C11 − GAL

6.05.2022

Geometrie analitică euclidiană

$(\mathbb{R}^n, (\mathbb{R}^n, g_0), \varphi)$ sp. punctual euclidian, cu str. canonică

$\mathcal{R} = \{O; e_1, .., e_n\}$ reper cartezian ortonormat

• Ec. unui plan afin (subsp. afin 2-dim)

a) $A(a_1, .., a_n) \in \pi$, $V_\pi = \langle\{u, v\}\rangle$, $\{u, v\}$ SLI

$\forall M(x_1, .., x_n) \in \pi \Rightarrow \overrightarrow{AM} \in V_\pi$

$\exists t, s \in \mathbb{R}$ aî $\overrightarrow{AM} = tu + sv$

$\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$, $\quad u = \sum_{i=1}^{n} u_i e_i$

$\overrightarrow{OM} = \sum_{i=1}^{n} x_i e_i$, $\quad v = \sum_{i=1}^{n} v_i e_i$

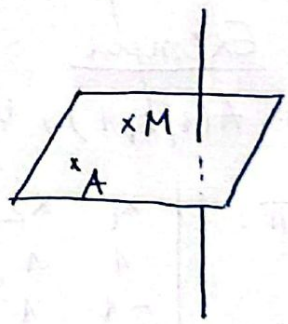
$\pi: \ x_i - a_i = t u_i + s v_i, \ \forall i = \overline{1,n}$ ec. parametrice

b) $A(a_1, .., a_n)$, $B(b_1, .., b_n)$, $C(c_1, .., c_n) \in \pi$ (necoliniare)

$V_\pi = \langle\{\overrightarrow{AB}, \overrightarrow{AC}\}\rangle$

$\pi: \ x_i - a_i = t(b_i - a_i) + s(c_i - a_i), \ \forall i = \overline{1,n}$

c) $A(a_1, .., a_n) \in \pi$, $\mathcal{D} \perp \pi$



$\mathcal{D}: \ \dfrac{x_1 - x_1^0}{u_1} = .. = \dfrac{x_n - x_n^0}{u_n}$

$u = (u_1, .., u_n) = N$.

$\forall M \in \pi: \ \langle \overrightarrow{AM}, N \rangle = 0$

$u_1(x_1 - a_1) + \ldots + u_n(x_n - a_n) = 0$.

Cazul $n = 3$

a) $A(a_1, a_2, a_3) \in \pi$, $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3)$

$\pi: \begin{vmatrix} x_1 - a_1 & u_1 & v_1 \\ x_2 - a_2 & u_2 & v_2 \\ x_3 - a_3 & u_3 & v_3 \end{vmatrix} = 0$. $\quad N = u \times v$

$\langle \overrightarrow{AM}, N \rangle = 0$.

Exemplu $A(1,-1,2) \in \Pi$, $V_{\Pi} = \langle \{ u = (2,3,1), v = (4,1,3)$

$$\Pi : \begin{vmatrix} x_1 - 1 & 2 & 4 \\ x_2 + 1 & 3 & 1 \\ x_3 - 2 & 1 & 3 \end{vmatrix} = 0$$

$$N = u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & 3 & 1 \\ 4 & 1 & 3 \end{vmatrix} = (8, -2, -10) = 2(\underline{4, -1, -5})$$

$$\langle \overrightarrow{AM}, N \rangle = 0$$

$$\Pi : 4(x_1 - 1) - 1 \cdot (x_2 + 1) - 5(x_3 - 2) = 0$$

$$\Pi : 4x_1 - x_2 - 5x_3 \underbrace{- 4 - 1 + 10}_{5} = 0 \quad \text{ec. generală.}$$

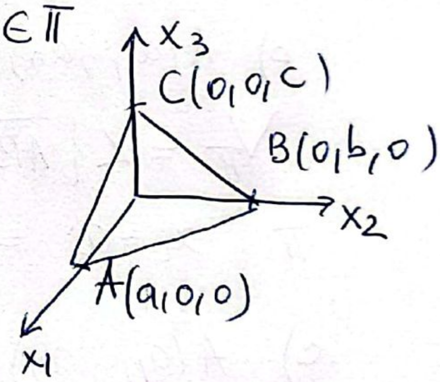
b) $A(a_1, a_2, a_3), B(b_1, b_2, b_3), C(c_1, c_2, c_3) \in \Pi$

$$\Pi : \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = 0$$

$A(a,0,0)$, $B(0,b,0)$, $C(0,0,c) \in \Pi$

$$\Pi : \frac{x_1}{a} + \frac{x_2}{b} + \frac{x_3}{c} = 1$$

(ec. prin tăieturi)

