11.25 Хохлов Сергей А-07-23

$$\int_0^{\ln 2} \frac{dx}{e^x(3+e^{-x})} = \int_0^{\ln 2} \frac{e^{-x}\,dx}{e^{-x}+3} = \left| \begin{matrix} u=-x & u_1=0 \\ x=-u & u_2=-\ln 2 \\ dx=-du & \end{matrix} \right| =$$

$$= -\int_0^{-\ln 2} \frac{e^u\,du}{e^u+3} = \left| \begin{matrix} v=e^u & v_1=1 \\ dv=e^u du & v_2=\frac{1}{2} \end{matrix} \right| = -\int_1^{\frac{1}{2}} \frac{1}{v+3}\,dv =$$

$$= -\ln(v+3) \Big|_1^{\frac{1}{2}} = -\ln(3{,}5) + \ln(4) = \ln\left(\frac{8}{7}\right)$$

12.25

$$\int_0^1 (x+1)\ln^2(x+1)\,dx = \left| \begin{matrix} x+1=u & u_1=1 \\ dx=du & u_2=2 \end{matrix} \right| =$$

$$= \int_1^2 u\ln^2 u\,du = \left| \begin{matrix} v=\ln u & v_1=\ln 1 \\ dv=\frac{1}{u}du & v_2=\ln 2 \\ du=u\,dv & \end{matrix} \right| =$$

$$= \int_{\ln 1}^{\ln 2} v^2 e^{2v}\,dv =$$

(Начнем брать по частям, сверху допишу результат)

$$\int v^2 e^{2v}\,dv = v^2\cdot\frac{1}{2}e^{2v} - \int 2v\,\frac{1}{2}e^{2v}\,dv =$$

$$\left| \begin{matrix} z=v^2 & y'=e^{2v} \\ z'=2v & y=\frac{1}{2}e^{2v} \end{matrix} \right|$$

$$\Downarrow^{*} \quad = v^2\cdot\frac{1}{2}e^{2v} - \left(\frac{1}{2}ve^{2v} - \frac{1}{4}e^{2v}\right) =$$

(2 раз по частям, допишу результат) (след стр)

$$* \int v e^{2v}\,dv = v\cdot\frac{1}{2}e^{2v} - \frac{1}{2}\int e^{2v} = \frac{1}{2}ve^{2v} - \frac{1}{4}e^{2v} + C$$

$$\begin{matrix} z=v & y=\frac{1}{2}e^{2v} \\ z'=1 & y'=e^{2v} \end{matrix}$$

14.25 продолжение   Хохлов Сергей А-07-23

$= \frac{1}{6}\int (\cos u + 1)^2 du = \frac{1}{6}\int \cos^2 u + 2\cos u + 1\, du =$

$= \frac{1}{6}\left(\int \cos^2 u\, du + 2\int \cos u\, du + \int 1\, du\right) =$

$= \frac{\sin 2u}{24} + \frac{\sin u}{3} + \frac{u}{4}\Big|_0^{\pi} = \frac{\sin 2\pi}{24} + \frac{\sin \pi}{3} + \frac{\pi}{4} =$

$= \frac{\pi}{4}$

№15.25

$y = \frac{1}{2}\ln\cos 2x \quad 0 \le x \le \frac{\pi}{12}$

$L = \int_a^b \sqrt{1 + y'^2}\, dx$

$y' = \frac{1}{2}[\ln(\cos 2x)]' = \frac{1}{2} \cdot \frac{-2\sin 2x}{\cos 2x} = -\operatorname{tg} 2x$

