Красешкова Д. Д
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11 Вариант

<u>11.11</u>

$$\int_0^{\pi/2} \cos^3 x \sqrt{1+\sin x}\, dx = \left\{ \begin{array}{ll} t = \sin x & x = \arcsin t \\ dt = \cos dx & \cos^2 x = 1 - \sin^2 x \\ dx = \frac{dt}{\cos x} & x = \frac{\pi}{2} \Rightarrow t = 1 \\ & x = 0 \Rightarrow t = 0 \end{array} \right\} =$$

$$= \int_0^1 (1-t^2)\sqrt{1+t}\, dt = \int_0^1 (\sqrt{t+1} - t^2\sqrt{1+t}\, dt) =$$

$$= \int_0^1 \sqrt{t+1}\, dt - \int_0^1 t^2\sqrt{1+t}\, dt = \Big|_0^1 \ominus$$

$$\int_0^1 \sqrt{t+1}\, dt = \frac{2}{3}(1+t)^{3/2}\Big|_0^1$$

$$\int_0^1 t^2\sqrt{1+t}\, dt = \left\{ \begin{array}{l} 1+t = u \\ du = dt \\ t^2 = (u-1)^2 \end{array} \right\} =$$

$$= \int_1^2 (u-1)^2 \sqrt{u}\, du = \int_1^2 (u^2\sqrt{u} - 2u\sqrt{u} + \sqrt{u})\, du =$$

$$= \int_1^2 u^2 \sqrt{u}\, du - 2\int_1^2 u\sqrt{u}\, du + \int_1^2 \sqrt{u}\, du =$$

$$= \int_1^2 u^{5/2} du - 2\int_1^2 u^{3/2} du + \int_1^2 u^{1/2} du =$$

$$= \frac{2}{7} u^{7/2}\Big|_1^2 - 2\cdot\frac{2}{5}\cdot u^{5/2}\Big|_1^2 + \frac{2}{3}u^{3/2}\Big|_1^2 =$$

$$= \frac{2}{7}\cdot 8\sqrt{2} - \frac{2}{7} - \frac{4}{5}\cdot 4\sqrt{2} + \frac{4}{5} + \frac{2}{3}\cdot 2\sqrt{2} - \frac{2}{3} =$$

$= \frac{44\sqrt{2}}{105} - \frac{16}{105}$

$\textcircled{=} \frac{2}{3} \cdot 2\sqrt{2} - \frac{2}{3} - \frac{44\sqrt{2}}{105} + \frac{16}{105} = \frac{32\sqrt{2}}{35} -$

$- \frac{18}{35} = \frac{32\sqrt{2} - 18}{35}$

12.11

$\int_0^{\pi/4} (2x-1)\cos 2x\, dx = \left\{ \begin{array}{l} t = 2x \\ dt = 2dx \end{array} \right\} =$

$= \frac{1}{2} \int_0^{\pi/2} (t-1)\cos t\, dt = \left\{ \begin{array}{l} u = t-1 \\ du = dt \\ dv = \cos t\, dt \\ v = \sin t \end{array} \right\} =$

$= \frac{1}{2}(t-1)\sin t \Big|_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2} \sin t\, dt =$

$= \frac{(t-1)\sin t}{2}\Big|_0^{\pi/2} + \frac{\cos t}{2}\Big|_0^{\pi/2} = \frac{t \sin t}{2}\Big|_0^{\pi/2} -$

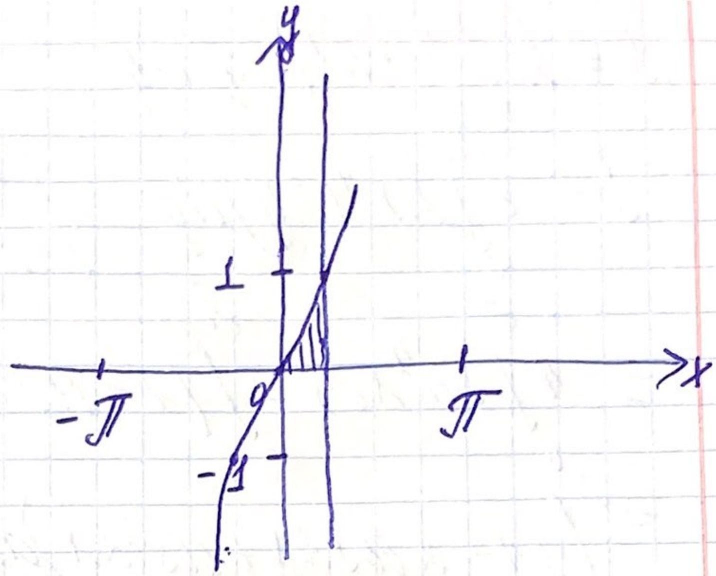
$- \frac{\sin t}{2}\Big|_0^{\pi/2} + \frac{\cos t}{2}\Big|_0^{\pi/2} = \frac{\pi}{4} - \frac{1}{2} - \frac{1}{2} =$