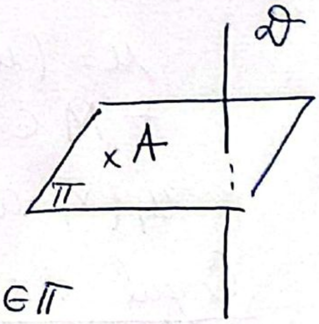


Exemplu

Π: $A(1,1,1)$, $B(-1,1,1)$, $C(2,0,0)$

$$\Pi : \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \end{vmatrix} = 0.$$

c) $\mathcal{D} \perp \Pi$, $A \in \Pi$



$$\mathcal{D} : \frac{x_1 - 1}{2} = \frac{x_2 - 1}{1} = \frac{x_3}{2} \quad , \quad A(1,0,3) \in \Pi$$

$$N_{\Pi} = u_{\mathcal{D}} = (2,1,2) \qquad \Pi : \langle \overrightarrow{AM}, N \rangle = 0$$

$$\Pi : 2(x_1 - 1) + 1 \cdot (x_2 - 0) + 2(x_3 - 3) = 0 \Rightarrow 2x_1 + x_2 + 2x_3 - 8 = 0$$

⊛ Ec. unui hiperplan afin (ssp. afin $(n-1)$-dim)

$A(a_1,..,a_n) \in \mathcal{H}$, $V_{\mathcal{H}} = \langle \{u_1,..,u_{n-1}\} \rangle$

$\{u_1,..,u_{n-1}\}$ SLI.

$\forall M(x_1,..,x_n) \in \mathcal{H} \Rightarrow \overrightarrow{AM} \in V_{\mathcal{H}}$.

$\exists\, t_1,..,t_{n-1} \in \mathbb{R}$ a.î. $\overrightarrow{AM} = \sum_{i=1}^{n-1} t_i u_i$

$x_i - a_i = t_1 u_1^i + \ldots + t_{n-1} u_{n-1}^i$, $i = \overline{1,n}$

$$\mathcal{H} : \begin{vmatrix} x_1 - a_1 & u_1^1 & \ldots & u_{n-1}^1 \\ \vdots & \vdots & & \vdots \\ x_n - a_n & u_1^n & & u_{n-1}^n \end{vmatrix} = 0$$

$\mathcal{H}$: $\underline{A_1 x_1 + \ldots + A_n x_n + A_0 = 0}$, $A_1^2 + \ldots + A_n^2 > 0$

$N = (A_1,..,A_n)$

<u>Poz relativă a 2 hiperplane</u>

1) $\mathcal{H}_1 \parallel \mathcal{H}_2 \iff \dfrac{A_1}{A_1'} = .. = \dfrac{A_n}{A_n'} \neq \dfrac{A_0}{A_0'}$
$(\mathcal{H}_1 \neq \mathcal{H}_2)$

$\mathcal{H}_1$: $A_1 x_1 + \ldots + A_n x_n + A_0 = 0$

$\mathcal{H}_2$: $A_1' x_1 + .. + A_n' x_n + A_0' = 0$

2) $\mathcal{H}_1 = \mathcal{H}_2 \iff \dfrac{A_1}{A_1'} = .. = \dfrac{A_n}{A_n'} = \dfrac{A_0}{A_0'}$

3) $\mathcal{H}_1 \cap \mathcal{H}_2 \neq \emptyset$

$\mathcal{H}_1 \cap \mathcal{H}_2$ = subsp. afin $(n-2)$-dim.

