$\boxed{1.24}$ $u=\frac{\sqrt{x}}{y}-\frac{yz}{x+\sqrt{y}}$; $\vec{l}=2\vec{i}+\vec{k}$, $M(4;1;-2)$

Градиент функции u:

$$\overrightarrow{\operatorname{grad} u}=\frac{\partial u}{\partial x}\vec{i}+\frac{\partial u}{\partial y}\vec{j}+\frac{\partial u}{\partial z}\vec{k}$$

$$\frac{\partial u}{\partial x}=\frac{1}{2\sqrt{x}\,y}+\frac{yz}{(x+\sqrt{y})^2}=\frac{1}{4}-\frac{2}{2,5}=0,17$$

$$\frac{\partial u}{\partial y}=-\frac{\sqrt{x}}{y^2}-\frac{z(x+\sqrt{y})-yz\cdot\frac{1}{2\sqrt{y}}}{(x+\sqrt{y})^2}=-\frac{\sqrt{x}}{y^2}-\frac{z\left(x+\frac{\sqrt{y}}{2}\right)}{(x+\sqrt{y})^2}=-2+\frac{2\cdot 4,5}{25}=-1,64$$

$$\frac{\partial u}{\partial z}=-\frac{y}{x+\sqrt{y}}=-\frac{1}{4+1}=-0,2$$

$$\overrightarrow{\operatorname{grad} u}=0,17\vec{i}+(-1,64)\vec{j}+(-0,2)\vec{k}=\{0,17;-1,64;-0,2\}$$

Производное поле по направлению:

$$\frac{\partial u}{\partial l}=\frac{(\operatorname{grad} u,\vec{l})}{|\vec{l}|}=\frac{0,34-0,2}{\sqrt{2^2+1^2}}=\frac{0,14}{\sqrt{4+1}}=\frac{0,14}{\sqrt{5}}=0,028\sqrt{5}$$

Ответ: $0,028\sqrt{5}$

$\boxed{2.24}$ $v=9\sqrt{2}x^3-\frac{y^3}{2\sqrt{2}}-\frac{4z^3}{\sqrt{3}}$

$u=\frac{xy^2}{z^3}$; $M\left(\frac{1}{3};2;\sqrt{\frac{3}{2}}\right)$

$$\frac{\partial u}{\partial x}=\frac{y^2}{z^3}=\frac{8\sqrt{2}}{3\sqrt{3}}\ ;\ \frac{\partial u}{\partial y}=\frac{2xy}{z^3}=\frac{8\sqrt{2}}{9\sqrt{3}}\ ;\ \frac{\partial u}{\partial z}=-\frac{3xy^2}{z^4}=-\frac{3\cdot4\cdot4}{3\cdot9}$$

$$\overrightarrow{\operatorname{grad} u}=\frac{8\sqrt{2}}{3\sqrt{3}}\vec{i}+\frac{8\sqrt{2}}{9\sqrt{3}}\vec{j}-\frac{16}{9}\vec{k}$$

$$\frac{\partial v}{\partial x}=27\sqrt{2}x^2=\frac{27\sqrt{2}}{9}\ ;\ \frac{\partial v}{\partial y}=-\frac{3y^2}{2\sqrt{2}}=-\frac{3\cdot4}{2\sqrt{2}}\ ;\ \frac{\partial v}{\partial z}=-\frac{12z^2}{\sqrt{3}}=-\frac{12\cdot3}{\sqrt{3}\cdot2}$$

$$\overrightarrow{\operatorname{grad} v}=\frac{27\sqrt{2}}{9}\vec{i}-3\sqrt{2}\vec{j}-6\sqrt{3}\vec{k}$$

$$\cos\alpha=\frac{(\operatorname{grad} u,\operatorname{grad} v)}{|\operatorname{grad} u|\cdot|\operatorname{grad} v|}=\frac{\frac{8\sqrt{2}}{3\sqrt{3}}\cdot3\sqrt{2}-\frac{8\sqrt{2}}{9\sqrt{3}}\cdot3\sqrt{2}+\frac{16}{9}\cdot6\sqrt{3}}{\sqrt{\frac{64\cdot2}{9\cdot3}+\frac{64\cdot2}{81\cdot3}+\frac{16\cdot16}{81}}\cdot\sqrt{9\cdot2+9\cdot2+36\cdot3}}=$$

$$=\frac{\frac{128}{3\sqrt{3}}}{\frac{8}{3}\sqrt{\frac{18+2+12}{27}}\cdot12}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

