№9.14

$\vec{a} = xy^2\vec{i} + x^2y\vec{j} + z\vec{k}$ $\quad S: \begin{cases} x^2+y^2=1 \\ z=0, z=1 \\ x=0, y=0 \end{cases}$

$a_x = xy^2;\ a_y = x^2y;\ a_z = z$

$\operatorname{div}\vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = y^2 + x^2 + 1$

$\Pi = \iint\limits_S (\vec{a}, \vec{n})\, dS = \iiint\limits_V \operatorname{div}\vec{a}\, dV = \iiint\limits_V (x^2+y^2+1)\,dx\,dy\,dz =$

$= \int\limits_0^{\pi/2} d\varphi \int\limits_0^1 \rho\, d\rho \int\limits^1 (\rho^2+1)\, dz = \int\limits_0^{\pi/2} d\varphi \int\limits_0^1 (\rho^3+\rho)\, d\rho = \frac{\pi}{2}\left(\frac{\rho^4}{4} + \frac{\rho^2}{2}\right)\Big|_0^1 =$

$= \frac{\pi}{2}\left(\frac{1}{4}+\frac{1}{2}\right)^0 = \frac{3\pi}{8}$

Ответ: $\frac{3\pi}{8}$

№10.14

$\vec{F} = y\vec{i} - x\vec{j};\ L: 2x^2 + y^2 = 1;\ (y \geq 0)$

$M\left(\frac{1}{\sqrt{2}}; 0\right);\ N\left(-\frac{1}{\sqrt{2}}; 0\right)$

$x = \frac{\cos t}{\sqrt{2}};\ \}\ dx = -\frac{\sin t}{\sqrt{2}}dt$

$dy = \cos t\, dt$

$A = \int\limits_L (\vec{F}, d\vec{s}) = \int\limits_0^{\pi} y\,dx - x\,dy = \int\limits_0^{\pi} \left(\sin t\left(-\frac{\sin t}{\sqrt{2}}\right) - \frac{\cos t}{\sqrt{2}}\cos t\right)dt =$

$= -\frac{1}{\sqrt{2}} \int\limits_0^{\pi} (\sin^2 t + \cos^2 t)\, dt = -\frac{1}{\sqrt{2}} \int\limits_0^{\pi} dt = -\frac{1}{\sqrt{2}}\, t \Big|_0^{\pi} = -\frac{\pi}{\sqrt{2}}$

Ответ: $-\frac{\pi}{\sqrt{2}}$

№7.14

$\vec{a} = (3x-2z)\vec{i} + (z-2y)\vec{j} + (1+2z)\vec{k}$, $S: z^2 = 4(x^2+y^2),\ z=2$

$$\iint_S (\vec{a}, d\vec{S}) = \iiint_V \operatorname{div}\vec{a}\, dV$$

$$\operatorname{div}\vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = 3-2+2=3$$

$$\Pi = \iint_S (\vec{a}, d\vec{S}) = 3\iiint_V dV = 3V,$$ где $V = \frac{1}{3}\pi R^2 H = \frac{2\pi}{3} \Rightarrow$

$$\Rightarrow \Pi = 3\cdot\frac{2\pi}{3} = 2\pi$$

Ответ: $\Pi = 2\pi$

№8.14

$\vec{a} = (z+y)\vec{i} + (x-z)\vec{j} + z\vec{k}$

$$\operatorname{div}\vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial a_y} + \frac{\partial a_z}{\partial z} = 0+0+1=1$$

$$\Pi = \iint_S (\vec{a}, d\vec{S}) = \iiint_V \operatorname{div}\vec{a}\, dV = \int_0^{2\pi} d\varphi \int_0^1 2\rho\, d\rho \int_1^{12-6\rho\cos\varphi-4\rho\sin\varphi} dz =$$

$$= \int_0^{2\pi} d\varphi \int_0^1 (2\rho(12-6\rho\cos\varphi - 4\rho\sin\varphi - 1))\, d\rho = \int_0^{2\pi} d\varphi \int (22\rho - 12\rho^2\cos\varphi - 8\rho^2\sin\varphi)\, d\rho = \int_0^{2\pi} d\varphi \left(11\rho^2 - 4\rho^3\cos\varphi - \frac{8}{3}\rho^3\sin\varphi\right)\Big|_0^1 =$$

$$= \int_0^{2\pi} \left(11 - 4\cos\varphi - \frac{8}{3}\sin\varphi\right) d\varphi = \left(11\varphi - 4\sin\varphi + \frac{8}{3}\cos\varphi\right)\Big|_0^{2\pi} =$$

$$= \left(22\pi - 0 + \frac{8}{3}(1-1)\right) = 22\pi$$

Ответ: 22π

N6.14

$\bar{a} = \pi x \vec{i} + \frac{\pi}{2} y \vec{j} + (4-2z)\vec{k}$

$\vec{N} = \{1; \frac{1}{3}; \frac{1}{4}\}; \ |\vec{N}| = \sqrt{1+\frac{1}{9}+\frac{1}{16}} = \frac{13}{12}; \ \vec{n} = \frac{\vec{N}}{|N|} = \{\frac{12}{13}; \frac{4}{13}; \frac{3}{13}\}$

$(\vec{a}, \vec{n}) = \frac{12\pi x + 2\pi y + 12 - 6z}{13} = \frac{12\pi x + 2\pi y + 12 - 24 + 24x + 8y}{13} =$

$= \frac{12(\pi+2)x + 2(\pi+4)y - 12}{13}$

$z'_x = -4; \ z'_y = -\frac{4}{3}; \ \sqrt{1+(z'_x)^2+(z'_y)^2} = \sqrt{1+16+\frac{16}{9}} = \frac{13}{3}$

$\Pi = \iint_{\sigma} (\vec{a}, \vec{n})\, d\sigma = \frac{1}{13}\iint_{D_{xy}} (12(\pi+2)x + 2(\pi+4)y - 12)\frac{13}{3}\, dx\, dy =$

$= \frac{1}{3}\int_0^1 dx \int_0^{3-3x} (12(\pi+2)x + 2(\pi+4)y - 12)\, dy = \frac{1}{3}\int_0^1 dx\, (12(\pi+2)x(3-3x) + (\pi+4)(3-3x)^2 - 12(3-3x)) = \frac{1}{3}\cdot\int_0^1 (12(3-3x)(\pi+1)x + (\pi+4)9\cdot(x-1)^2\, dx = \frac{1}{3}(9(\pi+4)\frac{(x-1)^3}{3} + 36(\pi+1)(\frac{x^2}{2} - \frac{x^3}{3}))\Big|_0^1 =$

$= \frac{1}{3}(3(\pi+4)(0+1) + 36(\pi+1)\frac{1}{6}) = \frac{1}{3}(3\pi+12+6\pi+6) =$

$= \frac{9\pi+18}{3} = 3\pi+6$

Ответ: $3\pi+6$

№5.14

$a = 2x\vec{i} + y\vec{j} + 4z\vec{k}$

$P - x/3 + y + z/2 = 1$

$div\,\bar{a} = -2 + 1 + 4 = 3$

$\Pi = \iiint div\,\vec{a}\,dx\,dy\,dz$

$V = \begin{cases} 0 \le y \le 1 \\ 0 \le x \le 3-3y \\ 0 \le z \le 2-2y - \frac{2x}{3} \end{cases}$

$\Pi = 3\int_0^1 dy \int_0^{3-3y} dx \int_0^{2-2y-\frac{2x}{3}} dz = 3\int_0^1 dy \int_0^{3-3y} (2-2y-\frac{2x}{3})dx =$

$= 3\int_0^1 (2x - 2yx - \frac{x^2}{3}\Big|_0^{3-3y})dy = 3\int_0^1 (2(3-3y) - 2y(3-3y) - \frac{(3-3y)^2}{3})dy = 3\int_0^1 (6-6y-6y+6y^2-3+6y-3y^2)dy =$

$= 3\int_0^1 (3y^2-6y+3)dy = 9\int_0^1 (y-1)^2 dy =$

$\left| \begin{matrix} t = y-1 \\ dt = dy \\ t = 0, t = -1 \end{matrix} \right| = 9\int_{-1}^0 t^2 dt = 3t^3\Big|_{-1}^0 = 3$

Ответ: 3

$|\operatorname{grad} v| = \sqrt{(-3)^2+(-1)^2+(\sqrt{6})^2} = 4$;

$|\operatorname{grad} u| = \sqrt{\left(-\frac{27}{2}\right)^2+\left(\frac{27}{2}\right)^2+\left(-9\sqrt{\frac{3}{2}}\right)^2} = \sqrt{\frac{3^6}{2}+\frac{3^5}{2}} = \sqrt{2\cdot 3^5} = 9\sqrt{6}$