<u>Exemplu</u>

$\pi_1 : x_1 + x_2 + x_3 = 1$

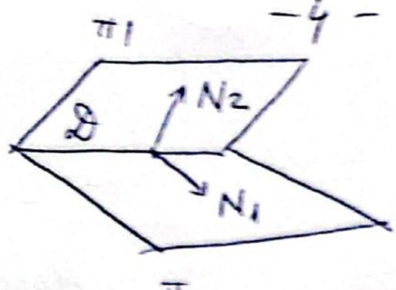
$\pi_2 : 2x_1 \quad - x_3 = 0.$

$$\left(\begin{array}{cc|c} \boxed{\begin{matrix} 1 & 1 \\ 2 & 0 \end{matrix}} & \begin{matrix} 1 \\ -1 \end{matrix} \end{array}\right) \left| \begin{matrix} 1 \\ 0 \end{matrix} \right.$$

$\pi_1 \cap \pi_2 = \mathcal{D}$

(M1) $x_3 = t \Rightarrow \begin{cases} x_1 + x_2 = 1 - t \\ 2x_1 = t \end{cases}$; $\mathcal{D} : \begin{cases} x_1 = \dfrac{t}{2} \\ x_2 = 1 - t - \dfrac{t}{2} = 1 - \dfrac{3}{2}t \\ x_3 = t \end{cases}$

M_2 — 4 —



$$u_D = N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{vmatrix} = (-1, 3, -2)$$

• Intersectia unei drepte cu un ~~plan~~ hiperplan

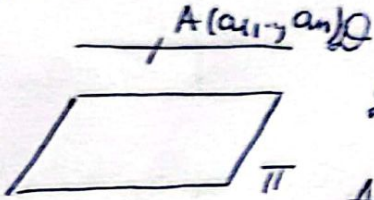
$$D : \frac{x_1 - a_1}{u_1} = \frac{x_2 - a_2}{u_2} = \dots = \frac{x_n - a_n}{u_n} = t \Rightarrow \begin{cases} x_1 = u_1 t + a_1 \\ \vdots \\ x_n = u_n t + a_n \end{cases}$$

$$\pi : A_1 x_1 + \dots + A_n x_n + A_0 = 0$$

$$D \cap \pi : t (A_1 u_1 + \dots + A_n u_n) + A_1 a_1 + \dots + A_n a_n + A_0 = 0 \quad (*)$$

1) $u = (u_1, \dots, u_n) = u_D$.

$N = (A_1, \dots, A_n) = N_\pi$



$D \not\subset \pi$

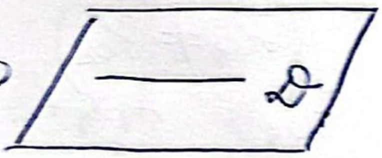
$A \notin \pi$

$D \parallel \pi : \langle N, u \rangle = 0 \Rightarrow \sum_{i=1}^{n} A_i u_i = 0$

$D \not\subset \pi \qquad \sum_{i=1}^{n} A_i a_i + A_0 \neq 0$

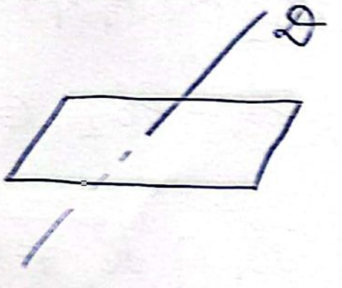
(*) nu are sol.

2) $\langle N, u \rangle = 0$, $\sum_{i=1}^{n} A_i a_i + A_0 = 0$



(*) o inf sol.

3) (*) o sol



$$t = - \frac{\sum_{i=1}^{n} A_i a_i + A_0}{\sum_{i=1}^{n} A_i u_i}$$

$D \cap \pi \neq \emptyset$

Exemplu

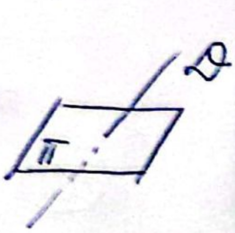
$\pi : x_1 + x_2 + x_3 = 2$, $D : \frac{x_1 - 3}{1} = \frac{x_2}{3} = \frac{x_3}{1} = t$

$D \cap \pi = ?$

$$\begin{cases} x_1 = t + 3 \\ x_2 = 3t \\ x_3 = t \end{cases}$$

$t + 3 + 3t + t = 2$

$5t = -1 \Rightarrow t = -\frac{1}{5}$



$P\left(3 - \frac{1}{5}, -\frac{3}{5}, -\frac{1}{5}\right) \Rightarrow P\left(\frac{14}{5}, -\frac{3}{5}, -\frac{1}{5}\right)$

