T. P. 1.12

1.12

$$\int_0^1 dy \int_0^{\sqrt[3]{y}} f\,dx + \int_1^2 dy \int_0^{2-y} f\,dx =$$

$$= \int_0^1 dx \int_{x^3}^{2-x} f\,dy$$

Ответ: $\int_0^1 dx \int_{x^3}^{2-x} f\,dy$

$D_1: \begin{cases} 0 \le y \le 1 \\ 0 \le x \le \sqrt[3]{y} \end{cases}$

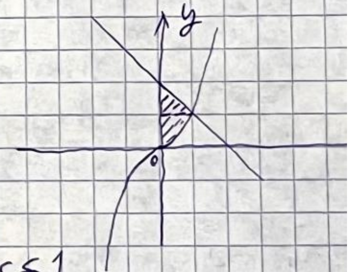
$D_2: \begin{cases} 1 \le y \le 2 \\ 0 \le x \le 2-y \end{cases}$

$x = \sqrt[3]{y}$

$y = x^3$

$x = 2-y$

$y = 2-x$



$D: \begin{cases} 0 \le x \le 1 \\ x^3 \le y \le 2-x \end{cases}$

T. P. 2.12

2.12

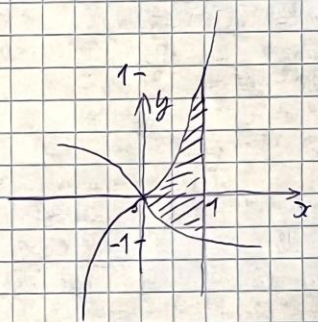
$$\iint_D (24xy + 18x^2y^2)\,dx\,dy =$$

$$= \int_0^1 dx \int_{-\sqrt[3]{x}}^{x^3} (24xy + 18x^2y^2)\,dy =$$

$$= \int_0^1 dx \left(24x\frac{y^2}{2} + 18x^2\frac{y^3}{3}\right)\Bigg|_{-\sqrt[3]{x}}^{x^3} =$$

$$= \int_0^1 \left(12x\cdot x^6 + 6x^2x^9 - 12x\cdot x^{\frac{2}{3}} + 6x^2\cdot x\right)dx = \int_0^1 \left(12x^7 + 6x^{11} - 12x^{\frac{5}{3}} + 6x^3\right)dx =$$

$$= \left(12\frac{x^8}{8} + 6\frac{x^{12}}{12} - 12\frac{x^{\frac{8}{3}}}{\frac{8}{3}} + 6\frac{x^4}{4}\right)\Bigg|_0^1 = \frac{3}{2} + \frac{1}{2} - \frac{9}{2} + \frac{3}{2} = -1$$



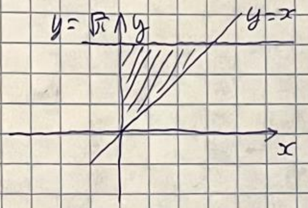
$D: \begin{cases} 0 \le x \le 1 \\ -\sqrt[3]{x} \le y \le x^3 \end{cases}$

Ответ: -1

T. P. 3.12

3.12

$$\iint\limits_D y^2 \cos xy \, dx\, dy =$$



$$= \int_0^{\sqrt{\pi}} y^2\, dy \int_0^{y} \cos xy \, dx =$$

$$D: \begin{cases} 0 \le x \le y \\ 0 \le y \le \sqrt{\pi} \end{cases}$$

$$= \int_0^{\sqrt{\pi}} y\, dy \int_0^{y} \cos(xy)\, d(xy) = \int_0^{\sqrt{\pi}} y \cdot (\sin xy)\Big|_0^{y} dy =$$

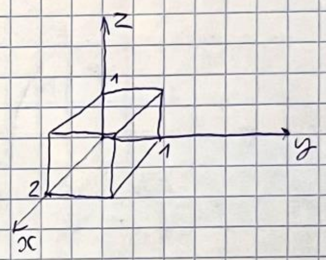
$$= \int_0^{\sqrt{\pi}} y \sin y^2 \, dy = \int_0^{\sqrt{\pi}} \frac{\sin y^2}{2}\, dy^2 = -\frac{\cos y^2}{2}\Big|_0^{\sqrt{\pi}} = -\frac{1}{2}(-1-1) = 1$$

