Т. Р. 1.12

$$\int \cos 3x \sqrt[5]{4+\sin 3x}\, dx$$

$$\cos 3x\, dx = d(\frac{\sin 3x}{3})$$

$$\frac{\sin 3x}{3} = t$$

$$\int (4+3t)^{\frac{1}{5}} dt = (4+3t)^{\frac{6}{5}} \cdot \frac{5}{6} \cdot \frac{1}{3} + C = (4+3t)^{\frac{6}{5}} \cdot \frac{5}{18} + C = (4+\sin 3x)^{\frac{6}{5}} \cdot \frac{5}{18} + C$$

$$\textit{Ответ}: (4+\sin 3x)^{\frac{6}{5}} \cdot \frac{5}{18} + C$$

Т. Р. 2.12

$$\int (3x+1)e^{2x} dx$$

$$\begin{cases} u = 3x+1 \\ u' = 3 \\ v' = e^{2x} \\ v = \frac{e^{2x}}{2} \end{cases}$$

$$\int (3x+1)e^{2x} dx = (3x+1)\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot 3\, dx = \frac{(3x+1)e^{2x}}{2} - \frac{3}{2}\int e^{2x} dx = \frac{(3x+1)e^{2x}}{2} - \frac{3e^{2x}}{4} + C$$

$$\textit{Ответ}: \frac{(3x+1)e^{2x}}{2} - \frac{3e^{2x}}{4} + C$$

Т. Р. 3.12

$$\int \frac{10-7x}{\sqrt{2x^2-6x+5}}dx$$

$$t=\sqrt{2x^2-6x+5}$$

$$t^2=2x^2-6x+5$$

$$t^2=(2x+\frac{3}{2})^2+\frac{11}{4}$$

$$\sqrt{t^2-\frac{11}{4}}=2x+\frac{3}{2}$$

$$x=\frac{\sqrt{t^2-\frac{11}{4}}}{2}-\frac{3}{4}$$

$$dx=\frac{2t}{4\sqrt{t^2-\frac{11}{4}}}dt=\frac{t}{2\sqrt{t^2-\frac{11}{4}}}dt$$

$$\int \frac{10-7x}{\sqrt{2x^2-6x+5}}dx=\int \frac{10}{\sqrt{2x^2-6x+5}}dx-\int \frac{7x}{\sqrt{2x^2-6x+5}}dx=$$

$$=10\int \frac{t}{t\cdot 2\sqrt{t^2-\frac{11}{4}}}dt-7\int \frac{(\frac{\sqrt{t^2-\frac{11}{4}}}{2}-\frac{3}{4})t}{t\cdot 2\sqrt{t^2-\frac{11}{4}}}dt=$$

$$=5\int \frac{dt}{\sqrt{t^2-\frac{11}{4}}}-\frac{7}{4}\int dt\ +\frac{21}{8}\int \frac{dt}{\sqrt{t^2-\frac{11}{4}}}=$$

$$=\frac{61}{8}\ln\left|t+\sqrt{t^2-\frac{11}{4}}\right|-\frac{7}{4}t+C=$$

$$=\frac{61}{8}\ln\left|\sqrt{2x^2-6x+5}+\sqrt{2x^2-6x+\frac{9}{4}}\right|-\frac{7}{4}\sqrt{2x^2-6x+5}+C$$

$$Ответ:\ \frac{61}{8}\ln\left|\sqrt{2x^2-6x+5}+\sqrt{2x^2-6x+\frac{9}{4}}\right|-\frac{7}{4}\sqrt{2x^2-6x+5}+C$$

Т. Р. 4.12

$$\int \frac{x^4-6x}{(x+1)(x+2)^3}dx$$

$$(x+1)(x+2)^3 = x^4+7x^3+18x^2+20x+8$$

$$\int \frac{x^4-6x}{(x+1)(x+2)^3}dx = \int dx + \int \frac{-7x^3-18x^2-26x-8}{(x+1)(x+2)^3}dx$$

$$\frac{-7x^3-18x^2-26x-8}{(x+1)(x+2)^3} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2} + \frac{D}{(x+2)^3} =$$

$$= \frac{A(x+2)^3 + B(x+1)(x+2)^2 + C(x+1)(x+2) + D(x+1)}{(x+1)(x+2)^3} =$$

$$= \frac{A(x^3+6x^2+12x+8) + B(x^3+5x^2+8x+4) + C(x^2+3x+2) + D(x+1)}{(x+1)(x+2)^3} =$$