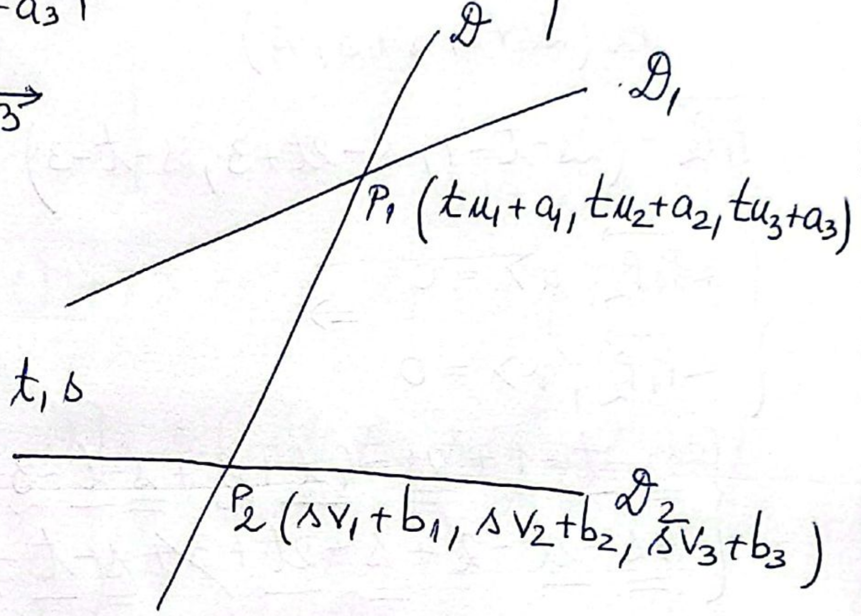
• Perpendiculara comună a 2 drepte necoplanare

$(n=3)$

$$\mathcal{D}_1 : \frac{x_1-a_1}{u_1} = \frac{x_2-a_2}{u_2} = \frac{x_3-a_3}{u_3} = t,\ u=(u_1,u_2,u_3),\ A(a_1,a_2,a_3)$$

$$\mathcal{D}_2 : \frac{x_1-b_1}{v_1} = \frac{x_2-b_2}{v_2} = \frac{x_3-b_3}{v_3} = s,\ v=(v_1,v_2,v_3),\ B(b_1,b_2,b_3)$$

$$\begin{array}{c}\begin{vmatrix} u_1 & v_1 & b_1-a_1 \\ u_2 & v_2 & b_2-a_2 \\ u_3 & v_3 & b_3-a_3 \end{vmatrix} \neq 0 \Rightarrow \mathcal{D}_1, \mathcal{D}_2 \text{ necoplanare.} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ u & v & \overrightarrow{AB} \end{array}\end{array}$$



(M1)
$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow t, s$$

$\Rightarrow P_1, P_2$

(M2) • π_1 plan det. de $\mathcal{D}$ și $\mathcal{D}_1$.

$N = u \times v = u_{\mathcal{D}}$

$N_1 = N \times u = N_{\pi_1}$, $A(a_1,a_2,a_3) \in \pi_1$

• π_2 plan det de $\mathcal{D}$ și $\mathcal{D}_2$.

$N_2 = N \times v = N_{\pi_2}$, $B(b_1,b_2,b_3) \in \pi_2$

$\mathcal{D} = \pi_1 \cap \pi_2$.

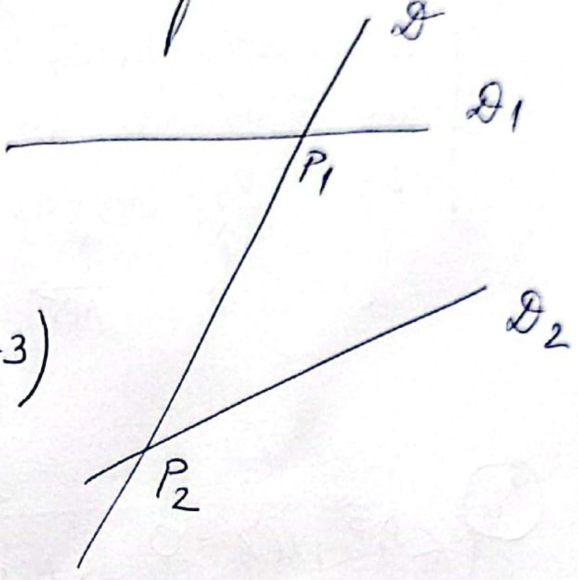
Exemplu

$\mathcal{D}_1: \frac{x_1-2}{1}=\frac{x_2}{2}=\frac{x_3-3}{1}=t \quad u=(1,2,1),\ A(2,0,3)$

