КУЛИКОВ ДАНИИЛ А-07-23

6.13) $$\int \frac{x+2\sqrt{3x-1}-4}{\sqrt{3x-1}+3}\,dx = \left| \begin{array}{ll} u=\sqrt{3x-1} & \\ x=\frac{u^2+1}{3} & dx=\frac{2u}{3}du \end{array} \right| =$$

$$= \int \frac{\left(\frac{u^2+1}{3}+2u-4\right)\frac{2u}{3}du}{u+3} = \frac{2}{3}\int \frac{u\left(\frac{u^2+1}{3}+2u-4\right)}{u+3}du =$$

$$= \left| \begin{array}{l} v=u+3 \\ u=v-3 \\ dv=du \end{array} \right| = \frac{2}{3}\int \frac{(v-3)\left(\frac{(v-3)^2+1}{3}+2(v-3)-4\right)dv}{v} =$$

$$= \frac{2}{3}\int \frac{(v-3)\left(\frac{v^2-6v+9+1+6v-18-12}{3}\right)dv}{v} =$$

$$= \frac{2}{3}\int \frac{v^3-3v^2-20v+60}{3v}dv = \frac{2}{9}\int v^2dv - 3\int v\,dv - 20\int dv +$$

$$+60\int \frac{dv}{v} = \frac{2v^3}{27} - \frac{v^2}{3} - \frac{40v}{9} + \frac{40\ln|v|}{3} =$$

$$= \frac{2(u+3)^3}{27} - \frac{(u+3)^2}{3} - \frac{40(u+3)}{9} + \frac{40\ln|u+3|}{3} =$$

1 стр

$$= \frac{2(\sqrt{3x-1}+3)^3}{27} - \frac{(\sqrt{3x-1}+3)^2}{3} - \frac{40(\sqrt{3x-1}+3)}{9} +$$

$$+ \frac{40\ln|\sqrt{3x-1}+3|}{3} = \frac{40\ln|\sqrt{3x-1}+3|}{3} + \frac{2(\sqrt{3x-1}+3)^3}{27} -$$

$$- \frac{3x-1+6\sqrt{3x-1}+9}{3} - \frac{40\sqrt{3x-1}+120}{9} = \frac{40\ln|\sqrt{3x-1}+3|}{3} +$$

$$+ \frac{2(\sqrt{3x-1}+3)^3}{27} - \frac{58\sqrt{3x-1}}{9} - x - 16 + C$$

7.13) $$\int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x\cdot\sqrt[12]{x^5}}dx = \int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x^{\frac{17}{12}}}dx = \left| \begin{array}{l} u=\sqrt[12]{x} \\ x=u^{12} \end{array} \right. \ldots$$

$$\ldots \left. dx = 12u^{11}du \right| = \int \frac{12\sqrt[4]{u^4+1}}{u^6}du = \left| \begin{array}{l} v=\frac{\sqrt[4]{u^4+1}}{u} \\ u=\frac{1}{\sqrt[4]{v^4-1}} \end{array} \right. \ldots$$

$$\ldots \left. du = -\frac{v^3}{(v^4-1)^{\frac{5}{4}}}dv \right| = -12\int \frac{v\cdot(v^4-1)^{\frac{5}{4}}\cdot v^3}{(v^4-1)^{\frac{5}{4}}}dv =$$

$$= -12\int v^4 dv = -2,4v^5 = -2,4\frac{(u^4+1)^{\frac{5}{4}}}{u^5} =$$

$$= -\frac{2,4\sqrt[4]{u^4+1}(u^4+1)}{u^5} = -\frac{2,4\sqrt[4]{\sqrt[3]{x}+1}(\sqrt[3]{x}+1)}{\sqrt[12]{x^5}} + C$$

стр 2

$$10.13)\ \int \frac{dx}{2+\cos x+2\sin x} = \left| \begin{matrix} \sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} & \\ \cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} & u = tg\frac{x}{2} \\ & du = \frac{dx}{2\cos^2\frac{x}{2}} \end{matrix} \right| =$$

$$= \int \frac{du}{\left(2+\frac{1-u^2}{1+u^2}+\frac{4u}{1+u^2}\right)\left(\frac{1}{2}+\frac{1}{2}u^2\right)} = \int \frac{2du}{\left(\frac{3+u^2+4u}{1+u^2}\right)(1+u^2)} =$$

$$= 2\int \frac{du}{u^2+4u+3} = 2\int \frac{du}{(u+1)(u+3)} = \int \frac{du}{u+1} - \int \frac{du}{u+3} =$$