Ответ: 1

T. P. 4.12

4.12

$$\iiint\limits_V x^2 z\, \mathrm{sh}(xyz)\, dx\, dy\, dz =$$



$$= \int_0^2 dx \int_0^1 dy \int_0^1 x^2 z\, \mathrm{sh}(xyz)\, dz =$$

$$V: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 1 \\ 0 \le z \le 1 \end{cases}$$

$$= \int_0^2 x^2 dx \int_0^1 z\, dz \int_0^1 \mathrm{sh}(xyz)\, dy =$$

$$= \int_0^2 x^2 dx \int_0^1 z\, dz \cdot \frac{\mathrm{ch}(xyz)}{xz}\Big|_0^1 =$$

$$= \int_0^2 \frac{x^2 dx}{x} \int_0^1 \frac{z\, dz}{z} \cdot \mathrm{ch}(xyz)\Big|_0^1 = \int_0^2 x\, dx \int_0^1 (\mathrm{ch}(xz) - 1)\, dz =$$

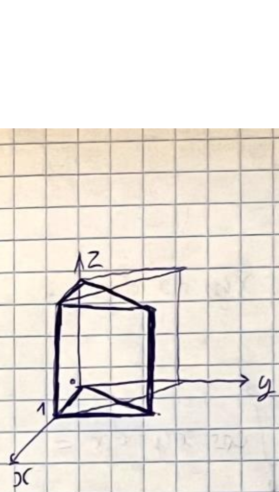
$$= \int_0^2 x\, dx \left(\frac{\mathrm{sh}(xz)}{x} - z\right)\Big|_0^1 = \int_0^2 x\left(\frac{\mathrm{sh}\, x}{x} - 1\right) dx = \int_0^2 (\mathrm{sh}\, x - x)\, dx =$$

$$= \left(\mathrm{ch}\, x - \frac{x^2}{2}\right)\Big|_0^2 = \mathrm{ch}\, 2 - 2 - 1 = \mathrm{ch}\, 2 - 3$$

Ответ: ch 2 − 3

T. P. 5.12

5.12



$$\iiint\limits_V (1+2x^3)\,dx\,dy\,dz =$$

$$= \int_0^1 dx \int_0^{36x} dy \int_0^{\sqrt{xy}} (1+2x^3)\,dz =$$

$$= \int_0^1 (1+2x^3)\,dx \int_0^{36x} dy \int_0^{\sqrt{xy}} dz =$$

$$V: \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 36x \\ 0 \le z \le \sqrt{xy} \end{cases}$$

$$= \int_0^1 (1+2x^3)\,dx \int_0^{36x} \sqrt{xy}\,dy = \int_0^1 (1+2x^3)\sqrt{x}\cdot \frac{y^{3/2}}{\frac{3}{2}}\Bigg|_0^{36x} =$$

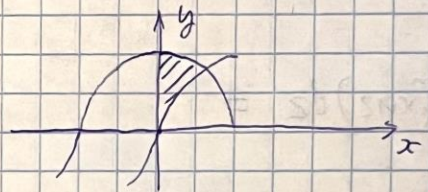
$$= \frac{2}{3}\int_0^1 (1+2x^3)\sqrt{x}\left(\sqrt{36x}\right)^3 = \frac{2}{3}\int_0^1 (1+2x^3)\cdot 216x^2\,dx = 144\int_0^1 (x^2+2x^5)\,dx =$$

$$= 144\left(\frac{x^3}{3}+\frac{2x^6}{6}\right)\Bigg|_0^1 = 144\cdot\frac{2}{3} = 96$$

Ответ: 96

T. P. 6.12

6.12



$$\iint\limits_D dx\,dy = \int_0^{\frac{\pi}{4}} dx \int_{\sin x}^{\cos x} dy =$$

$$D: \begin{cases} 0 \le x \le \frac{\pi}{4} \\ \sin x \le y \le \cos x \end{cases}$$

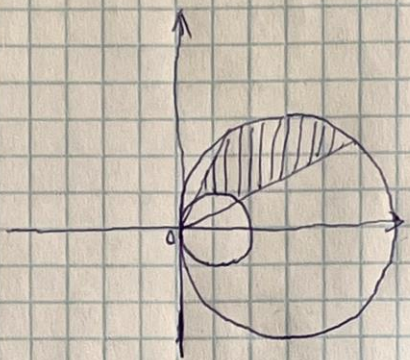
$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx = (\sin x + \cos x)\Bigg|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 = \sqrt{2} - 1$$

