ x=0 \end{cases} \quad y^2-4y+4+x^2=4 \quad (y-2)^2+x^2=2^2$$

$$y^2-6y+9+x^2=9 \quad (y-3)^2+x^2=3^2 \quad \begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \end{cases}$$



$y=x$

$\cancel{\rho}\sin\theta=\cancel{\rho}\cos\theta$

$tg\,\theta=1$

$\theta=\frac{\pi}{4}$

$x=0$

$\rho\cos\theta=0$

$\begin{cases} \rho=0 \\ \cos\theta=0 \end{cases} \Rightarrow \theta=\frac{\pi}{2}$

$y^2-4x+x^2=0$

$\rho^2\sin^2\theta-4\rho\sin\theta+\rho^2\cos^2\theta=0$

$\rho^2-4\rho\sin\theta=0$

$\rho=4\sin\theta$

$y^2-6y+x^2=0$

$\rho^2\sin^2\theta-6\rho\sin\theta+\rho^2\cos^2\theta=0$

$\rho=6\sin\theta$

$$S: \begin{cases} \frac{\pi}{4}\le\theta\le\frac{\pi}{2} \\ 4\sin\theta\le\rho\le6\sin\theta \end{cases}$$

$$S=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}d\theta\int_{4\sin\theta}^{6\sin\theta}\rho\,d\rho=\int_{\pi/4}^{\frac{\pi}{2}}\frac{\rho^2}{2}\Big|_{4\sin\theta}^{6\sin\theta}d\theta=\int_{\pi/4}^{\frac{\pi}{2}}\left(\frac{36\sin^2\theta}{2}-\frac{16\sin^2\theta}{2}\right)d\theta=$$

$$=\int_{\pi/4}^{\pi/2}10\sin^2\theta\,d\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{10(1-\cos2\theta)}{2}d\theta=\int_{\pi/4}^{\frac{\pi}{2}}(5-5\cos2\theta)d\theta=$$

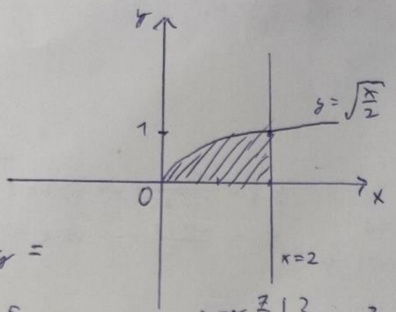
$$=5\theta\Big|_{\pi/4}^{\pi/2}-\cancel{5}\frac{\sin2\theta}{2}\Big|_{\pi/4}^{\pi/2}=5\left(\frac{\pi}{2}-\frac{\pi}{4}-\frac{1}{2}\left(\sin\pi-\sin\frac{\pi}{2}\right)\right)=5\left(\frac{\pi}{4}-\frac{1}{2}(0-1)\right)=$$

$$=5\left(\frac{\pi}{4}+\frac{1}{2}\right)=\frac{5\pi}{4}+\frac{5}{2}$$

Ответ: $\frac{5\pi}{4}+\frac{5}{2}$

ТР №8.7

$D: x=2;\ y=0;\ y^2=\frac{x}{2}\ (y\geq 0);$
$\cancel{y\geq 0;}\ \mu = \frac{7x^2}{2}+6y$



$y^2=\frac{x}{2}$
$y=\sqrt{\frac{x}{2}}$

$m=\iint\limits_D \mu\, dx\, dy = \int_0^2 dx \int_0^{\sqrt{\frac{x}{2}}}\left(\frac{7x^2}{2}+6y\right)dy =$

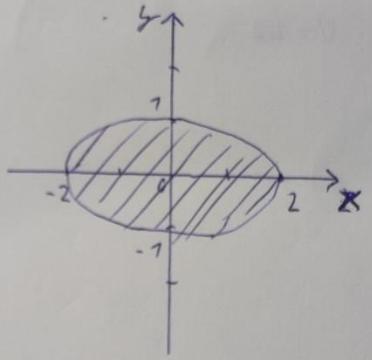
$=\int_0^2 dx\left(\frac{7}{2}x^2y+3y^2\right)\Big|_0^{\sqrt{\frac{x}{2}}} = \int_0^2\left(\frac{7}{2\sqrt2}\cdot x^{\frac52}+\frac32\cdot x\right)dx = \frac{2\cdot 7x^{\frac72}}{7\cdot 2\sqrt2}\Big|_0^2+\frac34 x^2\Big|_0^2 =$

