w1.16

$$\int e^{4\sin^2 x}\sin 2x\,dx = 2\int e^{4\sin^2 x}\cos x\sin x\,dx = 2\int \frac{e^{4t}\cdot\cancel{\cos x\sin x}\,dt}{2\cdot\cancel{\cos x\sin x}}$$

$\sin^2 x = t$

$dt = 2\sin x\cos x$

$$= 2\int\frac{e^{4t}dt}{2} = \int e^{4t}dt =$$

$$= \frac{1}{4}e^{4\sin^2 x} + C$$

w2.16

$$\int 3^{-x}\cdot(x+2)\,dx = \int\frac{1}{3^x}\cdot(x+2)\,dx$$

$u = x+2$ $\quad dv = \frac{1}{3^x}$

$du = 1$ $\quad v = -\frac{1}{\ln 3\cdot 3^x}$

$$\int\frac{1}{3^x}\cdot(x+2)\,dx = -(x+2)\cdot\frac{1}{\ln 3\cdot 3^x} + \int\left(\frac{1}{\ln 3\cdot 3^x}dx\right) = -(x+2)\cdot\frac{1}{\ln 3\cdot 3^x} - \frac{1}{\ln(3)}\cdot\frac{1}{\ln(3)3^x}$$

$$\frac{1}{\ln 3}\int\frac{1}{3^x}dx = \frac{1}{\ln 3}\cdot\int 3^{-x}dx = \frac{1}{\ln 3}\cdot -\frac{1}{\ln(3)3^x}$$

w3.16

$$\int \frac{24x^3-12x^2+11x-2}{4x^2-2x+1}\,dx = \int \frac{5x-2}{4x^2-2x+1} + 6x\,dx$$

$$\begin{array}{l|l} 24x^3-12x^2+11x-2 & 4x^2-2x+1 \\ -(24x^3-12x^2+6x) & 6x \\ \hline \quad 5x-2 & \end{array}$$

$$\frac{4x^2-2x+1}{4x^2-2x+1} = 1$$

$$\cancel{(24x^3-12x^2+11x-2)-(4x^2-2x+1)} =$$

$$= \cancel{24x^3-16x^2+13x+1}$$

$$\cancel{\int\frac{5x-2}{4x^2-2x+1}dx}$$

$$\int\frac{5x-2}{4x^2-2x+1}dx \overset{*}{=}$$

$$5x-2 = \frac58(8x-2)-\frac34$$

$$\frac58\int\frac{8x-2}{4x^2-2x+1}dx - \int\frac{3}{4(4x^2-2x+1)}dx$$

$$t = 4x^2-2x+1$$

$$dt = (8x-2)dx$$

$$\frac58\int\frac{\cancel{(8x-2)}}{t\cdot\cancel{(8x-2)}}dt = \frac58\int\frac1t\,dt = \frac58\ln t = \frac58\ln(4x^2-2x+1)$$

$$-\int\frac{3}{4(4x^2-2x+1)}dx = -\frac34\int\frac{1}{4x^2-2x+1}dx =$$

$$\int\frac{1}{4(x^2-\frac x2+\frac14)}dx = \frac14\int\frac{1}{x^2-\frac x2+\frac14}dx \quad \frac{3}{16}\int\frac{1}{(x-\frac14)^2+\frac{3}{16}}dx = -\frac{1}{\cancel{4}16}\int\frac{1}{t^2+\frac{3}{16}} =$$

$$= -\frac{3}{16}\cdot\frac{4\,\text{arctg}(\frac{4t}{\sqrt3})}{\sqrt3}$$

$$= \frac{\sqrt3\,\text{arctg}(\frac{4t}{\sqrt3})}{4}$$

$$(x-\tfrac14)^2 = x^2-\frac x2+\frac1{16}$$

$$(x-\tfrac14) = t$$
$$dt = dx$$

$$* = \frac58\ln(4x^2-2x+1) - \frac{\sqrt3\,\text{arctg}(\frac{4x-1}{\sqrt3})}{4} + C$$

w4.16

$$\int \frac{5x^2+5x-7}{(x+1)^2(x^2+4)}\,dx$$

$\backslash(x+1)^2$ $\backslash(x^2+4)(x+1)$

$\backslash x^2+4$ $\cancel{Ax^3}$

$$\frac{5x^2+5x-7}{(x+1)^2(x^2+4)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1} + \frac{D}{(x+1)^2} =$$

$$= \frac{Ax+B(x^2+2x+1) + C(x^3+x^2+4x+4) + D(x^2+4)}{(x+1)^2(x^2+4)} = \cancel{Ax^3+2Ax^2+Ax+Bx^2+2Bx}$$

$$= \frac{Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^3+Cx^2+4Cx+4C+Dx^2+4D}{(x+1)^2(x^2+4)} =$$

$$= \frac{(A+C)x^3+(2A+B+C+D)x^2+(\cancel{4C}A+2B+4C)x+B+4C+4D}{(x+1)^2(x^2+4)}$$

