w1.16

$$\int e^{4\sin^2 x}\sin 2x\,dx = 2\int e^{4\sin^2 x}\cos x\sin x\,dx = 2\int \frac{e^{4t}\cdot \cancel{\cos x\sin x}\,dt}{2\cdot\cancel{\cos x\sin x}}$$

$\sin^2 x = t$

$dt = 2\sin x\cos x$

$$= 2\int\frac{e^{4t}dt}{2} = \int e^{4t}dt =$$

$$= \frac{1}{4}e^{4\sin^2 x} + C$$

w2.16

$$\int 3^{-x}\cdot(x+2)\,dx = \int\frac{1}{3^x}\cdot(x+2)\,dx$$

$u = x+2$ $\quad dv = \frac{1}{3^x}$

$du = 1$ $\quad v = -\frac{1}{\ln 3\cdot 3^x}$

$$\int\frac{1}{3^x}\cdot(x+2)\,dx = -(x+2)\cdot\frac{1}{\ln 3\cdot 3^x} + \int\left(\frac{1}{\ln 3\cdot 3^x}dx\right) = -(x+2)\cdot\frac{1}{\ln 3\cdot 3^x} + \frac{1}{\ln(3)}\cdot\frac{1}{\ln(3)3^x}$$

$$\frac{1}{\ln 3}\int\frac{1}{3^x}dx = \frac{1}{\ln 3}\cdot\int 3^{-x}dx = \frac{1}{\ln 3}\cdot -\frac{1}{\ln(3)3^x}$$

w3.16

$$\int \frac{24x^3-12x^2+11x-2}{4x^2-2x+1}dx = \int \frac{5x-2}{4x^2-2x+1} + 6x\,dx$$

$$\begin{array}{l} \;\; 24x^3-12x^2+11x-2 \;\lfloor\, 4x^2-2x+1 \\ -\underline{24x^3-12x^2+6x} \qquad 6x \\ \qquad\qquad 5x-2 \end{array}$$

$$\frac{4x^2-2x+1}{4x^2-2x+1}=1 \qquad \cancel{(24x^3-12x^2+11x-2)-(4x^2-2x+1)} =$$

$$= \cancel{24x^3-16x^2+13x+1}$$

$$\int \frac{5x-2}{4x^2-2x+1}dx \,{}^{*}=$$

$$5x-2 = \frac{5}{8}(8x-2)-\frac{3}{4}$$

$$\frac{5}{8}\int \frac{8x-2}{4x^2-2x+1}dx - \int \frac{3}{4(4x^2-2x+1)}dx$$

$$t = 4x^2-2x+1$$

$$dt = (8x-2)dx$$

$$\frac{5}{8}\int \frac{\cancel{(8x-2)}}{t\cdot\cancel{(8x-2)}}dt = \frac{5}{8}\int \frac{1}{t}dt = \frac{5}{8}\ln t = \frac{5}{8}\ln(4x^2-2x+1)$$

$$-\int \frac{3}{4(4x^2-2x+1)}dx = -\frac{3}{4}\int \frac{1}{4x^2-2x+1}dx =$$

$$\int \frac{1}{4(x^2-\frac{x}{2}+\frac{1}{4})}dx = \frac{1}{4}\int \frac{1}{(x^2-\frac{x}{2}+\frac{1}{4})}dx = \frac{\frac{3}{16}}{\cancel{4}}\int \frac{1}{(x-\frac{1}{4})^2+\frac{3}{16}}dx = \frac{1}{\cancel{4}16}\int \frac{1}{t^2+\frac{3}{16}} =$$

$$(x-\tfrac{1}{4})^2 = x^2-\frac{x}{2}+\frac{1}{16}$$

$$(x-\tfrac{1}{4}) = t \qquad dt = dx$$

$$= -\frac{3}{16}\cdot\frac{4\operatorname{arctg}(\frac{4t}{\sqrt3})}{\sqrt3}$$

$$= \frac{\sqrt3\operatorname{arctg}(\frac{4t}{\sqrt3})}{4}$$

$$* = \frac{5}{8}\ln(4x^2-2x+1) - \frac{\sqrt3\operatorname{arctg}(\frac{4x-1}{\sqrt3})}{4} + C$$

w4.16

$$\int \frac{5x^2+5x-7}{(x+1)^2(x^2+4)}\,dx$$

$\backslash (x+1)^2 \quad \backslash (x^2+4)(x+1)$

$\backslash x^2+4$ $\cancel{Ax^3}$

$$\frac{5x^2+5x-7}{(x+1)^2(x^2+4)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1} + \frac{D}{(x+1)^2} =$$

