$$\rho = \{ (x,y) \in \mathbb{N} \times \mathbb{N} \mid y = 2x+1 \} \subseteq \mathbb{N} \times \mathbb{N}$$

$$\sigma = \{ (1,4), (3,5), (2,6) \} \subseteq \mathbb{N} \times \mathbb{N}$$

$\rho \circ \sigma, \sigma \circ \rho = ?$

$\rho \subseteq A \times B, \quad \sigma \subseteq B \times C$

$$\sigma \circ \rho = \{ (a,c) \mid \exists b \in B : (a,b) \in \rho \ \wedge \ (b,c) \in \sigma \} \subseteq A \times C$$

$$\sigma \circ \rho = \{ (a,c) \mid \exists b \in \mathbb{N} : (a, \underbrace{b}_{=2a+1}) \in \rho, (\underbrace{b}_{=2a+1}, c) \in \sigma \}$$

$\Leftrightarrow a \in \{0,1\}$

$$= \{ (0,c) \mid (1,c) \in \sigma \} \cup \{ (1,c) \mid (3,c) \in \sigma \}$$

$$= \{ (0,4), (1,5) \}$$

$id_B = (B, B, \Delta_B)$, $\Delta_B = \{(b,b) \mid b \in B\} \subseteq B \times B$ $\quad (B \to B)$

$\alpha = (A, B, \rho)$, $\rho \subseteq A \times B$ $(A \not\to B)$

$A \xrightarrow{f} B \xrightarrow{g} C$, $g \circ f$

$id_B \circ \rho = \{(a,b) \mid \exists c \text{ a.i. } (a,c) \in \rho \text{ și } (c,b) \in \Delta_B\}$ $\quad id_B \circ \rho \subseteq A \times B$

$A \times B$ $\quad \Leftrightarrow c = b$

$= \{(a,b) \mid (a,b) \in \rho\} = \rho$

$A \times B$

De fel, $\rho \circ id_A = \rho$. $\quad (\rho \subseteq A \times B,\ \Delta_A \subseteq A \times A$

$\rho \circ \Delta_A$ $\quad A \not\to A$ $\quad A \to B$

$(\alpha = (A, B, \rho))$

$(\rho \subseteq A \times B)$

Exp:

1) $\rho = \{(1,2); (5,6); (1,9)\} \subseteq \mathbb{Z} \times \mathbb{Z}$

$\rho^{-1} = \{(2,1); (6,5), (9,1\} \subseteq \mathbb{Z} \times \mathbb{Z}$

2) $\sigma = \{(a, 5a+2) \mid a \in \mathbb{N}\} \subseteq \mathbb{N} \times \mathbb{N}$

$\sigma^{-1} = \{(5a+2, a) \mid a \in \mathbb{N}\} \subseteq \mathbb{N} \times \mathbb{N}$

3) cu σ ca la 2):

$5a+2 \quad a$

$\sigma \circ \sigma^{-1} = \{(x,y) \mid \exists z \text{ a.i. } (x,z) \in \sigma^{-1}$

$(z,y) \in \sigma\}$

$a \quad 5a+2$

$= \{(5a+2, 5a+2) \mid a \in \mathbb{N}\} \neq \Delta_{\mathbb{N}} = \{(b,b) \mid b \in \mathbb{N}\}$