$=\frac{8\sqrt2}{\sqrt2}+\frac34\cdot4 = 8+3=11$

Ответ: 11

ТР №9.7

$D: \frac{x^2}{4}+y^2\leq 1;\ \mu=4y^4$



$\frac{x^2}{2^2}+\frac{y^2}{1^2}\leq 1$ - эллипс $\Rightarrow$ $0\leq\varphi\leq 2\pi$

$\Rightarrow \begin{cases} x = 2R\cos\varphi \\ y = R;\ \varphi=2\pi; \end{cases}$ $\rho = 2R$, $I = 2R$

$m=\int_0^{2\pi}d\varphi\int_0^1 I\mu\, dR$

$\mu = 4(R\sin\varphi)^4 = 4R^4\sin^4\varphi$

$m=\int_0^{2\pi}d\varphi\int_0^1 2R\,4R^4\sin^4\varphi\, dR = \int_0^{2\pi}d\varphi\int_0^1 8R^5\sin^4\varphi\, dR =$

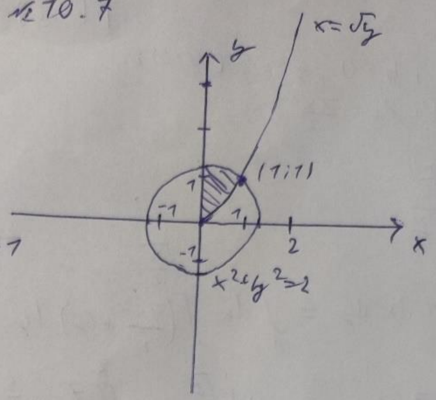
$=\int_0^{2\pi}\frac{8R^6}{6}\Big|_0^1\sin^4\varphi\, d\varphi = \frac43\int_0^{2\pi}\left(\frac{1-\cos2\varphi}{2}\right)^2 d\varphi = \frac13\int_0^{2\pi}(1-2\cos2\varphi+\cos^2 2\varphi)\,d\varphi$

$=\frac13\left(\varphi\Big|_0^{2\pi}-\sin2\varphi\Big|_0^{2\pi}+\int_0^{2\pi}\frac{1+\cos4\varphi}{2}d\varphi\right)=\frac13\left(2\pi+\frac{\varphi}{2}\Big|_0^{2\pi}+\frac{\sin4\varphi}{8}\Big|_0^{2\pi}\right)=$

$=\frac13(2\pi+\pi+0)=\pi$

Ответ: $m=\pi$

ТР №10.7



$x^2+y^2=2;\ x=\sqrt{y};\ x=0,$

$z=0,\ z=30y$

$x^2+y^2=2$

$\underline{x^2=y}$

$y+y^2=2$

$y^2+y-2=0$

$D=1-4\cdot(-2)=9$

$y_1=\frac{-1+3}{2}=1 \Rightarrow x=1$

$y_2=\frac{-1-3}{2}=-2$ н.к.

$y^2=2-x^2$

$y=\sqrt{2-x}$

$$V=\int_0^1 dx\int_{x^2}^{\sqrt{2-x^2}} 30y\,dy=\int_0^1 dx\ \frac{30y^2}{2}\Big|_{x^2}^{\sqrt{2-x^3}}=\int_0^1 15(2-x^2-x^4)\,dx=$$

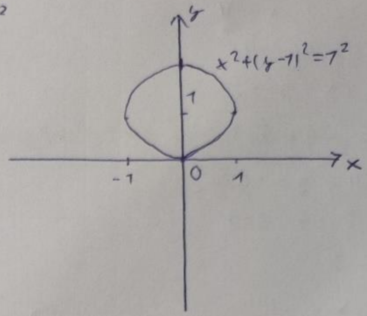
$$=15\int_0^1(2-x^2-x^4)\,dx=15\left(2x-\frac{x^3}{3}-\frac{x^5}{5}\right)\Big|_0^1=15\left(2-\frac{1}{3}-\frac{1}{5}\right)=30-5-3=22$$