$(\operatorname{grad} u, \operatorname{grad} v) = -\frac{27}{2}\cdot(-3) + \frac{27}{2}\cdot(-1) - 9\sqrt{\frac{3}{2}}\cdot\sqrt{6} =$

$= \frac{81}{2} - \frac{27}{2} - 27 = 0$

$\operatorname{grad} u \perp \operatorname{grad} v$, т.е $\alpha = 90^0$

Ответ: $\alpha = 90^0$

№ 3.14

$\bar{a} = 2z\vec{j} + 3y\vec{k}$; $a_x = 0$; $a_y = 2z$; $a_z = 3y$

$\frac{dx}{a_x} = \frac{dy}{a_y} = \frac{dz}{a_z}$ $\quad \frac{dx}{0} = \frac{dy}{2z} = \frac{dz}{3y} \Rightarrow dx = 0$; $\frac{dy}{2z} = \frac{dz}{3y} \Rightarrow$

$3\int y\,dy = 2\int z\,dz \Rightarrow \frac{3}{2}y^2 = z^2 + C \Rightarrow \frac{y^2}{2} - \frac{z^2}{3} = C$ - гипербола

№4.14

$\bar{a} = xz\vec{i} + y^2\vec{j} + (z^2-1)\vec{k}$, $S: x^2+y^2=z^2$, $(z\geq 0)$

$D: z = 4$)

$F(x,y,z) = x^2+y^2-z^2$

$\frac{\partial F}{\partial x} = 2x$; $\frac{\partial F}{\partial y} = 2y$; $\frac{\partial F}{\partial z} = -2z$

$\vec{N}\left\{\frac{\partial F}{\partial x}; \frac{\partial F}{\partial y}; \frac{\partial F}{\partial z}\right\} = \{2x; 2y; -2z\} \Rightarrow |\vec{N}| = \sqrt{4x^2+4y^2+4z^2} =$

$= 2\sqrt{x^2+y^2+z^2} = 2\sqrt{2z^2} = 2\sqrt{2}\cdot z$

$\bar{n} = \frac{\bar{N}}{|N|} = \left\{\frac{x}{\sqrt{2}z}; \frac{y}{\sqrt{2}z}; -\frac{1}{\sqrt{2}}\right\} \Rightarrow (\bar{a},\bar{n}) = \frac{xz\cdot x}{\sqrt{2}z} + \frac{yz\cdot y}{\sqrt{2}z} - \frac{z^2\cdot 1}{\sqrt{2}} =$

$= \frac{x^2+y^2-z^2+1}{\sqrt{2}} = \frac{z^2-z^2+1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Pi = \iint\limits_S (\bar{a},\bar{n})\,dS = \iint\limits_S \frac{1}{\sqrt{2}}\,dS = \frac{1}{\sqrt{2}}\iint dS = \frac{1}{\sqrt{2}}|S| =$

$= \frac{1}{\sqrt{2}} S_{бок} = \frac{1}{\sqrt{2}}(\pi R l) = \frac{1}{\sqrt{2}}\cdot\pi\cdot 4\cdot 4\sqrt{2} = 16\pi$

Ответ: 16π

ТР N 1.14

$u = \ln(1+x^2+y^2) - \sqrt{x^2+z^2}$; $S: x^2-6x+9y^2+z^2=4z+4$; $M(3;0;4)$

$\frac{\partial u}{\partial x} = \frac{2x}{1+x^2+y^2} - \frac{x}{\sqrt{x^2+z^2}} = \frac{6}{1+9+0} - \frac{3}{\sqrt{9+16}} = 0,6-0,6=0$

$\frac{\partial u}{\partial y} = \frac{2y}{1+x^2+y^2} = 0$,

$\frac{\partial u}{\partial z} = -\frac{z}{\sqrt{x^2+z^2}} = \frac{4}{\sqrt{9+16}} = 0,8$; $grad\, u = \{0;0;0,8\}$

$F = x^2-6x+9y^2+z^2-4z-4$; $F'_x = 2x-6=0$; $F'_y = 18y = 0$; $F'_z = 2z-4$

$\vec{N} = \{F'_x; F'_y; F'_z\} = \{0;0;-12\}$ $\vec{n} = \{0;0;1\}$

$\frac{\partial u}{\partial} = \frac{(grad\,u, \bar{n})}{|n|} = \frac{0+0+0,8}{1} = 0,8$

Ответ: 0,8

N2.14

$v = \frac{2}{x} + \frac{3}{2y} - \frac{\sqrt{6}}{4z}$, $u = \frac{y^3}{x^2 z}$, $M\left(\sqrt{\frac{2}{3}}; \sqrt{\frac{3}{2}}; \frac{1}{2}\right)$

$\frac{\partial v}{\partial x} = -\frac{2}{x^2}$; $\frac{\partial v}{\partial y} = -\frac{3}{2y^2}$; $\frac{\partial v}{\partial z} = \frac{\sqrt{6}}{4z^2}$; $\frac{\partial u}{\partial x} = \frac{-2y^3}{x^3 z}$; $\frac{\partial u}{\partial y} = \frac{3y^2}{x^2 z}$;

$\frac{\partial u}{\partial z} = \frac{-y^3}{x^2 z^2}$; В точке М: $x = \sqrt{\frac{2}{3}}$, $y = \sqrt{\frac{3}{2}}$; $z = \frac{1}{2} \Rightarrow$

$\frac{\partial v}{\partial x} = -\frac{2}{\frac{2}{3}} = -3$; $\frac{\partial v}{\partial y} = \frac{-3}{2 \cdot \frac{3}{2}} = -1$, $\frac{\partial v}{\partial z} = \frac{\sqrt{6}}{4 \cdot \frac{1}{4}} = \sqrt{6}$;

$= -13,5$;

$\frac{\partial u}{\partial x} = \frac{-2 \cdot \frac{3}{2}\sqrt{\frac{3}{2}}}{\frac{2}{3} \cdot \sqrt{\frac{2}{3}} \cdot \frac{1}{2}} = -4 \cdot \frac{27}{8} = -\frac{27}{2}$

$\frac{\partial u}{\partial y} = \frac{3 \cdot \frac{3}{2}}{\frac{2}{3} \cdot \frac{1}{2}} = \frac{27}{2} = 13,5$; $\frac{\partial u}{\partial z} = \frac{-\frac{3}{2}\sqrt{\frac{3}{2}}}{\frac{2}{3} \cdot \frac{1}{4}} = -9\sqrt{\frac{3}{2}}$

$grad\, u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k} = -\frac{27}{2}\vec{i} + \frac{27}{2}\vec{j} - 9\sqrt{\frac{3}{2}}\vec{k}$

$grad\, v = \frac{\partial v}{\partial x}\vec{i} + \frac{\partial v}{\partial y}\vec{j} + \frac{\partial v}{\partial z}\vec{k} = -3\vec{i} - \vec{j} + \sqrt{6}\vec{k}$

№10.14

$x^2+y^2=\frac{z^2}{25}$, $x^2+y^2=\frac{z}{5}$; $x=0, y=0, (x\geq 0, y\geq 0)$ $\mu=14yz$

$$\Omega: \begin{cases} 0\leq \varphi \leq \pi/2 \\ 0\leq r\leq 1 \\ 5r^2\leq z\leq 5r \end{cases} \qquad V=\iiint\limits_{\Omega} dx\,dy\,dz=\int_0^{\pi/2} d\varphi \int_0^1 r\,dr\int_{5r^2}^{5r} dz =$$

$$= 5\cdot\frac{\pi}{2}\int_0^1 (r^2-r^3)\,dr=\frac{\pi}{2}\left(\frac{r^3}{3}-\frac{r^4}{4}\right)\Big|_0^1=\frac{5}{24}$$

$$m=\iiint\limits_{\Omega}\mu\,dx\,dy\,dz=14\int_0^{\pi/2} d\varphi\int_0^1 r\,dr\int_{5r^2}^{5r} r\sin\varphi\cdot z\cdot z\,dz=$$

$$=14\int_0^{\pi/2}\sin\varphi\,d\varphi\int_0^1 r^2\frac{z^2}{2}\Big|_{5r^2}^{5r}dr = 2(-\cos\varphi)\Big|_0^{\pi/2}25\int_0^1(r^4-r^6)\,dr=$$

$$=175\cdot\left(\frac{r^5}{5}-\frac{r^7}{7}\right)\Big|_0^1=\frac{175\cdot 2}{35}=10$$