$$= \frac{x^3(A+B) + x^2(6A+5B+C) + x(12A+8B+3C+D) + 8A+4B+2C+D}{(x+1)(x+2)^3}$$

$$\begin{cases} A+B=-7 \\ 6A+5B+C=-18 \\ 12A+8B+3C+D=-26 \\ 8A+4B+2C+D=-8 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & -7 \\ 6 & 5 & 1 & 0 & -18 \\ 12 & 8 & 3 & 1 & -26 \\ 8 & 4 & 2 & 1 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 & -7 \\ 0 & -1 & 1 & 0 & 24 \\ 0 & -4 & 3 & 1 & 58 \\ 0 & -4 & 2 & 1 & 48 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 & -7 \\ 0 & -1 & 1 & 0 & 24 \\ 0 & 0 & -1 & 1 & -38 \\ 0 & 0 & -2 & 1 & -48 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 & -7 \\ 0 & -1 & 1 & 0 & 24 \\ 0 & 0 & -1 & 1 & -38 \\ 0 & 0 & 0 & -1 & 28 \end{pmatrix}$$

$$\begin{cases} A+B=-7 \\ -B+C=24 \\ -C+D=-38 \\ -D=28 \end{cases}$$

$$\begin{cases} -D=28 \\ C=10 \\ B=-14 \\ A=7 \end{cases}$$

$$x + C + \int \frac{7}{x+1}dx + \int \frac{-14}{x+2}dx + \int \frac{10}{(x+2)^2}dx + \int \frac{-28}{(x+2)^3}dx =$$

$$= x + 7\ln|x+1| - 14\ln|x+2| - \frac{10}{x+2} + \frac{14}{(x+2)^2} + C$$

$$\textit{Ответ}: x + 7\ln|x+1| - 14\ln|x+2| - \frac{10}{x+2} + \frac{14}{(x+2)^2} + C$$

T. P. 5.12

$$\int \frac{dx}{(x-1)\sqrt{3x^2-2x-5}}$$

$$t=\frac{1}{x-1}$$

$$x=\frac{1}{t}$$

$$dx=-\frac{1}{t^2}dt$$

$$\int \frac{dx}{(x-1)\sqrt{3x^2-2x-5}}=-\int \frac{dt}{t^2(\frac{t+1}{t}-1)\sqrt{\frac{3(t+1)^2}{t^2}-\frac{2(t+1)}{t}-5}}=-\int \frac{dt}{t\sqrt{\frac{3t^2+6t+3}{t^2}-\frac{2t+2}{t}-5}}=$$

$$=-\int \frac{dt}{\sqrt{3t^2+6t+3-2t^2+2t-5t^2}}=-\int \frac{dt}{\sqrt{-4t^2+8t+3}}=-\int \frac{dt}{\sqrt{(-2t+3)(2t+1)}}$$

$$v=\sqrt{\frac{3-2t}{2t+1}}$$

$$v^2=\frac{3-2t}{2t+1}$$

$$v^2(2t+1)=3-2t$$

$$2tv^2+2t=3-v^2$$

$$t=\frac{3-v^2}{2v^2+2}=\frac{-v^2-1+4}{2v^2+2}=\frac{2}{v^2+1}-\frac{1}{2}$$

$$dt=\frac{4v}{(v^2+1)}dv$$

$$-\int \frac{dt}{\sqrt{(-2t+3)(2t+1)}}=\int \frac{4vdv}{(v^2+1)^2\cdot v(\frac{4}{v^2+1}-1+1)}=\int \frac{dv}{(v^2+1)^2\cdot\frac{1}{v^2+1}}=\int \frac{dv}{(v^2+1)}=$$

$$=arctg\, v\ +C=arctg\sqrt{\frac{3-t}{2t+1}}+C=arctg\sqrt{\frac{3x-3-1}{2+x-1}}+C=arctg\sqrt{\frac{3x-4}{x+1}}+C$$