Ответ: $V=22$

ТР № 11.7

$x^2+y^2=2y \Rightarrow x^2+y^2-2y=0 \Rightarrow x^2+(y-1)^2=1^2$

$z=\frac{9}{4}-x^2;\ z=0$



Перейдем к полярным координатам:

$0 \le \alpha \le \frac{\pi}{2}$ $\qquad I=R$

$R^2=2R\sin\alpha \Rightarrow R=2\sin\alpha$

(наше ур-е в пол. коорд.)

$0 \le R \le 2\sin\alpha$

$0 \le z \le \frac{9}{4}-R^2\cos^2\alpha$

$$V=\iiint R\,dR\,d\alpha\,dz = 2\int_0^{\frac{\pi}{2}} d\alpha \int_0^{2\sin\alpha} R\,dR \int_0^{\frac{9}{4}-R^2\cos^2\alpha} dz = 2\int_0^{\frac{\pi}{2}} d\alpha \cdot \int_0^{2\sin\alpha} R\left(\frac{9}{4}-R^2\cos^2\alpha\right)dR =$$

$$=2\int_0^{\frac{\pi}{2}} \frac{9}{4}\cdot\frac{R^2}{2}-\frac{R^4}{4}\cdot\cos^2\alpha)\Big|_0^{2\sin\alpha} d\alpha = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{9}{2}\cdot 4\sin^2\alpha-16\sin^4\alpha\cdot\cos^2\alpha)d\alpha =$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}(18\sin^2\alpha-16\sin^4\alpha\cos^2\alpha)d\alpha = \int_0^{\frac{\pi}{2}}(9\sin^2\alpha-8\sin^4\alpha\cos^2\alpha)d\alpha = \int_0^{\frac{\pi}{2}}\frac{9}{2}-\frac{9}{2}\cos 2\alpha -$$

$$-(1-\cos 2\alpha)^2(1+\cos 2\alpha))\,d\alpha = \int_0^{\frac{\pi}{2}}\left(\frac{9}{2}-\frac{9}{2}\cos 2\alpha-(1-\cos^2 2\alpha)(1-\cos 2\alpha)\right)d\alpha =$$

$$=\int_0^{\frac{\pi}{2}}\frac{9}{2}-\frac{9}{2}\cos 2\alpha-(1-\cos^2 2\alpha-\cos 2\alpha+\cos^3 2\alpha))\,d\alpha = \int_0^{\frac{\pi}{2}}\left(\frac{9}{2}-\frac{9}{2}\cos 2\alpha -\right.$$

$$-1+\frac{1}{2}+\frac{\cos 4\alpha}{2}+\cos 2\alpha+(1-\sin^2 2\alpha)\cdot\cos 2\alpha)\,d\alpha =$$

$$=\int_0^{\frac{\pi}{2}}\left(4-\frac{7}{2}\cos\alpha+\frac{1}{2}\cos 4\alpha\right)d\alpha+\frac{1}{2}\int_0^{\frac{\pi}{2}}(1-\sin^2 2\alpha)\,d(\sin 2\alpha) = (4\alpha-\frac{7}{4}\sin 2\alpha+\frac{1}{2}\cdot$$

$$\cdot\frac{\sin 4\alpha}{4}+\frac{1}{2}\sin^2\alpha-\frac{1}{2}\frac{\sin^3 2\alpha}{3})\Big|_0^{\frac{\pi}{2}} = 4\,\frac{\pi}{2}-0+0-0=2\pi$$