$\mathcal{D}_2: \frac{x_1-1}{2}=\frac{x_2-3}{1}=\frac{x_3}{1}=s \quad v=(2,1,1),\ B(1,3,0)$

$\overrightarrow{AB}=(-1,3,-3)$

$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & -3 \end{vmatrix} \neq 0 \Rightarrow \mathcal{D}_1, \mathcal{D}_2$ necoplanare.



M1 $P_1(t+2, 2t, t+3)$

$P_2(2s+1, s+3, s)$

$\overrightarrow{P_1P_2}=(2s-t-1, s-2t+3, s-t-3)$

$\begin{cases} \langle \overrightarrow{P_1P_2}, u\rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v\rangle = 0 \end{cases} \Rightarrow$

$\begin{cases} 2s-t-1+2s-4t+6+s-t-3=0 \\ 4s-2t-2+s-2t\cancel{+3}+s-t\cancel{-3}=0 \end{cases} \Rightarrow \begin{cases} -6t+5s=-2 \\ -5t+6s=2 \end{cases}$

$11(-t+s)=0$

$t=s=2.$

$P_1(4,4,5),\ P_2(5,5,2),\ \overrightarrow{P_1P_2}=(1,1,-3)$

$\mathcal{D}: \frac{x_1-4}{1}=\frac{x_2-4}{1}=\frac{x_3-5}{-3}$

$dist(\mathcal{D}_1,\mathcal{D}_2)=dist(P_1,P_2)=\|\overrightarrow{P_1P_2}\|=\sqrt{11}.$

M2 π_1 plan det de $\mathcal{D}$ şi $\mathcal{D}_1$

$A(2,0,3)\in\pi_1 \quad N=u\times v=\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}=$

$N=(1,1,-3)=u_{\mathcal{D}}$

$N_1=N\times u=\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & -3 \\ 1 & 2 & 1 \end{vmatrix}=(7,-4,1)$

$\pi_1: 7(x_1-2)-4(x_2-0)+1\cdot(x_3-3)=0 \Rightarrow \boxed{7x_1-4x_2+x_3-17=0}$

π_2 plan det de $\mathcal{D}_2$ și $\mathcal{D}$, $B(1,3,0) \in \pi_2$.

$N = (1,1,-3) = u_{\mathcal{D}}$

$$N_2 = N \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & -3 \\ 2 & 1 & 1 \end{vmatrix} = (4,-7,-1)$$

$$\pi_2 : 4(x_1-1) - 7(x_2-3) - 1(x_3-0) = 0 \Rightarrow 4x_1 - 7x_2 - x_3 + 17 = 0$$

$$\mathcal{D} = \pi_1 \cap \pi_2 : \begin{cases} 7x_1 - 4x_2 + x_3 - 17 = 0 \\ 4x_1 - 7x_2 - x_3 + 17 = 0. \end{cases}$$

$x_3 = t$

$$\begin{cases} 7x_1 - 4x_2 = -t + 17 \\ 4x_1 - 7x_2 = t - 17 \end{cases} \begin{vmatrix} 7 \\ -4 \end{vmatrix}$$

$$x_1 \underbrace{(49-16)}_{33} = t(-11) + 17 \cdot 11 \Rightarrow x_1 = -\frac{1}{3} t + \frac{17}{3}.$$

$$x_2 = \frac{1}{4}\left[7x_1 + t - 17\right] = \frac{1}{4}\left[-\frac{7}{3}t + \frac{7 \cdot 17}{3} + t - \frac{17}{3}\right]$$

$$= \frac{1}{4}\left(-\frac{4}{3}t + \frac{17 \cdot 4}{3}\right) = -\frac{1}{3}t + \frac{17}{3}.$$

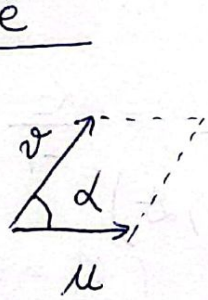
$$\mathcal{D} : \begin{cases} x_1 = -\frac{1}{3}t + \frac{17}{3} \\ x_2 = -\frac{1}{3}t + \frac{17}{3} \\ x_3 = t \end{cases}$$

$P_1, P_2 \in \mathcal{D}$.