№14.14

$z = 30[(x+1)^2+y^2]+1;\ z = 60x+61$

$$\Omega: \begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le r \le 1 \\ 30[(x+1)^2+y^2]+1 \le z \le 60x+61 \\ 30r^2+60r\cos\varphi+31 \le z \le 60r\cos\varphi+61 \end{cases}$$

$$V = \iiint\limits_{\Omega} d\varphi\, dz\, dr = \int_0^{2\pi} d\varphi \int_0^1 r\,dr \int_{30r^2+60r\cos\varphi+31}^{60r\cos\varphi+61} dz = \int_0^{2\pi} d\varphi \int_0^1 r(30-30r^2)dr =$$

$$= 30 \cdot 2\pi \int_0^1 (r - r^3)dr = 60\pi\left(\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^1 = 60\pi\left(\frac{1}{2} - \frac{1}{4}\right) = 15\pi$$

Ответ: 15π

№15.14

$36 \le x^2+y^2+z^2 \le 121;\ z \le -\sqrt{\frac{x^2+y^2}{99}};\ y \ge \sqrt{3}x;\ y \ge 0$

$dx\,dy\,dz = r^2 \sin\varphi\, dr\, d\theta\, d\varphi$

$$\Omega: \begin{cases} \frac{\pi}{3} \le \theta \le \pi \\ \pi - \operatorname{arctg}\sqrt{99} \le \varphi \le \pi \end{cases}$$

$$V = \iiint\limits_{\Omega} dx\,dy\,dz =$$

$$= \int_{\pi/3}^{\pi} d\theta \int_{\pi-\operatorname{arctg}\sqrt{99}}^{\pi} \sin\varphi\, d\varphi \int_6^{11} r^2 dr = \frac{2\pi}{3}(-\cos\varphi)\Big|_{\pi-\operatorname{arctg}\sqrt{99}}^{\pi} \cdot \frac{r^3}{3}\Big|_6^{11} = \frac{2\pi}{3} \cdot \frac{1115}{3}(\cos(\pi - \operatorname{arctg}\sqrt{99}) - \cos\pi) = \frac{2230}{9}(-\cos(\operatorname{arctg}\sqrt{99})$$

$$+1) = \frac{2230\pi}{9}\left(1 - \frac{1}{\sqrt{1+\operatorname{tg}^2(\operatorname{arctg}\sqrt{99})}}\right) = \frac{2230\pi}{9}\left(1 - \frac{1}{\sqrt{1+99}}\right) =$$

$$= \frac{2230\pi}{9}\left(1 - \frac{1}{10}\right) = 223\pi$$

Ответ: 223π

№12.14

$x = y^2 - 2,\ x = -4y^2 + 3,\ z = \sqrt{16-x^2-y^2}+2 \quad z = \sqrt{16-x^2-y^2}-1$

$$\Omega: \begin{cases} -1 \le y \le 1 \\ y^2 - 2 \le x \le -4y^2 + 3 \\ \sqrt{16-x^2-y^2} - 1 \le z \le \sqrt{16-x^2-y^2}+2 \end{cases}$$

$$V = \iiint_\Omega dx\,dy\,dz = \int_{-1}^{1} dy \int_{y^2-2}^{-4y^2+3} dx \int_{\sqrt{16-x^2-y^2}-1}^{\sqrt{16-x^2-y^2}+2} dz = \int_{-1}^{1} dy \int_{y^2-2}^{-4y^2+3} (\sqrt{16-x^2-y^2}+2 -$$

$$- \sqrt{16-x^2-y^2}+1)\,dx =$$

$$= 3\int_{-1}^{1} dy\ x \Big|_{y^2-1}^{-4y^2+3} = 3\int_{-1}^{1} (-4y^2+3-y^2+2)\,dy = 3\int_{-1}^{1} (5-5y^2)\,dy$$

$$= 15\left(y - \frac{y^3}{3}\right)\Big|_{-1}^{1} = 15\left(1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right)\right) = 20$$

Ответ: 20

№13.14

$z = 6\sqrt{x^2+y^2},\ z = 16 - x^2 - y^2$

$\left(\frac{z}{6}\right)^2 + z - 16 = 0$

$z^2 + 36z - 576 = 0$

$D = 36^2 + 4 \cdot 576 = 3600,\ z > 0$

$z = \frac{-36+60}{2} = 12$, $\quad \Omega: \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le \rho \le 2 \\ 6\rho \le z \le 16 - \rho^2 \end{cases}$

$$V = \int_0^{2\pi} d\theta \int_0^2 \rho\, d\rho \int_{6\rho}^{16-\rho^2} dz = 2\pi \int_0^2 \rho(16 - \rho^2 - 6\rho)\,d\rho = 2\pi \int_0^2 (16\rho - \rho^3 - 6\rho^2)\,d\rho$$

$$= 2\pi\left(8\rho^2 - \frac{\rho^4}{4} - 2\rho^3\right)\Big|_0^2 = 2\pi(32-4-16) = 24\pi$$

Ответ: 24π

№10.14

$x = 19\sqrt{2y}$; $x = 4\sqrt{2y}$; $z = 0$; $z + y = 2$

Тело Ω: $\begin{cases} 0 \le y \le 2 \\ 4\sqrt{2y} \le x \le 19\sqrt{2y} \\ 0 \le z \le 2-y \end{cases}$ $V = \iiint\limits_{\Omega} dx\,dy\,dz =$

$= \int_0^2 dy \int_{4\sqrt{2y}}^{19\sqrt{2y}} dx \int_0^{2-y} dz = \int_0^2 (19\sqrt{2y} - 4\sqrt{2y})\cdot(2-y-0)\,dy = \int_0^2 15\sqrt{2y}(2-y)\,dy =$

$= 30\sqrt{2}\int_0^2 y^{1/2}\,dy - 15\sqrt{2}\int_0^2 y^{3/2}\,dy = 30\sqrt{2}\,\frac{y^{3/2}}{3/2}\Big|_0^2 - 15\sqrt{2}\,\frac{y^{5/2}}{5/2}\Big|_0^2 =$

$= 20\sqrt{2}\left((\sqrt{2})^3\right) - 6\sqrt{2}(\sqrt{2})^5 = 20\cdot 4 - 6\cdot 8 = 32$

Ответ: 32

№11.14

$x^2 + y^2 = 3y$; $x^2 + y^2 = 6y$; $z = \sqrt{x^2+y^2}$; $z = 0$

$r^2 = 6r\sin\varphi$; $r = 6\sin\varphi$

$x^2 + y^2 = 3y$; $x^2 + (y-3)^2 = 3^2$; $r^2 = 3r\sin\varphi$; $r = 3\sin\varphi$

$x^2 + (y - \frac{3}{2})^2 = (\frac{3}{2})^2$; $dx\,dy\,dz = r\,dr\,d\varphi\,dz$

Ω: $\begin{cases} 0 \le \varphi \le \pi \\ 3\sin\varphi \le r \le 6\sin\varphi \\ 0 \le z \le r \end{cases}$

$V = \int_0^{\pi} d\varphi \int_{3\sin\varphi}^{6\sin\varphi} r\,dr \int_0^{r} dz = \int_0^{\pi} d\varphi \int_{3\sin\varphi}^{6\sin\varphi} r^2\,dr = \frac{1}{3}\int_0^{\pi} (6^3 - 3^3)\sin^3\varphi\,d\varphi =$

$= -63\int_0^{\pi} (1-\cos^2\varphi)\,d\cos\varphi = 63\left(\frac{\cos^3\varphi}{3} - \cos\varphi\right)\Big|_0^{\pi} =$

$= 63\left(-\frac{1}{3} - \frac{1}{3} - (-1-1)\right) = 63\left(\frac{4}{3}\right) = 84$

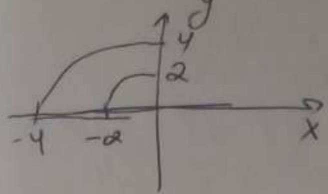
Ответ: 84

№8.14

$D: x^2+y^2=4;\ x^2+y^2=16$

$x=0,\ y=0.\ (x\le 0,\ y\ge 0)$

$\mu = \dfrac{(2y-3x)}{x^2+y^2}$



$m=\iint\limits_D \mu\, dx\,dy = \iint\limits_D \mu\rho\, d\varphi\, d\rho$

$D: \begin{cases} 2\le\rho<4 \\ \pi/2\le\varphi\le\pi \end{cases}$ $\quad \mu = 2\rho\sin\varphi - 3\rho\cos\varphi$

$m=\int\limits_{\frac{\pi}{2}}^{\pi} d\varphi \int\limits_{2}^{4}\dfrac{(2\rho\sin\varphi-3\rho\cos\varphi)\rho\, d\rho}{\rho^2} = \int\limits_{\pi/2}^{\pi}(2\sin\varphi-3\cos\varphi)\,d\varphi\cdot$