Ответ: $V=2\pi$

ТР №12.7

$x = 5y^2 - 9;\ x = -4$

$z = x^2 + 4x - y^2 - 4$

$z = x^2 + 4x - y^2 + 2$

$x = -4 \Rightarrow -4 = 5y^2 - 9 \Rightarrow 5 = 5y^2 \Rightarrow y = \pm 1$

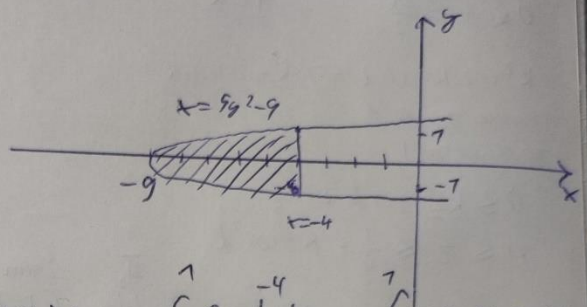
$z = -y^2 - 4;\ z = -y^2 + 2$

$y = \pm 1;\ z = -5;\ z = 1$

$-1 \le y \le 1$

$5y^2 - 9 \le x \le -4$

$x^2+4x-y^2-4 \le z \le x^2+4x-y^2+2$



$$V = \int_{-1}^{1} dy \int_{5y^2-9}^{-4} dx \int_{x^2+4x-y^2-4}^{x^2+4x-y^2+2} dz = \int_{-1}^{1} dy \int_{5y^2-9}^{-4} (2+4)\,dx = \int_{-1}^{1} 6x\Big|_{5y^2-9}^{-4} dy = \int_{-1}^{1} (-24-30y^2+54)\,dy =$$

$$= \int_{-1}^{1} (30 - 30y^2)\,dy = 30\int_{-1}^{1} (1-y^2)\,dy = 30\left(y - \frac{y^3}{3}\right)\Big|_{-1}^{1} = 30\left(1 - \frac{1}{3} - \left(-1 - \frac{(-1)^3}{3}\right)\right) =$$

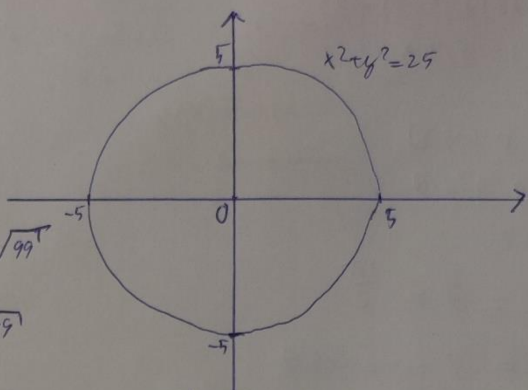
$$30\left(2 - \frac{2}{3}\right) = 30 \cdot \frac{4}{3} = 40$$

Ответ: 40

ТР №13.7

$z=\sqrt{25-x^2-y^2} \Rightarrow z^2+x^2+y^2=25$

$z=\sqrt{\dfrac{x^2+y^2}{99}}$



$x=\rho\cos\varphi\cos\theta$ $\quad 0\le\varphi\le 2\pi$

$y=\rho\sin\varphi\,\cancel{\cos}\sin\theta$ $\quad 0\le\rho\le 5$

$\cancel{z=\rho\ldots\cos\theta}$

$z=\rho\cos\theta$ $\quad \dfrac{\sqrt{x^2+y^2}}{z}=\operatorname{tg}\theta=\sqrt{99}$

$I=\rho^2\sin\theta$

$0\le\theta\le\operatorname{arctg}\sqrt{99}$

$$V=\int_0^{2\pi}d\varphi\int_0^{\operatorname{arctg}\sqrt{99}}\sin\theta\,d\theta\int_0^5\rho^2\,d\rho=-2\pi\cos\theta\Big|_0^{\operatorname{arctg}\sqrt{99}}\frac{\rho^3}{3}\Big|_0^5=-\frac{250}{3}\pi\cos\theta\Big|_0^{\operatorname{arctg}\sqrt{99}}\ominus$$