$y'^2 = (\operatorname{tg} 2x)^2$

$L = \int_0^{\frac{\pi}{12}} \sqrt{1 + (\operatorname{tg} 2x)^2}\, dx = \left| \begin{array}{l} u_1 = 0 \quad u_2 = \frac{\pi}{6} \\ u = 2x \\ x = \frac{u}{2} \\ dx = \frac{1}{2} du \end{array} \right| =$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \sqrt{\operatorname{tg}^2(u) + 1}\, du = \frac{1}{2}\int_0^{\frac{\pi}{6}} \sqrt{\frac{\sin^2 u}{\cos^2 u} + 1}\, du =$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \sqrt{\frac{1 - \cos^2 u}{\cos^2 u} + 1}\, du = \left| \begin{array}{ll} v = \operatorname{tg} u & \cos^2 u = \frac{1}{v^2 + 1} \\ dv = \frac{1}{\cos^2 u} du & v_1 = 0 \\ & v_2 = \frac{\sqrt{3}}{3} \end{array} \right| =$

Хохлов Сергей

$$= \frac{1}{4}\int_0^{\frac{\sqrt{3}}{3}} \frac{1}{\sqrt{v^2+1}}\,dv = \left.\frac{\ln\left(\sqrt{v^2+1}+v\right)}{4}\right|_0^{\frac{\sqrt{3}}{3}} =$$

$$= \frac{\ln\left(\sqrt{\left(\frac{\sqrt{3}}{3}\right)^2+1}+\frac{\sqrt{3}}{3}\right)}{4} =$$

$$= \frac{\ln\left(\sqrt{\frac{12}{9}}+\frac{\sqrt{3}}{3}\right)}{4} = \frac{\ln\left(\sqrt{\frac{4}{3}}+\frac{\sqrt{3}}{3}\right)}{4} = \frac{\ln\sqrt{3}}{4} = \frac{\ln 3}{8}$$

№16.25

$$\int_0^{+\infty} \frac{4-x^2}{x^2+9} = \lim_{b\to\infty} \left.\frac{13\operatorname{arctg}\left(\frac{x}{3}\right)}{3} - x\right|_0^b = -\infty \Rightarrow$$ интеграл расходится

$$\int \frac{4-x^2}{x^2+9}\,dx = -\int \frac{x^2-4}{x^2+9}\,dx = -\left(\int 1\,dx - 13\int \frac{1}{x^2+9}\,dx\right) =$$

$$= \frac{13\operatorname{arctg}\left(\frac{x}{3}\right)}{3} - x + C$$

результат интегрирования $\int\limits_{\ln 1}^{\ln 2} v^2 e^{2v} dv =$

Хохлов Сергей
А-07-23

$$= \frac{v^2 e^{2v}(v-1)}{2} + \frac{e^{2x}}{4} \Bigg|_{\ln 1}^{\ln 2} =$$

$$= \underbrace{\left(\frac{\ln 2\, e^{2\ln 2}((\ln 2)-1)}{2} + \frac{e^{2\ln 2}}{4}\right)}_{2\ln(2)^2 - 2\ln(2)+1} - \underbrace{\left(\frac{\ln 1\, e^{2\ln 1}((\ln 1)-1)}{2}\right)}_{0} \underbrace{- \frac{e^{2\ln 1}}{4}}_{-\frac{1}{4}} =$$

$$= 2\ln(2)^2 - 2\ln(2) + 1 - \frac{1}{4} = \boxed{2 \cdot \ln(2)^2 - 2\ln 2 + \frac{3}{4}}$$

№13.25

$y = x^3$

$y = 1$

$x = 0$

$$S = \int_0^1 x^3 - 1\, dx = \frac{x^4}{4} - x \Bigg|_0^1 = \frac{1}{4} - 1 = -\frac{3}{4}$$

№14.25

$\rho = 1 + \cos 3\varphi$ $\quad$ $1 + \cos 3\varphi = 0$

$\varphi = \frac{\pi}{3}$

$$S = \frac{1}{2}\int_0^{\frac{\pi}{3}} (1+\cos 3\varphi)^2 d\varphi = \left| \begin{matrix} u = 3\varphi & d\varphi = \frac{1}{3}du \\ \varphi = \frac{u}{3} & u_1 = 0 \\ & u_2 = \pi \end{matrix} \right| =$$

= продолжение след лист