$I - \int 5x-2$ ...

w 5.16

$$\int \frac{1}{(x+2)^2\sqrt{8x^2+26x+11}}\,dx = -\int t\,\frac{(x+2)^{\cancel{2}}\,dt}{(x+2)\sqrt{8x^2+26x+11}} =$$

$$t = \frac{1}{(x+2)} \qquad dt = \frac{-1}{(x+2)^2}\,dx$$

$$x = -\frac{2t-1}{t}$$

$$8x^2+26x+11 =$$

$$= 8\cdot\left(\frac{2t-1}{t}\right)^2 - 26\cdot\frac{2t-1}{t} + 11 \qquad \backslash t \ \backslash t^2$$

$$= \frac{8\cdot(4t^2-4t+1)}{t^2} - \frac{52t-26}{t} + 11$$

$$= \frac{32t^2-32t+8-52t^2+26t+11t^2}{t^2}$$

9

$$= \frac{-\cancel{20}9t^2-6t+8}{t^2}$$

$$= -\int \frac{t^{\cancel{2}}\,dt}{t\cdot\frac{1}{\cancel{t}}\sqrt{-9t^2-6t+8}} = -\int \frac{t\,dt}{\sqrt{9-(3t+1)^2}} =$$

$$\sqrt{-9t^2-6t+8} = \sqrt{-(3t+1)^2+9}$$

$$3t+1 = u$$

$$du = 3dt$$

$$t = \frac{u-1}{3}$$

$$= -\frac{1}{9}\int \frac{u-1\,du}{\sqrt{9-u^2}} =$$

$$= -\frac{1}{9}\left(\int\frac{u\,du}{\sqrt{9-u^2}} - \int\frac{1\,du}{\sqrt{9-u^2}}\right)$$

$$= -\frac{1}{9}\sqrt{9-u^2} - \frac{\arcsin\left(\frac{u}{3}\right)}{9}$$

$$= -\frac{1}{9}\sqrt{9-(3t+1)} - \frac{\arcsin\left(\frac{3t+1}{3}\right)}{9}$$

$$= -\frac{\sqrt{9-\left(\frac{3}{(x+2)}+1\right)}}{9} - \frac{\arcsin\left(\frac{3}{(x+2)}+1\right)}{9} + C$$

Ew 6.16

$$\int \frac{\sqrt{3x+2}}{(3x+2)^2\sqrt{3x+1}}dx = \int \frac{1}{\sqrt{3x+1}\,(3x+2)^{\frac{3}{2}}} = \frac{1}{3}\int \frac{1}{\sqrt{t+1}\,(t+2)^{\frac{3}{2}}} =$$

$t = 3x$, $dt = 3dx$

$$= \frac{1}{3}\int 2du = \frac{1}{3}\cdot 2u + C =$$

$u = \sqrt{\frac{t+1}{t+2}}$

$$= \frac{1}{3}\cdot 2\left(\sqrt{\frac{t+1}{t+2}}\right) + C =$$

$t+1 = (t+2)\cdot u^2$

$t = -\frac{1}{u^2-1} - 2$

$$= \frac{2}{3}\cdot\sqrt{\frac{3x+1}{3x+2}} + C$$

$dt = \frac{2u}{(u^2-1)^2}du$

8.16

$$\int \sqrt{5x^2-4x+7}\,dx = \int \sqrt{\left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}}\,dx = \int \frac{\sqrt{t^2+\frac{31}{5}}}{\sqrt{5}}dt =$$

$5x^2-4x+7 = \left(\sqrt{5}x - \frac{2}{\sqrt{5}}\right)^2 + \frac{31}{5}$

$$= \frac{1}{\sqrt{5}}\int \sqrt{t^2+\frac{31}{5}}\,dt =$$

$$\frac{1}{\sqrt{5}}\left(\frac{t}{2}\sqrt{t^2+\frac{31}{5}} + \frac{31}{10}\ln\left(t+\sqrt{t^2+\frac{31}{5}}\right.\right.$$

$t = \sqrt{5}x - \frac{2}{\sqrt{5}}$

$x = \frac{\sqrt{5}t+2}{5}$ $\quad dx = \frac{1}{\sqrt{5}}dt$

$$= \frac{1}{\sqrt{5}}\circ\left(\frac{31\ln\left(t+\sqrt{t^2+\frac{31}{5}}\right)}{10} + \frac{t\sqrt{t^2+\frac{31}{5}}}{2}\right)$$

$$= \frac{1}{\sqrt{5}}\cdot\left(\frac{31\ln\left(\sqrt{5}x-\frac{2}{\sqrt{5}}+\sqrt{\left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}}\right)}{10} + \frac{\sqrt{5}x-\frac{2}{\sqrt{5}}\sqrt{\left(\sqrt{5}x-\frac{2}{\sqrt{5}}\right)^2+\frac{31}{5}}}{2}\right)$$

$+C$