Ответ: $\frac{\sqrt{2}}{2}$

3.24 $\vec a = 9z\vec i - 4x\vec k \Rightarrow a_x = 9z,\ a_y = 0,\ a_z = -4x$

Уравнение векторной линии

$$\frac{dx}{a_x} = \frac{dy}{a_y} = \frac{dz}{a_z} \Rightarrow \begin{cases} \frac{dx}{9z} = \frac{dz}{-4x} \\ dy = 0 \end{cases} \Rightarrow \begin{cases} 4x\,dx + 9z\,dz = 0 \\ C_1 = y \end{cases}$$

$$\int 4x\,dx + \int 9z\,dz = 2x^2 + \frac{9}{2}z^2 = C_2 \quad ; \quad \frac{x^2}{9} + \frac{z^2}{4} = C_2$$

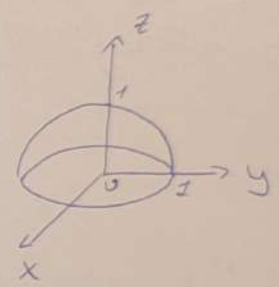
Ответ: $C_1 = y;\ \frac{x^2}{9} + \frac{z^2}{4} = C_2$

4.24 $\vec a = (x + xy)\vec i + (y - x^2)\vec j + z\vec k$; $S: x^2+y^2+z^2 = 1$; $P: z = 0$ $(z \geq 0)$

$F(x,y,z) = x^2 + y^2 + z^2 - 1$

$\vec N = \left\{\frac{\partial F}{\partial x}; \frac{\partial F}{\partial y}; \frac{\partial F}{\partial z}\right\} = \{2x; 2y; 2z\}$

$\vec n_0 = \frac{\vec N}{|\vec N|} = \frac{\{2x; 2y; 2z\}}{2\sqrt{x^2+y^2+z^2}} = \frac{\{x; y; z\}}{\sqrt{x^2+y^2+z^2}} = \{x; y; z\}$



Поток векторного поля:

$$\Pi = \iint_S (\vec a, \vec n)\,dS = \iint_S (x^2 + x^2y + y^2 - x^2y + z^2)\,dS = \iint_S (x^2+y^2+z^2)\,dS = \iint_S dS = \frac{1}{2}4\pi R^2$$

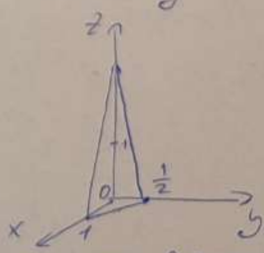
$R = 1$. $\Pi = 2\pi \cdot 1^2 = 2\pi$

Ответ: 2π

5.24 $\vec a = x\vec i + 4y\vec j + 5z\vec k$; $P: x + 2y + \frac{z}{2} = 1 \Rightarrow z = 2 - 2x - 4y$

$F(x,y,z) = x + 2y + \frac{z}{2} - 1$; $\vec N = \left\{\frac{\partial F}{\partial x}; \frac{\partial F}{\partial y}; \frac{\partial F}{\partial z}\right\} = \left\{1; 2; \frac{1}{2}\right\}$

$\vec n_0 = \frac{\vec N}{|\vec N|} = \frac{\{1; 2; \frac{1}{2}\}}{\sqrt{1+4+\frac{1}{4}}} = \left\{\frac{2}{\sqrt{21}}; \frac{4}{\sqrt{21}}; \frac{1}{\sqrt{21}}\right\}$



$$\Pi = \iint_S (\vec a, \vec n_0)\,dS = \iint_S \left(\frac{2x}{\sqrt{21}} + \frac{16y}{\sqrt{21}} + \frac{5z}{\sqrt{21}}\right) dS = \iint_S \frac{2x + 16y + 5z}{\sqrt{21}}\,dS = \quad D = \begin{cases} 0 \le x \le 1 \\ 0 \le y \le \frac{1-x}{2} \end{cases}$$

$$= \frac{1}{\sqrt{21}}\iint_S (2x + 16y + 5z)\,dS = \frac{1}{\sqrt{21}}\iint_D (2x + 16y + 5(2 - 2x - 4y)) \cdot \sqrt{21}\,dx\,dy =$$

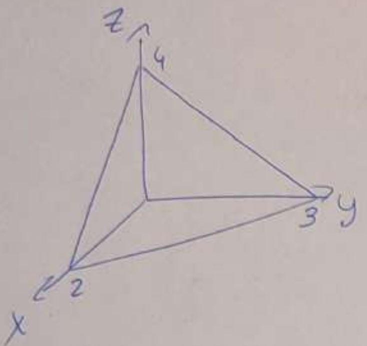
$$= \iint_D (10 - 8x - 4y)\,dx\,dy = \int_0^1 dx \int_0^{\frac{1-x}{2}} (10 - 8x - 4y)\,dy = \int_0^1 dx\,(10y - 8xy - 2y^2)\Big|_0^{\frac{1-x}{2}} =$$

$$\int_0^1 \left(5(1-x) - 4x(1-x) - \frac{(1-x)^2}{2}\right)dx = \int_0^1 (3{,}5x^2 - 8x + 4{,}5)\,dx =$$

$$= \left(3{,}5\cdot\frac{x^3}{3} - 4x^2 + 4{,}5x\right)\Big|_0^1 = \frac{3{,}5}{3} + \frac{1{,}5}{3} = \frac{5}{3}$$