$$= \int \frac{d(u+1)}{u+1} - \int \frac{d(u+3)}{u+3} = \ln|u+1| - \ln|u+3| =$$

$$= \ln\left|tg\frac{x}{2}+1\right| - \ln\left|tg\frac{x}{2}+3\right| + C$$

$$11.13)\ \int_3^8 \frac{dx}{x\sqrt{(1+x)^3}} = \int_3^8 \frac{dx}{x(x+1)^{\frac{3}{2}}} = \left| \begin{matrix} u = \sqrt{x+1} \\ x = u^2-1 \\ dx = 2udu \end{matrix} \right| =$$

$$= \int_3^8 \frac{2udu}{(u^2-1)u^3} = \int_3^8 \frac{2du}{u^2(u^2-1)} =$$

$$\frac{1}{u^2(u^2-1)} = \frac{Au+B}{u^2-1} + \frac{C}{u} + \frac{D}{u^2} = \frac{Au^3+Bu^2+Cu^3-Cu+Du^2-D}{u^2(u^2-1)} =$$

$$= \frac{(C+A)u^3+(D+B)u^2-Cu-D}{u^2(u^2-1)}$$

$$\begin{cases} -D=1 \\ -C=0 \\ D+B=0 \\ C+A=0 \end{cases} \quad \begin{cases} D=-1 \\ C=0 \\ B=1 \\ A=0 \end{cases} \qquad \frac{Au+B}{u^2-1} + \frac{C}{u} + \frac{D}{u^2} = \frac{1}{u^2-1} - \frac{1}{u^2}$$

$$= 2\int_3^8 \frac{1}{u^2-1} - \frac{1}{u^2}\,du = 2\int_3^8 \frac{du}{u^2-1} - 2\int_3^8 \frac{du}{u^2} =$$

$$\cancel{\left(-\ln\left|\frac{1+u}{1-u}\right| + \frac{2}{u}\right)\Big|_3^8 = \left(-\ln\left|\frac{1+\sqrt{x+1}}{1-\sqrt{x+1}}\right| + \frac{2}{\sqrt{x+1}}\right)\Big|_3^8} =$$

$$= \cancel{-\ln \frac{u}{}}$$

$$\int \frac{du}{u^2-1} = \int \frac{du}{(u+1)(u-1)} = \frac{1}{2}\int\left(\frac{1}{(u-1)} - \frac{1}{u+1}\right)du =$$

$$= \frac{1}{2}\ln|u-1| - \frac{1}{2}\ln|u+1| + C$$

$$= \left(\ln|u-1| - \ln|u+1| + \frac{2}{u}\right)\Big|_3^8 =$$

$$= \left(\ln\left|\sqrt{x+1}-1\right| - \ln\left|\sqrt{x+1}+1\right| + \frac{2}{\sqrt{x+1}}\right)\Bigg|_3^8 =$$

$$= \ln 2 - \ln 4 + \frac{2}{3} + \ln 3 - 1 = -\ln 4 + \ln 3 + \ln 2 - \frac{1}{3}$$

12.13) $$\int_0^1 (x^2-x)\sin\frac{\pi x}{2}\,dx = \begin{vmatrix} u = x^2 - x & du = (2x-1)dx \\ v = -\frac{2\cos\frac{\pi x}{2}}{\pi} & dv = \sin\frac{\pi x}{2} \end{vmatrix} =$$

$$= -\frac{2(x^2-x)\cos\frac{\pi x}{2}}{\pi}\Bigg|_0^1 + \int_0^1 \frac{2(2x-1)\cos\frac{\pi x}{2}}{\pi}\,dx =$$

$$\int \frac{2(2x-1)\cos\frac{\pi x}{2}}{\pi}\,dx = \begin{vmatrix} u = 2x-1 & v = \frac{2\sin\frac{\pi x}{2}}{\pi} \\ du = 2 & dv = \cos\frac{\pi x}{2}\,dx \end{vmatrix} =$$

$$= \frac{2}{\pi}\left(\frac{2(2x-1)\sin\frac{\pi x}{2}}{\pi} - \frac{2}{\pi}\int \frac{4\sin\frac{\pi x}{2}}{\pi}\,d\left(\frac{\pi x}{2}\right)\right) = \frac{2}{\pi}\left(\frac{2(2x-1)\sin\frac{\pi x}{2}}{\pi} + \right.$$

$$\left. + \frac{8\cos\frac{\pi x}{2}}{\pi^2}\right) = \frac{8x\sin\frac{\pi x}{2} - 4\sin\frac{\pi x}{2}}{\pi^2} + \frac{\cancel{8}16\cos\frac{\pi x}{2}}{\pi^3} + C$$

$$= \left(-\frac{2(x^2-x)\cos\frac{\pi x}{2}}{\pi} + \frac{8x\sin\frac{\pi x}{2}}{\pi^2} - \frac{4\sin\frac{\pi x}{2}}{\pi^2} + \frac{16\cos\frac{\pi x}{2}}{\pi^3}\right)\Bigg|_0^1 =$$

$$= \frac{8}{\pi^2} - \frac{4}{\pi^2} - \frac{16}{\pi^3} = \frac{4}{\pi^2} - \frac{16}{\pi^3}$$