T. P. 6.12

6.12

$$\int \frac{(5-2x)}{4+\sqrt{3-5x}}\,dx = -\int \frac{5-\frac{6-2t}{5}}{20+5\sqrt{t}}\,dt =$$

$$t = 3-5x \qquad x = \frac{3-t}{5} \qquad dx = -\frac{1}{5}dt$$

$$= -\int \frac{\left(5-\frac{6-2s^2}{5}\right)2s}{20+5s}\,ds =$$

$$s = \sqrt{t} \qquad t = s^2 \qquad dt = 2s\,ds$$

$$= -\frac{1}{5}\int \frac{2s\left(\frac{2(s^2-3)}{5}+5\right)}{s+4}\,ds =$$

$$v = s+4 \qquad s = v-4 \qquad ds = dv$$

$$= -\frac{2}{5}\int \frac{\left(\frac{2((v-4)^2-3}{5}+5\right)(v-4)}{v}\,dv =$$

$$= -\frac{2}{5}\int \left(\frac{2v^2-16v+32-6}{5}+5\right)\left(1-\frac{4}{v}\right)dv = -\frac{2}{5}\int\left(\frac{2v^2}{5}-\frac{16v}{5}+\frac{51}{5}\right)\left(1-\frac{4}{v}\right)dv =$$

$$= -\frac{2}{25}\int\left(2v^2-16v+51-8v+64-\frac{204}{v}\right)dv = -\frac{2}{25}\int\left(2v^2-24v+115-\frac{204}{v}\right)dv =$$

$$= -\frac{4}{25}\int v^2\,dv + \frac{48}{25}\int v\,dv - \frac{46}{5}\int dv + \frac{408}{25}\int\frac{1}{v}\,dv =$$

$$= -\frac{4v^3}{75} + \frac{24v^2}{25} - \frac{46v}{5} + \frac{408}{25}\ln v + C = -\frac{4\left(\sqrt{3-5x}+4\right)^3}{75} + \frac{24\left(\sqrt{3-5x}+4\right)^2}{25} -$$

$$- \frac{46\left(\sqrt{3-5x}+4\right)}{5} + \frac{408}{5}\ln\left(\sqrt{3-5x}+4\right) + C$$

Ответ: $-\frac{4\left(\left(\sqrt{3-5x}\right)+4\right)^3}{75} + \frac{24\left(\sqrt{3-5x}+4\right)^2}{25} - \frac{46\left(\sqrt{3-5x}+4\right)}{5} + \frac{408}{5}\ln\left(\sqrt{3-5x}+4\right) + C$

T. P. 7.12

7.12

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{(1+x^5)^5}}\,dx =$$

$$t = \sqrt[4]{x} \qquad x = t^4 \qquad dx = 4t^3\,dt$$

$$= \int \frac{4t^4}{\sqrt[4]{(t^{20}+1)^5}}\,dt = 4\int -\frac{\frac{1}{\sqrt[20]{(v^4+1)^4}}\cdot v^3}{\sqrt[4]{\left(\frac{1}{v-1}+1\right)^5}\cdot 5\sqrt[20]{(v^4-1)^{21}}}\,dv =$$

$$v = \frac{\sqrt[4]{t^{20}+1}}{t^5} \qquad t = \frac{1}{\sqrt[20]{v^4-1}} \qquad dt = -\frac{v^3}{5(v^4-1)^{\frac{21}{20}}}$$

$$= -4\int \frac{v^3\,dv}{\sqrt[4]{\left(\frac{v^4}{v^4-1}\right)^5}\cdot 5\sqrt[20]{(v^4-1)^{25}}} =$$

$$= -4\int \frac{v^3\,dv}{\frac{v^5}{(v^4-1)^{\frac{5}{4}}}\cdot 5\sqrt[4]{(v^4-1)^5}} = -4\int\frac{dv}{5v^2} =$$

$$= -\frac{4}{5}\cdot\left(-\frac{1}{v}\right) + C = \frac{4}{5v} + C = \frac{4t^5}{5\sqrt[4]{t^{20}+1}} + C =$$

$$= \frac{4\sqrt[4]{x^5}}{5\sqrt[4]{x^5+1}} + C$$

Ответ: $\frac{4\sqrt[4]{x^5}}{5\sqrt[4]{x^5+1}} + C$

T. P. 8.12

8.12

$$\int\sqrt{-2x^2+5x+4}\,dx = \int\sqrt{-\left(\sqrt{2}x-\frac{5}{2\sqrt{2}}\right)^2+\frac{7}{8}}\,dx =$$

$t=\sqrt{2}x-\frac{5}{2\sqrt{2}}$

$x=\dfrac{t+\frac{5}{2\sqrt{2}}}{\sqrt{2}}$

$dx=\frac{1}{\sqrt{2}}dt$

$$=\frac{1}{\sqrt{2}}\int\sqrt{\left(\frac{\sqrt{7}}{2\sqrt{2}}\right)^2-t^2}\,dt =$$

$$=\frac{1}{\sqrt{2}}\left(\frac{t}{2}\sqrt{\frac{7}{8}-t^2}+\frac{7}{16}\arcsin\frac{t\cdot 2\sqrt{2}}{\sqrt{7}}\right)+C=$$

$$=\left(\frac{\sqrt{2}}{\sqrt{2}\,2}x-5\right)\sqrt{\frac{7}{8}-\left(\sqrt{2}x-\frac{5}{2\sqrt{2}}\right)^2}+\frac{7}{16}\arcsin\frac{4x-5}{\sqrt{7}}+C$$