$$= \frac{Ax+B(x^2+2x+1) + C(x^3+x^2+4x+4) + D(x^2+4)}{(x+1)^2(x^2+4)} = \cancel{Ax^3+2Ax^2+Ax+Bx^2+2Bx}$$

$$= \frac{Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^3+Cx^2+4Cx+4C+Dx^2+4D}{(x+1)^2(x^2+4)} =$$

$$= \frac{(A+C)x^3+(2A+B+C+D)x^2+(\cancel{4}A+2B+4C)x+B+4C+4D}{(x+1)^2(x^2+4)}$$

w 5.16

$$\int \frac{1}{(x+2)^2\sqrt{8x^2+26x+11}}dx = -\int t\frac{(x+2)^{\cancel{2}}dt}{(x+2)\sqrt{8x^2+26x+11}} =$$

$$t = \frac{1}{(x+2)} \quad dt = \frac{-1}{(x+2)^2}dx$$

$$x = -\frac{2t-1}{t}$$

$$8x^2+26x+11 =$$

$$= 8\cdot\left(\frac{2t-1}{t}\right)^2 - 26\cdot\frac{2t-1}{t} + 11 \quad \backslash t \backslash t^2$$

$$= \frac{8\cdot(4t^2-4t+1)}{t^2} - \frac{52t-26}{t} + 11$$

$$= \frac{32t^2-32t+8-52t^2+26t+11t^2}{t^2}$$

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$$= \frac{-\cancel{20}t^2-6t+8}{t^2}$$

$$= -\int \frac{t\,dt}{t\cdot\frac{1}{t}\sqrt{-9t^2-6t+8}} = -\int\frac{t\,dt}{\sqrt{9-(3t+1)^2}} =$$

$$\sqrt{-9t^2-6t+8} = \sqrt{-(3t+1)^2+9}$$

$$3t+1 = u$$

$$du = 3dt$$

$$t = \frac{u-1}{3}$$

$$= -\frac{1}{9}\int\frac{u-1\,du}{\sqrt{9-u^2}} =$$

$$= -\frac{1}{9}\left(\int\frac{u\,du}{\sqrt{9-u^2}} - \int\frac{1\,du}{\sqrt{9-u^2}}\right)$$

$$= -\frac{1}{9}\sqrt{9-u^2} - \frac{\arcsin\left(\frac{u}{3}\right)}{9}$$

$$= -\frac{1}{9}\sqrt{9-(3t+1)} - \frac{\arcsin\left(\frac{3t+1}{3}\right)}{9}$$

$$= -\frac{\sqrt{9-\left(\frac{3}{(x+2)}+1\right)}}{9} - \frac{\arcsin\left(\frac{3}{(x+2)}+1\right)}{9} + C$$

Ew 6.16

$$\int \frac{\sqrt{3x+2}}{(3x+2)^2\sqrt{3x+1}}dx = \int \frac{1}{\sqrt{3x+1}\,(3x+2)^{\frac{3}{2}}} = \frac{1}{3}\int \frac{1}{\sqrt{t+1}\,(t+2)^{\frac{3}{2}}} =$$

$t = 3x \quad dt = 3dx$

$u = \sqrt{\frac{t+1}{t+2}}$

$t+1 = (t+2)\cdot u^2$

$t = -\frac{1}{u^2-1} - 2$

$dt = \frac{2u}{(u^2-1)^2}du$

$$= \frac{1}{3}\int 2du = \frac{1}{3}\cdot 2u + C =$$

$$= \frac{1}{3}\cdot 2\left(\sqrt{\frac{t+1}{t+2}}\right) + C =$$

$$= \frac{2}{3}\cdot\sqrt{\frac{3x+1}{3x+2}} + C$$

8.16

$$\int\sqrt{5x^2-4x+7}\,dx = \int\sqrt{\left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}}\,dx = \int\frac{\sqrt{t^2+\frac{31}{5}}}{\sqrt{5}}dt =$$