Ответ: $\sqrt{2} - 1$

T. P. 7.12

7.12

$$\begin{cases} x^2-2x+y^2=0 \\ x^2-6x+y^2=0 \\ y=\frac{x}{\sqrt{3}} \\ y=\sqrt{3}\,x \end{cases}$$



$$\begin{cases} x^2-2x+1+y^2=1 \\ x^2-6x+9+y^2=9 \end{cases}$$

$$\begin{cases} (x-1)^2+y^2=1 \\ (x-3)^2+y^2=9 \end{cases}$$

$x=\rho\cos\theta \qquad dx\,dy=\rho\,d\rho\,d\theta$

$y=\rho\sin\theta \qquad x^2+y^2=\rho^2$

$$\begin{cases} \rho^2-2\rho\cos\theta=0 \\ \rho^2-6\rho\cos\theta=0 \end{cases} \qquad \begin{cases} \rho=2\cos\theta \\ \rho=6\cos\theta \end{cases}$$

$x=3 \Rightarrow y=\frac{3}{\sqrt{3}}=\sqrt{3}$

$tg\,\theta=\frac{y}{x}=\frac{\sqrt{3}}{3} \Rightarrow \theta=\frac{\pi}{6}$

$tg\,\theta=\frac{y}{x}=\frac{\sqrt{3}\,x}{x} \Rightarrow \theta=\frac{\pi}{3}$

$$D: \begin{cases} \frac{\pi}{6}\le\theta\le\frac{\pi}{3} \\ 2\cos\theta\le\rho\le 6\cos\theta \end{cases}$$

$$S=\iint_D dx\,dy=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\rho^2}{2}\Big|_{2\cos\theta}^{6\cos\theta} d\theta=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{36\cos^2\theta-4\cos^2\theta}{2}\,d\theta=$$

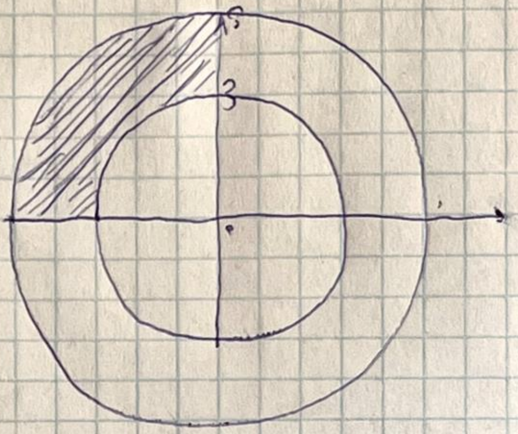
$$=16\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2\theta\,d\theta=16\cdot\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1+\cos 2\theta)\,d\theta=8\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}-\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)=\frac{4\pi}{3}$$

Ответ: $\frac{4\pi}{3}$

T. P. 8.12

8.12

$$D = \begin{cases} x^2+y^2=9 \\ x^2+y^2=25 \\ x=0,\ y=0 \\ x\le 0,\ y\ge 0 \end{cases} \qquad \mu = \frac{2y-x}{x^2+y^2}$$



$x=\rho\cos\theta$

$y=\rho\sin\theta$

$dx\,dy=\rho\,d\rho\,d\theta$

$x^2+y^2=\rho^2$

$$\begin{cases} \rho^2=9 \\ \rho^2=25 \end{cases} \qquad \begin{cases} \rho=3 \\ \rho=5 \end{cases}$$

$$D: \begin{cases} \frac{\pi}{2}\le\theta\le\pi \\ 3\le\rho\le 5 \end{cases}$$

$$m=\iint_D \mu\,dx\,dy=\int_{\frac{\pi}{2}}^{\pi}d\theta\int_3^5\frac{2\rho\sin\theta-\rho\cos\theta}{\rho^2}\rho\,d\rho=$$

$$=\int_{\frac{\pi}{2}}^{\pi}d\theta\int_3^5(2\sin\theta-\cos\theta)\,d\rho=\int_{\frac{\pi}{2}}^{\pi}(2\sin\theta-\cos\theta)\,d\theta\int_3^5 d\rho=$$

$$=2\int_{\frac{\pi}{2}}^{\pi}(2\sin\theta-\cos\theta)\,d\theta=2(-2\cos\theta-\sin\theta)\Big|_{\frac{\pi}{2}}^{\pi}=$$

$$=-2\left(2\cos\pi+\sin\pi-2\cos\frac{\pi}{2}-\sin\frac{\pi}{2}\right)=-2(-2-1)=6$$