$\cdot\int\limits_0^4 d\rho = 2\int\limits_{\pi/2}^{\pi}(2\sin\varphi-3\cos\varphi)\,d\varphi = -4\cos\varphi-6\sin\varphi\Big|_{\frac{\pi}{2}}^{\pi} = 4+6=10$

Ответ: 10

№9.14

$D: \dfrac{x^2}{16}+y^2\le 1;\ x\ge 0,\ y\ge 0,\ \mu = 5xy^7$

$\begin{cases} x=4\rho\cos\varphi \\ y=\rho\sin\varphi \end{cases}$ $\quad \mu = 5\cdot 4\rho\cos\varphi\cdot\rho^7\sin^7\varphi$

$0\le\varphi\le\frac{\pi}{2}$

$\dfrac{16\rho^2\cos^2\varphi}{16}+\rho^2\sin^2\varphi\le 1 \Rightarrow \rho\le 1$

$m=\int\limits_0^{\frac{\pi}{2}} d\varphi\int\limits_0^1 4\rho\cdot 5\cdot 4\rho\cos\varphi\cdot\rho^7\sin^7\varphi\, d\rho = 80\int\limits_0^{\frac{\pi}{2}}\cos\varphi\sin^7\varphi\, d\varphi\cdot$

$\cdot\int\limits_0^1\rho^9 d\rho = 80\int\limits_0^{\pi/2}\cos\varphi\sin^7\varphi\, d\varphi\cdot\frac{1}{10} = 8\int\limits_0^{\frac{\pi}{2}}\sin^7\varphi\, d(\sin\varphi) =$

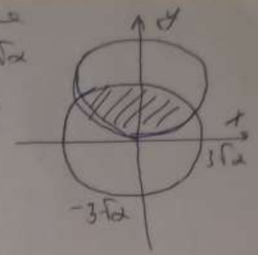
$= 8\cdot\dfrac{\sin^8\varphi}{8}\Big|_0^{\pi/2} = 8\cdot\frac{1}{8}=1$

Ответ: $m=1$

№6.14

$y=\sqrt{18-x^2}$; $y=3\sqrt{2}-\sqrt{18-x^2}$; | Пересечение: $2\sqrt{18-x^2}=3\sqrt{2}$, $x=\pm 3\sqrt{3/2}$, $y=\frac{3\sqrt{2}}{2}$

$R=\sqrt{18}=3\sqrt{2}$



$$D:\begin{cases}-3\sqrt{\frac{3}{2}}\le x\le 3\cdot\sqrt{\frac{3}{2}}\\ 3\sqrt{2}-\sqrt{18-x^2}\le y\le\sqrt{18-x^2}\end{cases}$$

$$S=\iint_D dx\,dy=\int_{-3\sqrt{1,5}}^{3\sqrt{1,5}}dx\int dy=\int_{-3\sqrt{1,5}}^{3\sqrt{1,5}}\left(2\sqrt{18-x^2}-3\sqrt{2}\right)dx \ (=)$$

$dx=3\sqrt{2}\cos t\,dt;\ t_1=-\frac{\pi}{3};\ t_2=\frac{\pi}{3}$

$$(=)\ S=2\int_{-\pi/3}^{\pi/3}\sqrt{18-18\sin^2 t}\ 3\sqrt{2}\cos t\,dt-3\sqrt{2}(3\sqrt{1,5}+3\sqrt{1,5})=$$

$$=2\int_{-\pi/3}^{\pi/3}18\cos^2 t\,dt-18\sqrt{3}=18\int_{-\pi/3}^{\pi/3}(1+\cos 2t)\,dt-18\sqrt{3}=$$

$$=18\left(t+\frac{\sin 2t}{2}\right)\Big|_{-\pi/3}^{\pi/3}-18\sqrt{3}=18\left(\frac{2\pi}{3}+\frac{1}{2}\left(\sin\frac{2\pi}{3}+\sin\frac{2\pi}{3}\right)\right)-18\sqrt{3}=$$

$$=12\pi-9\sqrt{3}$$

Ответ: $12\pi-9\sqrt{3}$

№7.14

$x^2-2x+y^2=0$; $x^2-8x+y^2=0$; $y=x/\sqrt{3}$; $y=\sqrt{3}x$

$x^2-2x+y^2=0$, $(x-4)^2+y^2=4^2$

$(x-1)^2+y^2=1$;

$$D:\begin{cases}\frac{\pi}{6}\le\varphi\le\frac{\pi}{3}\\ 2\cos\varphi\le r\le 8\cos\varphi\end{cases}$$

$$\int_{\pi/6}^{\pi/3}d\varphi\int_{2\cos\varphi}^{8\cos\varphi}\rho\,d\rho=\int_{\pi/6}^{\pi/3}\frac{1}{2}\rho^2\Big|_{2\cos\varphi}^{8\cos\varphi}d\varphi=\frac{1}{2}60\int_{\pi/6}^{\pi/3}\cos^2\varphi\,d\varphi=$$

$$=15\int_{\pi/6}^{\pi/3}(1+\cos 2\varphi)\,d\varphi=15\left(\varphi+\frac{\sin 2\varphi}{2}\right)\Big|_{\pi/6}^{\pi/3}=15\left(\frac{\pi}{3}-\frac{\pi}{6}+\frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)\right)=\frac{5\pi}{2}$$

Ответ: $\frac{5\pi}{2}$

№5.14

$$\iiint\limits_V \frac{dx\,dy\,dz}{\left(1+\frac{x}{10}+\frac{y}{8}+\frac{z}{3}\right)^6} \quad V: \begin{cases} \frac{x}{10}+\frac{y}{8}+\frac{z}{3}=1 \\ x=0,\ y=0,\ z=0 \end{cases}$$

$$V: \begin{cases} 0 \le x \le 10 \\ 0 \le y \le 8\left(1-\frac{x}{10}\right) \\ 0 \le z \le 3\left(1-\frac{x}{10}-\frac{y}{8}\right) \end{cases}$$

$$= \int_0^{10} dx \int_0^{8(1-\frac{x}{10})} dy \int^{3(1-\frac{x}{10}-\frac{y}{8})} \frac{dz}{\left(1+\frac{x}{10}+\frac{y}{8}+\frac{z}{3}\right)^6} = 3\int_0^{10} dx \int_0^{8(1-\frac{x}{10})} \left(\frac{-dy}{5\left(1+\frac{x}{10}+\frac{y}{8}+\frac{z}{3}\right)^5}\right)\Bigg|_0^{3(1-\frac{x}{10}-\frac{y}{8})} =$$

$$= -\frac{3}{5}\int_0^{10} dx \int_0^{8(1-\frac{x}{10})} \left(\frac{1}{2^5} - \frac{1}{\left(1+\frac{x}{10}+\frac{y}{8}\right)^5}\right) dy = -\frac{3}{5}\int_0^{10} \Bigg[\frac{y}{32}\Bigg|_0^{8(1-\frac{x}{10})} +$$

$$+ \frac{8}{4\left(1+\frac{x}{10}+\frac{y}{8}\right)^4}\Bigg|_0^{8(1-\frac{x}{10})} \Bigg] dx = -\frac{3}{5}\int_0^{10} \left(\frac{1}{4}\left(1-\frac{x}{10}\right) + 2\left(\frac{1}{16} - \frac{1}{\left(1-\frac{x}{10}\right)^4}\right)\right) dx =$$

$$= -\frac{3}{5}\int_0^{10} \left[\frac{3}{8} - \frac{x}{10} - \frac{2}{\left(1+\frac{x}{10}\right)^4}\right] dx = -\frac{3}{5}\left[\frac{3}{8}x - \frac{x^2}{80} + \frac{20}{3}\frac{1}{\left(1+\frac{x}{10}\right)^3}\right]\Bigg|_0^{10} =$$

$$= -\frac{3}{5}\left[\frac{30}{8} - \frac{5}{4} + \frac{20}{3}\left(\frac{1}{8}-1\right)\right] = -\frac{3}{5}\left[\frac{15}{4} - \frac{5}{4} - \frac{20}{3}\cdot\frac{7}{8}\right] =$$

$$= -\frac{3}{5}\left[+\frac{5}{4} - \frac{35}{6}\right] = \frac{3}{5}\frac{(-15+35)}{6} = 2$$

Ответ: 2

№3.14

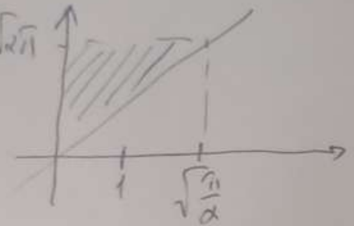
$\iint\limits_D 4y^2 \sin 2xy \, dx dy \ominus$ $\quad D: x=0;\ y=\sqrt{2\pi};\ y=2x$