Arii. Volume. Distanțe

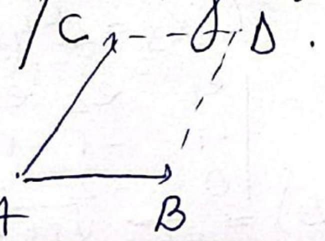
$n = 3$.

- Aria unui triunghi



$\{u, v\}$ SLI, $w = u \times v$

$\|w\| = \|u\| \cdot \|v\| \cdot \sin\alpha = A$ paralelogramului.



$$A_{\Delta ABC} = \frac{1}{2} \|\vec{AB} \times \vec{AC}\|$$

$$= \frac{1}{2}\sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ b_1-a_1 & b_2-a_2 & b_3-a_3 \\ c_1-a_1 & c_2-a_2 & c_3-a_3 \end{vmatrix} = (\Delta_1, \Delta_2, \Delta_3)$$

$A(a_1, a_2, a_3)$
$B(b_1, b_2, b_3)$
$C(c_1, c_2, c_3)$

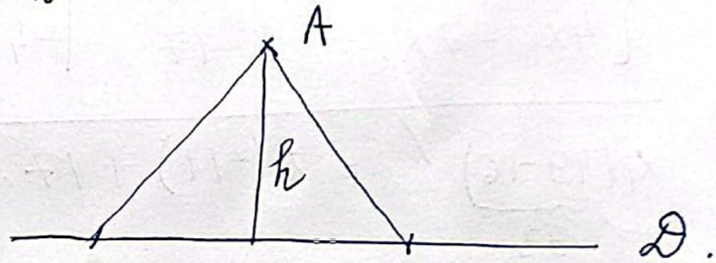
Ex. $A(1,0,1)$, $B(0,-1,0)$, $C(0,1,1)$

$A_{\Delta ABC}$.

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -1 & -1 \\ -1 & 1 & 0 \end{vmatrix} = (1, \pm, -2).$$

$$A_\Delta = \frac{1}{2}\sqrt{1+1+4} = \frac{\sqrt{6}}{2}.$$

- $Dist(A, \mathcal{D}) = ?$



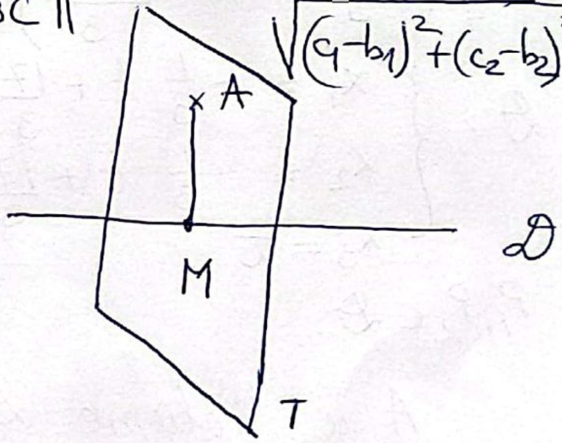
(M1) $A_{\Delta ABC} = \frac{1}{2}\| \vec{AB} \times \vec{AC} \| = \frac{h \cdot \|BC\|}{2} \Longrightarrow$ B C

$$dist(A, \mathcal{D}) = h = \frac{2\,\| \vec{AB} \times \vec{AC} \|}{\| \vec{BC} \|} = \frac{\sqrt{\Delta_1^2+\Delta_2^2+\Delta_3^2}}{\sqrt{(c_1-b_1)^2+(c_2-b_2)^2+(c_3-b_3)^2}}$$