13.13) $y = \sin\frac{x}{2}$, $y = \cos\frac{x}{2}$, $x = 0$

$$\sin\frac{x}{2} = \cos\frac{x}{2} \Rightarrow x = \frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{2}} \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)dx = 2\int_0^{\frac{\pi}{2}} \cos\frac{x}{2}\, d\frac{x}{2} - 2\int_0^{\frac{\pi}{2}} \sin\frac{x}{2}\, d\frac{x}{2} =$$

$$= \left(2\sin\frac{x}{2} + 2\cos\frac{x}{2}\right)\Bigg|_0^{\frac{\pi}{2}} = 2\sqrt{2} - 2$$

14.13) $\rho = 4\sin\frac{3\varphi}{2}$, $\rho = 2$ $(\rho \geq 2)$

$$2 = 4\sin\frac{3\varphi}{2} \Rightarrow \varphi = \frac{\pi}{9}, \frac{5\pi}{9}$$

$$\frac{1}{2}\int_{\frac{\pi}{9}}^{\frac{5\pi}{9}} \left(4\sin\frac{3\varphi}{2}\right)^2 d\varphi = 8\int_{\frac{\pi}{9}}^{\frac{5\pi}{9}} \frac{2}{3}\sin^2\frac{3\varphi}{2}\, d\frac{3\varphi}{2} = \frac{16}{3}\int_{\frac{\pi}{9}}^{\frac{5\pi}{9}} \frac{1-\cos 3\varphi}{2}\, d\frac{3\varphi}{2} =$$

$$\ominus \frac{8}{3}\int_{\frac{\pi}{9}}^{\frac{5\pi}{9}} d\left(\frac{3\varphi}{2}\right) - \frac{4}{3}\int_{\frac{\pi}{9}}^{\frac{5\pi}{9}} \cos\frac{3\varphi}{2}\, d\frac{3\varphi}{2} = \left(\frac{8}{3}\cdot\frac{3\varphi}{2} - \frac{4}{3}\sin 3\varphi\right)\Big|_{\frac{\pi}{9}}^{\frac{5\pi}{9}} =$$

$$= \left(4\varphi - \frac{4\sin 3\varphi}{3}\right)\Big|_{\frac{\pi}{9}}^{\frac{5\pi}{9}} = \frac{20\pi}{9} + \frac{2}{\sqrt{3}} - \frac{4\pi}{9} + \frac{2}{\sqrt{3}} = \frac{16\pi}{9} + \frac{4}{\sqrt{3}} =$$

$$= \frac{16\pi}{9} + \frac{4\sqrt{3}}{3}$$

15.13) $y = \ln x, \quad \sqrt{3} \le x \le \sqrt{8}$

$$L = \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}}\, dx = \frac{1}{\operatorname{sgn} x}\int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{x^2+1}}{|x|}\, dx = \left| \begin{array}{l} u = \sqrt{x^2+1} \\ x = \sqrt{u^2-1} \\ dx = \frac{u}{\sqrt{u^2-1}}\, du \end{array} \right| =$$

$$= \frac{1}{\operatorname{sgn} x}\int_{\sqrt{3}}^{\sqrt{8}} \frac{u^2}{u^2-1}\, du = \frac{1}{\operatorname{sgn} x}\int_{\sqrt{3}}^{\sqrt{8}} \left(1 + \frac{1}{u^2+1}\right) du = \frac{1}{\operatorname{sgn} x}\int_{\sqrt{3}}^{\sqrt{8}} du + \frac{1}{\operatorname{sgn} x}\int_{\sqrt{3}}^{\sqrt{8}} \frac{du}{u^2+1} =$$

$$= -\frac{(\ln|u+1| - 2u - \ln|u-1|) +}{2|x|}\Bigg|_{\sqrt{3}}^{\sqrt{8}} \ominus$$

7 стр

$$\ominus \left. \frac{x \ln|\sqrt{x^2+1} - 1| - x\ln|\sqrt{x^2+1}+1| + 2x\sqrt{x^2+1}}{2|x|} \right|_{\sqrt{3}}^{\sqrt{8}} =$$

$$= \frac{-2\sqrt{2}\ln 4 + 2\sqrt{2}\ln 2 + 12\sqrt{2}}{4\sqrt{2}} - \frac{4\sqrt{3} - \sqrt{3}\ln 3}{2\sqrt{3}} =$$

$$= -\frac{\ln 4}{2} + \frac{\ln 2}{2} + 1 + \frac{\ln 3}{2}$$