$D: \begin{cases} 0 \le y \le \sqrt{2\pi} \\ 0 \le x \le \frac{y}{2} \end{cases}$

$\ominus \int\limits_0^{\sqrt{2\pi}} 4y^2 dy \int\limits_0^{y/2} \sin 2xy \, dx = 4\int\limits_0^{\sqrt{2\pi}} y^2 \frac{\cos 2xy}{(-2y)} \Big|_0^{y/2} dy = -2\int\limits_0^{\sqrt{2\pi}} y(\cos y^2 - \cos 0) dy =$

$= y^2 \Big|_0^{\sqrt{2\pi}} - \int\limits_0^{\sqrt{2\pi}} \cos y^2 dy^2 = 2\pi - \sin y^2 \Big|_0^{\sqrt{2\pi}} = 2\pi$

Ответ: 2π



№4.14

$\iiint\limits_V y^2 z \cos \frac{xyz}{3} \, dx dy dz \ominus$ $\quad V: \begin{cases} x=3,\ y=1,\ z=2\pi \\ x=0,\ y=0,\ z=0 \end{cases}$

$V: \begin{cases} 0 \le x \le 3 \\ 0 \le y \le 1 \\ 0 \le z \le 2\pi \end{cases}$ $\quad \ominus \int\limits_0^3 dx \int\limits_0^1 dy \int\limits_0^{2\pi} y^2 z \cos \frac{xyz}{3} \cdot dz =$

$= \int\limits_0^1 y^2 dy \int\limits_0^{2\pi} z \, dz \int\limits_0^3 \cos \frac{xyz}{3} dx = \int\limits_0^1 y^2 dy \int\limits_0^{2\pi} z \, dz \cdot \frac{3}{yz} \sin \frac{xyz}{3} \Big|_0^3 =$

$= 3\int\limits_0^1 y \, dy \int\limits_0^{2\pi} \sin(yz) dz = -3\int\limits_0^1 y \frac{\cos yz}{y} \Big|_0^{2\pi} dy = -3\int\limits_0^1 (\cos(2\pi y) - \cos 0) dy =$

$= \left(3y - \frac{3\sin 2\pi y}{2\pi}\right)\Big|_0^1 = 3 - \frac{3\sin 2\pi}{2\pi} - 0 + 0 = 3$

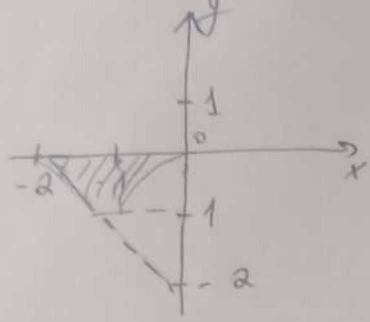
Ответ: 3

ТР №1.14

$$\int_{-2}^{-1} dx \int_{-(2+x)}^{0} f(x,y)\,dy + \int_{-1}^{0} dx \int_{\sqrt[3]{x}}^{0} f(x,y)\,dy \ominus$$

Область интегрирования $D_1: \begin{cases} -2 \le x \le -1 \\ -2-x \le y \le 0 \end{cases}$, $D_2: \begin{cases} -1 \le x \le 0 \\ \sqrt[3]{x} \le y \le 0 \end{cases}$

$$D: \begin{cases} -1 \le y \le 0 \\ -2-y \le x \le y^3 \end{cases} \Rightarrow \ominus \iint_D f(x,y)\,dx\,dy = \int_{-1}^{0} dy \int_{-(2+y)}^{y^3} f(x,y)\,dx$$



№2.14

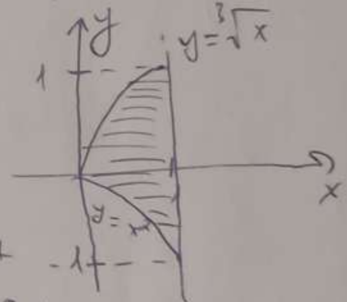
$$\iint_D (8xy+18x^2y^2)\,dx\,dy \ominus \quad D: x=1,\ y=\sqrt[3]{x},\ y=-x^2$$

$$D: \begin{cases} 0 \le x \le 1 \\ -x^2 \le y \le x^{1/3} \end{cases} \ominus \int_0^1 dx \int_{-x^2} (8xy+18x^2\cdot y)\,dy =$$

$$= \int_0^1 (4xy^2+6x^2y^3)^2 \Big|_{-x^2}^{x^{1/3}} dx = \int_0^1 (4x^{5/3} - 4x^5 + 6x^3 + 6x^8)\,dx = \left(4\cdot\frac{x^{8/3}}{8/3} - 4\frac{x^6}{6} + 6\frac{x^4}{4} + 6\frac{x^9}{9}\right)\Big|_0^1 =$$

$$= \left(\frac{3}{2} - \frac{2}{3} + \frac{3}{2} + \frac{2}{3}\right) = 3$$

Ответ: 3



N15.14

$$\begin{cases} x = 2\cos^2 t & 0 \le t \le \frac{\pi}{4} \\ y = 2\sin^2 t \end{cases}$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

1. $\frac{dx}{dt} = \frac{d}{dt}(2\cos^2 t) = 2 \cdot 2\cos t$
$(-\sin t) = -4\cos t \sin t$

2. $\frac{dy}{dt} = \frac{d}{dt}(2\sin^2 t) = 2 \cdot 2\sin t$
$\cos t = 4\sin t \cos t$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{(4\cos t \sin t)^2 + (4\cos t \sin t)^2} = \int_0^{\frac{\pi}{4}} \sqrt{2(4\cos t \sin t)^2}\, dt =$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{2(4\cos t \sin t)^2}\, dt = \int_0^{\frac{\pi}{4}} \sqrt{2} \cdot 4\cos t \sin t\, dt = 4\sqrt{2} \int_0^{\frac{\pi}{4}} \cos t \sin t\, dt$$

$$= 4\sqrt{2} \int_0^{\frac{\pi}{4}} \frac{1}{2} \sin 2t\, dt = 2\sqrt{2} \int_0^{\frac{\pi}{4}} \sin 2t\, dt = .$$

$$\int \sin 2t\, dt = -\frac{1}{2}\cos 2t$$

$$L = 2\sqrt{2}\left(-\frac{1}{2}\cos(2t)\right)\Big|_0^{\frac{\pi}{4}} = 2\sqrt{2}\left(-\frac{1}{2}\cos\frac{\pi}{2} + \frac{1}{2}\cos 0\right) = 2\sqrt{2}\left(-\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 1\right) =$$

$$= 2\sqrt{2} \cdot \frac{1}{2} = \sqrt{2}$$

Ответ: $\sqrt{2}$

N14.14

$\rho = 2\cos\varphi,\ \rho = \cos\varphi\ (\rho \geq \cos\varphi)$

$S = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta$

1. $\rho = 2\cos\varphi$

2. $\rho = \cos\varphi$

Уравнения пересекаются, когда $\rho_1 = \rho_2$:

$2\cos\varphi = \cos\varphi$

$2\cos\varphi - \cos\varphi = 0$

$\cos\varphi = 0$

$\varphi = \frac{\pi}{2},\ \varphi = \frac{3\pi}{2}$

Площадь фигуры, ограниченной линией $\rho = 2\cos\varphi$:

$$S_1 = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (2\cos\varphi)^2\, d\varphi = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 4\cos^2\varphi\, d\varphi = 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2\varphi\, d\varphi =$$

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1+\cos 2\varphi}{2}\, d\varphi = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1+\cos 2\varphi)\, d\varphi = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1\, d\varphi + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos 2\varphi\, d\varphi =$$

$$= [\varphi]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \left[\frac{\sin 2\varphi}{2}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + \left(\frac{\sin 3\pi}{2} - \frac{\sin\pi}{2}\right) = \pi + (0-0)$$

$$S_2 = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\cos\varphi)^2\, d\varphi = \frac{1}{2}\cdot\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1+\cos 2\varphi}{2}\, d\varphi = \frac{1}{4}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1+\cos 2\varphi)\, d\varphi =$$

$$= \frac{1}{4}\left(\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1+\cos 2\varphi)\, d\varphi = \frac{1}{4}\left(\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1\, d\varphi + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos 2\varphi\, d\varphi\right) =\right.$$

$$= \frac{1}{4}\left([\varphi]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \left[\frac{\sin 2\varphi}{2}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right) = \frac{1}{4}\left(\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) + \left(\frac{\sin 3\pi}{2} - \frac{\sin\pi}{2}\right)\right)$$

$$= \frac{1}{4}(\pi + (0-0)) = \frac{\pi}{4}$$

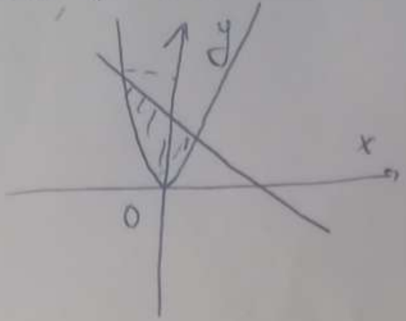
$$S = S_1 - S_2 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Ответ: $\frac{3\pi}{4}$