(M2) $\pi \perp \mathcal{D}$, $A \in \pi$

$\pi \cap \mathcal{D} = \{M\}$

$dist(A, \mathcal{D}) = dist(A, M)$



Exemplu

$\mathcal{D}: \frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3-1}{2} = t$, $A(1,1,0)$

$dist(A, \mathcal{D})$

SOL (M1) $\mathcal{D}: \begin{cases} x_1 = t \\ x_2 = -t \\ x_3 = 2t+1, t \in \mathbb{R} \end{cases}$

$B(0,0,1)$
$C(1,-1,3)$
$\vec{BC} = (1,-1,2)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -1 & 1 \\ 0 & -2 & 3 \end{vmatrix} = (-1, 3, 2)$$

$$dist(A, \mathcal{D}) = \frac{1}{\sqrt{6}}\sqrt{1+9+4} = \sqrt{\frac{14}{6}} = \sqrt{\frac{7}{3}} = \frac{\sqrt{21}}{3}$$

(M2) Construim $\pi \perp \mathcal{D}$, $A \in \pi$

$N_\pi = u_{\mathcal{D}} = (1,-1,2)$, $A(1,1,0) \in \pi$

$\pi: 1(x_1 - 1) - 1(x_2 - 1) + 2(x_3 - 0) = 0$.

sau $\pi: x_1 - x_2 + 2x_3 + \alpha = 0$

$A(1,1,0) \in \pi \Rightarrow 1 - 1 + 0 + \alpha = 0 \Rightarrow \alpha = 0$

$\pi: x_1 - x_2 + 2x_3 = 0$

$\{M\} = \mathcal{D} \cap \pi: t + t + 4t + 2 = 0 \Rightarrow 6t = -2 \Rightarrow t = -\frac{1}{3}$.

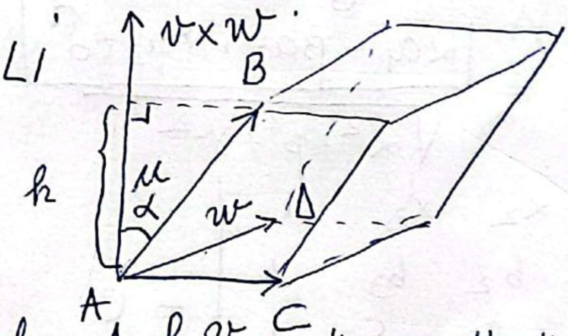
$M\left(-\frac{1}{3}, \frac{1}{3}, -\frac{2}{3} + 1\right) \Rightarrow M\left(-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$Dist(A, \mathcal{D}) = dist(A, M) = \sqrt{\left(1 + \frac{1}{3}\right)^2 + \left(1 - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} =$

$= \sqrt{\frac{16 + 4 + 1}{9}} = \sqrt{\frac{21}{9}} = \sqrt{\frac{7}{3}} = \frac{\sqrt{21}}{3}$

Volume

$\{u, v, w\}$ SLI



$V_{paralelipiped} = A_b \cdot h \overset{v}{=} \|v \times w\| \cdot \|u\| \cdot \cos\alpha$

$= \langle u, v \times w \rangle = |u \wedge v \wedge w|$

$$= \left| \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \right| = |\Delta|$$

$$V_{ABC\Delta} = \frac{1}{6} |\Delta| = \frac{1}{6} \left| \begin{vmatrix} b_1 - a_1 & b_2 - a_2 & b_3 - a_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \\ d_1 - a_1 & d_2 - a_2 & d_3 - d_3 \end{vmatrix} \right| =$$

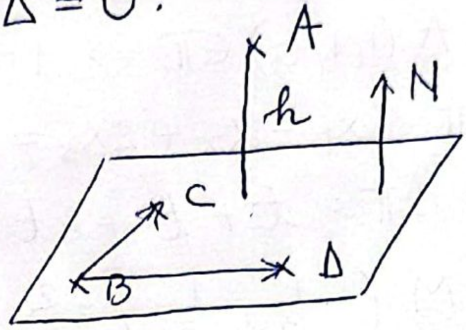
$u = \overrightarrow{AB}$

$v = \overrightarrow{AC}$

$w = \overrightarrow{A\Delta}$

$$= \frac{1}{6} \left| \begin{vmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix} \right| = \frac{1}{6} |\Delta|$$

<u>OBS</u> A, B, C, D coplanare $\Leftrightarrow \Delta = 0$.