Ответ: $\left(\frac{x}{2}-5\right)\sqrt{\frac{7}{8}-\left(\sqrt{2}x-\frac{5}{2\sqrt{2}}\right)^2}+\frac{7}{16}\arcsin\frac{4x-5}{\sqrt{7}}+C$

T. P. 9.12

9.12

$$\int\sin^4 3x\,dx=\frac{1}{3}\int\sin^4 t\,dt=$$

$t=3x$

$x=\frac{t}{3}$

$dx=\frac{1}{3}dt$

$$=\frac{1}{3}\int\frac{(1-\cos 2t)^2}{4}dt=\frac{1}{12}\int\frac{(1-\cos v)^2}{2}dv=$$

$v=2t$

$t=\frac{v}{2}$

$dt=\frac{1}{2}dv$

$$=\frac{1}{24}\left(\int dv-\int 2\cos v\,dv+\int\cos^2 v\,dv\right)=$$

$$=\frac{1}{24}v-\frac{\sin v}{12}+C+\frac{1}{24}\int\frac{\cos 2v+1}{2}dv=$$

$$=\frac{1}{24}v-\frac{\sin v}{12}+C+\frac{\sin 2v}{96}+\frac{1}{48}v=$$

$$=\frac{3}{8}x-\frac{\sin 6x}{12}+\frac{\sin 12x}{96}+C$$

Ответ: $\frac{3}{8}x-\frac{\sin 6x}{12}+\frac{\sin 12x}{96}+C$

T. P 10.12

10.12

$$\int \frac{dx}{2\sin^2 x - 11\cos^2 x + 2} = \int \frac{dx}{-13\cos^2 x + 4} =$$

$$t = tg\,x$$
$$dt = \frac{1}{\cos^2 x}dx$$
$$\cos^2 x = \frac{1}{t^2+1}$$

$$= \int \frac{\frac{1}{t^2+1}\cdot(t^2+1)\,dt}{4t^2-9} = \int \frac{dt}{4t^2-9} =$$

$$v = 4t$$
$$t = \frac{v}{4}$$
$$dv = 4dt$$
$$dt = \frac{1}{4}dv$$

$$= \int \frac{4}{(4t-6)(4t+6)}dt = \int \frac{1}{(v-6)(v+6)}dv =$$

$$= \int \frac{\frac{1}{12}((v+6)-(v-6))}{(v-6)(v+6)}dv = \int \frac{\frac{1}{v-6} - \frac{1}{v+6}}{12}dv =$$

$$= \frac{1}{12}\ln|v-6| - \frac{1}{12}\ln|v+6| = \frac{1}{12}\ln|4tg\,x-6| - \frac{1}{12}\ln|4tg\,x+6| + C$$

Ответ: $\frac{1}{12}\ln|4tg\,x-6| - \frac{1}{12}\ln|4tg\,x+6|$

T. P. 11.12

11.12

$$\int_0^2 \frac{x\,dx}{x^2+3x+2} = \int_0^2 \frac{x\,dx}{(x+1)(x+2)}$$

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+2} + \frac{B}{x+1} = \frac{(A+B)x + 2B + A}{(x+1)(x+2)}$$

$$\begin{cases} A+B = 1 \\ 2B+A = 0 \end{cases} \qquad \begin{cases} A = 2 \\ B = -1 \end{cases}$$

$$\int_0^2 \frac{x\,dx}{(x+1)(x+2)} = \int_0^2 \frac{2dx}{x+2} + \int_0^2 \frac{-dx}{x+1} = 2\ln|x+2|\Big|_0^2 - \ln|x+1|\Big|_0^2 =$$

$$= 2\ln 4 - 2\ln 2 - \ln 3 + 0 = 2\ln 2 - \ln 3$$

Ответ: $2\ln 2 - \ln 3$

T. P. 12.12

12.12

$$\int_0^1 \ln(x^2+1)\,dx =$$

$$\begin{cases} u = \ln(x^2+1) \\ u' = \frac{2x}{x^2+1} \\ v' = 1 \\ v = x \end{cases}$$

$$x\ln(x^2+1)\Big|_0^1 - \int_0^1 \frac{2x^2}{x^2+1}\,dx =$$

$$= x\ln(x^2+1)\Big|_0^1 - 2\int_0^1 \left(1 - \frac{1}{x^2+1}\right)dx =$$

$$= x\ln(x^2+1)\Big|_0^1 - 2x\Big|_0^1 + 2\,\mathrm{arctg}\,x\Big|_0^1 = 2\ln(2) - 2 + \frac{\pi}{2}$$