$\cos\theta=\dfrac{1}{\sqrt{1+\operatorname{tg}^2\theta}}$

$$\ominus-\frac{250}{3}\pi\cdot\frac{1}{\sqrt{1+\operatorname{tg}^2\theta}}\Big|_0^{\operatorname{arctg}\sqrt{99}}=-\frac{250}{3}\pi\cdot\left(\frac{1}{\sqrt{1+99}}-1\right)=-\frac{250\pi}{3}\left(\frac{1}{10}-\frac{10}{10}\right)=$$

$$=\frac{250\pi}{3}\cdot\frac{9}{10}=75\pi$$

Ответ: 75π

ТР №14.7

$z = 32(x^2+y^2)+3$ | $\Rightarrow$ $32x^2+32y^2+3 = 3-64x$

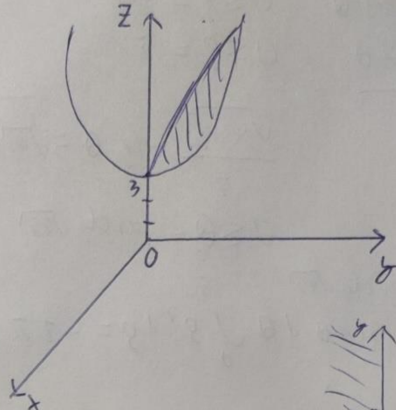
$z = 3-64x$ $\qquad$ $x^2+y^2 = -2x$

параболоид

плоскость

$\rho^2\cos^2\theta + \rho^2\sin^2\theta = -2\rho\cos\theta$

$\rho = -2\cos\theta$

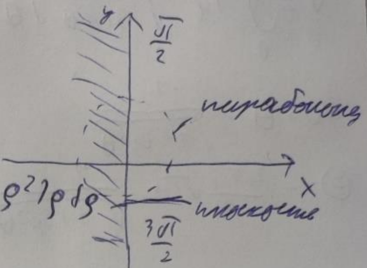


$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases}$$

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$0 \le \rho \le -2\cos\theta$

$32\rho^2+3 \le z \le 3-64\rho\cos\theta$



$$V = \int_{\pi/2}^{3\pi/2} d\theta \int_0^{-2\cos\theta} \rho\, d\rho \int_{32\rho^2+3}^{3-64\rho\cos\theta} dz = \int_{\pi/2}^{3\pi/2} d\theta \int_0^{-2\cos\theta} (-64\rho\cos\theta - 32\rho^2)\rho\, d\rho$$

$$= \int_{\pi/2}^{3\pi/2} \left(-64\cos\theta\,\frac{\rho^3}{3} - 32\,\frac{\rho^4}{4}\right)\Big|_0^{-2\cos\theta} = \int_{\pi/2}^{3\pi/2}\left(\frac{64\cdot 8}{3}\cos^4\theta - 8\cdot 16\cos^4\theta\right) d\theta =$$

$$= \frac{8\cdot 16}{3}\int_{\pi/2}^{3\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta = \frac{32}{3}\int_{\pi/2}^{3\pi/2}\left(1+2\cos 2\theta + \frac{1}{2} + \frac{1}{2}\cos 4\theta\right) d\theta =$$

$$= \frac{32}{3}\left(\frac{3}{2}\theta + \sin 2\theta + \frac{\sin 4\theta}{8}\right)\Big|_{\pi/2}^{3\pi/2} = \frac{32}{\cancel{3}}\cdot\frac{\cancel{3}}{2}\pi = 16\pi$$

Ответ: 16π

ТР №19.7

$1 \le x^2+y^2+z^2 \le \cancel{76}49$

$0 \le z \le \sqrt{\frac{x^2+y^2}{24}}$

$y \le -\frac{x}{\sqrt{3}}, \; y \le -\sqrt{3}x$

$\begin{cases} x = r\cos\varphi\cos\theta \\ y = r\sin\varphi\cos\theta \\ z = r\sin\theta \end{cases}$

$\cancel{0 \le r^2\sin^2\theta \le r^2\cos^2\theta\cos^2\varphi +}$

1. $0 \le r^2\sin^2\theta \le \frac{r^2\cos^2\varphi\cos^2\theta + r^2\sin^2\varphi\cos^2\theta}{24}$

$0 \le \sin^2\theta \le \frac{\cos^2\theta}{24}$

$0 \le tg^2\theta \le \frac{1}{24}$

$0 \le tg\,\theta \le \frac{1}{\sqrt{24}}$

$0 \le \theta \le arctg\frac{1}{\sqrt{24}}$

2. $\cancel{r}\sin\varphi\cos\theta \le -\frac{\cancel{r}\cos\varphi\cancel{\cos\theta}}{\sqrt{3}}$