Ответ: $\frac{5}{3}$

$\boxed{6.24}$ $\vec{a} = \pi x\vec{i} + 2\pi y\vec{j} + 2\vec{k}$; P: $\frac{x}{2} + \frac{y}{4} + \frac{z}{3} = 1$



Уравнение плоскости:

$z = 3 - \frac{3}{2}x - \frac{3}{4}y$

Нормальный вектор:

$\vec{N} = \left\{\frac{1}{2}; \frac{1}{4}; \frac{1}{3}\right\}$

Единичный нормальный вектор:

$$\vec{n}_0 = \frac{\vec{N}}{|\vec{N}|} = \frac{\left\{\frac{1}{2}; \frac{1}{4}; \frac{1}{3}\right\}}{\sqrt{\frac{1}{4}+\frac{1}{16}+\frac{1}{9}}} = \left\{\frac{6}{\sqrt{61}}; \frac{3}{\sqrt{61}}; \frac{4}{\sqrt{61}}\right\}$$

$$D = \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 4-2x \end{cases}$$

Поток векторного поля:

$$\Pi = \iint_S (\vec{a},\vec{n})\,dS = \iint_S \frac{6\pi x + 6\pi y + 8}{\sqrt{61}}\,dS = \frac{1}{\sqrt{61}}\iint_S (6\pi x + 6\pi y + 8)\,dS =$$

$$= \frac{1}{\sqrt{61}}\iint_D (6\pi x + 6\pi y + 8)\sqrt{1 + (z'_x)^2 + (z'_y)^2}\,dx\,dy = \frac{1}{\sqrt{61}}\iint_D (6\pi x + 6\pi y + 8)\sqrt{1 + \frac{9}{4} + \frac{9}{16}}\,dx\,dy =$$

$$= \frac{1}{\sqrt{61}}\cdot\frac{\sqrt{61}}{4}\iint_D (6\pi x + 6\pi y + 8)\,dx\,dy = \frac{1}{4}\int_0^2 dx\int_0^{4-2x}(6\pi x + 6\pi y + 8)\,dy = \frac{1}{4}\int_0^2 dx\left(6\pi xy + \frac{6\pi y^2}{2} + 8y\right)\Big|_0^{4-2x} =$$

$$= \int_0^2 (12\pi + 8 - 4x - 6\pi x)\,dx = \left(12\pi x + 8x - 2x^2 - 3\pi x^2\right)\Big|_0^2 = 12\pi + 8$$

Ответ: $12\pi + 8$

$\boxed{7.24}$ $\vec{a} = (\sqrt{z} + 1 + x)\vec{i} + (2x+y)\vec{j} + (\sin x + z)\vec{k}$

$S: \begin{cases} z^2 = x^2 + y^2 \\ z = 1 \end{cases}$

$x^2 + y^2 = 1 \Rightarrow \rho = 1$

$z = \rho$; Перейдём в цилиндрическую систему координат

$$\begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le \rho \le 1 \\ \rho \le z \le 1 \end{cases}$$

$$\operatorname{div}\vec{a} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 3$$

Поток векторного поля:

$$\Pi=\iiint\limits_V div\vec{a}\,dV=\iiint\limits_V 3dV=3\int\limits_0^{2\pi}d\varphi\int\limits_0^1\rho d\rho\int\limits_\rho^1 dz=3\int\limits_0^{2\pi}d\varphi\int\limits_0^1\rho(1-\rho)d\rho=$$

$$=3\int\limits_0^{2\pi}d\varphi\int\limits_0^1(\rho-\rho^2)d\rho=3\int\limits_0^{2\pi}d\varphi\left(\frac{\rho^2}{2}-\frac{\rho^3}{3}\right)\Big|_0^1=3\cdot\frac{1}{6}\int\limits_0^{2\pi}d\varphi=\frac{1}{2}\cdot 2\pi=\pi$$