№13.14

$y=x^2,\ x+y=2,\ x=0\ (x\geq 0)$

$\begin{cases} y=x^2 \\ y=2-x \end{cases}$



$x^2=2-x$

$x^2+x-2=0$

$D=1+8=9$

$x_1=\frac{-1+3}{2}=1$

$x_2=\frac{-1-3}{2}=-2$

$\int\limits_{-2}^{1}dx\int\limits_{x^2}^{2-x}dy$

(1) $\int\limits_{x^2}^{2-x}dy=y\Big|_{x^2}^{2-x}=2-x-x^2$

(2) $\int\limits_{-2}^{1}(2-x-x^2)dx=2x-\frac{x^2}{2}-\frac{x^3}{3}\Big|_{-2}^{1}=2-\frac{1}{2}-\frac{1}{3}-$

$-\left(-4-2+\frac{8}{3}\right)=2-\frac{1}{2}-\frac{1}{3}+4+2-\frac{8}{3}=8-\frac{3-2}{6}-\frac{8}{3}=$

$=8-\frac{1}{6}-\frac{8}{3}=8-\frac{1+16}{6}=8-\frac{17}{6}=\frac{48-17}{6}=\frac{31}{6}=5\frac{1}{6}$

Ответ: $5\frac{1}{6}$

№12.14

$$\int_0^{\frac{1}{2}} x \operatorname{arctg} 2x \, dx$$

$u = \operatorname{arctg}(2x)$ | $dv = x\,dx$

$du = \frac{2}{4x^2+1}\,dx$ | $v = \int x\,dx = \frac{x^2}{2}$

$$x^2\frac{\operatorname{arctg}(2x)}{2} - \int \frac{x^2}{4x^2+1}\,dx = x^2\frac{\operatorname{arctg} 2x}{2} + \frac{\operatorname{arctg} 2x}{8} - \frac{x}{4}$$

$$\int \frac{x^2}{4x^2+1}\,dx = \int \frac{1}{4} - \frac{1}{4(4x^2+1)}\,dx = \frac{1}{4}\int 1\,dx - \frac{1}{4}\int \frac{1}{4x^2+1}\,dx =$$

$$= \frac{1}{4}\cdot(2) - \frac{1}{4}\cdot(3) = \frac{x}{4} - \frac{\operatorname{arctg}(2x)}{8}$$

(2) $\int 1\,dx = x$

$u = 2x$ | $x = \frac{u}{2}$, $dx = \frac{1}{2}\,du$

(3) $\int \frac{1}{4x^2+1}\,dx$

$$\int \frac{1}{2(u^2+1)}\,du = \frac{1}{2}\operatorname{arctg} u = \frac{\operatorname{arctg} u}{2}$$

$$x^2\frac{\operatorname{arctg} 2x}{2} + \frac{\operatorname{arctg} 2x}{8} - \frac{x}{4}\Bigg|_0^{\frac{1}{2}} = \frac{\pi}{16} - \frac{1}{8}$$

Ответ: $\frac{\pi}{16} - \frac{1}{8}$

№9.14 №11.14

$\int \frac{\sin^2 x\,dx}{\cos^4 x}$ $\int_1^{\sqrt{3}} \frac{x\,dx}{x^4-2x^2+5}$

$u = x^2$

$\frac{1}{2}du = x\,dx$

$\int \frac{1}{2u^2-4u+10}du = \int \frac{1}{2(u^2-2u+5)}du = \frac{1}{2}\int \frac{1}{(u-1)^2+4}du =$

$v = u-1 \left| \begin{array}{l} u = v+1 \\ du = dv \end{array} \right.$

$= \frac{1}{2}\int \frac{1}{v^2+4}dv =$ $t = \frac{v}{2} \left| \begin{array}{l} v = 2t \\ dv = 2dt \end{array} \right.$

$= \frac{1}{2}\int \frac{2}{4t^2+4}dt = \frac{1}{2}\int \frac{1}{2(t^2+1)}dt = \frac{1}{4}\operatorname{arctg} t = \frac{\operatorname{arctg} t}{4}$

$t = \frac{v}{2} \mid v = 2t$

$\frac{\operatorname{arctg}(\frac{v}{2})}{4} = \frac{\operatorname{arctg}(\frac{u-1}{2})}{4} = \frac{\operatorname{arctg}(\frac{x^2-1}{2})}{4}$

$\frac{\operatorname{arctg}(\frac{x^2-1}{2})}{4}\Bigg|_1^{\sqrt{3}} = \frac{\operatorname{arctg}(\frac{3-1}{2})}{4} - \frac{\operatorname{arctg}(0)}{4} = \frac{\pi}{16}$

Ответ: $\frac{\pi}{16}$

№ 9.14

$$\int \frac{\sin^7 x\, dx}{\cos^4 x} = \int \frac{(1-\cos^2 x)^3 \sin x}{\cos^4 x} = \int \frac{(1-u^2)^3}{u^4}\, du =$$

$u = \cos x$

$du = -\sin x\, dx$

$$= -\int \frac{-u^6+3u^4-3u^2+1}{u^4}\, du =$$

$$= -\int -u^2 - \frac{3}{u^2} + \frac{1}{u^4} + 3\, du = -\left(-\left(\int u^2\, du - 3\int \frac{1}{u^2}\, du + \int \frac{1}{u^4}\, du + 3\int 1\, du\right) = -(-(*) - 3\cdot(**) + (***) + 3\cdot(****) \underline{\underline{=}} *$$

$(*)\ \frac{\cos^6 x - 9\cos^2 x + 1}{3\cos^3 x} - 3\cos x = \int u^2\, du$

$(**)\ \int \frac{1}{u^2}\, du;\ (***)\int \frac{1}{u^4}\, du;\ \int 1\, du\ (****)$

$$\underline{\underline{*}}\ \frac{\cos^6 x - 9\cos^2 x + 1}{3\cos^3 x} - 3\cos x + C$$

№ 10.14

$$\int \frac{dx}{\sin^2 x - 11\cos^2 x + 2} = \int \frac{1}{3-12\cos^2 x}\, dx = \int \frac{1}{3(1-4\cos^2 x)}\, dx =$$

$u = tg\, x$

$du = \frac{1}{\cos^2 x}\, dx$

$\cos^2 x = \frac{1}{u^2+1}$

$$\frac{1}{3}\int \frac{1}{u^2-3}\, du = \frac{1}{3}\int \frac{1}{(u-\sqrt{3})(u+\sqrt{3})}\, du =$$

$$= \frac{1}{3}\int \frac{\frac{1}{-u+\sqrt{3}} - \frac{1}{u-\sqrt{3}}}{2\sqrt{3}}\, du = \frac{1}{3}\left(-\frac{1}{2\sqrt{3}}\int \frac{1}{u+\sqrt{3}}\, du + \frac{1}{2\sqrt{3}}\int \frac{1}{u-\sqrt{3}}\, du\right.$$

$$= \frac{1}{3}\left(-\frac{1}{2\sqrt{3}}\ (1) + \frac{1}{2\sqrt{3}}\ (2)\right) = \frac{\ln(|u-\sqrt{3}|)}{2\cdot 3\sqrt{3}} - \frac{\ln(|u+\sqrt{3}|)}{2\cdot 3\sqrt{3}}$$

Ответ: $\frac{\ln(|tg x - \sqrt{3}|)}{2\cdot 3\sqrt{3}} - \frac{\ln(|tg x + \sqrt{3}|)}{2\cdot 3\sqrt{3}} + C$

Озарков Илья Вариант №14

№7.14

$$\int \frac{\sqrt[5]{(1+\sqrt[4]{x^3})^4}}{x^2 \cdot \sqrt[20]{x^7}}\, dx$$

№8.14

$$\int \sqrt{3x^2+2x+3}\, dx = \int \sqrt{(\sqrt{3}x+\frac{1}{\sqrt{3}})^2+\frac{8}{3}}\, dx \overset{*}{=}$$