Ответ: 6

T. P. 9.12

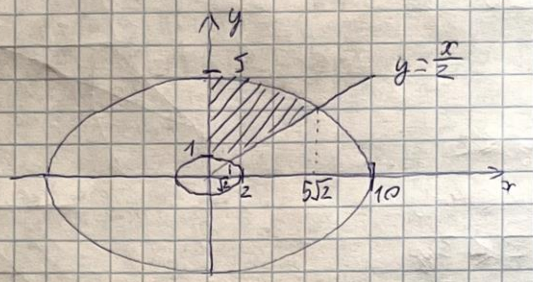
9.12

D: $1 \le \frac{x^2}{4} + y^2 \le 25 \quad x \ge 0 \quad y \ge \frac{x}{2} \quad M = \frac{x}{y^3}$

$\frac{x^2}{4} + y^2 = 1$

$\frac{x^2}{4} + y^2 = 25$

$\frac{x^2}{100} + \frac{y^2}{25} = 1$



D:

$0 \le x \le \sqrt{2} \qquad \sqrt{2} \le x \le 5\sqrt{2}$

$\frac{\sqrt{4-x^2}}{2} \le y \le \frac{\sqrt{100-x^2}}{2} \qquad \frac{x}{2} \le y \le \frac{\sqrt{100-x^2}}{2}$

$$m = \int_0^{\sqrt{2}} dx \int_{\frac{\sqrt{4-x^2}}{2}}^{2} \frac{x}{y^3}\,dy + \int_{\sqrt{2}}^{5\sqrt{2}} dx \int_{\frac{x}{2}}^{\sqrt{100-x^2}} \frac{x}{y^3}\,dy =$$

$$= \int_0^{\sqrt{2}} -\frac{x}{2y^2}\Bigg|_{\frac{\sqrt{4-x^2}}{2}}^{\frac{\sqrt{100-x}}{2}} dx + \int_{\sqrt{2}}^{5\sqrt{2}} -\frac{x}{2y^2}\Bigg|_{\frac{x}{2}}^{\frac{\sqrt{100-x^2}}{2}} dx =$$

$$= \int_0^{\sqrt{2}} \left(-\frac{x \cdot 2}{100-x^2} + \frac{2x}{4-x^2}\right) dx + \int_{\sqrt{2}}^{5\sqrt{2}} \left(-\frac{2x}{100-x^2} + \frac{2x}{x^2}\right) dx =$$

$$= \int_0^{\sqrt{2}} \frac{d(100-x^2)}{100-x^2} - \int_0^{\sqrt{2}} \frac{d(4-x^2)}{4-x^2} + \int_{\sqrt{2}}^{5\sqrt{2}} \frac{d(100-x^2)}{100-x^2} + \int_{\sqrt{2}}^{5\sqrt{2}} \frac{2\,dx}{x} =$$

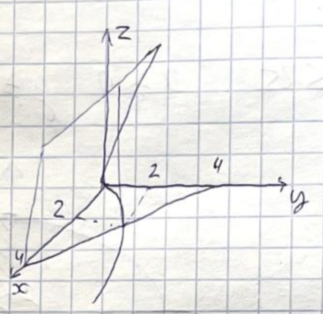
$$= \ln(100-x^2)\Big|_0^{5\sqrt{2}} - \ln(4-x^2)\Big|_0^{\sqrt{2}} + 2\ln x\Big|_{\sqrt{2}}^{5\sqrt{2}} = \ln 50 - \ln 100 - \ln 2 + \ln 4 + \ln 50 - \ln 2 =$$

$$= 2\ln 50 - \ln 100 = \ln 25 = 2\ln 5$$

Ответ: $2\ln 5$

10.12

$$\begin{cases} x + y = 4 \\ y = \sqrt{2x} \\ z = 3y, \; z = 0 \end{cases}$$



$$V: \begin{cases} 0 \leq y \leq 2 \\ \frac{y^2}{2} \leq x \leq 4 - y \\ 0 \leq z \leq 3y \end{cases}$$

$$V = \iiint\limits_V dx\,dy\,dz = \int_0^2 dy \int_{\frac{y^2}{2}}^{4-y} dx \int_0^{3y} dz =$$

$$= \int_0^2 dy \int_{\frac{y^2}{2}}^{4-y} 3y\,dx = 3\int_0^2 y\,dy \cdot x \Big|_{\frac{y^2}{2}}^{4-y} = 3\int_0^2 y\left(4 - y - \frac{y^2}{2}\right) dy =$$

$$= 3\int_0^2 \left(4y - y^2 - \frac{y^3}{2}\right) dy = 3\left(2y^2 - \frac{y^3}{3} - \frac{y^4}{8}\right)\Big|_0^2 = 3\left(8 - \frac{8}{3} - 2\right) = 10$$

Ответ: 10