$= \frac{\pi}{4} - 1$

13.11 $y = tg\,x$

$y = 0$

$x = \frac{\pi}{4}$

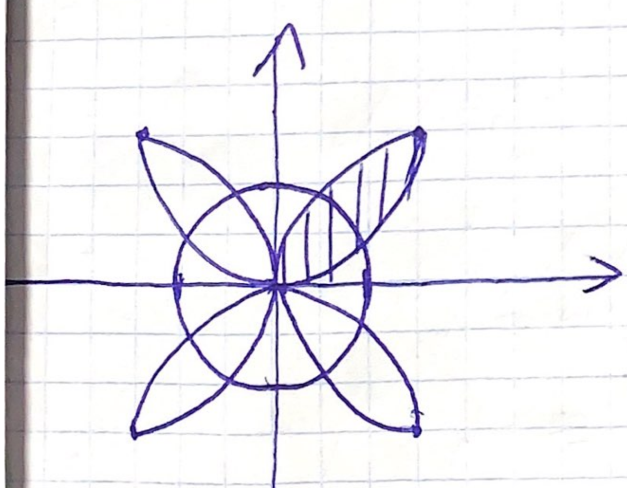


$$S = \int_0^{\pi/4} tg\,x\,dx = \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx = \left\{ \begin{matrix} t = \cos x \\ dt = -\sin x\,dx \end{matrix} \right\} =$$

$$= -\int_0^{\sqrt{2}/2} \frac{dt}{t} = -\ln|t| \Big|_0^{\frac{\sqrt{2}}{2}} = \frac{\ln 2}{2}$$

14.11

$\rho = 2\sin^2(2\varphi)$



$\varphi = 0 \Rightarrow \rho = 0$

$\varphi = \frac{\pi}{4} \Rightarrow \rho = 2$

$\varphi = \frac{\pi}{2} \Rightarrow \rho = 0$

$\varphi = \frac{3\pi}{4} \Rightarrow \rho = 2$

$\varphi = \pi \Rightarrow \rho = 0$

$\varphi = -\frac{\pi}{4} \Rightarrow \rho = 2$

$\varphi = -\frac{\pi}{2} \Rightarrow \rho = 0$

$\varphi = -\frac{3\pi}{4} \Rightarrow \rho = 2$

$$S = \frac{1}{2}\int_0^{\pi/2} 4\sin^4(2\varphi)\,d\varphi$$

$$S = 8\int_0^{\pi/2} \sin^4(2\varphi)\,d\varphi = \left\{\begin{matrix} x = 2\varphi \\ dx = 2d\varphi \end{matrix}\right\} =$$

$$= 4\int_0^{\pi} \sin^4 x\,dx = \int_0^{\pi}(1-\cos 2x)^2\,dx =$$

$$= \int_0^{\pi}(1 - 2\cos 2x + \cos^2 2x)\,dx =$$

$$= \int_0^{\pi} dx - 2\int_0^{\pi}\cos 2x\,dx + \int_0^{\pi}\cos^2 2x\,dx =$$

$$= x\Big|_0^{\pi} - \sin(2x)\Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi}(1+\cos 4x)\,dx =$$

$$= x\Big|_0^{\pi} - \sin 2x\Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi} dx + \frac{1}{2}\int_0^{\pi}\cos 4x\,dx =$$

$$= x\Big|_0^{\pi} - \sin 2x\Big|_0^{\pi} + \frac{x}{2}\Big|_0^{\pi} + \frac{\sin 4x}{8}\Big|_0^{\pi} =$$

$$= \pi - 0 + \frac{\pi}{2} + 0 = \frac{3\pi}{2}$$

15.11

$$\begin{cases} x = \sqrt{3}\,t^2 \\ y = t - t^3 \end{cases} \qquad 0 \le t \le 1$$

$$l = \int_0^1 \sqrt{x'(t)^2 + y'(t)^2}\,dt = \int_0^1 \sqrt{12t^2 + (1-3t^2)^2} =$$

15.11

$$= \int_0^1 \sqrt{12t^2 + 1 - 6t^2 - 9t^4}\, dt \ominus$$

$$x'(t) = (\sqrt{3}t^2)' = 2\sqrt{3}t$$

$$y'(t) = (t - t^3) = 1 - 3t^2$$

$$\ominus \int_0^1 \sqrt{1 + 6t^2 + 9t^4}\, dt = \int_0^1 (1 + 3t^2)\, dt =$$

$$= \int_0^1 dt + 3\int_0^1 t^2 dt = t\Big|_0^1 + t^3\Big|_0^1 = 1 + 1 = 2$$