$u = \sqrt{3}x + \frac{1}{\sqrt{3}}$ | $x = \frac{\sqrt{3}u-1}{3}$, $dx = \frac{1}{\sqrt{3}}du$

$$\overset{*}{=} \int \frac{\sqrt{u^2+\frac{8}{3}}}{\sqrt{3}}\, du \overset{*}{=}$$

$v = \frac{\sqrt{3}u}{2\sqrt{2}}$ | $u = \frac{2\sqrt{2}v}{\sqrt{3}}$, $du = \frac{2\sqrt{2}}{\sqrt{3}}dv$

$$\overset{*}{=} \frac{1}{\sqrt{3}} \int \frac{2\sqrt{2}\sqrt{\frac{8v^2}{3}+\frac{8}{3}}}{\sqrt{3}}\, dv = \frac{1}{\sqrt{3}} \int \frac{8\sqrt{v^2+1}}{3}\, dv \overset{*}{=}$$

$t = \arctg(v)$ | $v = tg(t)$, $dv = \frac{1}{\cos^2 t}dt$

$\frac{1}{\cos^2(t)} = v^2+1$

$$\overset{*}{=} \frac{8}{3\sqrt{3}} \int \frac{1}{\cos^3(t)}\, dt = \frac{8}{3\sqrt{3}} \int \sec^3(t)\, dt \cdot \int \sec^n(t)\, dt \overset{*}{=}$$

$$\overset{*}{=} \frac{\sec^{n-2}(t)\, tg\, t}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} t\, dt \overset{*}{=} \frac{8}{3\sqrt{3}}\left(\frac{\sec t\, tg\, t}{2} + \frac{1}{2}\cdot (*)\right)$$

$$\int \sec t\, dt\Big) = \frac{8}{3\sqrt{3}}\left(\frac{\sec(t)\, tg\, t}{2} + \frac{1}{2}\right.$$

$$(*) \int \sec t\, dt = \int \frac{\sec t\, tg\, t + \sec^2 t}{tg\, t + \sec t}\, dt$$

$$\frac{tg\, t + \sec t}{tg\, t + \sec(t)}$$

$z = tg(t) + \sec t$

$dz = (\sec(t)\, tg\, t + \sec^2 t)\, dt$

$$\int \frac{1}{z}\, dz \overset{*}{=},\quad z = tg(t) + \sec t$$

$$\overset{*}{=} \ln(|tg(t) + \sec(t)|)$$

$$\overset{*}{=} \frac{4\ln(|\sqrt{2}\sqrt{1+tg} + \sec t|)}{3\sqrt{3}} + \frac{4\sec t\, tg\, t}{3\sqrt{3}}$$

№7.14

$$\int \frac{\sqrt[5]{(1+\sqrt[4]{x^3})^4}}{x^2 \cdot \sqrt[20]{x^7}}\, dx$$

$$u = \sqrt[10]{x} \quad \Bigg| \begin{array}{l} x = u^{10} \\ dx = 10u^9\, du \end{array}$$

$$\int \frac{10\sqrt[5]{(u^{\frac{15}{2}}+1)^4}}{u^{\frac{29}{2}}}\, du = 10\int \frac{2\sqrt[5]{(v^{15}+1)^4}}{v^{28}}\, dv \overset{*}{=}$$

$$t = \frac{\sqrt[5]{v^{15}+1}}{v^3} \quad \Bigg| \begin{array}{l} v = \frac{1}{\sqrt[15]{t^5-1}} \\ dv = -\frac{t^4}{3(t^5-1)^{\frac{18}{15}}}\, dt \end{array}$$

$$\overset{*}{=} 20\int -\frac{t^8}{3}\, dt = -\frac{20}{3}\cdot\frac{t^9}{9} = -\frac{20t^9}{27} = -\frac{\sqrt[5]{v^{15}+1}}{v^3} = -\sqrt[5]{(v^{15}+1)^4}\cdot$$

$$\cdot\frac{(20v^{15}+20)}{27v^{27}} = -\frac{\sqrt[5]{(u^{\frac{15}{2}}+1)^4}(20u^{\frac{15}{2}}+20)}{27u^{\frac{27}{2}}} = \frac{-\sqrt[5]{(\sqrt[4]{x^3}+1)^4}(20x^{\frac{27}{20}}+20\sqrt[5]{x^3})}{27x^{\frac{29}{20}}} + C$$

Ответ: $\frac{\sqrt[5]{(\sqrt[4]{x^3}+1)^4}(20x^{\frac{27}{20}}+20\sqrt[5]{x^3})}{27x^{\frac{29}{20}}} + C$

N6.14

$$\int \frac{2\sqrt{5-x}-5\sqrt{x+1}}{(7\sqrt{x+1}+3\sqrt{5-x})(x+1)^2}dx = \int \frac{2\sqrt{5-x}-5\sqrt{x+1}}{\sqrt{5-x}\,(3x^2+6x+3)+\sqrt{x+1}}\cdot$$

$$\cdot(x^2+2x+1)\,dx = \int \frac{17\sqrt{5-x}}{\sqrt{5-x}\,(3x^2+6x+3)+\sqrt{x+1}\,(x^2+2x+1)} -$$

$$-\frac{5}{x^2+2x+1}\,dx = 17\int \frac{\sqrt{5-x}}{\sqrt{5-x}\,(3x^2+6x+3)+\sqrt{x+1}\,(x^2+2x+1)} \quad (*)$$

$$-5\int \frac{1}{x^2+2x+1}dx \quad (**) = 17\cdot(*)-5\cdot(**)$$

$$(*)\quad u=5-x \;\Big|\; \begin{matrix} x=5-u \\ dx=-du \end{matrix} = \int -\frac{\sqrt{u}}{(6(5-u)+3(5-u)^2+3)\sqrt{u}+}$$

$$+\overline{(2(5-u)+(5-u)^2+1)\sqrt{6-u}}\,du \qquad v=\sqrt{u}\;\Big|\;\begin{matrix} u=v^2 \\ du=2v\,dv \end{matrix}$$

$$-\int \frac{2v^2}{v(3(5-v^2)^2+6(5-v^2)+3)+\sqrt{6-v^2}\,((5-v^2)^2+2(5-v^2)+1)}\,dv$$

$$=-2\int \frac{3v^3}{10v^6-126v^4+432v^2-216}-\frac{1}{\sqrt{6-v^2}(90v^2-(4)}\cdot\frac{1}{\sqrt{6-v^2}(90v^2-(4)}\,dv$$

$$=17\ln\left(\left|\frac{1}{\sqrt{x+1}}\left(1\sqrt{x+1}-3\sqrt{5-x}\right|\right)\right. - 17\ln\left(\sqrt{x+1}\,\Big|\frac{1}{\sqrt{x+1}}+3\sqrt{5-x}\right.$$

$$\qquad 162$$

$$\left|\frac{1}{\sqrt{x+1}}\right|\Big)\Big|+ \sqrt{x+1}\,((-17-17)\cdot\ln(|5x-22|)\,(17x+17)\ln(|v+1|)-910$$

$$162$$

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№4.14

$$\int \frac{7x^2+22x+3}{(x+5)(x^2+2x+2)}\,dx =$$

$$\frac{7x^2+22x+3}{(x+5)(x^2+2x+2)} = \frac{Ax+B}{x^2+2x+2} + \frac{C}{x+5}$$

$$\frac{7x^2+22x+3}{(x+5)(x^2+2x+2)} = \frac{(C+A)x^2+(2C+B+5A)x+2C+5B}{(x+5)(x^2+2x+2)}$$

$$\begin{matrix} x^0: \\ x^1: \\ x^2: \end{matrix} \begin{cases} 2C+5B=3 \\ 2C+B+5A=22 \\ C+A=7 \end{cases} \Rightarrow \begin{cases} A=3 \\ B=-1 \\ C=4 \end{cases}$$

$$\frac{Ax+B}{x^2+2x+2} + \frac{C}{x+5}$$

$$\int \frac{3x-1}{x^2+2x+2} + \frac{4}{x+5}\,dx = \int \frac{3x-1}{x^2+2x+2}\,dx + 4\int \frac{1}{x+5}\,dx =$$

$$= \frac{3}{2}\int \frac{2x+2}{x^2+2x+2}\,dx + \int -\frac{4}{x^2+2x+2}\,dx = 4\ln(|x+5|) + \frac{3\ln(x^2+2x+2)}{2} -$$

$$- 4\,\mathrm{arctg}(x+1) + C$$

№5.14

$$\int \frac{dx}{x^2\sqrt{18x^2-10x+1}} = \int -\frac{u}{\sqrt{u^2-10u+18}}\,du = -\int \frac{u}{\sqrt{(u-5)^2-7}}\,du =$$

$$u=\frac{1}{x},\quad x=\frac{1}{u},\quad dx=-\frac{1}{u^2}\,du\ ;\quad v=u-5 \ \Big|\ \begin{matrix} u=v+5 \\ du=dv \end{matrix}$$

$$= -\int \frac{v+5}{\sqrt{v^2-7}}\,dv = -\int \frac{v}{\sqrt{v^2-7}} + \frac{5}{\sqrt{v^2-7}}\,dv =$$

$$= -\left(\int \frac{v}{\sqrt{v^2-7}}\,dv + 5\int \frac{1}{\sqrt{v^2-7}}\,dv\right) = -5\ln(|\sqrt{7}\sqrt{v^2-7}+$$

$$+\sqrt{7}v|) - \sqrt{v^2-7} + 5\ln(7) = -5\ln(|\sqrt{7}\sqrt{u^2-10u+18}+\sqrt{7}u-$$

$$-5\sqrt{7}|) - \sqrt{u^2-10u+18} + 5\ln 7 = -5(\ln(|\sqrt{7}x\sqrt{18x^2-10x+1}$$

$$+x(\sqrt{7}-5\sqrt{7}x)|) - 2\ln x - \frac{\sqrt{18x^2-10x+1}}{x} + 5\ln 7 + C$$