• $Dist(A, \pi)$

(M1) $N = \vec{BC} \times \vec{BD} = (\alpha, \beta, \gamma)$

$B(b_1, b_2, b_3)$

$\pi : \alpha(x_1 - b_1) + \beta(x_2 - b_2) + \gamma(x_3 - b_3) = 0$.

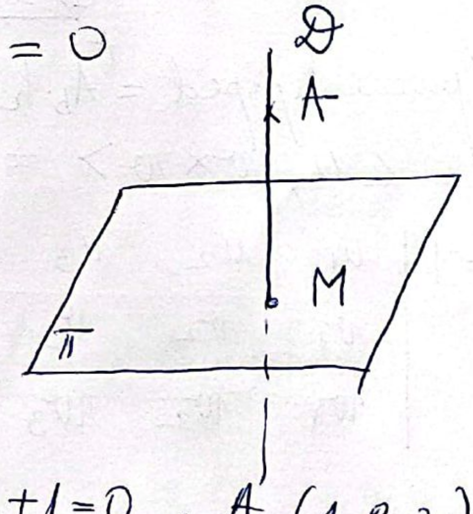
$\alpha x_1 + \beta x_2 + \gamma x_3 + \delta = 0$

$$V_{ABCD} = \frac{h \cdot A_{BCD}}{3} = \frac{1}{3} h \cdot \frac{1}{2} \|\vec{BC} \times \vec{BD}\|$$

$$= \frac{1}{6} h \, \|\vec{N}\| = \frac{1}{6} |\Delta|$$

$$h = \frac{|\Delta|}{\|\vec{N}\|} = \frac{|\alpha a_1 + \beta a_2 + \gamma a_3 + \delta|}{\sqrt{\alpha^2 + \beta^2 + \gamma^2}}$$

$$\pi : \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \\ d_1 & d_2 & d_3 & 1 \end{vmatrix} = 0$$



(M2) Construim $\mathcal{D} \ni A$, $\mathcal{D} \perp \pi$

$\mathcal{D} \cap \pi = \{M\}$

$Dist(A, \pi) = AM$.

<u>Exemplu</u> $\pi : x_1 - 2x_2 + 3x_3 + 1 = 0$, $A(1, 2, 3)$

$dist(A, \pi)$

(M1) $$dist(A, \pi) = \frac{|1 - 4 + 9 + 1|}{\sqrt{1 + 4 + 9}} = \frac{7}{\sqrt{14}} = \sqrt{\frac{7}{2}} = \frac{\sqrt{14}}{2}.$$

M2 $\mathcal{D} \perp \pi$, $A(1,2,3) \in \mathcal{D}$.

$u_{\mathcal{D}} = N_{\pi} = (1,-2,3)$

$$\mathcal{D}: \frac{x_1-1}{1} = \frac{x_2-2}{-2} = \frac{x_3-3}{3} = t \Rightarrow \begin{cases} x_1 = t+1 \\ x_2 = -2t+2 \\ x_3 = 3t+3, t \in \mathbb{R} \end{cases}$$

$\mathcal{D} \cap \pi = \{M\}$: $\underline{t}+1+\underline{4t}-4+\underline{9t}+9+1=0$

$3 - \frac{3}{2}$

$14t = -7 \Rightarrow t = -\frac{1}{2}$

$M\left(-\frac{1}{2}+1, 1+2, -\frac{3}{2}+3\right) \Rightarrow M\left(\frac{1}{2}, 3, \frac{3}{2}\right)$

$\text{dist}(A,\pi) = \text{dist}(A,M) = \sqrt{\frac{1}{4}+1+\frac{9}{4}} = \sqrt{\frac{1+4+9}{4}} = \frac{\sqrt{14}}{2}$

- $\text{dist}(\mathcal{D}_1, \mathcal{D}_2)$ (drepte necoplanare)

paralelipiped det $\overrightarrow{AB}, u, v$ (baza este generată de u, v)

$N = u \times v$

$$h = \frac{|\Delta|}{\|N\|} = \frac{|\overrightarrow{AB} \wedge u \wedge v|}{\|N\|} = \frac{|\langle \overrightarrow{AB}, N \rangle|}{\|N\|}$$