Ответ: π

$\boxed{8.24}$ $\vec{a}=(2x+y)\vec{i}+(y+2z)\vec{k}$

$$S:\begin{cases} z=2-4(x^2+y^2)\\ z=4(x^2+y^2)\end{cases}$$

$div\vec{a}=2+2=4$

$2-4\rho^2=4\rho^2$

$\rho=\frac{1}{2}$

$$\Pi=\iiint\limits_V 4dV=4\int\limits_0^{2\pi}d\varphi\int\limits_0^{\frac{1}{2}}\rho d\rho\int\limits_{4\rho^2}^{2-4\rho^2}dz=4\int\limits_0^{2\pi}d\varphi\int\limits_0^{\frac{1}{2}}(2\rho-8\rho^3)d\rho=$$

$$=4\int\limits_0^{2\pi}d\varphi(\rho^2-2\rho^4)\Big|_0^{\frac{1}{2}}=4\int\limits_0^{2\pi}\left(\frac{1}{4}-\frac{1}{8}\right)d\varphi=\frac{4}{8}\cdot 2\pi=\pi$$

Ответ: π

$\boxed{9.24}$ $\vec{a}=x^2\vec{i}$; $S:\begin{cases} z=1-x-y\\ x=0,\ y=0, z=0\end{cases}$

$$div\vec{a}=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}=2x$$

$$\Pi=\int\limits_0^1dx\int\limits_0^{1-x}dy\int\limits_0^{1-x-y}2x\,dz=\int\limits_0^1dx\int\limits_0^{1-x}2x(1-x-y)dy=2\int\limits_0^1xdx\left[\int\limits_0^{1-x}dy-\int\limits_0^{1-x}xdx-\int\limits_0^{1-x}ydy\right]=$$

$$=2\int\limits_0^1x\left(y-xy-\frac{y^2}{2}\right)\Big|_0^{1-x}dx=\int\limits_0^1(x^3-2x^2+x)dx=\left(\frac{x^4}{4}-\frac{2x^3}{3}+\frac{x^2}{2}\right)\Big|_0^1=\frac{1}{12}$$

Ответ: $\frac{1}{12}$

$\boxed{10.24}$ $\vec{F}=xy\vec{i}$; L: $y=\sin x$; $M(\pi,0)$, $N(0,0)$

$$A=\int\limits_L\vec{F}d\vec{s}=\int\limits_{MN}xydx=\int\limits_\pi^0 x\sin x\,dx=\left|\begin{matrix}u=x\\ dv=\sin x\\ du=dx\\ v=-\cos x\end{matrix}\right|=-x\cos x\Big|_\pi^0+\int\limits_\pi^0\cos x\,dx=$$

$$=-(0-\pi\cos\pi)+\sin x\Big|_\pi^0=-\pi$$

Ответ: $-\pi$

$\boxed{11.24}$ $\vec{a} = xy\vec{i} + x\vec{j} + y^2\vec{k}$; $\Gamma: \begin{cases} x = \cos t, \ y = \sin t \\ z = \sin t \end{cases}$

$$\begin{cases} dx \overset{*}{=} \sin t \\ dy = \cos t \\ dz = \cos t \end{cases}$$

$$C = \int_0^{2\pi} (-\cos t \sin^2 t + \cos^2 t + \sin^2 t \cos t)\, dt = \int_0^{2\pi} -\sin^2 t\, d\sin t + \int_0^{2\pi} \frac{1+\cos 2t}{2} dt +$$

$$+ \int_0^{2\pi} \sin^2 t\, d\sin t = 0 + \int_0^{2\pi} \frac{1}{2} dt + \frac{1}{4}\int_0^{2\pi} \cos 2t\, d2t + 0 = \frac{1}{2}\cdot 2\pi + \frac{1}{4}\sin 2t\Big|_0^{2\pi} = \pi$$

Ответ: π

$\boxed{12.24}$ $\vec{a} = -y\vec{i} + x\vec{j} + 3z^2\vec{k}$; $\Gamma: \begin{cases} x^2+y^2+z^2 = 9 \\ x^2+y^2 = 1 \end{cases}$, $z > 0$

$x^2+y^2+z^2 = 9$ - сфера, $R = 3$

$x^2+y^2 = 1$ - цилиндр, $r = 1$

$$\begin{cases} x = \cos t \\ y = \sin t \\ z = \sqrt{R^2 - r^2} \end{cases} \quad \begin{cases} x = \cos t \\ y = \sin t \\ z = \sqrt{8} \end{cases} \Rightarrow \begin{cases} dx = -\sin t \\ dy = \cos t \\ dz = 0 \end{cases}$$

$$C = \oint_\Gamma a_x dx + a_y dy + a_z dz = \oint_\Gamma (-y dx + x dy + 3z^2 dz) =$$

$$= \int_0^{2\pi} (-\sin t(-\sin t) + \cos t \cdot \cos t + 0)\, dt = \int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt = \int_0^{2\pi} dt = t\Big|_0^{2\pi} = 2\pi$$

Ответ: 2π