N3.14

$$\int \frac{4x^4+8x^3-11x^2-17}{4x^2+4x+5}dx = \int \frac{15x+8}{4x^2+4x+5}+x^2+x-5\,dx =$$

$$= \int \frac{15x+8}{4x^2+4x+5}dx + \int x^2 dx + \int x\,dx - 5\int 1\,dx \overset{*}{=}$$

(1) $\int \frac{15x+8}{4x^2+4x+5}dx = \frac{15}{8}\int \frac{8x+4}{4x^2+4x+5}dx + \int \frac{1}{2(4x^2+4x+5)}dx =$

$= \frac{15}{8}\cdot(2)\cdot(3)$

(2) $v = x+\frac{1}{2}$ $\left| \begin{array}{l} x = \frac{2v-1}{2} \\ dx = dv \end{array}\right.$

$\frac{1}{8}\int \frac{1}{v^2+1}dv = \cancel{\frac{1}{8}\int \frac{1}{v^2+1}}dv = \frac{1}{8}\cdot \operatorname{arctg} v = \frac{\operatorname{arctg} v}{8}$

$v = x+\frac{1}{2}$ $\left| x = \frac{2v-1}{2}\right.$ $\left(\frac{\operatorname{arctg}\left(\frac{2x+1}{2}\right)}{8}\right)$

(3) $\int \frac{1}{2(4x^2+4x+5)}dx = \frac{1}{2}\int \frac{1}{4(x^2+x+\frac{5}{4})}dx = \frac{1}{8}\int \frac{1}{(x+\frac{1}{2})^2+1}dx$

(4) $\int x^2 dx = \frac{x^3}{3}$

(5) $\int x\,dx = \frac{x^2}{2}$

$$\overset{*}{=} \frac{15\ln(4x^2+4x+5)}{8} + \frac{\operatorname{arctg}\left(\frac{2x+1}{2}\right)}{8} + \frac{x^3}{3} + \frac{x^2}{2} - 5x + C$$

Ответ: $\frac{15\ln(4x^2+4x+5)}{8} + \frac{\operatorname{arctg}\left(\frac{2x+1}{2}\right)}{8} + \frac{x^3}{3} + \frac{x^2}{2} - 5x + C$

№1. 14

$\int \sqrt[4]{3+5x^3}\, x^2 dx$

$u = 5x^3 + 3$

$\frac{1}{15} du = x^2 dx$

$\int \frac{\sqrt[4]{u}}{15} du = \frac{1}{15}\int u^{\frac{1}{4}} = \frac{1}{15} \cdot \frac{4u^{\frac{5}{4}}}{5} = \frac{4u^{\frac{5}{4}}}{75} = \frac{4x^3\sqrt[4]{5x^3+3}}{15} +$

$+ \frac{4\sqrt[4]{5x^3+3}}{25} + C$

Ответ: $\frac{4x^3\sqrt[4]{5x^3+3}}{15} + \frac{4\sqrt[4]{5x^3+3}}{25}$

№2. 14

$\int \sin(\ln 4x) dx$

1) $u = 4x$ | $x = \frac{u}{4}$, $dx = \frac{1}{4} du$

2) $v = \ln(u)$, $u\,dv = du$ | $u = e^{\ln(u)}$

$\int \frac{\sin(\ln(u))}{4} du = \frac{1}{4}\int e^v \sin(v) dv = \frac{1}{4}(-e^v \cos(v) + \int e^v \cos v\, dv) =$

$= \frac{1}{4}\left(\frac{e^v \sin v - e^v \cos v}{2}\right) = \frac{e^v \sin v}{8} - \frac{e^v \cos v}{8} = \frac{4\sin(\ln u)}{8} -$

$- \frac{u\cos(\ln u)}{8} = \frac{x\sin(\ln(4x))}{2} - \frac{x\cos(\ln(4x))}{2} + C$

Ответ: $\frac{x\sin(\ln(4x))}{2} - \frac{x\cos(\ln(x) + \ln 4)}{2} + C$

№11.14

$\vec{a} = x\vec{i} + 2z^2\vec{j} + y\vec{k}$ Г: $\begin{cases} x = \cos t,\ y = 3\sin t \\ z = 2\cos t - 3\sin t - 2 \end{cases}$

$0 < t \le 2\pi$

$dx = -\sin t\, dt,\ dy = 3\cos t\, dt,$

$dz = (-2\sin t - 3\cos t)\, dt$

$\oint_Г (\vec{a}, d\vec{s}) = \oint_Г x dx + 2z^2 dy + y dz = \int_0^{2\pi} (\cos t(-\sin t) + 2(2\cos t - 3\sin t - 2)^2 \cdot 3\cos t + 3\sin t(-2\sin t - 3\cos t))dt = \int_0^{2\pi} (-\cos t \sin t + 6(4\cos^2 t + 9\sin^2 t + 4 - 8\cos t + 12\sin t - 12\cos t \sin t)\cos t - 6\sin^2 t - 3\sin t \cdot \cos t)dt = \int_0^{2\pi} (-5\sin 2t - 3(1 - \cos 2t) + 48\cos t + 3\sin^2 t \cos t - 48\cos^2 t + 72\sin t \cos t - 72\cos^2 t \sin t)dt - \int_0^{2\pi} (31\sin 2t - 72\cos^2 t \sin t + 30\sin^2 t \cos t - 72\cos^2 t \sin t)dt =$

$-3 + 3\cos 2t + 48\cos t - 24 - 24\cos 2t + 30\sin^2 t \cos t - 24t - 12\sin 2t + 10\sin^3 t + 48\sin t - 24t - 12\sin 2t + 10\sin^3 t +$

$= (-\frac{31}{2}\cos 2t - 3t + \frac{3}{2}\sin 2t + 48\sin t + \frac{72}{3}\cos^3 t)\Big|_0^{2\pi} = -\frac{31}{2}(\cos 4\pi - \cos 0) - 27 \cdot 2\pi - 10{,}5(\sin 4\pi - \sin 0) + \frac{72}{3}(\cos^3 2\pi - \cos^3 0) = -54\pi$

$+48(\sin 2\pi - \sin 0) + 10(\sin^3 2\pi - \sin^3 0)$

Ответ: -54π

№12.14

$\vec{a} = 2y\vec{i} - 3x\vec{j} + z^2\vec{k}$; Г: $\begin{cases} x^2 + y^2 = z \\ z = 1 \end{cases}$

$\left.\begin{array}{l} x = \cos t \\ y = \sin t \end{array}\right\}\ 0 \le t \le 2\pi$

$z = 1$

$dx = -\sin t\, dt$

$dy = \cos t\, dt$

$dz = 0$

$\mathcal{U} = \oint_Г (\vec{a}, d\vec{s}) = \oint_Г (2y dx - 3x dy + z^2 dz) = \int_0^{2\pi}(2\sin t(-\sin t) - 3\cos t \cdot \cos t + 1^2 \cdot 0)dt = \int_0^{2\pi}(-2\sin^2 t - 3\cos^2 t)dt = \int_0^{2\pi}(-2 - \cos^2 t)dt =$

$= -\int_0^{2\pi}\left(2 + \frac{1 + \cos 2t}{2}\right)dt = \left(-2{,}5t - 0{,}5\frac{\sin 2t}{2}\right)\Big|_0^{2\pi} =$

$= -2{,}5 \cdot 2\pi - \frac{1}{4}(\sin 4\pi - \sin 0) = -5\pi$

$|\mathcal{U}| = 5\pi$

Ответ: 5π