Типовые расчеты

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Вариант - 19.

N 11.

$$\int\limits_0^{\sqrt{\frac{\pi}{2}}} \frac{x\,dx}{\cos^2 x^2} = \frac{1}{2}\int\limits_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t} = \frac{1}{2}\,tg\,t\Big|_0^{\frac{\pi}{4}} = \frac{1}{2}\,tg\,\frac{\pi}{4} = \frac{1}{2}$$

Ответ: $\frac{1}{2}$

N 12.

$$\int\limits_0^{\pi} e^{-x}\sin 2x\,dx = \left|\begin{matrix} u=\sin 2x & dv = e^{-x}dx \\ du = 2\cos 2x\,dx & v=-e^{-x}\end{matrix}\right| = -\sin 2x\cdot e^{-x}\Big|_0^{\pi} + \int\limits_0^{\pi} 2e^{-x}\cos 2x\,dx =$$

$$= \left|\begin{matrix} u=2\cos 2x & dv = e^{-x}dx \\ du = -4\sin 2x\,dx & v=-e^{-x}\end{matrix}\right| = -e^{-x}\sin 2x\Big|_0^{\pi} - \left(2e^{-x}\cos 2x\Big|_0^{\pi} - \int\limits_0^{\pi} -4e^{-x}\sin 2x\,dx\right) =$$

$$= -e^{-x}\sin 2x\Big|_0^{\pi} - 2e^{-x}\cos 2x\Big|_0^{\pi} - 4\int\limits_0^{\pi} e^{-x}\sin 2x\,dx = \left(\frac{-e^{-x}\sin 2x - 2e^{-x}\cos 2x}{5}\right)\Bigg|_0^{\pi} =$$

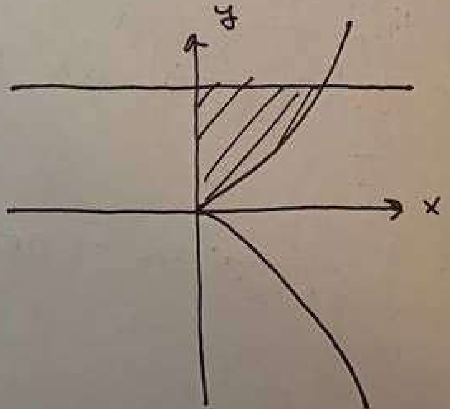
$$= \frac{2}{5} - \frac{2e^{-\pi}}{5}$$

Ответ: $\frac{2}{5} - \frac{2e^{-\pi}}{5}$

N 13.

$y^2 = x^3$, $y = 4$, $x = 0$ $\qquad 16 = x^3 \Rightarrow x = \sqrt[3]{16}$ $\qquad y = \sqrt{x^3}$

$$S = \int\limits_0^{\sqrt[3]{16}} x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Bigg|_0^{\sqrt[3]{16}} = \frac{2}{5}\cdot 16^{5/6} = \frac{2}{5}\cdot 4^{5/3} = \frac{8}{5}\cdot 4^{2/3}$$



Ответ: $\frac{8}{5}\cdot 4^{2/3}$

N14.

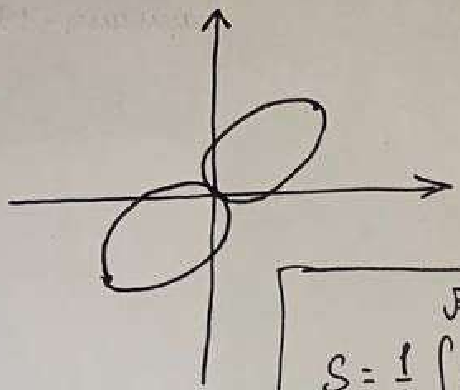
$\rho = 1+\sin 2\varphi$

$\varphi = 0 \quad \rho = 1 \qquad \varphi = \frac{\pi}{4} \quad \rho = 2$

$\varphi = \frac{\pi}{2} \quad \rho = 1 \qquad \varphi = \frac{3\pi}{4} \quad \rho = 0$

$\varphi = \pi \quad \rho = 1 \qquad \varphi = -\frac{\pi}{4} \quad \rho = 0$

$\varphi = -\frac{\pi}{2} \quad \rho = 1 \qquad \varphi = -\frac{3\pi}{4} \quad \rho = 2$



$$\boxed{S = \frac{1}{2}\int_{\alpha}^{\beta} \rho^2 \, d\varphi}$$

$$S = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1+\sin 2\varphi \, d\varphi = \frac{1}{2}\cdot\left(\int_0^{\frac{\pi}{2}} 1 \, d\varphi + \int_0^{\frac{\pi}{2}} \sin 2\varphi \, d\varphi\right) = \frac{1}{2}\left(\varphi - \frac{\cos 2\varphi}{2}\right)\Bigg|_0^{\frac{\pi}{2}} =$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - \frac{\cos\left(2\cdot\frac{\pi}{2}\right)}{2} - \left(0 - \frac{\cos(2\cdot 0)}{2}\right)\right) = \frac{1}{2}\left(\frac{\pi}{2}+1\right) = \frac{\pi}{4}+\frac{1}{2}$$

<u>Ответ:</u> $\frac{\pi}{4}+\frac{1}{2}$

N15

$y = -\sqrt{1-x^2} \qquad 0 \le x \le \frac{\sqrt{3}}{2} \qquad l = \int_{\alpha}^{\beta} \sqrt{1+(y')^2} \, dx$

$y' = \frac{x}{\sqrt{1-x^2}}$

$$l = \int_0^{\sqrt{3}/2} \sqrt{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2} \, dx = \int_0^{\frac{\sqrt{3}}{2}} \sqrt{1+\frac{x^2}{1-x^2}} \, dx = \int_0^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1\cancel{-x^2}\cancel{+x^2}}{1-x^2}} \, dx =$$

$$= \int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x \Bigg|_0^{\frac{\sqrt{3}}{2}} = \frac{\pi}{3} - 0 = \frac{\pi}{3}$$

<u>Ответ:</u> $\frac{\pi}{3}$