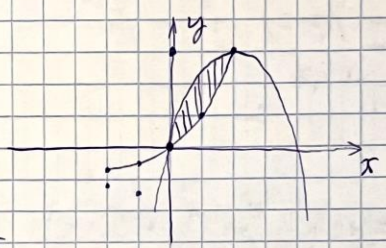
Ответ: $2\ln 2 - 2 + \frac{\pi}{2}$

T. P. 13.12

13.12

$$y = 2^x - 1$$

$$y = \frac{3}{4}x(4-x)$$



$$\iint_G dx\,dy = \int_0^2 dx \int_{2^x-1}^{3x-\frac{3x^2}{4}} dy =$$

$$= \int_0^2 dx\left(3x - \frac{3x^2}{4} - 2^x + 1\right) = \frac{3x^2}{2}\Big|_0^2 - \frac{x^3}{4}\Big|_0^2 - \frac{2^x}{\ln 2}\Big|_0^2 + x\Big|_0^2 =$$

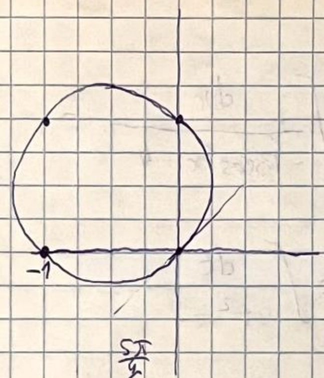
$$= 6 - 2 - \frac{4}{\ln 2} + \frac{1}{\ln 2} + 2 = 6 - \frac{3}{\ln 2}$$

Ответ: $6 - \frac{3}{\ln 2}$

T. P. 14.12

14.12

$\rho = \sin\varphi - \cos\varphi$



$$S = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin\varphi - \cos\varphi)^2\, d\varphi = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 - 2\sin 2\varphi)\, d\varphi =$$

$$= d\varphi \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + \frac{\cos 2x}{2}\cdot 2 \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = \pi + 0 - 0$$

Ответ: π

T. P. 15.12

15.12

$\rho = 1 + \cos\varphi \qquad 0 \le \varphi \le \frac{\pi}{3}$

$$S = \int_0^{\frac{\pi}{3}} \sqrt{(1+\cos\varphi)^2 + \sin^2\varphi}\, d\varphi = \int_0^{\frac{\pi}{3}} \sqrt{1 + 2\cos\varphi + 1}\, d\varphi =$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{2 + 2\cos\varphi}\, d\varphi = \int_0^{\frac{\sqrt{3}}{3}} \sqrt{2 + \frac{2-2t^2}{t^2+1}} \cdot 2 \cdot \frac{1}{t^2+1}\, dt =$$

$t = tg\left(\frac{\varphi}{2}\right)$

$dt = \frac{1}{2\cos^2\left(\frac{\varphi}{2}\right)}\, d\varphi$

$\cos\varphi = \frac{1-t^2}{t^2+1}$

$\cos^2\left(\frac{\varphi}{2}\right) = \frac{1}{t^2+1}$

$$= \int_0^{\frac{\sqrt{3}}{3}} \sqrt{\frac{2t^2+2+2-2t^2}{(t^2+1)^3}}\, dt = \int_0^{\frac{\sqrt{3}}{3}} \frac{4}{(t^2+1)^{\frac{3}{2}}}\, dt =$$

$$= \int_0^{\frac{\pi}{6}} \frac{4}{(tg^2 v + 1)^{\frac{3}{2}} \cos^2 v}\, dv = 4\int_0^{\frac{\pi}{6}} \frac{1}{\left(\frac{1}{\cos^2 v}\right)^{\frac{3}{2}} \cos^2 v}\, dv =$$

$v = arctg\, t$

$\frac{1}{\cos^2 v} = t^2 + 1$

$t = tg\, v$

$dt = \frac{1}{\cos^2 v}\, dv$

$$= 4\int_0^{\frac{\pi}{6}} \cos v\, dv = 4\sin v \Big|_0^{\frac{\pi}{6}} = 2$$

Ответ: 2