$\sin\varphi \le -\frac{\cos\varphi}{\sqrt{3}}$

$tg\varphi \le -\frac{1}{\sqrt{3}}$

$-\frac{\pi}{2} < \varphi \le arctg\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

3. $1 \le r \le 7$

$V = \iiint\limits_V r^2\cos\theta\,dr\,d\varphi\,d\theta = \int\limits_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} d\varphi \int\limits_0^{arctg\frac{1}{\sqrt{24}}} d\theta \int\limits_1^7 r^2\cos\theta\,dr = \int\limits_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} d\varphi \int\limits_0^{arctg\frac{1}{\sqrt{24}}} d\theta \Big(\cos\theta \cdot$

$\cdot\left(\frac{7^3}{3} - \frac{1}{3}\right)\Big) = 114 \int\limits_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} d\varphi \int\limits_0^{arctg\frac{1}{\sqrt{24}}} \cos\theta\,d\theta = 114 \cdot \int\limits_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} d\varphi \cdot \sin\left(arctg\frac{1}{\sqrt{24}}\right) =$

$= 114 \cdot \sin\left(arctg\frac{1}{\sqrt{24}}\right)\left(-\frac{\pi}{6} + \frac{\pi}{2}\right) = 114\sin\left(arctg\frac{1}{\sqrt{24}}\right)\frac{\pi}{3} = \frac{114\pi}{5 \cdot 3} = \frac{144\pi}{15} = \frac{48\pi}{\cancel{115}}$

Ответ: $\frac{48\pi}{5}$

ТР №16.7

$x^2+y^2+z^2=16$, $x^2+y^2=4$

сфера

$(x^2+y^2)\le 4$

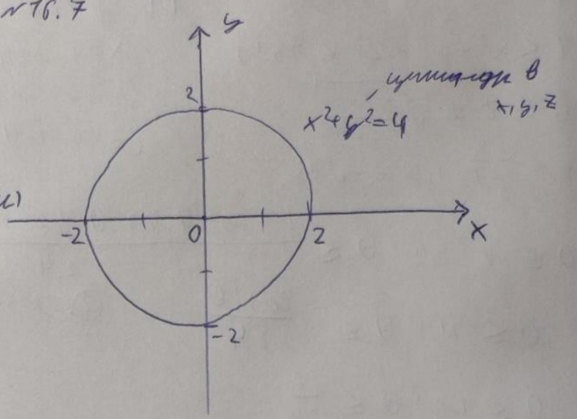
$\mu = 2|z|$

Перейдём к ~~цил~~ коорд

$0 \le \varphi \le \frac{\pi}{2}$ (в силу симметрии)

$0 \le \rho \le 2$

$0 \le z \le \sqrt{16-\rho^2}$



$\rho^2\cos^2\varphi + \rho^2\sin^2\varphi + z^2 = 16$

$\cancel{z^2 = \frac{16}{\rho^2} \Rightarrow z = \sqrt{\frac{4}{\rho}}}$

$z^2 = 16-\rho^2 \Rightarrow z = \sqrt{16-\rho^2}$

$$m = 4\cdot 2\int_0^{\frac{\pi}{2}} d\varphi \int_0^2 \rho\, d\rho \int_0^{\sqrt{16-\rho^2}} 2z\, dz = 16\int_0^{\frac{\pi}{2}} d\varphi \int_0^2 \rho\, \frac{z^2}{2}\Big|_0^{\sqrt{16-\rho^2}} d\rho = 8\int_0^{\frac{\pi}{2}} d\varphi \int_0^2 (\rho\cdot 16-\rho^3)\, d\rho =$$

$$= 8\int_0^{\frac{\pi}{2}} \left(\rho^2\cdot 8 - \frac{\rho^4}{4}\right)\Big|_0^2 d\varphi = 8\int_0^{\frac{\pi}{2}} \left(\rho^2\cdot 8 - \frac{\rho^4}{4}\right)\Big|_0^2 d\varphi = 8\int_0^{\frac{\pi}{2}} \left(32 - \frac{16}{4}\right) d\varphi = 8\cdot 28\cdot \frac{\pi}{2} =$$

$= 112\pi$

Ответ: 112π