$5x^2-4x+7 = \left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}$

$t = \sqrt{5}x - \frac{2}{\sqrt{5}}$

$x = \frac{\sqrt{5}t+2}{5} \quad dx = \frac{1}{\sqrt{5}}dt$

$$= \frac{1}{\sqrt{5}}\int\sqrt{t^2+\frac{31}{5}}\,dt =$$

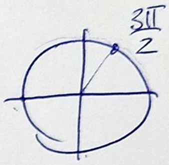
$$\frac{1}{\sqrt{5}}\left(\frac{t}{2}\sqrt{t^2+\frac{31}{5}} + \frac{31}{10}\ln\left(t+\sqrt{t^2+\frac{31}{5}}\right)\right)$$

$$= \frac{1}{\sqrt{5}}\cdot\left(\frac{31\ln\left(t+\sqrt{t^2+\frac{31}{5}}\right)}{10} + \frac{t\sqrt{t^2+\frac{31}{5}}}{2}\right)$$

$$= \frac{1}{\sqrt{5}}\cdot\left(\frac{31\ln\left(\sqrt{5}x-\frac{2}{\sqrt{5}}+\sqrt{\left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}}\right)}{10} + \frac{\sqrt{5}x-\frac{2}{\sqrt{5}}\sqrt{\left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}}}{2}\right)$$

$+C$

w14.16



$\rho = 2\sin\frac{2\varphi}{3}$ $\qquad 2\sin\frac{2\varphi}{3} \geq 0 \qquad \sin\frac{2\varphi}{3} \geq 0$

$0 \leq \frac{2\varphi}{3} \leq \pi$

$S = \frac{1}{2}\int_{0}^{\frac{3\pi}{2}} 4\sin^2\frac{2\varphi}{3}\,d\varphi$ $\qquad 0 \leq 2\varphi \leq 3\pi \qquad 0 \leq \varphi \leq \frac{3\pi}{2}$

$\int 4\sin^2\frac{2\varphi}{3}\,d\varphi = 4\int \frac{3\sin^2 t}{2}\,dt = 6\int \sin^2 t\,dt = 6\int \frac{1-\cos 2t}{2}\,dt = 6\left(\frac{1}{2}\left(\cdot\frac{2\varphi}{3}\right) - \frac{\sin\frac{4\varphi}{3}}{2}\right)$

$S = \frac{1}{2}\left(\left(2\varphi - 3\sin\frac{4\varphi}{3}\right)\right)\Big|_{0}^{\frac{3\pi}{2}}$

$S = \frac{1}{\cancel{3}2}\left(\left(3\pi - 3\sin\frac{\cancel{4}\cdot\cancel{3}\pi}{2\cdot\cancel{3}}\right) - (0)\right)$

$S = \frac{3\pi}{2} - \frac{3}{2}\sin 2\pi$ $\qquad S = \frac{3\pi}{2}$

$\sin 2\pi \to 0$

w15.16

$x = \frac{2}{3} y^{3/2}, \ 0 \leq y \leq 3$

$f'(y) = \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}} \cdot \sqrt{y}$

$$S = \int_0^3 \sqrt{1 + (f'(y))^2}\, dy = \int_0^3 \sqrt{1+y}\, dy = \cancel{\left. \frac{2}{3} (1+y)^{\frac{3}{2}} \right|_0^3} = \cancel{\frac{2}{3} \cdot (\sqrt{4})^3} =$$

$$\int \sqrt{1+y}\, dy = \int \sqrt{t}\, dt = \frac{2}{3} \cdot t^{\frac{3}{2}} = \frac{2}{3} \cdot (1+y)^{\frac{3}{2}} = \cancel{\frac{2y\sqrt{1+y}}{3} + \frac{2\sqrt{1+y}}{3}}$$

$1+y = t$

$dt = dy$

$$\cancel{\left. \left(\frac{2y\sqrt{1+y}}{3} + \frac{2\sqrt{1+y}}{3} \right) \right|_0^3} = \cancel{\frac{6 \cdot \sqrt{4}}{3} + \frac{2 \cdot \sqrt{4}}{3}} =$$

$\underline{12 + 4}$

$$\left. \frac{2}{3} (1+y)^{\frac{3}{2}} \right|_0^3 =$$

$$= \frac{2}{3} \sqrt{4^3} - \frac{2}{3} \cdot \sqrt{1} =$$

$$= \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

w13.16

$y=\sqrt{x}$, $y=x^3$

$S=\int_0^1 \sqrt{x}-x^3dx=$

$=\int_0^1 \sqrt{x}dx-\int_0^1 x^3dx=$

$=\frac{2}{3}\cdot x^{\frac{3}{2}}\Big|_0^1-\frac{1}{4}x^4\Big|_0^1=$

$=\frac{2}{3}\cdot 1^{\frac{3}{2}}-0-\frac{1}{4}1^4-0=$

$=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$